



Applications of the Projective Space PG (3,8) in Coding Theory

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Abstract

The main objective of this research is to present the relationship between the subject of coding theory and three-dimensional projection space in the eighth field. We found the points, lines, and planes of the Galois field of order 8 using algebraic equations. Then, we formed a projective matrix with a binary system of zero and one. We collect the elements of the Galois field of order 8 with the projective matrix. We have seven projective matrices, and we found the shortest distance between two different points of the matrices where the highest distance that we got is 585, and the shortest distance is 73. And we test the code. Hence the maximum value of code size on an eighth-order finite domain and an incidence matrix with parameters generated were, n (code length), d (minimum code), and e (correction of an error in the code). We test the code in coding theory as the code length is 581, the minimum code is 73 and error correction in the code is 36. We apply the coding theory to see if it is perfect or not perfect.

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Introduction

In coding theory, many mathematicians have studied the application of projective space over the Galois field for example[1]. The researcher dealt with arcs and complete surfaces in a three-dimensional projective space, the Galois field[2]. The (representation of PG(3,5) by r -Subspaces, $r=1,2$ [3] An arc and a blocking set are both geometrical objects linked with linear codes[4]. And studied bounds the minimum distance of the linear codes of GF(q) [5] Application Geometry of space in PG(3,P)[6] . And new application of coding theory in the projective space of order three [7] .application of the projective plane in coding theory[8] a number of researchers studied the projective space finite, and now in this research, we study the three-dimensional projective space of the Galois field of order 8 in coding theory, we find points, planes and lines using algebraic equations and we form a projective matrix with a binary system of zero and one and we find linear symbols $(n, k, d, 8)$ We apply the coding theory theorem and note that it is incomplete

Theorem1.[9] sphere packing or Hamming bound)

A q -ary $(n, M, 2e + 1)$ - code C satisfies

$$M \left\{ \binom{n}{0} + \binom{n}{1}(q-1) + \dots + \binom{n}{e}(q-1)^e \right\} \leq q^n$$

Corollary1.2 [6]A q -ary $(n, M, 2e + 1)$ code C is perfect if and only if equality holds in Theorem 1.1

Definition 1.3[10]A q -ary code C of length n is a subset of $(F_q)^n$

Definition1.4 .Linear Codes[4]

The minimum distance d of a non-trivial code C is given by

$$d = \min\{d(x, y) \mid x \in C, y \in C, x \neq y\}$$

The classification of PG(3,8)

Let $GF(8)=F_8 = (0,1,a,a^2,a^3,a^4,a^5,a^6,a^7=1)$ be the Galois field of order eight the polynomial $F(\chi) = \chi^4 - a^5 \chi^3 - \chi^2 - a^3 \chi - a^5 [11]$ (where a be the primitive element of F_8) is primitive polynomial over F_8 of degree four .

$$P(i) = [1, 0, 0, 0] \cdot \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ a^5 & a^3 & 1 & a^5 \end{bmatrix}^i, i=0, \dots, 584$$

Table (1) The point of PG (3,8)

1 = [1, 0, 0, 0]	41 = [a ⁶ ,a ⁵ ,a ² , 1]	81 = [a ⁵ ,a ³ , a ² , 1]
2 = [0, 1, 0, 0]	42 = [a ² , a, a, 1]	82 = [a ² ,a ⁶ , a ⁵ , 1]
3 = [0, 0, 1, 0]	43 = [a ⁶ , a ⁶ , a ⁴ , 1]	83 = [a ³ , a ³ , 1, 0]
4 = [0, 0, 0, 1]	44 = [a ⁵ , a ⁴ , a ² , 1]	84 = [0, a ³ , a ³ , 1]
5 = [1, a ⁵ ,a ² , 1]	45 = [a ² , a ⁶ , a ² , 1]	85 = [a ³ ,a, a ⁶ , 1]
6 = [a ² , a ⁵ ,a, 1]	46 = [a ² , a ² , a ⁶ , 1]	86 = [a ⁴ , 0, a ² , 1]
7 = [a ⁶ , a ⁶ , a ⁵ , 1]	47 = [a ⁴ , a ⁴ , a ⁵ , 1]	87 = [a ² , a ³ , a ⁴ , 1]
8 = [a ³ , a ² , 1, 0]	48 = [1, a, 1, 0]	88 = [a ⁵ , a ⁵ , a, 1]
9 = [0, a ³ ,a ² , 1]	49 = [0, 1, a, 1]	89 = [a ⁶ , a ³ ,a ⁵ , 1]
10 = [a ² , 1,a ⁵ , 1]	50 = [a ⁶ , a ⁴ , 0, 1]	90 = [a ⁴ , a ³ , 1, 0]
11 = [1, 1, 0, 0]	51 = [1, a ⁶ , 1, 1]	91 = [0, a ⁴ , a ³ , 1]
12 = [0, 1, 1, 0]	52 = [a, a ⁴ , a ⁵ , 1]	92 = [a ³ , a, a ³ , 1]
13 = [0, 0, 1, 1]	53 = [1, a ² , 1, 0]	93 = [a ³ , 0, a, 1]
14 = [a, a ⁶ ,a ³ , 1]	54 = [0, 1, a ² , 1]	94 = [a ⁶ , 0, a, 1]
15 = [a ³ ,a ⁵ , 1, 1]	55 = [a ² , 1, 0, 1]	95 = [a ⁶ , a ⁵ , a, 1]
16 = [a, 0, 1, 1]	56 = [1, 1, 0, 1]	96 = [a ⁶ ,a ⁵ , a ⁵ , 1]
17 = [a, a ³ , a ³ , 1]	57 = [1, a ³ , 0, 1]	97 = [a, 1, 1, 0]
18 = [a ³ , a ⁵ , a ⁶ , 1]	58 = [1, a ³ ,a ³ , 1]	98 = [0, a, 1, 1]
19 = [a ⁴ , 0, a ³ , 1]	59 = [a ³ , a ⁶ , a ⁶ , 1]	99 = [a, a ⁶ , a ⁶ , 1]
20 = [a ³ , a ⁴ , a ⁵ , 1]	60 = [a ⁴ , 0, a, 1]	100 = [a ⁴ , a ⁶ , a, 1]
21 = [1, 0, 1, 0]	61 = [a ⁶ , 1, a, 1]	101 = [a ⁶ , 1, a ³ , 1]
22 = [0, 1, 0, 1]	62 = [a ⁶ , a ⁵ , 0, 1]	102 = [a ³ , a ² , 0, 1]
23 = [1, a ⁵ , 0, 1]	63 = [1,a ⁶ , a ⁶ , 1]	103 = [1, 0, a, 1]
24 = [1, a ³ , a ⁶ , 1]	64 = [a ⁴ , 1,a, 1]	104 = [a ⁶ , a ² , a, 1]
25 = [a ⁴ , 1, 1, 1]	65 = [a ⁶ , 1, 0, 1]	105 = [a ⁶ , a ⁵ , 1, 1]
26 = [a, a ² , 0, 1]	66 = [1, a ⁶ , 0, 1]	106 = [a, 1, 1, 1]
27 = [1, a ² , a, 1]	67 = [1, a ³ , a ⁴ , 1]	107 = [a,a ³ , 0, 1]
28 = [a ⁶ ,a ² , 1, 1]	68 = [a ⁵ ,a, a, 1]	108 = [1, a ² , a ³ , 1]
29 = [a, 1, a ² , 1]	69 = [a ⁶ , a ³ ,a ⁴ , 1]	109 = [a ³ ,a ⁶ , a ⁴ , 1]
30 = [a ² , a ⁴ , 0, 1]	70 = [a ⁵ , a ⁴ , a, 1]	110 = [a ⁵ , 0,a ² , 1]
31 = [1, 1, 1, 1]	71 = [a ⁶ ,a ³ ,a ⁶ , 1]	111 = [a ² , a ⁶ , a ⁴ , 1]

