Construction robust simple linear regression profile Monitoring (A simulation study)

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تكوين لوحات الانحدار الخطي البسيط الحصينة (دراسة محاكاة)

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Abstract:

In this research suggest the construction of quality control charts for parameters of simple linear regression and the mean squares error (Linear Profile Monitoring) depending on robust estimators (M-Estimator), which was estimated weights through a bi-square function and then compare them with the classical charts that depend on the method of ordinary least squares (OLS) when there an outlier values for randomly generated data (Simulation) have a normal distribution, for this purpose, the program language MATLAB design and research concluded that the proposed charts more accurate than of classical charts.

Keywords: Linear Profile Monitoring, Quality control charts, Robust estimators

الملخص:

تم في هذا البحث اقتراح تكوين لوحات سـيطرة نوعية لمعلمات الانحدار الخطي البسـيط ومتوسط مربعات الخطأ بالاعتماد على المقدرات الحصينة (M-Estimator) التي تم تقدير أوزانها من خلال الدالة التربيعية ومن ثم مقارنتها مع اللوحات التقليدية التي تعتمد على طريقة المربعات الصغرى الاعتيادية (OLS) عند وجود قيم شاذة لبيانات مولدة عشوائياً (Simulation) لها توزيع طبيعي، ولهذا الغرض تم تصـميم برنامج بلغة ماتلاب وتوصـل البحث إلى أن اللوحات المقترحة أكثر دقة من اللوحات التقليدية.

1. Introduction

Control charts are known to be effective tools for monitoring the quality of processes and are applied in many industries. Data occur sequentially in time and often data are reduced to a statistic or two which represent the current state of the process. If the successively observed chart statistics are plotted within the upper and lower control limits (UCL and LCL), the process is deemed stable or in control. Chart statistics that are plotted outside of the control limits are signals that the process may be out of control and corrective action on the process may be needed.

Every process is affected by random fluctuations. These random fluctuations can be due to chance causes or assignable causes. An assignable cause is a result of an external change in the process and can be corrected by taking appropriate actions. A chance cause is due to the inherent variability in the process and it is difficult to eliminate or sometimes control. The primary aim of statistical process control is to identify the assignable cause variability in the process and to signal to the operating personnel to take appropriate actions. One tool that is used as a quick visual detection aid is a control chart. The research in the field of statistical process monitoring and control was initiated by the emergence of control charts in 1924, when

Dr. W. A. Shewhart proposed the concept of a visual monitoring scheme with control limits to detect changes in the process mean over time, Shewhart (1925, 1931). This formed the basis of the Shewhart control chart for monitoring process mean and variance. Since then, significant contributions have been made in the field and new charting schemes with improved performances have been proposed.

In many practical situations, the quality of a process or product is characterized by a relationship (or profile) between two or more variables instead of by the distribution of a single quality characteristic. It has also been used to determine optimum calibration frequency and to avoid errors due to over-calibration. Rosenblatt and Spiegelman, (1981) discuss these issues in calibration and suggest the use of control charts to determine the need for recalibration. Various control charts have been proposed to monitor measurement gauges and calibration curves thus obtained, see Mestek & et. al., (1994), Stover and Brill (1998), Kang and Albin (2000), and Chang and Gan (2007).

On the other hand estimators are used when unknown parameters in agiven mathematical model must be determined from available measurements. Usually, there are more measurements than are strictly needed to define the unknowns and the problem is called over-determined. This type of problem is variously referred to as parameter estimation, multivariate regression, and curve fitting. All these terms essentially describe the same computational process. This paper illustrates and explains some robust parameters estimation methods.

2. Methodology

Responses are ordered and the relationship between the quality variable over the range of explanatory variable is of interest. Profiles are of interest in various situations from food production, manufacturing, testing or calibration, process industries. [Croarkin and Varner (1982)], one of the initial applications of profile monitoring was in calibration to ascertain performance of the measurement method and to verify that it remained unchanged over time.

2.1. Classical Linear Profile Monitoring

Phase-I– the set of historical data is available, interest is on understanding process variation, assessing process stability, and estimating in-control process parameters.

In simple linear regression case, the i^{th} profile is modeled as [Azadeh, (2013)]:

$$y_{ij} = \beta_{i0} + \beta_{i1} x_i + \varepsilon_{ij} \qquad \cdots \quad (1)$$

Where y_{ij} is the j^{th} measurement (j=1,2, …, k) represent the dependent (response) variable for observations i=1,2, … ,n, ε_{ij} is the j^{th} random error, and x_i is the value of the explanatory (independent) variable corresponding to the i^{th} profile. It is assumed that the values of x_i are fixed for all i. The most common method for estimating the model parameters is the least square which finds the parameter estimates that minimize the sum of the squares of the difference between the fitted and observed profiles.

