Applied Stochastic Processing on tooth decay during the period 2013-2016 in Erbil City of Kurdistan Region of Iraq

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المعالجة العشوائية التطبيقية على تسوس الأسنان خلال المدة (٢٠١٣-٢٠١٦) في مدينة أربيل بمنطقة كوردستان العراق

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Abstract:

In this paper the researcher attempts to illustrate the use of stochastic process method, to formulate a general rule for its application in different fields that allow us to use primary information.

For this purpose the transition matrices of rank 8 * 8 were used, conditional probability matrix and case rate estimation. The time series data of 200 patients from the Republican Hospital in Erbil have been used, who had tooth decay or loss during the period 2013-2016. The result shows that, the probability of total (patients) losing their teeth equals 0.66. While, the probability of path which lids to ending this process, tooth decay (losing teeth) equals 0.68. Which are caused by either decay in premolar or canine surface of the vertical or in lateral, status (*abb*). The researcher proves that the method of stochastic process (Markov chains) is the appropriate method for this type of application.

Key words:- **Tooth decay, Markov chain process and Data from Erbil hospital**

المستخلص:

في هذا البحث يحاول الباحث توصيح استخدام طريقة عمليات تاعشوائية (سلاسل ماركوف)، لصياغة قاعدة عامة من أجل تطبيقة في العديد من المجالات العملية التي تسمح لنا باستخدام المعلومات الأولية. لغرض تطبيق هذة الطريفة ، قمنا بأيجاد المصفوفات الانتقالية ذات رتبة (٨*٨) مصفوفة الاحتمالية الشرطية و تقدير معدل الحالات، وقد استخدمنا بيانات السلاسل الزمنية و المكونة من (٢٠٠) مريضا من المستشفى الجمهوري في مدينة اربيل، والذين كانوا يعانون من و جود تسوس في الأسنان أو فقدانة خلال المدة ٢٠١٣-٢٠١٦ . أظهرت النتائج أن هناك احتمالية (٠,٦٦) من مجموع الحالات (المرضى) الى أن يفقدوا اسنانهم، استنتجنا أيضا المسار الذي يؤدي الي انهاء هذة العقلية، عملية تسوس الأسنان (فقدان الأسنان)، تساوي (٠,٦٨) . التي تسبب بها الحاليتين وجود التسوس على السطحين الأنسي و الاوحشي، أي الحالة (٠١١). وأثبت الباحث أخيرا بأن طريقة عمليات سلاسل ماركوف هي الطريقة المناسبة لهذا النوع من التطبيقات.

الكلمات المفتاحية: تسوس الاسنان، عملية سلاسل ماركوف، البيانات المستشفى اربيل المراقبة.

Introduction

Stochastic processes are of great importance in statistical applications because of its numerous uses in the fields of life, time and risk. There are three basic elements in the randomization processes involved in biostatistics applications. They are risk of death, risk of disease, and birth risk. And other risks which is the most recent human infection continuously and various degrees in abundance for a long time.

Markov chain is a special case of the stochastic process involving a number of states with time parameter. Markov (1907-1906) conceders the pioneer of creating the basic concepts of Markov chain, later on it developed by other mathematicians. Markov's chains have been used in many areas, including economics, agriculture, sociology, and many other studies in various fields (Crowder, 2012).

Therefore, the researcher applies Markov chains and notes the transitional conditions of patients suffering from decay in the teeth and the impact of time on this decay in the patient.

The Basic Concepts

1- **Markov process**

Generally the Markov chain can be describe as follow: Suppose *Zt* denoted the state of the unit at time t and assume a finite stat space, with states labeled as $\{z_0, z_1, \ldots, z_m\}$. State z_0 is an initial state and state z_m is a absorbing state. So assuming that the unit is monitored from birth, it starts in state z_0 and then, after a journey visiting none, some or all of the intermediate states, comes to its final resting place in state z_m by the probability of p_{ij}, and this probability does not depend on which states were in the chain before the current situation (Dobrow, 2016).