$32 = [a, a^4, 0, 1]$	$72 = [a^4, a^3, 1, 1]$	$112 = [a^5, a^5, a^2, 1]$
$33 = [1, a^2, 1, 1]$	$73 = [a, a^2, a^4, 1]$	$113 = [a^2, a^6, a, 1]$
$34 = [a, a^4, a^2, 1]$	$74 = [a^5, 1, a^6, 1]$	$114 = [a^6, a^6, a^3, 1]$
$35 = [a^2, a^4, a^2, 1]$	$75 = [a^4, a, 0, 1]$	$115 = [a^3, a^2, 1, 1]$
$36 = [a^2, a^2, a^2, 1]$	$76 = [1, a, a^5, 1]$	$116 = [a, 0, a^2, 1]$
$37 = [a^2, a^2, a^3, 1]$	$77 = [a^2, a^5, 1, 0]$
$38 = [a^3, a^3, a^4, 1]$	$78 = [0, a^2, a^5, 1]$
$39 = [a^5, 0, a, 1]$	$79 = [a^6, a^4, 1, 0]$
$40 = [a^6, a^3, a, 1]$	$80 = [0, a^6, a^4, 1]$	$585 = [a^6, a^6, a^6, 1]$

We found that the number of points in the projective space is 585 from the projective Matrix .Here, write the table of the plane and then the table of lines.

The points in the projective four-coordinate space, i.e. the point of the last element is zero. Let it be a plane, so that we have 585 planes, and each plane contains 73 points.

Table (2) the planes of PG(3,8)

1	2	3	4	5	6	.	.	585
2	3	4	5	6	7	.	.	1
3	4	5	6	7	8	.	.	2
8	9	10	11	12	13	.	.	7
11	12	13	14	15	15	.	.	10
12	13	14	15	16	17	.	.	11
21	22	23	24	25	26	.	.	20
48	49	50	51	52	53	.	.	47
53	54	55	56	57	58	.	.	52
77	78	79	80	81	82	.	.	76
79	80	81	82	83	84	.	.	78
83	84	85	86	87	88	.	.	82
90	91	92	93	94	95	.	.	89
97	98	99	100	101	102	.	.	96
119	120	121	122	123	124	.	.	118
121	122	123	124	125	126	.	.	120
127	128	129	130	131	132	.	.	126
135	136	137	138	138	139	.	.	134
139	140	141	142	143	144	.	.	138
142	143	144	145	146	147	.	.	141
156	157	158	159	160	161	.	.	155
158	159	160	161	162	163	.	.	157
175	176	177	178	179	180	.	.	174
176	177	178	179	180	181	.	.	175
185	186	187	188	189	190	.	.	184
187	189	190	191	192	193	.	.	186
189	190	191	192	193	194	.	.	188
190	191	192	193	194	195	.	.	189
199	200	201	202	203	204	.	.	198
202	203	204	205	206	207	.	.	201
205	206	207	208	209	210	.	.	204