The control limits for monitoring the intercept are:

$$UCL_{OLS} = \beta_0 + Z_{\alpha/2}\sqrt{\sigma^2\left(\frac{1}{n}-\frac{\bar{x}^2}{S_{xx}}\right)}$$
$$CL_{OLS} = \beta_0 \qquad \cdots \quad (2)$$
$$LCL_{OLS} = \beta_0 + Z_{\alpha/2}\sqrt{\sigma^2\left(\frac{1}{n}-\frac{\bar{x}^2}{S_{xx}}\right)}$$

The control limits for monitoring the slope are:

$$UCL_{OLS} = \beta_1 + Z_{\alpha/2}\sqrt{\frac{\sigma^2}{S_{xx}}}$$
$$CL_{OLS} = \beta_1 \qquad \cdots \quad (3)$$
$$LCL_{OLS} = \beta_1 + Z_{\alpha/2}\sqrt{\frac{\sigma^2}{S_{xx}}}$$

Where S_{xx} is defined as $\sum_{i=1}^{n}(x_i-\bar{x})^2$ and $z_{\alpha/2}$ is the upper $\alpha/2$ percentage point of the standard normal distribution.

The control limits for monitoring the error variance are:

$$UCL_{OLS} = \frac{\sigma^2}{n-2}\chi^2_{\alpha/2,(n-2)}$$
$$CL_{OLS} = \sigma^2 \qquad \cdots \quad (4)$$
$$LCL_{OLS} = \frac{\sigma^2}{n-2}\chi^2_{(1-\alpha/2),(n-2)}$$

The ordinary Least Squares (OLS) point estimates for three parameters (intercept, slop and σ^2) are [Junjia, and Dennis (2010)]:

$$\hat{\beta}_{0j} = \bar{y}_j - \hat{\beta}_{1j}\bar{x}$$
$$\hat{\beta}_{1j} = \frac{S_{xx(j)}}{S_{xy(j)}} \qquad \cdots \quad (5)$$
$$\text{MSE}_j = \hat{\sigma}_j^2 = \frac{\sum_{i=1}^{n}\left(\hat{y}_{ij} - y_{ij}\right)^2}{n-2}$$

For all samples (j=1,2, … , k), It is the values that draw on the profile charts and we have:

$$\hat{\beta}_0 = \frac{\sum_{j=1}^{k}\hat{\beta}_{0j}}{k}$$
$$\hat{\beta}_1 = \frac{\sum_{j=1}^{k}\hat{\beta}_{1j}}{k} \qquad \cdots \quad (6)$$
$$\text{MSE} = \hat{\sigma}^2 = \frac{\sum_{j=1}^{k} MSE_j}{k}$$

When the preliminary sample intercept, slope and MSE are plotted on these charts, no indication of an out of control condition is observed. Therefore, since all the charts exhibit control, we would conclude that the process is in control at the stated levels and adopt the trial control limits for use in phase II, where monitoring of future data is of interest.

2.2. Proposed Linear Profile Monitoring

Classical estimation methods for multivariate control charts will not yield appropriate control limits if there is instability in the data. Robust estimation methods have a distinct advantage over classical methods in that they are not unduly influenced by unusual data points. [3] Consequently, they are much more effective in detecting such points and ensuring that the control limits are reasonable. The term robustness refers to methods that are insensitive to departures from one's assumptions, which in our case are independence and identically distributed normal data.

The most common general method of robust regression is M-Estimation by uses iteratively re-weighted least squares with the squares weighting function [Holland and Welsch, (1977)].

$$w_i = (|r_i| < 1).\left(1 - r_i^2\right)^2 \qquad \cdots \quad (7)$$

For i=1,2, …,n, The value r_i in the weight function is:

$$r_i = \frac{(\hat{y}_i - y_i)}{t.S.\sqrt{1-h_i}} \qquad \cdots \quad (8)$$

Where residuals are calculated from the previous iteration, h is the leverage values from a least-squares fit, and s is an estimate of the standard deviation of the error term given by S = MAD/0.6745, Here MAD is the median absolute deviation of the residuals from their median. The constant 0.6745 makes the estimate unbiased for the normal distribution. t is a tuning constant (4.685). If tune is unspecified, the default value (4.685) is used. Default tuning constants give coefficient estimates that are approximately 95% as statistically efficient as the ordinary least-squares estimates, provided the response has a normal distribution with no outliers. Decreasing the tuning constant increases the down weight assigned to large residuals; increasing the tuning constant decreases the down weight assigned to large residuals.

