

Modified Adomian Decomposition Method with genetic algorithm for solving Nonlinear Belousov- Zhabofinkii System

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Abstract

In this study, the optimal parameters for the nonlinear (BZS) Belousov-Zhabofinkii system using the developed Adomian analysis method with the genetic algorithm abbreviated as GA MADM were found. The fit function was used to solve the nonlinear system. It is discovered that choosing the optimal parameters lead to solutions that are Extensively close solutions. The results showed that the approximate solutions were sufficiently close to the exact solutions, and the proposed method was more accurate and efficient than the normal MADM, which traditionally uses pre-selected parameters.

Keywords: Adomian decomposition method (ADM); Belousov-Zhabofinkii system (BZS); genetic algorithm (GA).

طريقة تحليل Adomian المعدلة باستخدام الخوارزمية الجينية لحل نظام Belousov-Zhabofinkii غير الخطي

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الملخص

في هذا البحث، وجدنا المعلمات المثلى لنظام Belousov-Zhabofinkii (BZS) غير الخطي باستخدام طريقة تحليل Adomian المطورة مع الخوارزمية الجينية والتي تختصر بـ GA MADM - تم استخدام وظيفة اللياقة المناسبة لحل النظام غير الخطي، اكتشفنا أن اختيار المعلمات المثلى تؤدي إلى حلول قريبة بشكل كبير. حيث أظهرت النتائج أن الحلول التقريبية كانت قريبة بدرجة كافية من الحلول الدقيقة، وأظهرت النتائج أيضاً أن الطريقة المقترحة أكثر دقة وكفاءة من MADM الاعتيادية، والتي تستخدم المعلمات التي تم اختيارها مسبقاً بشكل تقليدي.

1. Introduction:

Partial differential equations can explain most things that happen in the physical world. It is used for how air flows around airplanes, how spacecraft move, how sound signals are processed under water, how crystals grow, how fast a population grows, e-marketing, the stock market, and even nuclear explosions. Partial nonlinear differential equations make up most of mathematical physics. Even though abstraction turns the equations being studied into ordinary differential equations, the full description of these systems falls into the general area of partial differential equations.

In the year 1980 [1], a technique that would later be known as the Adomian decomposition method (ADM) was created. This method has been utilized to solve a wide variety of differential equations, both linear and nonlinear, in a quick, straightforward, and precise manner. Some research has recommended that the regular Adomian decomposition approach for initial value issues in ordinary and partial differential equations [2,3] be altered. This has been proposed by the findings of these investigations. [4] presented an error analysis of Adomian series solutions to a class of nonlinear differential equations, and [5] utilized the ADM to solve coupled systems of nonlinear physical equations and coupled systems of diffusion-reaction equations.

A Genetic Algorithm (GA) is a chance-based global search method that is part of a large group of algorithms called evolutionary algorithms [6]. Genetics and natural selection are the ideas that GA is based on. It is smart to use random search in this way. GA is like "survival of the fittest," or the

process of natural selection. This means species that can adjust to environmental changes are more likely to live, reproduce, and pass on to the next generation. Each generation is made up of many different people. Each person is a possible answer and a point in the search space. Each person is shown as a string of letters, numbers, floats, or bits. This string is like a piece of DNA. On each chromosome, a possible answer to the optimization problem will be written in code. Coding with real values will be used. With the fitness function, you can see how well a chromosome does its job. Crossover is when two chromosomes, which are the parents, come together to make two new chromosomes, which are the children. When random changes happen in a population, this is called a mutation. GA's algorithm is made up of the following steps: [6]:

- Step1: show the variable domain of the problem as a chromosome with a fixed length.
- Step2: define a fitness function.
- Step3: randomly generate an initial population.
- Step4: the fitness of each chromosome is calculated.
- Step5: Choose a pair of chromosomes from the current population to mate with. Parents are chosen with a chance that is based on how fit they are.
- Step6: using genetic operators, you can make a pair of offspring chromosomes (crossover & mutation).
- Step7: place the chromosomes of the new offspring in the new population.
- Step8: Repeat step 5 until the number of chromosomes in the new group is the same as the number in the first group.
- Step9: Replace the parent group of chromosomes with the new group of chromosomes from the offspring.
- Step10: Move on to step 4 and repeat the process until the end condition is met. Fig. 1 shows how the genetic algorithm works.