207	208	209	210	211	212	.	.	206
225	226	227	228	229	230	.	.	224
246	247	248	249	250	251	.	.	245
272	273	274	275	276	277	.	.	271
279	280	281	282	283	384	.	.	278
280	281	282	283	284	385	.	.	279
289	290	291	292	293	394	.	.	288
296	297	298	299	300	301	.	.	295
302	303	304	305	306	307	.	.	301
324	325	326	327	328	329	.	.	323
337	338	339	340	341	342	.	.	336
341	342	343	344	345	346	.	.	340
349	350	351	352	353	354	.	.	348
352	353	354	355	356	357	.	.	351
363	364	365	366	367	368	.	.	362
377	378	379	380	381	382	.	.	376
402	403	404	405	406	407	.	.	401
421	422	423	424	425	426	.	.	420
425	426	427	428	429	430	.	.	424
437	438	439	440	441	442	.	.	436
440	441	442	443	444	445	.	.	439
448	449	450	451	452	453	.	.	447
453	454	455	456	457	458	.	.	452
461	462	463	464	465	466	.	.	460
467	468	469	470	471	472	.	.	466
472	473	474	475	476	477	.	.	471
478	479	480	481	482	483	.	.	477
494	495	496	497	498	499	.	.	493
495	496	497	498	499	500	.	.	494
499	500	501	502	503	504	.	.	498
504	505	506	507	508	509	.	.	503
520	421	422	423	424	425	.	.	519
232	533	534	535	536	537	.	.	531
533	534	536	537	538	539	.	.	532
535	536	537	538	539	540	.	.	534
542	543	544	545	546	547	.	.	541
553	554	555	556	557	558	.	.	552
554	556	557	558	559	560	.	.	553
557	558	559	560	561	562	.	.	556
561	562	563	564	565	566	.	.	560
563	564	565	566	567	568	.	.	562
581	582	583	584	585	1	.	.	580

Which two planes of the above table intersect $q+1$ points where $q=8$

Table (3) The line of $PG(3,8)$ are:

1 = [1, 2, 11, 175, 189, 279, 494, 532, 553]
2 = [1, 3, 21, 349, 377, 402, 478, 520, 557]
3 = [1, 4, 151, 239, 334, 356, 395, 526, 574]
4 = [1, 5, 41, 112, 168, 218, 370, 454, 528]
5 = [1, 6, 88, 95, 140, 277, 294, 540, 559]
6 = [1, 7, 82, 126, 204, 301, 466, 477, 562]
7 = [1, 8, 53, 190, 207, 453, 472, 499, 504]
8 = [1, 9, 81, 154, 223, 322, 335, 435, 470]
9 = [1, 10, 174, 188, 278, 493, 531, 552, 585]
10 = [1, 12, 97, 121, 127, 202, 246, 324, 421]
11 = [1, 13, 16, 163, 251, 346, 368, 407, 538]
12 = [1, 14, 114, 149, 265, 273, 345, 418, 487]
.....
.....
734 = [317, 345, 370, 446, 488, 525, 554, 556, 574]
4735 = [318, 346, 371, 447, 489, 526, 555, 557, 575]
4736 = [319, 347, 372, 448, 490, 527, 556, 558, 576]
4737 = [320, 348, 373, 449, 491, 528, 557, 559, 577]
4738 = [321, 349, 374, 450, 492, 529, 558, 560, 578]
4739 = [322, 350, 375, 451, 493, 530, 559, 561, 579]
4740 = [323, 351, 376, 452, 494, 531, 560, 562, 580]
4741 = [324, 352, 377, 453, 495, 532, 561, 563, 581]
4742 = [325, 353, 378, 454, 496, 533, 562, 564, 582]
4743 = [326, 354, 379, 455, 497, 534, 563, 565, 583]
4744 = [327, 355, 380, 456, 498, 535, 564, 566, 584]
4745 = [328, 356, 381, 457, 499, 536, 565, 567, 585]

In the following theorem the parameters n, M, and d, are constructed

$$n=q^3+q^2+q+1 \quad , q = 8$$

$$d=2e+1$$

$$M= q^3+q^2+q+1- k \quad , k = 4$$

Theorem (2.1) the projective space of order eight is a code C with a parameter

$$[n = 585 , M \leq 8^{581} , d = 73, e = 36]$$

Proof: The plane π_8 has an incidence matrix A = (aij), where

$$a_{ij} = \begin{cases} 1 & \text{if } p_j \in \ell_i \\ 0 & \text{if } p_j \notin \ell_i \end{cases}$$

$$1 \leq j \leq 585, \quad 1 \leq i \leq 585$$

Construct a table of points in PG(3,8) so that we the point that lies on line we put one and for the point that does not lie on the line we put zero Table(4)