On the other hand, the random process with the discrete time $\{Z_i; t = 0,1,2,...\}$or random process with the continuous time $\{Z_i; t >= 0\}$is called the Markov process for any set of time periods$\{t_1 < t_2 < ... < t_n\}$. For a set of time series $\{t_1 < t_2 < ... < t_n\}$, if the distribution of $\{Z_m\}$conditional to values of$\{Z_{t1}, Z_{t2}, ..., Z_{tn}\}$, then the parameter depends only on$\{Z_{m-1}\}$, i.e., for any real number$(Z_1, Z_2, ..., Z_n)$.

$$P[Z_{tn} \leq Z_t / Z_{t1} = Z_1, ..., Z_{tn-1} = Z_{n-1}] = P[Z_{tn} < Z_n / Z_{tn-1} = Z_{n-1}] \qquad (1)$$

Equation (1) explains the case what if the state of the process is given at the present time; its future status does not depend on the movement of the past process.

Markov's process is characterized by the following characteristics, if a value Z_t is given then the value of Z_E, does not depends on the value of Z_n , for all $E>t$ and $n<t$. If the disease is known, the probability of any future state related to the process is not related to the state in the past period. Independent attempts can be defined as a set of finite or infinite possible outcomes, E_1, E_2,.. , each is associated with a probability of P_Z (Florescu, 2015)..

$$P[Z_{j0}, Z_{j1}, ..., Z_{jn}] = P_{j0} ... P_{jn} \qquad (2)$$

2 Classifications of Markov processes

The Markov Processes classifies based on:
1- The nature of the process where if they are discrete parameter or continuous parameter.

2- State space process, the real number Z is said to be a state of the random parameter $\{Z_t, t \in T\}$ if t contain in T, So that $P[Z-h<(t)<Z+h)]$ has positive probability for each (h>0). For this case a set of possible values called State-Space. And it is called discrete state-space if it contains a finite or infinite number can be calculated by trials, while the not discrete one is denoted as continuous state-space. The Markov process with discrete state-space identifies as Markov chain with a set of real numbers [0,1,2,..] (Herzog, 2016).

Table 1; Illustrated the state-space

	Countable state space	**Continuous or general state space**
Discrete-time	(discrete-time) Markov chain on a countable or finite state space	(Markov chain on a general state space)
Continuous-time	Continuous-time Markov process or Markov jump process	Any continuous stochastic process with the Markov property.

3 Markov chains

A Markov chain is a stochastic process with the Markov property. The term "Markov chain" refers to the sequence of random variables based on a process that defines Markov's property only between adjacent times (as in a "chain") series as a function of time. A stochastic process has the Markov property when the conditional probability distribution of future states of the process (provisional on current and past states), not on the sequence of advanced events. A process with this feature is called a Markov process (Crowder, 2012).

Let $\{E_1, E_2, \ldots\}$ be a sequence set of multiple outcomes, the probability of this sequence is determined by the following equation:

$$P[E_{j0},,...,E_{jn}] = P_{j0\,j1}...P_{jn-2}P_{jn-1}P_{jn} \qquad (3)$$

Or the stochastic process is called Markov chain if it met the following condition:

$$P\lfloor Z_{n+1=j} / Z_0,...,Z_n \rfloor = P\lfloor Z_{n+1=j} / Z_n \rfloor \qquad (4)$$

For all values ($n \epsilon N$, $j \epsilon J$), the second condition in equation (3) is known as the Markov property, which shows that the Markov chain is the series of random variables, so that for all $n \epsilon N$ is a future state Z_{n+1} and it is independent from the previous state. The Markov chains is a special case of the random process characterized by specific properties which represents Markov process with finite or infinite of states. The t parameter is denoted for the time. It supposed to be a set of non negative integer numbers, based on that we faced two states: the discreet and continuous time. For the first case we realized regularity between the time periods (discrete time series), and Z_n represents a random variable its value denotes state at time (n). While the sequence of (n=1, 2 …) variables stand for all states. This situation will be correct for all discrete random variable and for all states (j_1, j_2, …,j_n) (Avery, et al., 1999) . And it can be represents as:

$$P\lfloor Z_n = j_n / Z_{n-1} = j_{n-1}, Z_{n-2} = j_{n-2}...Z_j = j_1 \rfloor = P[Z_n = j_n / Z_{n-1} = n-1] \qquad (5)$$