Bachir N.K and et. [7] used the Numerical Solution of Singular Boundary Value Problems Using a Genetic Algorithm. Using a genetic algorithm to find a numerical solution to Laplace's equation in two dimensions in [8]. Eman, A. H. Yaseen, M., [9,10], Accelerated Genetic Algorithm Solutions of Some Parametric Families of Stochastic Differential Equations with Solving Differential Equations by Modified Genetic Algorithms. Syarif and et [11], studied Genetic Algorithm (GA) Approaches for the Solution of Flow Shop Scheduling Issues (FSSP).

The organization of this paper looks like this. In Section 2, both the mathematical model and the accompanying work are presented for examination. The core concepts of the modified Adomian decomposition technique (MADM) are broken down and illustrated in section 3. In the fourth section

of this article, MADM is put to use in order to solve a nonlinear Belousov-Zhabofinkii system. In section 5, a method that combines GA and MADM is explained. In Section 6, there are some final thoughts.

2. Mathematics Model and Related Works

In the last few years, there has been a lot of interest in biological and biochemical systems that keep oscillating. This happens in the belousove-Zhabotinskii reaction [12], where malonic acid is oxidized by acid bromate in the presence of ferritin. In practice, waves of red and blue colures spread out from the point where the chemical reaction begins:

$$\begin{aligned} \frac{\partial u}{\partial t} &= Lrv + u(1 - u - rv) + \frac{\partial^2 v}{\partial x^2}, \\ \frac{\partial u}{\partial t} &= Mv - buv + \frac{\partial^2 v}{\partial x^2}, \end{aligned} \quad (1)$$

Where r is a made-up parameter added to the problem to show that the amount of bromide changes can be made far ahead of the wave front.

The constant solution of Eq. (1) are given by $(u, v) = (1, 0)$ and $(u, v) = (0, a)$ for each $a \in R$. if a solution (u, v) is found which satisfies $(u, v) \rightarrow (0, a)$ as $x \rightarrow \infty$ (or $x \rightarrow -\infty$) or some $r_a > 0$, then set $r = ar_a$ and it follows that $(u, \hat{v}) \equiv (u, v/a)$ satisfies (1) and $(u, \hat{v}) \rightarrow (0, 1)$ and $x \rightarrow +\infty$ (or $x \rightarrow -\infty$). So, without losing the generality, Murray looks into the boundary conditions:

$$u(-\infty, t) = v(\infty, t) = 0, \quad (2)$$

$$u(\infty, t) = v(-\infty, t) = 1, \quad (3)$$

For every fixed $t > 0$, He finds the right place to start so that there is only one way to solve the problem (1-3). Manokranjan, V.S., and Mitchell, A.R., study a Galerkin finite element method to find a numerical solution to the Belousov-Zhabofinkii reaction. Gene, A.K., et al. [13], found the solutions for the asymptotic behavior of a system of reaction-diffusion equations that models the Belousov-Zhabotinskii chemical reaction. Ying, Y., et al. [14], are interested in the Dynamics of the Stochastic Belousov-Zhabofinkii Chemical Reaction Model. Andrea C. and Marco P. [15], looked at how chemical systems like Belousove-Zhabotinsky reactions don't behave in a straight line.

3. The modified Adomian decomposition method (MADM)

Consider that the fundamental equation can be expressed as:

$$Lu + Nu + Ru = f(x) \quad (4)$$

where L is the invertible linear operator, N is the non-linear operator, and R is the linear part that can't be changed. From equation 4, we can see that

$$L u = f(x) - Nu - Ru,$$

Using initial conditions and inverse operator L^{-1} in equation 4, we find:

$$u = g(x) - L^{-1} Nu - L^{-1} Ru, \quad (5)$$

$L^{-1} = \int_0^x (\cdot) ds$, and $g(x)$ are the parts of the term $f(x)$ that are left after the initial conditions are used. The ADM thinks that the infinite approximation of the polynomial function given as could be used to break up the nonlinear manager $N(u)$.