	1	2	3	4	5	6	7	8	9	10	.	.	576	577	578	579	580	581	582	583	584	585
ℓ_1	1	1	1	0	0	0	0	1	0	0	.	.	0	0	0	0	0	1	0	0	0	0
ℓ_2	0	1	1	1	0	0	0	0	1	0	.	.	0	0	0	0	0	0	1	0	0	0
ℓ_3	0	0	1	1	1	0	0	0	0	1	.	.	0	0	0	0	0	0	0	1	0	0
ℓ_4	0	0	0	1	1	1	0	0	0	0	.	.	0	0	0	0	0	0	0	0	1	0
ℓ_5	0	0	0	0	1	1	1	0	0	0	.	.	0	0	0	0	0	0	0	0	0	1
ℓ_6	1	0	0	0	0	1	1	1	0	0	.	.	0	0	0	0	0	0	0	0	0	0
ℓ_7	0	1	0	0	0	0	1	1	1	0	.	.	0	0	0	0	0	0	0	0	0	0
ℓ_8	0	0	1	0	0	0	0	1	1	1	.	.	0	0	0	0	0	0	0	0	0	0
ℓ_9	0	0	0	1	0	0	0	0	1	1	.	.	0	0	0	0	0	0	0	0	0	0
ℓ_{10}	0	0	0	0	1	0	0	0	0	1	.	.	0	0	0	0	0	0	0	0	0	0
ℓ_{11}	0	0	0	0	0	1	0	0	0	0	.	.	0	0	0	0	0	0	0	0	0	0
ℓ_{12}	0	0	0	0	0	0	1	0	0	0	.	.	0	0	0	0	0	0	0	0	0	0
ℓ_{13}	0	0	0	0	0	0	0	1	0	0	.	.	0	0	0	0	0	0	0	0	0	0
ℓ_{14}	0	0	0	0	0	0	0	0	1	0	.	.	1	0	0	0	0	0	0	0	0	0
ℓ_{15}	0	0	0	0	0	0	0	0	0	1	.	.	0	1	0	0	0	0	0	0	0	0
.
.
ℓ_{571}	0	0	0	0	0	1	0	0	0	0	.	.	0	0	1	0	0	1	1	0	0	0
ℓ_{572}	0	0	0	0	0	0	1	0	0	0	.	.	0	0	0	1	0	0	1	1	0	0
ℓ_{573}	0	0	0	0	0	0	0	1	0	0	.	.	0	0	0	0	1	0	0	1	1	0
ℓ_{574}	0	0	0	0	0	0	0	0	1	0	.	.	1	0	0	0	0	1	0	0	1	1
ℓ_{575}	1	0	0	0	0	0	0	0	0	1	.	.	1	1	0	0	0	0	1	0	0	1
ℓ_{576}	1	1	0	0	0	0	0	0	0	0	.	.	1	1	1	0	0	0	0	1	0	0
ℓ_{577}	0	1	1	0	0	0	0	0	0	0	.	.	0	1	1	1	0	0	0	0	1	0
ℓ_{578}	0	0	1	1	0	0	0	0	0	0	.	.	0	0	1	1	1	0	0	0	0	1
ℓ_{579}	1	0	0	1	1	0	0	0	0	0	.	.	0	0	0	1	1	1	0	0	0	0
ℓ_{580}	0	1	0	0	1	1	0	0	0	0	.	.	0	0	0	0	1	1	1	0	0	0
ℓ_{581}	0	0	1	0	0	1	1	0	0	0	.	.	1	0	0	0	0	1	1	1	0	0
ℓ_{582}	0	0	0	1	0	0	1	1	0	0	.	.	0	1	0	0	0	0	1	1	1	0
ℓ_{583}	0	0	0	0	1	0	0	1	1	0	.	.	0	0	1	0	0	0	0	1	1	1
ℓ_{584}	1	0	0	0	0	1	0	0	1	1	.	.	0	0	0	1	0	0	0	0	1	1
ℓ_{585}	1	1	0	0	0	0	1	0	0	1	.	.	0	0	0	0	1	0	0	0	0	1

$$\begin{aligned}
 m_i &= [0,0,0,0,0, \dots, 0,0,0,0,0] \\
 v_i &= [1,1,1,1,1, \dots, 1,1,1,1,1] \\
 h_i &= [a,a,a,a,a, \dots, a,a,a,a,] \\
 k_i &= [a^2,a^2,a^2,a^2, \dots, a^2,a^2,a^2,a^2] \\
 o_i &= [a^3,a^3,a^3,a^3, \dots, a^3,a^3,a^3,a^3] \\
 w_i &= [a^4,a^4,a^4,a^4, \dots, a^4,a^4,a^4,a^4] \\
 p_i &= [a^5,a^5,a^5,a^5, \dots, a^5,a^5,a^5,a^5] \\
 s_i &= [a^6,a^6,a^6,a^6, \dots, a^6,a^6,a^6,a^6]
 \end{aligned}$$

Combine v_i with a binary system projective matrix we get a zero-one matrix $x_i = [1,1,1,1,1, \dots, 1,1,1,1,1]$
 Let $a_i = v_i + \ell_i \quad 1 \leq i \leq 585$
 Now the table of a_i