Then the estimating formulas may be written as [Huber, (1964)]:

$$\sum_{i=1}^{n} w_i \left(y_i - x_i'\hat{\beta}\right)x_i' = 0 \qquad and \qquad \underline{\hat{\beta}'} = \left[\hat{\beta}_0 \quad \hat{\beta}_1\right]$$

Solving the estimating formulas is a weighted least-squares problem, minimizing $\sum_{i=1}^{n} w_i^2 r_i^2$. The weights, however, depend upon the residuals; the residuals depend upon the estimated coefficients, and the estimated coefficients depend upon the weights. An iterative solution (called iteratively re-weighted least-squares, IRLS) is therefore required:

1. Select initial estimates $\underline{\hat{\beta}}^{(0)}$, such as the least-squares estimates.

2. At each iteration l, calculate residuals $r_i^{(l-1)}$ and associated weights $w_i^{(l-1)} = w[r_i^{(l-1)}]$ from the previous iteration.

3. Solve for new weighted-least-squares estimates

$$\underline{\hat{\beta}}^{(l)} = [\mathrm{X}'\mathrm{W}^{(l-1)}\mathrm{X}]^{-1}\,\mathrm{X}'\mathrm{W}^{(l-1)}\,\underline{y} \qquad \cdots \quad (9)$$

Where X is the model matrix, with x_i' as its i^{th} row, and:

$\mathrm{W}^{(l-1)} = diag\left\{w_i^{(l-1)}\right\}$ is the current weight matrix.

Steps 2 and 3 are repeated until the estimated coefficients converge.

The asymptotic covariance matrix of $\underline{\beta}^{Robust}$ is [John, (2014)]:

$$v\left(\underline{\beta}^{Robust}\right) = \frac{E\left(\psi^2\right)}{[E(\psi')]^2}(\mathrm{X}'\mathrm{X})^{-1} \qquad \cdots \quad (10)$$

Using $\sum[\psi(r_i)]^2$ to estimate $E\left(\psi^2\right)$, and $\sum[\psi'(r_i)/n]^2$ to estimate $[E(\psi')]^2$ produces the estimated asymptotic covariance matrix $v\left(\underline{\hat{\beta}}^{Robust}\right)$.

Using robust estimation will construct a proposed simple linear regression profile monitoring (Phase-I) for a three charts:

The control limits for monitoring the intercept are:

$$UCL_{Robust} = \beta_0^{Robust} + Z_{\alpha/2}\sqrt{v\left(\beta_0^{Robust}\right)}$$
$$CL_{Robust} = \beta_0^{Robust} \qquad \cdots \quad (11)$$
$$LCL_{Robust} = \beta_0^{Robust} - Z_{\alpha/2}\sqrt{v\left(\beta_0^{Robust}\right)}$$

The control limits for monitoring the slope are:

$$UCL_{Robust} = \beta_1^{Robust} + Z_{\alpha/2}\sqrt{v\left(\beta_1^{Robust}\right)}$$
$$CL_{Robust} = \beta_1^{Robust} \qquad \cdots \quad (12)$$
$$LCL_{Robust} = \beta_1^{Robust} - Z_{\alpha/2}\sqrt{v\left(\beta_1^{Robust}\right)}$$

$v\left(\beta_0^{Robust}\right)$ and $v\left(\beta_1^{Robust}\right)$ it is estimated by the formula (10).

The control limits for monitoring the error variance are:

$$UCL_{Robust} = \frac{\sigma^2_{Robust}}{n-2}\chi^2_{\alpha/2,(n-2)}$$
$$CL_{Robust} = \sigma^2_{Robust} \qquad \cdots \quad (13)$$
$$LCL_{Robust} = \frac{\sigma^2_{Robust}}{n-2}\chi^2_{(1-\alpha/2),(n-2)}$$

The robust estimates for three parameters (intercept, slop and σ^2_{Robust}) are:

$$\underline{\hat{\beta}}_j^{(l)} = [X'W_j^{(l-1)}X]^{-1}X'W_j^{(l-1)}\underline{y}_j \qquad \cdots \quad (14)$$

$$\text{MSE}(Robust)_j = \frac{\sum_{i=1}^{n}\left(\hat{y}_{ij}^{Robust} - y_{ij}\right)^2}{n-2} \qquad \cdots \quad (15)$$

For all samples (j=1,2, … , k), It is the values that draw on the profile charts and we have:

$$\hat{\beta}_0^{Robust} = \frac{\sum_{j=1}^{k}\hat{\beta}_{0j}^{l}}{k}$$
$$\hat{\beta}_1^{Robust} = \frac{\sum_{j=1}^{k}\hat{\beta}_{1j}^{l}}{k} \qquad \cdots \quad (16)$$
$$\text{MSE}_{Robust} = median\left(\text{MSE}\,(Robust)_j\right)$$

Estimated parameters in formula (16) it is used to calculate the limits of the charts in the formulas (11), (12) and (13) and when the preliminary sample intercept, slope and MSE are plotted on these charts, no indication of an out of control condition is observed. Therefore, since all the charts exhibit control, we would conclude that the process is in control at the stated levels and adopt the trial control limits for use in phase II, where monitoring of future data is of interest.