$N(u) = \sum_{n=0}^{\infty} A_n(u_0, u_1, \dots, u_n)$ and A_n are the Adomian's polynomials (AP) defined as [1]:

$$A_n = \frac{1}{n!} \frac{d^n}{d\lambda^n} [N(\sum_{i=0}^{\infty} \lambda^i u_i)]_{\lambda=0}, \quad n = 0, 1, 2, 3, \dots \quad (6)$$

El-Kalla [4] gave another recipe for AP and said that an Adomian agreement that uses this novel recipe meets faster than one that uses AP (6). Kalla polynomial is given in a structure that goes with:

$$A_n = N(S_n) - \sum_{i=0}^{n-1} A_i(u_0, u_1, \dots, u_{n-1}) \quad (7)$$

Where $S_n = u_0 + u_1 + \dots + u_n$ and A_n can be given as

$$\begin{aligned} A_0 &= N(u_0) \\ A_1 &= \frac{d}{dx} (N(u_0)) u_1 + \frac{1}{2} \frac{d^2}{dx^2} (N(u_0)) u_1^2 + \frac{1}{6} \frac{d^3}{dx^3} (N(u_0)) u_1^3 + \frac{1}{24} \frac{d^4}{dx^4} (N(u_0)) u_1^4 + \dots \\ A_2 &= \frac{d}{dx} (N(u_0)) u_2 + \frac{1}{2} \frac{d^2}{dx^2} (N(u_0)) [2u_1 u_2 + u_2^2] + \\ &\quad \frac{1}{6} \frac{d^3}{dx^3} (N(u_0)) [3u_1^2 u_2 + 3u_1 u_2^2 + u_2^3] + \dots \\ &\quad \vdots \end{aligned}$$

Where these formulas were used with a program Maple13.

4. Application MADM to Belousov- Zhabofinkii system

This section is about studying the analytical solution of the Belousov-Zhabofinkii system (1) – (2). Using (MADM) to get this solution is one way to do it.

$$\begin{aligned} L_t u &= L_{xx} u + u(1 - u - rv) + L_r u \\ L_t v &= L_{xx} v - buv + MV \end{aligned} \quad (8)$$

With

$$u(x, 0) = g_1(x), \quad v(x, 0) = g_2(x)$$

Where the notation $L_t = \frac{\partial}{\partial t}$, $L_x = \frac{\partial}{\partial x}$ and $L_{xx} = \frac{\partial^2}{\partial x^2}$ symbolize the linear differential operator.

Define $L_t^{-1} = \int_0^t dt$, Then system (8) becomes:

$$\begin{aligned}
u(x, t) &= g_1(x) + L_t^{-1}[L_{xx}u + L_r u + u - \vartheta_1(u) - r\vartheta_2(u, v)] \\
v(x, t) &= g_2(x) + L_t^{-1}[L_{xx}v + MV - b\vartheta_2(u, v)]
\end{aligned}
\tag{9}$$

Where

$$\vartheta_1(u) = u^2, \vartheta_2(u, v) = uv$$

MADM thinks that the unknown functions $u(x, t)$ and $v(x, t)$ have an infinite series in the form:

$$u(x, t) = \sum_{n=0}^{\infty} u_n(x, t), v(x, t) = \sum_{n=0}^{\infty} v_n(x, t) \tag{10}$$

We can write ϑ_1 and ϑ_2 by an endless series of adman polynomials of the type

$$\vartheta_1(u) = \sum_{n=0}^{\infty} A_n, \vartheta_2(u, v) = \sum_{n=0}^{\infty} B_n$$

Where A_n and B_n are the appreciated adman polynomials. that

$$A_n(u_0, u_1, \dots, u_n) = \frac{1}{n!} \frac{d^n}{dr^n} [\vartheta_1(\sum_{k=1}^{\infty} r^k u_k)]_{r=0}, n \geq 0 \tag{11}$$

$$B_n(u_0, u_1, \dots, u_n, v_0, v_1, \dots, v_n) = \frac{1}{n!} \frac{d^n}{dr^n} [\vartheta_2(\sum_{k=1}^{\infty} r^k u_k), \sum_{k=1}^{\infty} r^k v_k]_{r=0}, n \geq 0 \tag{12}$$