	1	2	3	4	5	6	7	8	9	10	.	.	576	577	578	579	580	581	582	583	584	585
a ₁	0	0	0	1	1	1	1	0	1	1	.	.	1	1	1	1	1	0	1	1	1	1
a ₂	1	0	0	0	1	1	1	1	0	1	.	.	1	1	1	1	1	1	0	1	1	1
a ₃	1	1	0	0	0	1	1	1	1	0	.	.	1	1	1	1	1	1	1	0	1	1
a ₄	1	1	1	0	0	0	1	1	1	1	.	.	1	1	1	1	1	1	1	1	0	1
a ₅	1	1	1	1	0	0	0	1	1	1	.	.	1	1	1	1	1	1	1	1	1	0
a ₆	0	1	1	1	1	0	0	0	1	1	.	.	1	1	1	1	1	1	1	1	1	1
a ₇	1	0	1	1	1	1	0	0	0	1	.	.	1	1	1	1	1	1	1	1	1	1
a ₈	1	1	0	1	1	1	1	0	0	0	.	.	1	1	1	1	1	1	1	1	1	1
a ₉	1	1	1	0	1	1	1	1	0	0	.	.	1	1	1	1	1	1	1	1	1	1
a ₁₀	1	1	1	1	0	1	1	1	1	0	.	.	1	1	1	1	1	1	1	1	1	1
a ₁₁	1	1	1	1	1	0	1	1	1	1	.	.	1	1	1	1	1	1	1	1	1	1
a ₁₂	1	1	1	1	1	1	0	1	1	1	.	.	1	1	1	1	1	1	1	1	1	1
a ₁₃	1	1	1	1	1	1	1	0	1	1	.	.	1	1	1	1	1	1	1	1	1	1
a ₁₄	1	1	1	1	1	1	1	1	0	1	.	.	0	1	1	1	1	1	1	1	1	1
a ₁₅	1	1	1	1	1	1	1	1	1	0	.	.	1	0	1	1	1	1	1	1	1	1
.
.
a ₅₇₁	1	1	1	1	1	0	1	1	1	1	.	.	1	1	0	1	1	0	0	1	1	1
a ₅₇₂	1	1	1	1	1	1	0	1	1	1	.	.	1	1	1	0	1	1	0	0	1	1
a ₅₇₃	1	1	1	1	1	1	1	0	1	1	.	.	1	1	1	1	0	1	1	0	0	1
a ₅₇₄	1	1	1	1	1	1	1	1	0	1	.	.	0	1	1	1	1	0	1	1	0	0
a ₅₇₅	0	1	1	1	1	1	1	1	1	0	.	.	0	0	1	1	1	1	0	1	1	0
a ₅₇₆	0	0	1	1	1	1	1	1	1	1	.	.	0	0	0	1	1	1	1	0	1	1
a ₅₇₇	1	0	0	1	1	1	1	1	1	1	.	.	1	0	0	0	1	1	1	1	0	1
a ₅₇₈	1	1	0	0	1	1	1	1	1	1	.	.	1	1	0	0	0	1	1	1	1	0
a ₅₇₉	0	1	1	0	0	1	1	1	1	1	.	.	1	1	1	0	0	0	1	1	1	1
a ₅₈₀	1	0	1	1	0	0	1	1	1	1	.	.	1	1	1	1	0	0	0	1	1	1
a ₅₈₁	1	1	0	1	1	0	0	1	1	1	.	.	0	1	1	1	1	0	0	0	1	1
a ₅₈₂	1	1	1	0	1	1	0	0	1	1	.	.	1	0	1	1	1	1	0	0	0	1
a ₅₈₃	1	1	1	1	0	1	1	0	0	1	.	.	1	1	0	1	1	1	1	0	0	0
a ₅₈₄	0	1	1	1	1	0	1	1	0	0	.	.	1	1	1	0	1	1	1	1	0	0
a ₅₈₅	0	0	1	1	1	1	0	1	1	0	.	.	1	1	1	1	0	1	1	1	1	0

hi=[a,a,a,,a,a,a.....a,a,a,a,a,]

Let $b_i = h_i + \ell_i \quad 1 \leq i \leq 585$.

Now the table of b_i

	1	2	3	4	5	6	7	8	9	10	.	.	576	577	578	579	580	581	582	583	584	585	
b ₁	a ⁵	a ⁵	a ⁵	a	a	a	a	a ⁵	a	a	.	.	a	a	a	a	a	a ⁵	a	a	a	a	