3. Application side

To illustrate the performance of our proposed method, Simulations were used to generate the data through program MATLAB is designed for this purpose (see Appendix-A) the underlying simple linear profile used in this paper is defined as:

$$y_{ij} = 3 + 0.75x_i + \varepsilon_{ij}$$

Where the errors, ε_{ij} are independent and identically distributed (i.i.d.) normal random variables with $\mu = 0$ and $\sigma^2 = 0.5$ with the addition of outlier to one of values. The x-values for each sample are initially fixed at x = 2, 4, 6, 8 and 10. The number of x-values, n, per sample are also constant and equal to (5) for (20) samples, (see Appendix-B).

In Phase I, a set of data is generated and analyzed, constructing trial control limits to determine if the process has been in control during the period which the data were generated, and to see if reliable control limits can be established for monitoring future production. In Phase I, it is typical to collect about 20 points. Using these generated data, control limits are calculated and data points are plotted on the control chart. If there is any point outside of the control limits, investigation is needed for potential causes. The operators/engineers work on the identified assignable causes to eliminate them. Then, points outside of the control limits are excluded and a new set of revised control limits are calculated. New data are collected again and compared to the revised control limits.

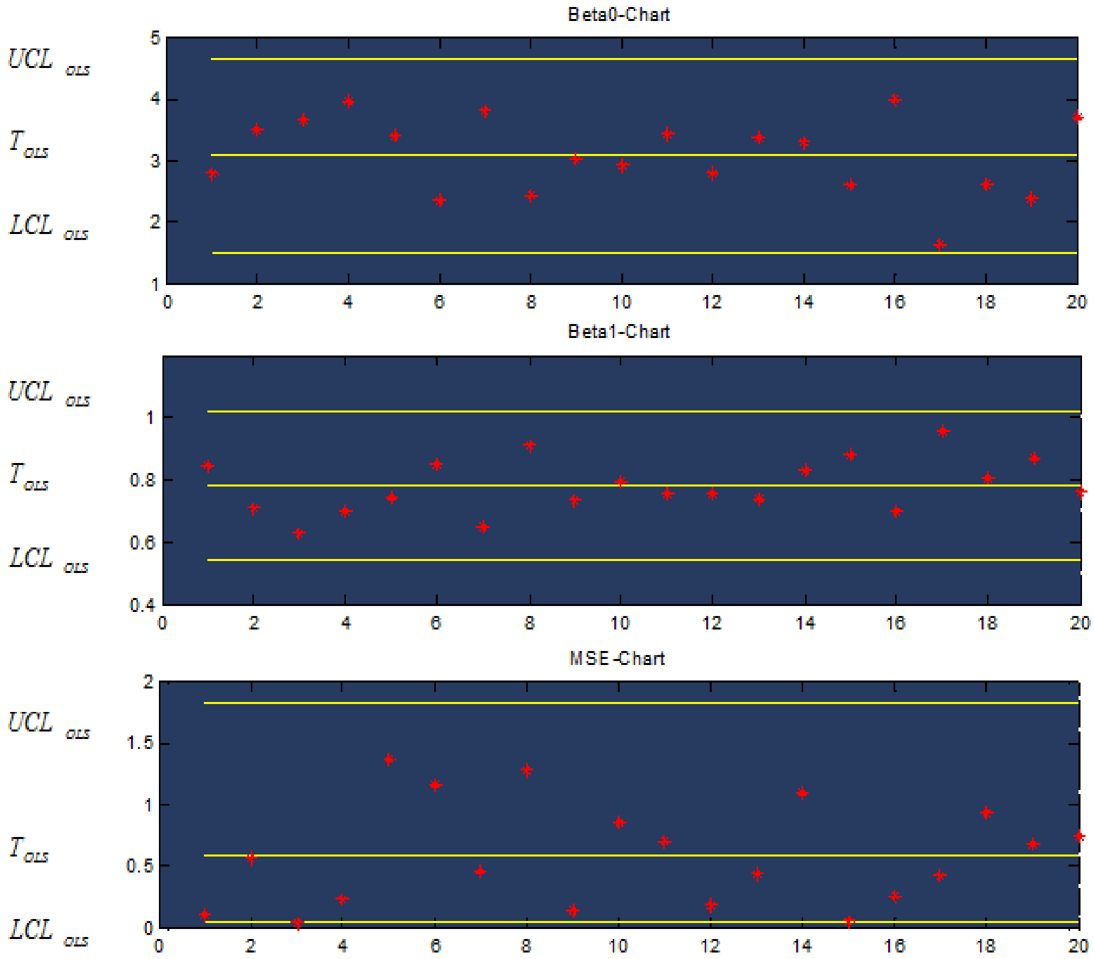
3.1: Classical method

Profile Monitoring is the utilization of control charts for checking the stability of the quality of a product over time when the product quality is characterized by a function at each time point. The profile can be presented by a simple linear regression model (formula-1), specifically intercept, slope and MSE for (20) samples (the data in Appendix-B) and estimation parameters by using OLS method (Depending on the formula 5) as in the following table:

Table nuber (1)
Estimated parameters using (OLS) classical method

Sample	$\hat{\beta}_0$	$\hat{\beta}_1$	MSE
1	2.801775	0.843431	0.111440
2	3.504743	0.707192	0.560694
3	3.674752	0.630598	0.042243
4	3.980747	0.699719	0.229908
5	3.397213	0.742510	1.365322
6	2.365829	0.847905	1.171745
7	3.821197	0.648417	0.459731
8	2.434472	0.909862	1.277712
9	3.027204	0.736979	0.141877
10	2.916312	0.792542	0.862238
11	3.439549	0.756399	0.703021
12	2.800055	0.755140	0.186850
13	3.381978	0.740953	0.436847
14	3.310197	0.829359	1.100407
15	2.605640	0.882676	0.056761
16	3.995703	0.698429	0.258459
17	1.637003	0.957360	0.421074
18	2.623086	0.805555	0.931233
19	2.383516	0.869578	0.684788
20	3.725316	0.758347	0.748918

Construct linear profile charts using classical method depending on the formulas 2, 3, 4 and 6 were as follows:



Fiq nuber 1
Classical Profile Monitoring Charts

The charts are shown in Fig. 1. When the preliminary sample intercept, slope and MSE are plotted on these charts, no indication of an out of control condition is observed. Therefore, since all the charts exhibit control, we would conclude that the process is in control at the stated levels and adopt the trial control limits for use in phase II, where monitoring of future data is of interest, the following table summarizes this charts with width of charts (the difference between UCL and LCL):

Table nuber (2)
Control Limits for Classical Profile

Chart	Target	UCL	LCL	Width
intercept	3.091314	4.667037	1.515592	3.151445
slope	0.780648	1.018197	0.543099	0.475098
MSE	0.587563	1.831240	0.042305	1.788935

3.2: Proposed (Robust) method

The profile can be represented by a simple linear regression model (formula-1) for same data (Appendix-B), specifically intercept, slope and

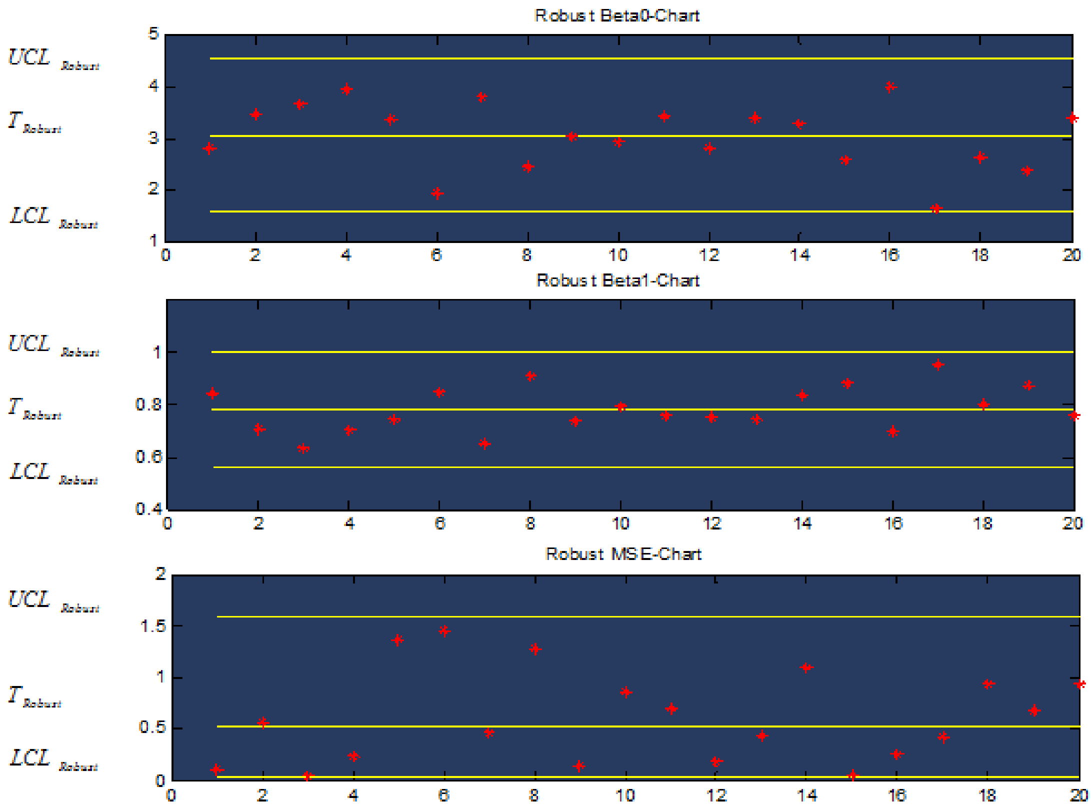
MSE for (20) samples and estimation parameters by using robust method (formulas-14 and 15) as in the following table:

Table nuber (3)

Estimated parameters using robust method

Sample		$\hat{\beta}_0^l$	$\hat{\beta}_1^l$	MSE (Robust)
1		2.812715	0.841868	0.111476
2		3.475056	0.709149	0.561282
3		3.665196	0.630711	0.042375
4		3.959218	0.701194	0.230205
5		3.371951	0.743891	1.365828
6		1.950397	0.847601	1.461920
7		3.806031	0.651343	0.459855
8		2.455996	0.908473	1.278028
9		3.036025	0.736404	0.141929
10		2.938946	0.790843	0.862535
11		3.433441	0.756710	0.703053
12		2.800137	0.753747	0.186990
13		3.385417	0.741468	0.436921
14		3.280161	0.835417	1.100963
15		2.593026	0.883923	0.056826
16		3.996695	0.696690	0.258648
17		1.654563	0.954647	0.421175
18		2.634849	0.803705	0.931280
19		2.368485	0.870669	0.684924
20		3.393341	0.758389	0.932315

Construct linear profile charts using proposed method depending on the formulas 11, 12 ,13 and 16 were as follows:



Fiq nuber (2)
Robust Profile Monitoring Charts

The charts are shown in Fig. 2. When the preliminary sample intercept, slope and MSE are plotted on these charts, no indication of an out of control condition is observed. Therefore, since all the charts exhibit control, we would conclude that the process is in control at the stated levels and adopt the trial control limits for use in phase II, where monitoring of future data is of interest, the following table summarizes this charts with width of charts (the difference between UCL and LCL):

Table nuber (4)
Control Limits for Robust Profile

Chart	Target	UCL	LCL	Width
intercept	3.050582	4.519439	1.581726	2.937713
slope	0.780542	1.002281	0.559404	0.442877
MSE	0.510568	1.591271	0.036761	1.554510

3.2: Comparison between the Classical and Robust method:

Through Table 2 and 4 notes that the robust estimates (3.050582, 0.780542 and 0.510568) were more accurate, better and closer than the classical method (3.091314, 0.780648 and 0.587563) to value assumed in the model (3, 0.75 and 0.5) for a three parameters (intercept, slope and σ^2

respectively), robust charts width were also more accurate and smaller than classical charts width.

Simulation was performed for the linear model is assumed and repeated 30 times and at several different values of the parameters and sample sizes and construction classical and robust profile Monitoring Charts and width account of charts and summarized in the following table (n=5, k=20, intercept=3, slope=0.75 and MSE=0.5):