As an illustration, the first polynomials utilizing equates (11) and (12) are computed as follows:

$$A_0 = u_0^2$$

$$A_1 = 2u_0u_1 + u_1^2$$

$$A_2 = 2u_0u_2 + 2u_1u_2 + u_2^2$$

⋮

$$B_0 = u_0v_0$$

$$B_1 = u_0v_0 + u_1v_0 + u_1v_1$$

$$B_2 = u_0v_2 + v_0u_2 + u_2v_1 + u_1v_2 + u_2v_2$$

⋮

And so on. The nonlinear system (8) are built as follows:

$$u_0(x, t) = g_1(x), v_0(x, t) = g_2(x)$$

$$u_{n+1}(x, t) = L_t^{-1}[L_{xx}u + L_r v + u - A_n - rB_n], n \geq 1 \quad (13)$$

$$v_{n+1}(x, t) = L_t^{-1}[L_{xx}v + MV - A_n - bB_n], n \geq 1 \quad (14)$$

Where $g_1(x)$ and $g_2(x)$ are the initial conditions and the construction for solutions $u(x, t)$ and $v(x, t)$ of:

$$u(x, t) = \lim_{n \rightarrow \infty} \varphi_n, v(x, t) = \lim_{n \rightarrow \infty} \varphi_n$$

Where

$$\varphi_n(x, t) = \sum_{k=0}^n u_k(x, t), \varphi_n(x, t) = \sum_{k=0}^n v_k(x, t), \quad (15)$$

To test how well MADM works for solving equations (13–14), we set the initial condition.

$$u_0(x, t) = e^x$$

$$v_0(x, t) = e^x$$

Then using the relation can be written as follows:

$$u_1(x, t) = Lre^x t + 2. e^x t - 1. e^{2.x} t - 1. re^{2.x} t$$

$$v_1(x, t) = te^x(M - 1be^x + 1.)$$

$$u_2(x, t) = 0.5000000000(Lre^x M - 1Lre^{2.x} b + 4. e^x - 9. e^{2.x} - 2. e^{2.x} Lr - 1. e^{2.x} Lr^2 - 1. re^{2.x} M + re^{3.x} b + 3. e^{3.x} r + e^{3.x} r^2 + 2e^{3.x} - 8. re^{2.x} + 3Lre^x)t^2$$

$$v_2(x, t) = (0.5000000000(-1. M^2 e^x + 2. e^{2.x} - 2. Me^{2.x} b - 2. Me^x + L. re^{2.x} b + 7. be^{2.x} - 1. be^{3.x} - 1. re^{3.x} b - 1. b^2 e^{3.x} - 1. e^x)t^2)$$

Then

$$u(x, t) = \sum_{k=0}^{\infty} u_n(x, t)$$

$$u(x, t) = e^x + Lre^x t + 2. e^x t - 1e^{2.x} t - 1. re^{2.x} t + 0.5000000000(Lre^x M - 1Lre^{2.x} b + 4. e^x - 9. e^{2.x} - 2. e^{2.x} Lr - 1. e^{2.x} Lr^2 - 1. re^{2.x} M + re^{3.x} b + 3. e^{3.x} r + e^{3.x} r^2 + 2e^{3.x} - 8. re^{2.x} + 3Lre^x)t^2 \quad (16)$$

And

$$v(x, t) = \sum_{k=0}^{\infty} v_n(x, t)$$

$$v(x, t) = e^x + te^x(M - 1be^x + 1.) - 0.50000000000(-1.M^2e^x + 2.e^{2.x} - 2.Me^{2.x}b - 2.Me^x + L.re^{2.x}b + 7.be^{2x} - 1.be^{3x} - 1.re^{3x}b - 1.b^2e^{3x} - 1.e^x)t^2$$

(17)

5. Proposed method (GA_MADM)

The proposed method is based on the idea that the GA and MADM can be used to find the best parameters for a nonlinear Belousov-Zhabofinkii system. Based on the results of the MADM solution series (25)–(26), the following equations are used to figure out the fitness function in the genetic algorithm:

$$U(c, L, r, M, b) = \sum_{i=1}^n \sum_{j=1}^m (u(x_i, t_j) - \hat{u}(x_i, t_j))^2 \tag{18}$$

$$V(c, L, r, M, b) = \sum_{i=1}^n \sum_{j=1}^m (v(x_i, t_j) - \hat{v}(x_i, t_j))^2 \tag{19}$$

$$F = \frac{1}{2} |U(c, L, r, M, b) + V(c, L, r, M, b)| \tag{20}$$

where n and m are the whole numbers of steps used in the domain of x and t , respectively, u and v are the solutions of the nonlinear Belousov-Zhabofinkii system (8), and \hat{u} and \hat{v} are the exact solutions for this system. The GA is used to solve the fitness function, which is shown by the letter F and stands for "mean square error." So, here's how to find the best values for the system (8):