b ₂	a	a ⁵	a ⁵	a ⁵	a	a	a	a	a ⁵	a	.	.	a	a	a	a	a	a	a ⁵	a	a	a	
b ₃	a	a	a ⁵	a ⁵	a ⁵	a	a	a	a	a ⁵	.	.	a	a	a	a	a	a	a	a ⁵	a	a	
b ₄	a	a	a	a ⁵	a ⁵	a ⁵	a	a	a	a	.	.	a	a	a	a	a	a	a	a	a ⁵	a	
b ₅	a	a	a	a	a ⁵	a ⁵	a ⁵	a	a	a	.	.	a	a	a	a	a	a	a	a	a	a ⁵	
b ₆	a ⁵	a	a	a	a	a ⁵	a ⁵	a ⁵	a	a	.	.	a	a	a	a	a	a	a	a	a	a	
b ₇	a	a ⁵	a	a	a	a	a ⁵	a ⁵	a ⁵	a	.	.	a	a	a	a	a	a	a	a	a	a	
b ₈	a	a	a ⁵	a	a	a	a	a ⁵	a ⁵	a ⁵	.	.	a	a	a	a	a	a	a	a	a	a	
b ₉	a	a	a	a ⁵	a	a	a	a	a ⁵	a ⁵	.	.	a	a	a	a	a	a	a	a	a	a	
b ₁₀	a	a	a	a	a ⁵	a	a	a	a	a ⁵	.	.	a	a	a	a	a	a	a	a	a	a	
b ₁₁	a	a	a	a	a	a ⁵	a	a	a	a	.	.	a	a	a	a	a	a	a	a	a	a	
b ₁₂	a	a	a	a	a	a	a ⁵	a	a	a	.	.	a	a	a	a	a	a	a	a	a	a	
b ₁₃	a	a	a	a	a	a	a	a ⁵	a	a	.	.	a	a	a	a	a	a	a	a	a	a	
b ₁₄	a	a	a	a	a	a	a	a	a ⁵	a	.	.	a ⁵	a	a	a	a	a	a	a	a	a	
b ₁₅	a	a	a	a	a	a	a	a	a	a ⁵	.	.	a	a ⁵	a	a	a	a	a	a	a	a	
.
.
b ₅₇₁	a	a	a	a	a	a ⁵	a	a	a	a	.	.	a	a	a ⁵	a	a	a ⁵	a ⁵	a ⁵	a	a	
b ₅₇₂	a	a	a	a	a	a	a ⁵	a	a	a	.	.	a	a	a	a ⁵	a	a	a ⁵	a ⁵	a	a	
b ₅₇₃	a	a	a	a	a	a	a	a ⁵	a	a	.	.	a	a	a	a	a ⁵	a	a	a ⁵	a ⁵	a	
b ₅₇₄	a	a	a	a	a	a	a	a	a ⁵	a	.	.	a ⁵	a	a	a	a	a ⁵	a	a	a ⁵	a ⁵	
b ₅₇₅	a ⁵	a	a	a	a	a	a	a	a	a ⁵	.	.	a ⁵	a ⁵	a	a	a	a	a ⁵	a	a	a ⁵	
b ₅₇₆	a ⁵	a ⁵	a	a	a	a	a	a	a	a	.	.	a ⁵	a ⁵	a ⁵	a	a	a	a	a ⁵	a	a	
b ₅₇₇	a	a ⁵	a ⁵	a	a	a	a	a	a	a	.	.	a	a ⁵	a ⁵	a ⁵	a	a	a	a	a ⁵	a	
b ₅₇₈	a	a	a ⁵	a ⁵	a	a	a	a	a	a	.	.	a	a	a ⁵	a ⁵	a ⁵	a	a	a	a	a ⁵	
b ₅₇₉	a ⁵	a	a	a ⁵	a ⁵	a	a	a	a	a	.	.	a	a	a	a ⁵	a ⁵	a ⁵	a	a	a	a	
b ₅₈₀	a	a ⁵	a	a	a ⁵	a ⁵	a	a	a	a	.	.	a	a	a	a	a ⁵	a ⁵	a ⁵	a	a	a	
b ₅₈₁	a	a	a ⁵	a	a	a ⁵	a ⁵	a	a	a	.	.	a ⁵	a	a	a	a	a ⁵	a ⁵	a ⁵	a	a	
b ₅₈₂	a	a	a	a ⁵	a	a	a ⁵	a ⁵	a	a	.	.	a	a ⁵	a	a	a	a	a ⁵	a ⁵	a ⁵	a	
b ₅₈₃	a	a	a	a	a ⁵	a	a	a ⁵	a ⁵	a	.	.	a	a	a ⁵	a	a	a	a	a ⁵	a ⁵	a ⁵	
b ₅₈₄	a ⁵	a	a	a	a	a ⁵	a	a	a ⁵	a ⁵	.	.	a	a	a	a ⁵	a	a	a	a	a ⁵	a ⁵	
b ₅₈₅	a ⁵	a ⁵	a	a	a	a	a ⁵	a	a	a ⁵	.	.	a	a	a	a	a ⁵	a	a	a	a	a ⁵	