Table nuber (5)
Width of Classical and Robust Profile Charts

Simulation	Intercept-Chart		Slope-Chart		MSE-Chart	
	Classical	Robust	Classical	Robust	Classical	Robust
1	3.8824	3.5682	0.5853	0.5379	2.7150	2.2934
2	3.9485	3.2026	0.5953	0.4828	2.8083	1.8475
3	3.1994	2.7946	0.4823	0.4213	1.8439	1.4067
4	3.3898	3.5016	0.5110	0.5279	2.0698	2.2085
5	3.1514	2.9378	0.4751	0.4429	1.7889	1.5545
6	2.9235	2.3197	0.4407	0.3497	1.5395	0.9693
7	3.2988	3.2278	0.4973	0.4866	1.9601	1.8767
8	3.2988	3.2278	0.4973	0.4866	1.9601	1.8767
9	3.6247	3.1089	0.5464	0.4687	2.3665	1.7410
10	3.8751	3.7390	0.5842	0.5637	2.7048	2.5182
11	3.3270	3.4220	0.5016	0.5159	1.9938	2.1093
12	3.5027	3.2974	0.5281	0.4971	2.2099	1.9585
13	3.5409	3.2458	0.5338	0.4893	2.2584	1.8977
14	3.3068	3.0753	0.4985	0.4636	1.9697	1.7035
15	4.0041	3.7536	0.6036	0.5659	2.8880	2.5379
16	3.8445	4.0158	0.5796	0.6054	2.6623	2.9048
17	3.5921	3.4751	0.5415	0.5239	2.3242	2.1753
18	3.5557	3.1356	0.5360	0.4727	2.2773	1.7710
19	3.7097	3.2929	0.5593	0.4964	2.4788	1.9532
20	3.4274	3.2981	0.5167	0.4972	2.1160	1.9593
21	3.8593	3.4457	0.5818	0.5194	2.6827	2.1385
22	3.7822	3.6919	0.5702	0.5566	2.5767	2.4551
23	3.7803	4.1359	0.5699	0.6235	2.5741	3.0812
24	3.4691	3.1149	0.5229	0.4696	2.1678	1.7477
25	3.8607	3.7979	0.5820	0.5725	2.6848	2.5981
26	3.8141	3.3824	0.5750	0.5099	2.6204	2.0608
27	3.3807	3.4248	0.5096	0.5163	2.0587	2.1128
28	3.1351	2.9495	0.4726	0.4446	1.7704	1.5670
29	3.2702	3.0017	0.4929	0.4525	1.9263	1.6229
30	3.4730	2.8387	0.5236	0.42795	2.1727	1.4515

Table (5) shows that width of proposed charts smaller than the width of classical charts, which means that the proposed charts more accurate than classical charts, the proposed method also got better results at several values of parameters and sizes and the number of different samples.

4. Conclusions

1. Robust estimates can be used in the construction linear profile Monitoring (phase-I) when there an outlier values.
2. Proposed charts more accurate than classical charts when there an outlier values.
3. Robust estimates for a three parameters (intercept, slope and σ^2) were more accurate, better and closer than the classical method to value assumed in the model when there an outlier values.

5. References

[1] Azadeh Adibi (2013), A P-value based Approach for Phase II Profile Monitoring, A Dissertation "Degree Doctor of Philosophy", Arizona state university.

[2] Chang Y. C., Lee Y. J., Chiu Y. N., Lin T. D., Wu S. Y., Chiu H. C., Kwo J., Wang Y. H., Hong M., and Cryst J., (2007) Growth 301/302, 390.

[3] Charles V. Sterwart, (1999), Robust Parameter Estimation in Computer Vision, Society for Industrial and Applied Mathematics, Vol. 41, No. 3, pp. 513–537.

[4] Croarkin, C. and Varner, R. (1982) ‘Measurement assurance for dimensional measurements on integrated-circuit photomasks’, NBS Technical Note 1164, US Department of Commerce, Washington, DC.

[5] Holland, P. W., and R. E. Welsch, (1977) "Robust Regression Using Iteratively Re weighted Least-Squares." Communications in Statistics: Theory and Methods, A6, pp. 813–827.

[6] Huber, P. J. (1964), “Robust Estimation of a Location Parameter.” Annals of Mathematical Statistics 35:73—101.

[7] John Fox, (2014), "Robust Regression", Appendix to an R Companion to Applied Regression, Second Edition.

[8] Junjia Zhu and Dennis K. J. Lin (2010), Monitoring the Slopes of Linear Profiles, Quality Engineering, 22:1-12.Taylor & francis Group, LLC.

[9] Kang, L., and Albin, S. L. (2000). “On-Line Monitoring When the Process Yields a Linear Profile”. Journal of Quality Technology 32, pp. 418-426.

[10] Mestek, O., Pavlik, J., Suchánek, M., Multivariate Control Charts: Control Charts for Calibration Curves. Fresen. J. Anal. Chem. 350 (6), 1994, pp. 344–351.

[11] Rosenblatt, J .R. & Spiegelman, C.H. (1981). Discussion of "A Bayesian analysis of the linear calibration problem" by William G. Hunter and Warren F. Lamboy, Technametrics 23, 329-333.

[12] Stover F.S., Brill R.V. (1998) Statistical Quality Control Applied to Ion Chromatography Calibrations, Journal of Chromatography A, 804: 37-43.