- $c = 1$
- $b = 0$
- $L = 1$
- $M = 0$
- $r = -1$

The proposed method is based on the structure of a first standard model, but some of the parameters are not known. The goal of this method is to find the best (c, L, r, M, b) parameters for a nonlinear Belousov-Zhabofinkii system so that the model output vector and the real response vector are as close as possible. The algorithm was made using the default settings of the GA code in Matlab (R2016a), with the exception of the settings in Table 1.

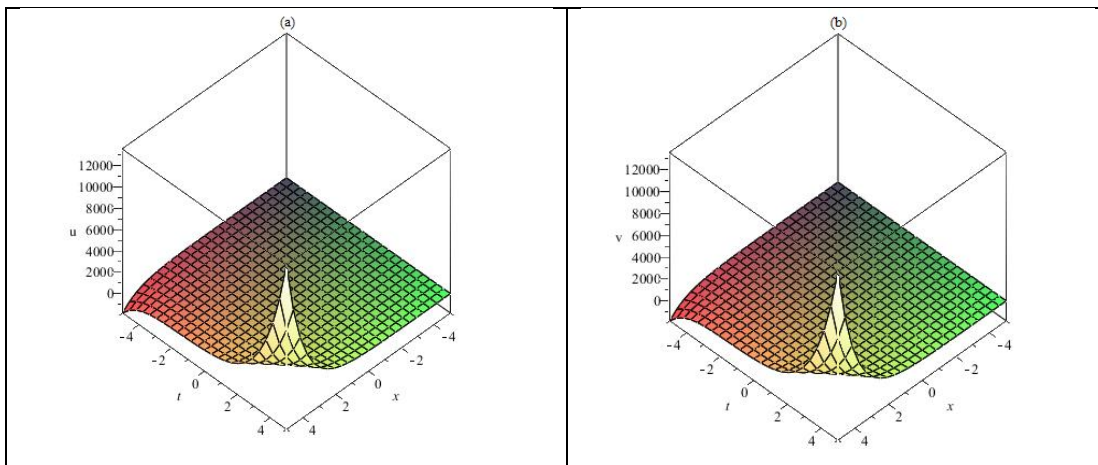
Table1: The parameter values for GA

Parameter Name	Values
Initial Population Range	[-10 10]
Max NO. of Iterations	500

Max Stall Generations	50
Swarm Size	30
Attraction Coefficient	2
Mutation Coefficient	0.2
Number of Decision Variables	3
Light Absorption Coefficient	1

Table 2: Comparison the absolute errors for various values of x for Belousov- Zhabofinkii system between GA_MADM and Exact solutions when $L = 1, b = 0, r = -1, M = 0, C = 1$.