$k_i = [a^2, a^2, a^2, a^2, \dots, a^2, a^2, a^2]$

Let $c_i = k_i + \ell_i \quad 1 \leq i \leq 585$

Now the table of c_i

	1	2	3	4	5	6	7	8	9	10	.	.	576	577	578	579	580	581	582	583	584	585		
C ₁	a ³	a ³	a ³	a ²	a ²	a ²	a ²	a ²	a ²	a ²	.	.	a ²	a ²	a ²	a ²	a ²	a ³	a ²	a ²	a ²	a ²		
C ₂	a ²	a ³	a ³	a ³	a ²	a ²	a ²	a ²	a ²	a ²	.	.	a ²	a ²	a ²	a ²	a ²	a ²	a ³	a ²	a ²	a ²	a ²	
C ₃	a ²	a ²	a ³	a ³	a ³	a ²	a ²	a ²	a ²	a ²	.	.	a ²	a ²	a ²	a ²	a ²	a ²	a ²	a ³	a ²	a ²	a ²	
C ₄	a ²	a ²	a ²	a ³	a ³	a ³	a ²	a ²	a ²	a ²	.	.	a ²	a ²	a ²	a ²	a ²	a ²	a ²	a ²	a ³	a ²	a ²	
C ₅	a ²	a ²	a ²	a ²	a ³	a ³	a ³	a ²	a ²	a ²	.	.	a ²	a ²	a ²	a ²	a ²	a ²	a ²	a ²	a ²	a ²	a ³	
C ₆	a ³	a ²	a ²	a ²	a ²	a ³	a ³	a ³	a ²	a ²	.	.	a ²	a ²	a ²	a ²	a ²	a ²	a ²	a ²	a ²	a ²	a ²	
C ₇	a ²	a ³	a ²	a ²	a ²	a ²	a ³	a ³	a ³	a ²	.	.	a ²	a ²	a ²	a ²	a ²	a ²	a ²	a ²	a ²	a ²	a ²	
C ₈	a ²	a ²	a ³	a ²	a ²	a ²	a ²	a ³	a ³	a ³	.	.	a ²	a ²	a ²	a ²	a ²	a ²	a ²	a ²	a ²	a ²	a ²	
C ₉	a ²	a ²	a ²	a ³	a ²	a ²	a ²	a ²	a ³	a ³	.	.	a ²	a ²	a ²	a ²	a ²	a ²	a ²	a ²	a ²	a ²	a ²	
C ₁₀	a ²	a ²	a ²	a ²	a ³	a ²	a ²	a ²	a ²	a ³	.	.	a ²	a ²	a ²	a ²	a ²	a ²	a ²	a ²	a ²	a ²	a ²	
C ₁₁	a ²	a ²	a ²	a ²	a ²	a ³	a ²	a ²	a ²	a ²	.	.	a ²	a ²	a ²	a ²	a ²	a ²	a ²	a ²	a ²	a ²	a ²	
C ₁₂	a ²	a ²	a ²	a ²	a ²	a ²	a ³	a ²	a ²	a ²	.	.	a ²	a ²	a ²	a ²	a ²	a ²	a ²	a ²	a ²	a ²	a ²	
C ₁₃	a ²	a ²	a ²	a ²	a ²	a ²	a ²	a ³	a ²	a ²	.	.	a ²	a ²	a ²	a ²	a ²	a ²	a ²	a ²	a ²	a ²	a ²	
C ₁₄	a ²	a ²	a ²	a ²	a ²	a ²	a ²	a ²	a ³	a ²	.	.	a ³	a ²	a ²	a ²	a ²	a ²	a ²	a ²	a ²	a ²	a ²	
C ₁₅	a ²	a ²	a ²	a ²	a ²	a ²	a ²	a ²	a ²	a ³	.	.	a ²	a ³	a ²	a ²	a ²	a ²	a ²	a ²	a ²	a ²	a ²	
.
C ₅₇₁	a ²	a ²	a ²	a ²	a ²	a ³	a ²	a ²	a ²	a ²	.	.	a ²	a ²	a ³	a ²	a ²	a ³	a ³	a ²	a ²	a ²	a ²	
C ₅₇₂	a ²	a ²	a ²	a ²	a ²	a ²	a ³	a ²	a ²	a ²	.	.	a ²	a ²	a ²	a ³	a ²	a ²	a ³	a ³	a ²	a ²	a ²	
C ₅₇₃	a ²	a ²	a ²	a ²	a ²	a ²	a ²	a ³	a ²	a ²	.	.	a ²	a ²	a ²	a ²	a ³	a ²	a ²	a ³	a ³	a ²	a ²	
C ₅₇₄	a ²	a ²	a ²	a ²	a ²	a ²	a ²	a ²	a ³	a ²	.	.	a ³	a ²	a ²	a ²	a ²	a ³	a ²	a ²	a ³	a ³	a ³	
C ₅₇₅	a ³	a ²	a ²	a ²	a ²	a ²	a ²	a ²	a ²	a ³	.	.	a ³	a ³	a ²	a ²	a ²	a ²	a ³	a ²	a ²	a ³	a ³	
C ₅₇₆	a ³	a ³	a ²	a ²	a ²	a ²	a ²	a ²	a ²	a ²	.	.	a ³	a ³	a ³	a ²	a ²	a ²	a ²	a ³	a ²	a ²	a ²	
C ₅₇₇	a ²	a ³	a ³	a ³	a ²	a ²	a ²	a ²	a ²	a ²	.	.	a ²	a ³	a ³	a ³	a ²	a ²	a ²	a ²	a ³	a ²	a ²	
C ₅₇₈	a ²	a ²	a ³	a ³	a ²	a ²	a ²	a ²	a ²	a ²	.	.	a ²	a ²	a ³	a ³	a ³	a ²	a ²	a ²	a ²	a ²	a ³	
C ₅₇₉	a ²	a ²	a ²	a ³	a ³	a ²	a ²	a ²	a ²	a ²	.	.	a ²	a ²	a ²	a ³	a ³	a ³	a ²	a ²	a ²	a ²	a ²	
C ₅₈₀	a ²	a ²	a ²	a ²	a ³	a ³	a ²	a ²	a ²	a ²	.	.	a ²	a ²	a ²	a ²	a ³	a ³	a ³	a ²	a ²	a ²	a ²	
C ₅₈₁	a ²	a ²	a ²	a ²	a ²	a ³	a ³	a ²	a ²	a ²	.	.	a ³	a ²	a ²	a ²	a ²	a ³	a ³	a ³	a ²	a ²	a ²	
C ₅₈₂	a ²	a ²	a ²	a ²	a ²	a ²	a ³	a ³	a ²	a ²	.	.	a ²	a ³	a ³	a ²	a ²	a ²	a ²	a ³	a ³	a ³	a ²	
C ₅₈₃	a ²	a ²	a ²	a ²	a ²	a ²	a ²	a ³	a ³	a ²	.	.	a ²	a ²	a ³	a ²	a ²	a ²	a ²	a ³	a ³	a ³	a ³	
C ₅₈₄	a ³	a ²	a ²	a ²	a ²	a ²	a ²	a ²	a ³	a ³	.	.	a ²	a ²	a ²	a ³	a ²	a ²	a ²	a ²	a ³	a ³	a ³	
C ₅₈₅	a ³	a ³	a ²	a ²	a ²	a ²	a ²	a ²	a ²	a ³	.	.	a ²	a ²	a ²	a ²	a ³	a ²	a ²	a ²	a ²	a ²	a ³	

$o_i = [a^3, a^3, a^3, a^3, \dots, a^3, a^3, a^3, a^3]$

Let $n_i = o_i + \ell_i \quad 1 \leq i \leq 585$.