On another hand, the second case is when the parameters are represents a continuous time periods, whereas, t process represents a set of non negative real numbers and the random process [Z_t; $t \epsilon R$] will satisfy Markov properties for all time $t_0<t_1<\ldots<t_n$ periods and all values n.

$$P[Z_t = j / Z_0 = j_0,...,Z_t = j_0] = P[Z_m = j_n] \qquad (6)$$

A Markov chain with *r* state called an ergodic chain if it is possible to go from every state to every state (not necessarily in one move) if there are $\{\pi_1, \pi_2, \ldots, \pi_r\}$ numbers, So that for any two cases(i, j).

$$\lim_{m\to\infty} P_{m(i,j)} = j \qquad (7)$$

If the Markov chain is ergodic, it arrive at statistically equilibrium after many states, then the unconditional probabilities $P_n(j)$ are approach to, see (Israel, et al.. 2001).

$$\lim_{n\to\infty} P_{n(j)} = \Pi_j \qquad (8)$$

4 Transition probability matrix

A square matrix used to describe the transitions of a Markov chain is called transition probability matrix. Each of its entries is a nonnegative real number representing a probability (Florescu, 2015).

The states spaces for discrete Markov chain, Zn, define as a set of specific values, where the discrete time T takes (T = 0,1, ...) values. Then Z_n is said to be in i state if ($Z_n=i$) with the probability of ($Z_{n+1}= J$) in state J, knowing that Zn in state i called one step probability transition and denotes as (P_{ij}) means that:

$$P_{ij} = P[Z_n = j / Z_{n-1} = i] \qquad (9)$$

The discrete Markov chain will be constant and continuous, if the transition probability from state to another independent from time in the same step where the state that has occurred and this is for all cases.

$$\{P_{ij} = P_{ij}\} \qquad (10)$$

Let P_{iJ} represents the probability of stochastic transaction from i to J and this conditional transition called a probability transaction in the series

Assuming that Z_{k1} is stationary Markov chain at discrete time with a specific states, [$E=1,2,3,\ldots, n$], in this case we have n^2 of probability transaction (Herzog, 2016).

$$P_{iJ}, i = 1,2,\ldots, n; J = 1,2,\ldots, n \qquad (11)$$

These values can be written in a transition probability matrix P:

$$P = \begin{bmatrix} P_{11} & P_{12} & P_{13} & . & . & . & P_{1n} \\ P_{21} & P_{22} & P_{23} & . & . & . & P_{2n} \\ . & . & . & & & & \\ . & . & . & & & & \\ . & . & . & & & & \\ P_{n1} & P_{n2} & P_{n3} & . & . & . & P_{nn} \end{bmatrix}$$

The transition probability matrix for Markov chain P , is a random matrix, and it requires the following properties:

1- All probabilities non-negative and not greater than unity, $P_{iJ} \geq 0$.

2- Sum of each row is unity.

$$p_{i1} + \ldots + p_{in} = P\left[Z_{k=1} / Z_{k-1=i}\right] + P\left[Z_{k=2} / Z_{k-1=i}\right] + \ldots + P\left[Z_{k=n} / Z_{k-1=i}\right] = P\left[Z_{k\in s} / Z_{k-1=i}\right] = 1 .$$

3- Probability of staying in present state may be non-zero (Florescu, 2015).