x	$u(x, t)$		$v(x, t)$	
	$t=0.4, n=3$	$t=0.4, n=5$	$t=0.4, n=3$	$t=0.4, n=5$
0	$1.27982200 \times 10^{-3}$	$6.66500000 \times 10^{-6}$	$1.27982200 \times 10^{-3}$	$6.66500000 \times 10^{-6}$
0.1	$1.41442200 \times 10^{-3}$	$7.36600000 \times 10^{-6}$	$1.41442200 \times 10^{-3}$	$7.36600000 \times 10^{-6}$
0.2	$1.56317700 \times 10^{-3}$	$8.13900000 \times 10^{-6}$	$1.56317700 \times 10^{-3}$	$8.13900000 \times 10^{-6}$
0.3	$1.72757800 \times 10^{-3}$	$8.99600000 \times 10^{-6}$	$1.72757800 \times 10^{-3}$	$8.99600000 \times 10^{-6}$
0.4	$1.90926900 \times 10^{-3}$	$9.94200000 \times 10^{-6}$	$1.90926900 \times 10^{-3}$	$9.94200000 \times 10^{-6}$
0.5	$2.11007000 \times 10^{-3}$	$1.09890000 \times 10^{-5}$	$2.11007000 \times 10^{-3}$	$1.09890000 \times 10^{-5}$
0.6	$2.33198800 \times 10^{-3}$	$1.21450000 \times 10^{-5}$	$2.33198800 \times 10^{-3}$	$1.21450000 \times 10^{-5}$
0.7	$2.57724600 \times 10^{-3}$	$1.34220000 \times 10^{-5}$	$2.57724600 \times 10^{-3}$	$1.34220000 \times 10^{-5}$
0.8	$2.84829600 \times 10^{-3}$	$1.48340000 \times 10^{-5}$	$2.84829600 \times 10^{-3}$	$1.48340000 \times 10^{-5}$
0.9	$3.14785400 \times 10^{-3}$	$1.63940000 \times 10^{-5}$	3.1785400×10^{-3}	$1.63940000 \times 10^{-5}$
MSE	4.726642×10^{-6}	1.281915×10^{-10}	4.726642×10^{-6}	1.281915×10^{-10}



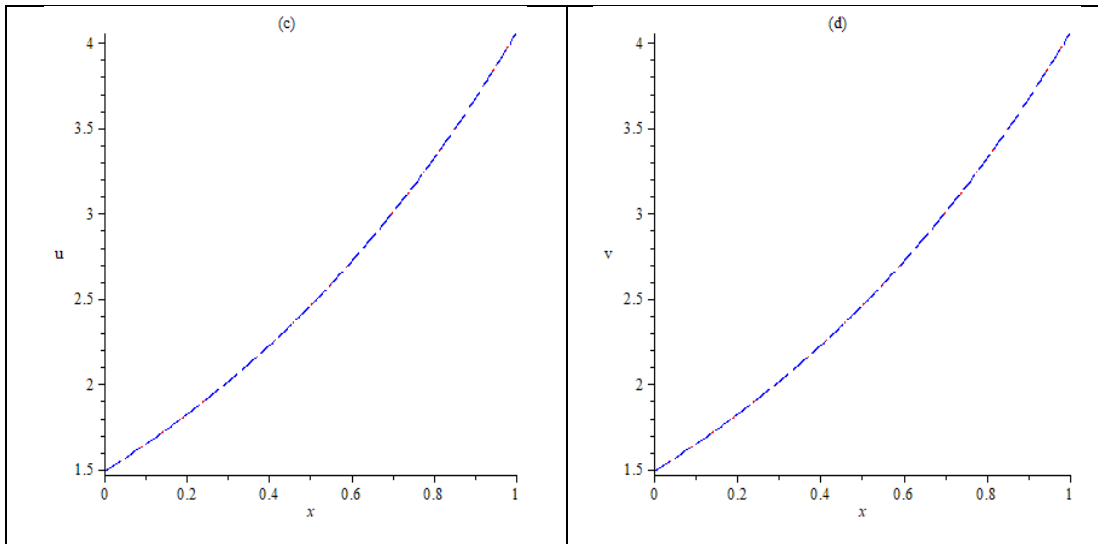


Figure1: The absolute error of the Belousov-Zhabofinkii system when (a) numerical solution of u_5 , (b) numerical solution of v_5 , (c) Comparison the absolute errors for u_5 between GA_MADM and Exact solutions when $t = 0.4$, and (d) Comparison the absolute errors for v_5 between GA_MADM and Exact solutions when $t = 0.4$.

6. Conclusions

In this paper, we applied the modified Adomian decomposition approach in conjunction with the Genetic algorithm to solve the nonlinear Belousov-Zhabofinkii system (GA). GA MADM's primary objective is determine the optimal parameters for the nonlinear Belousov-Zhabofinkii system (c, L, r, M, b) . In tables (2) and (3), our suggested approach GA-MADM ensures that the optimal parameters are determined precisely, even when the system is complex and has several values. Furthermore, it provides more accurate results than MADM. Through Figures (1), (2), and (3), our numerical results demonstrated that the approximate solutions generated by GA-MADM were quite close to the exact solutions. The computations for this paper were performed using the MAPLE 13 and Matlab® R2016a software packages.

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