Appendix-A

```
clc
k=20, x=[2 4 6 8 10]', n=length(x), E=[ones(size(x)) x], beta0=3,
beta1=0.75, Variance=.5, xbar=mean(x), v=(n-1)*var(x);
beta=[beta0 beta1]';
for j=1:k
   ebs=randn(1,n),  ebs(3)=rand*3,  ebs=sqrt(Variance)*ebs';
   y=E*beta+ebs, betahad=E\y, F(:,j)=y, b0(j)=betahad(1,1);
   b1(j)=betahad(2,1), Y=E*betahad, e=y-Y, MSE=e'*e/(n-2);
   MS(j)=MSE, [brob S]=robustfit(x,y), Yr=E*brob;
  br0(j)=brob(1,1), br1(j)=brob(2,1), Yr=E*brob, e1=y-Yr;
  MSEr=e1'*e1/(n-2), MSr(j)=MSEr;
end
MSE=mean(MS), MSErob=median(MSr)
A0=mean(b0), A1=mean(b1), Ar0=mean(br0), Ar1=mean(br1);
S0=sqrt(MSE*(1/n)), S1=sqrt(MSE*(1/v)), Vr=inv(E'*E)*MSErob;
for j=1:k
   Tb0(j)=A0,  Tb1(j)=A1, TMSE(j)=MSE;
   UCLb0(j)=A0+1.96*S0;
   LCLb0(j)=A0-1.96*S0;
   UCLbr0(j)=Ar0+1.96*sqrt(Vr(1,1))
   LCLbr0(j)=Ar0-1.96*sqrt(Vr(1,1))
   UCLbr1(j)=Ar1+1.96*sqrt(Vr(2,2))
   LCLbr1(j)=Ar1-1.96*sqrt(Vr(2,2))
   UCLb1(j)=A1+1.96*S1;
   LCLb1(j)=A1-1.96*S1;
   UCLMSE(j)=(MSE/(n-2))*9.35;
   LCLMSE(j)=(MSE/(n-2))*0.216;
   UCLMSEr(j)=(MSErob/(n-2))*9.35;
   LCLMSEr(j)=(MSErob/(n-2))*0.216;
   Tbr0(j)=Ar0;
   Tbr1(j)=Ar1;
   TMSEr(j)=MSErob;
end
t=1:k;
subplot(6,1,1), plot(t,UCLb0, '-',t,LCLb0,'-',t,Tb0,'-',t,b0,'*'),title('Beta0-Chart'),
```

```
subplot(6,1,2),plot(t,UCLb1,'-',t,LCLb1,'-',t,Tb1,'-',t,b1,'*'),title('Beta1-Chart')
subplot(6,1,3),plot(t,UCLMSE,'-',t,LCLMSE,'-',t,TMSE,'-',t,MS,'*'), title('MSE-Chart')
subplot(6,1,4), plot(t,UCLbr0,'-',t,LCLbr0,'-',t,Tbr0,'-',t,br0,'*'),title('Robust Beta0-Chart')
subplot(6,1,5), plot(t,UCLbr1,'-',t,LCLbr1,'-',t,Tbr1,'-',t,br1,'*'),title('Robust eta1-Chart')
subplot(6,1,6),plot(t,UCLMSEr,'-',t,LCLMSEr,'-',t,TMSEr,'-',t,MSr,'*'), title('Robust MSE-Chart')
```

Appendix-B

n	x	y_{i1}	y_{i2}	y_{i3}	y_{i4}	y_{i5}	y_{i6}	y_{i7}	y_{i8}	y_{i9}	y_{i10}
1	2	4.63	4.73	4.93	5.14	4.58	3.69	4.78	5.14	4.14	4.72
2	4	5.78	5.83	6.07	6.61	6.31	5.23	6.53	4.70	6.17	5.05
3	6	8.24	8.80	7.75	8.87	7.72	9.12	7.79	8.53	7.82	8.44
4	8	9.42	9.31	8.57	9.64	10.97	8.88	9.86	9.03	9.05	9.94
5	10	11.24	10.06	9.98	10.63	9.68	10.35	9.59	12.07	10.07	10.20
n	x	y_{i11}	y_{i12}	y_{i13}	y_{i14}	y_{i15}	y_{i16}	y_{i17}	y_{i18}	y_{i19}	y_{i20}
1	2	5.14	4.35	5.20	4.62	4.19	5.23	2.95	4.60	3.68	4.78
2	4	5.79	5.39	5.79	6.69	6.46	7.03	5.76	4.84	5.99	6.63
3	6	8.99	7.92	8.34	9.51	7.85	7.83	8.18	8.55	8.69	9.60
4	8	8.76	8.81	8.62	8.69	9.50	10.23	9.22	8.42	8.53	9.39
5	10	11.22	10.19	11.19	11.92	11.50	10.61	10.79	10.87	11.11	10.98