Theoretical Part

Before coming to application process the researcher see that it is necessary to explain some basic concepts relating to our study. First of all we have to rewrite the transition matrix to analyze finite Markov chain in our study, as illustrated by following equation:

$$P = [P_{ij}] \qquad (12)$$

$$P = \left[\begin{array}{cc|c} r-s & I & \begin{matrix} S \\ 0 \end{matrix} \\ \hline & R & Q \end{array}\right]$$

where:

S: is the transient state.
r-S: is the absorbing state.
I: is the Identity matrix, in order *(r-S)*(r-S)*.
0: is the zero matrix, in order *(r-S)*S*.
R: is the matrix in order *S*(r-S)*, its element represents the probability of transition from transient state to the absorbing state.

Q: is a matrix of rank $(S*S)$, its element (conditional probability values) represents the probability of transition P_{ij} the non absorbing states.
Before addressing the results of the theory, it is necessary to identify some terms that are closely related to the practical part, including:
E: is a vertical vector for each element.
A^2: is the matrix with which, each square elements of (A) matrix that is:
$A^2=[a^2_{ij}]$
$rot\ A^2$: is the matrix its elements represents the square root of the matrix A.
Diagonal A: is the matrix that its diagonal elements are a value of (A) matrix and the elements of its off diagonal are zero.
n_j: is the number time unites in which a process in (S_j) states before finishing. (S_j) represents a state of decay states.
t: is the number of total time unites steps, that the process follow before ending.
h_{ij}: is the possibility that a process stay in (S_j) state, when it started at (Si) state, where $h_{ij}>1$.
H: is the matrix with (h_{ij}) elements (see Allen, 2003: Lakatos, et al., 2013).

The following steps should be followed to achieve our aim (Holden et al., 2010: Knill, 2009).

1- The first step is to set the conditional probability matrix, as:

$$C=[C_{ij}] \qquad (13)$$

$$\left.\begin{aligned} C_{ij} &= p_{ij}/(1-p_{ij})IF \quad i \neq j \\ C_{ij} &= 1/(1-p_{ij})IF \quad i = j \end{aligned}\right] \qquad (14)$$

In order to move to second step the following definition should be first developed:

The sequence of process is

$$\underbrace{S_i - S -}_{m} S - - - - - S_j$$

where

$$1 \le i \le j \le r, 0 \le m \le j-i-1$$

Determining the path between pair of states (S_i, S_j), with number of interrelated states. As for any pair of states there may by more than one path.

We assume that (m_k) refers to (m) values for the (k) j-i-1 path, then the maximum number of paths between (S_i, S_j) is $J>=i$ and this can be obtained only if:

$$i < j \quad P_{i(i+1)} \neq 0$$

q_{ij} is the probability that the process begin in (S_i) state and ends in (S_j) state toward the path k.

$$q_{ij} = \prod_{k=0} c_{ij} \tag{15}$$

Whereas, the symbol Π_k indicates the multiplication of the conditional probability (C_{ij}) of the (m_{k-1}) number that connecting (S_i, S_j).

2 – It is possible that the process remain in transient state (S_i) before it finishes (second step). It can be achieved by the following relations:

$$H_{s*s} = [h_{ij}] \tag{16}$$

$$\left.\begin{aligned} h_{ij} &= \sum q_{ij}^{k} \quad if \;\; i < j \\ h_{ij} &= 1 \qquad\;\; if \;\; i = j \end{aligned}\right\} \tag{17}$$

In case of using basic matrix (N) then

$$N = N\, N_{dg}^{-1} \tag{18}$$

While the average number of stats, that the process take before ending it, can be obtain by adding rows in H matrix. The average number of cases taken by the process before they are completed can be obtained by pooling through H ranks (Yates and Goodman, 2014).

In other word by using the following relationships:

$$W = \left[\overline{W}_i\right] \tag{19}$$

$$\overline{W}_i = 1 + \sum_{i<j\leq S} h_{ij} = 1 + \sum_{K} m\, q_{ij} \tag{20}$$

If we want to express it in term of basic matrix N, then:

$$W = H_E = (N \overset{-1}{N} d_g)E \tag{21}$$

3- The average number of time units that a process takes in transient state (S_i) before ending it if it started at *(S_i)* with it, (the third step) caricaturized by *(n_{ij})* elements for the basic matrix *(N)*. i.e

$$N = [M_{(ni)}] = [I - Q]^{-1} \qquad (22)$$

Whilst, the average number of required steps that the process takes, which starts in specific state to be in ending state, is represents by:

$$[M_{i(t)}] = N_E \qquad (23)$$

Thought, the probability of transformation *(P_{ij})* , which means that the process of decay is moving from state *(i)* to *(J)* which is determined for each pair of existing cases within *(T)*, which can be estimated by the following formula (see Yang, 2015).

Total number of age with Sj test pattern at (Z + 1) of cases

The total number of years with an SJ test pattern at (x) of cases

$$P_{ij} = \frac{Total\, No. of\, tooth with\, S_j\, test\, at (Z+1)\, of\, states}{Total\, No. of\, tooth with\, test\, at\, (Z)\, of\, states} \qquad (24)$$

As show in the probability matrix (Parzen, 2015).

Practical Part

A classical stochastic process is applied on tooth decay on a group of patients with a special type of decays, which is called a (larynx), located in the upper jaw of the mouth. A sample of (200) observations were taken from Republic Erbil Hospital during 2013-2016, the patients were observed every year for three years. The objective of this study is to:

1- Estimate the conditional probability, which explain the entering the decay process to state (S_j) at specific time as previously was in state (S_i). Through it we can estimate the probability for each possible path for the decay process.

2- Estimate the probability that the process will always be in a specific situation, through that we can estimate the average states which is taken by decay process before reaching absorbing state.

Markov chains process is suitable for this study because:

- There is a limited group of cases [$T=(S_1, S_2, \ldots, S_r)$], there were 8 possible cases.

- Always there is a state to begin with, which is expressed by the symbol (aaa).

- Transition process from one state to another, (Step).

- The process can be transferred from one state to another on an irregular basis.

- The probability of process moves from (S_i) to (S_j) by (K) steps, depends only on (S_i) state which is now engaged with, and independent from all other states previously been occupied by it before.

- The limited and finite Markov chain is the processes that have at least one state to enter the process and cannot get out from it, state (S_8).

We can depend on three types of tooth surfaces:

1- The Occlusal Surface (Z_1).

2- Mesial Surface (Z_2).

3- Distal Surface (Z_3).

These three types were studied, to explain for two cases, as follows:

- In case of healthy tooth surface *(a)*.

- in case of decayed tooth surface *(b)*.

There are (2^3=8) possible state for decay on the tooth surface, It can be represented by a three-spaced binary number *(Z1, Z2, Z3)* to represent different surfaces of tooth *(Z_i=0,1, i=1,2,3)*. It has been observed that the decay processes have 8 states:

(aaa), (aab), (aba), (baa), (abb), (bab), (bba), (bbb)

It is known that the *(bbb)* states, represents the stat of losing tooth, which is the state where ending the decay process, so it is the absorbing state. Whilst the remaining states, were either transient or transient states.

However, the case *(abb)* represents the decay on the (premolars and canine) surfaces. These different states were numbered according to the binary number system.

The result shows that, existing a set of states which are symbolized by [$T=(S_1, S_2, S_3, \dots, S_r)$], there is a group of process, in case of decay on both premolars tooth and canine tooth there is a great chance for molars tooth to be infected, which leads to loosing tooth entirely. Due to the occurrence of (damage decays on the teeth, and this is probably due to the nature and composition of the evidence of tongues and trees).

Dependent on the flow chart, Figure (1), we can calculate the average number of transient states that are occupied by the decay process, prior to reaching the absorbing state. Based on the equations (19) and (20), and following the next possibilities.

For instance, the probability of the process may occupy the transient stats before absorbing state.

(0.01+0.08+0.10+0.03)=0.22

This means the multiplication of all conditional probabilities, and then collecting all the multiplied numbers with four transient states. Whereas the probability that the process occupies two transient states before ending is:

(0.02+0.02+0.07+0.10+0.02)=0.34

While the combination of multiplication of all conditional probabilities for two-step transient states are explained in Table 3.

So:

$\overline{W}$ (aaa) =1(0)+2(0.34+3(0.22)

$\overline{W}$ (aaa) =1.34

The average of each transition is the sum of multiplied number of steps by the conditional probability resulted from adding transient probabilities.

Table 2. Stochastic process (Markov chain)

State	aaa	aba	abb	aab	baa	bab	bba	bbb
aaa	180	5	3	4	2	1	2	3
aba	0	142	0	20	0	0	25	13
abb	0	7	153	12	0	8	0	20
aab	0	0	0	0	162	20	16	12
baa	0	0	0	182	0	8	0	10
bab	0	0	0	0	0	160	15	25
bba	0	0	0	0	0	0	105	95
bbb	0	0	0	0	0	0	0	200

(A)

State	aaa	aba	abb	aab	baa	bab	bba	bbb
aaa	0	0.25	0.15	0.2	0.1	0.05	0.1	0.15
aba	0	0	0	0.26	0	0.17	0	0.43
abb	0	0	0	0.34	0	0	0.43	0.22
aab	0	0	0	0	0	0.44	0	0.56
baa	0	0	0	0	0	0.53	0.16	0.32
bab	0	0	0	0	0	0.8	0.38	0.63
bba	0	0	0	0	0	0	0	1
bbb	0	0	0	0	0	0	0	1

(C)

State	aaa	aba	abb	aab	baa	bab	bba	bbb
aaa	0.9	0.03	0.02	0.02	0.01	0.01	0.01	0.15
aba	0	0.71	0	0.1	0	0	0.13	0.07
abb	0	0.04	0.77	0.06	0	0.04	0	0.0.1
aab	0	0	0	0	0.81	0.1	0.08	0.06
baa	0	0	0	0.91	0	0.04	0	0.05
bab	0	0	0	0	0	0.8	0.08	0.13
bba	0	0	0	0	0	0	0.53	0.48
bbb	0	0	0	0	0	0	0	1

(B)

Primary State	States			$\overline{W}_j$
	1	2	3	
aaa	0	0.34	0.22	1.34
aba	0.43	0.19	0	0.81
abb	0.22	0.539	0	1.30
aab	0.32	0.28	0	1.11
baa	0.56	0.49	0	1.31
bab	0.63	0.38	0	1.39
bba	1	0	0	1
bbb	1	0	0	1

(D)

Table 2. Represents the following: (A) Original patients Data, (B) Probability translate matrix, (C) Probability conditional matrix and (D) Estimating the rate of cases taken by the caries before reaching the absorbing state.

The probability value for the tooth be in (aab) state is (0.25) and the probability of being in state (aba) is (0.15). These possibilities can be illustrated in Figure 1, and we note from this flow-chart that at the end leads to absorbing state , (bbb), and the canine surface appears to be more disposed to decay than other surfaces.

The probability value of any path can be calculated as the multiplication of conditional probability of equation (17). Table (4) shows the probability of paths (aaa), (aab), (bba) and (bbb).

We also note that there are two paths: the first rate represent (0.66) of total states, which leads to lose teeth. And the second rate equals (0.68), which represents the path that leads to the end of the process (losing teeth), due to the presence of decay on premolars and canine surface, (abb). As shown in Figure 1.

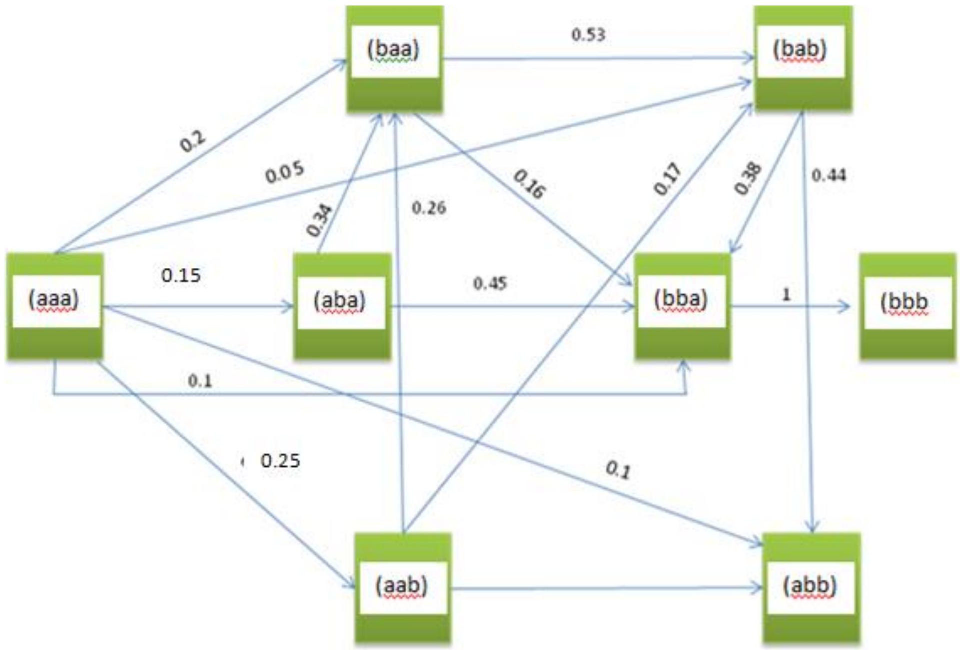

Figure 1: The flow-chart of the paths

The figure shows possible pathways for decays based on Table 4, where the arrows indicate the direction of the decay process and the numbers are conditional probabilities.

We can find the probability value for all paths through flow-chart, Figure 1, and Table 3. Which show probabilities for all possible paths. From state (aaa) to state (bbb) losing teeth.

Table 3: Represents the possibilities for all paths

State	Path					Wj	Li
1	(aaa)	(bba)	(bbb)			2	0.1
2	(aaa)	(bab)	(bba)	(bbb)		3	0.01
3	(aaa)	(bab)	(abb)	(bbb)		3	0.07
4	(aaa)	(baa)	(bba)	(bbb)		3	0.03
5	(aaa)	(aab)	(bab)	(bba)	(bbb)	4	0.02
6	(aaa)	(abb)	(bab)	(bba)	(bbb)	4	0.02
7	(aaa)	(aab)	(baa)	(bba)	(bbb)	4	0.01
8	(aaa)	(aba)	(baa)	(bba)	(bbb)	4	0.01
9	(aaa)	(baa)	(bab)	(bba)	(bbb)	4	0.04

Conclusions and recommendations

Depending on our results we come by some conclusions and recommendations:

First of all, we conclude that there are two paths: the first one (0.66) of total states leads to lose teeth. And the second one equals (0.68), which represents the path leading to the end of the process of decay or loss of teeth due to the presence of decay on the surface of the premolars and canine.

Secondly, in the three cases of infection: the premolars and the canine or the molars surfaces, the probability of getting dental overhaul will be very small, valued just 0.22. Unfortunately in this case the doctors advise their patient to not search for treatment, i.e. they are not providing them with treatments.

Thirdly, if the premolars and molars surfaces scorbutic by decay, then the probability of their dental overhaul equals to (0.34).

Based on that, we advise parents to not let their children to consumption a lot of sweets. To guaranty that they are not losing their teeth at early age.

It is necessary that the government provide hospitals with modern equipment and medical tools to detect some types of teeth decay, which does not appear by simple examination.

Finally, to prevent decay we recommend that every citizens follow-up the dental clinic regularly.

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