Now the table of o_i

	1	2	3	4	5	6	7	8	9	10	.	.	576	577	578	579	580	581	582	583	584	585	
n ₁	a ²	a ²	a ²	a ³	a ³	a ³	a ³	a ²	a ³	a ³	.	.	a ³	a ³	a ³	a ³	a ³	a ²	a ³	a ³	a ³	a ³	
n ₂	a ³	a ²	a ²	a ²	a ³	a ³	a ³	a ³	a ²	a ³	.	.	a ³	a ³	a ³	a ³	a ³	a ³	a ²	a ³	a ³	a ³	
n ₃	a ³	a ³	a ²	a ²	a ²	a ³	a ³	a ³	a ³	a ²	.	.	a ³	a ³	a ³	a ³	a ³	a ³	a ³	a ²	a ³	a ³	
n ₄	a ³	a ³	a ³	a ²	a ²	a ²	a ³	a ³	a ³	a ³	.	.	a ³	a ³	a ³	a ³	a ³	a ³	a ³	a ³	a ²	a ³	
n ₅	a ³	a ³	a ³	a ³	a ²	a ²	a ²	a ³	a ³	a ³	.	.	a ³	a ³	a ³	a ³	a ³	a ³	a ³	a ³	a ³	a ²	
n ₆	a ²	a ³	a ³	a ³	a ³	a ²	a ²	a ²	a ³	a ³	.	.	a ³	a ³	a ³	a ³	a ³	a ³	a ³	a ³	a ³	a ³	
n ₇	a ³	a ²	a ³	a ³	a ³	a ³	a ²	a ²	a ²	a ³	.	.	a ³	a ³	a ³	a ³	a ³	a ³	a ³	a ³	a ³	a ³	
n ₈	a ³	a ³	a ²	a ³	a ³	a ³	a ³	a ²	a ²	a ²	.	.	a ³	a ³	a ³	a ³	a ³	a ³	a ³	a ³	a ³	a ³	
n ₉	a ³	a ³	a ³	a ²	a ³	a ³	a ³	a ³	a ²	a ²	.	.	a ³	a ³	a ³	a ³	a ³	a ³	a ³	a ³	a ³	a ³	
n ₁₀	a ³	a ³	a ³	a ³	a ²	a ³	a ³	a ³	a ³	a ²	.	.	a ³	a ³	a ³	a ³	a ³	a ³	a ³	a ³	a ³	a ³	
n ₁₁	a ³	a ³	a ³	a ³	a ³	a ²	a ³	a ³	a ³	a ³	.	.	a ³	a ³	a ³	a ³	a ³	a ³	a ³	a ³	a ³	a ³	
n ₁₂	a ³	a ³	a ³	a ³	a ³	a ³	a ²	a ³	a ³	a ³	.	.	a ³	a ³	a ³	a ³	a ³	a ³	a ³	a ³	a ³	a ³	
n ₁₃	a ³	a ³	a ³	a ³	a ³	a ³	a ³	a ²	a ³	a ³	.	.	a ³	a ³	a ³	a ³	a ³	a ³	a ³	a ³	a ³	a ³	
n ₁₄	a ³	a ³	a ³	a ³	a ³	a ³	a ³	a ³	a ²	a ³	.	.	a ²	a ³	a ³	a ³	a ³	a ³	a ³	a ³	a ³	a ³	
n ₁₅	a ³	a ³	a ³	a ³	a ³	a ³	a ³	a ³	a ³	a ²	.	.	a ³	a ²	a ³	a ³	a ³	a ³	a ³	a ³	a ³	a ³	
.
.
n ₅₇₁	a ³	a ³	a ³	a ³	a ³	a ²	a ³	a ³	a ³	a ³	.	.	a ³	a ³	a ²	a ³	a ³	a ²	a ²	a ³	a ³	a ³	
n ₅₇₂	a ³	a ³	a ³	a ³	a ³	a ³	a ²	a ³	a ³	a ³	.	.	a ³	a ³	a ³	a ²	a ³	a ³	a ²	a ²	a ³	a ³	
n ₅₇₃	a ³	a ³	a ³	a ³	a ³	a ³	a ³	a ²	a ³	a ³	.	.	a ³	a ³	a ³	a ³	a ²	a ³	a ³	a ²	a ²	a ³	
n ₅₇₄	a ³	a ³	a ³	a ³	a ³	a ³	a ³	a ³	a ²	a ³	.	.	a ²	a ³	a ³	a ³	a ³	a ²	a ³	a ³	a ²	a ²	
n ₅₇₅	a ²	a ³	a ³	a ³	a ³	a ³	a ³	a ³	a ³	a ²	.	.	a ²	a ²	a ³	a ³	a ³	a ³	a ²	a ³	a ³	a ²	
n ₅₇₆	a ²	a ²	a ³	a ³	a ³	a ³	a ³	a ³	a ³	a ³	.	.	a ²	a ²	a ²	a ³	a ³	a ³	a ³	a ²	a ³	a ³	
n ₅₇₇	a ³	a ²	a ²	a ³	a ³	a ³	a ³	a ³	a ³	a ³	.	.	a ³	a ²	a ²	a ²	a ³	a ³	a ³	a ³	a ²	a ³	
n ₅₇₈	a ³	a ³	a ²	a ²	a ³	a ³	a ³	a ³	a ³	a ³	.	.	a ³	a ³	a ²	a ²	a ²	a ³	a ³	a ³	a ³	a ²	
n ₅₇₉	a ³	a ³	a ³	a ²	a ²	a ³	a ³	a ³	a ³	a ³	.	.	a ³	a ³	a ³	a ²	a ²	a ²	a ³	a ³	a ³	a ³	
n ₅₈₀	a ³	a ³	a ³	a ³	a ²	a ²	a ³	a ³	a ³	a ³	.	.	a ³	a ³	a ³	a ³	a ²	a ²	a ²	a ³	a ³	a ³	
n ₅₈₁	a ³	a ³	a ³	a ³	a ³	a ²	a ²	a ³	a ³	a ³	.	.	a ²	a ³	a ³	a ³	a ³	a ²	a ²	a ²	a ³	a ³	
n ₅₈₂	a ³	a ³	a ³	a ³	a ³	a ³	a ²	a ²	a ³	a ³	.	.	a ³	a ²	a ³	a ³	a ³	a ³	a ²	a ²	a ²	a ³	
n ₅₈₃	a ³	a ³	a ³	a ³	a ³	a ³	a ³	a ²	a ²	a ³	.	.	a ³	a ³	a ²	a ³	a ³	a ³	a ³	a ²	a ²	a ²	
n ₅₈₄	a ²	a ³	a ³	a ³	a ³	a ³	a ³	a ³	a ²	a ²	.	.	a ³	a ³	a ³	a ²	a ³	a ³	a ³	a ³	a ²	a ²	
n ₅₈₅	a ²	a ²	a ³	a ³	a ³	a ³	a ³	a ³	a ³	a ²	.	.	a ³	a ³	a ³	a ³	a ²	a ³	a ³	a ³	a ³	a ²	

w_i=[a⁴,a⁴,a⁴,a⁴.....a⁴,a⁴,a⁴,a⁴]

Let e_i= w_i+ ℓ_i 1 ≤ i ≤ 585.

Now the table of e_i

	1	2	3	4	5	6	7	8	9	10	.	.	576	577	578	579	580	581	582	583	584	585	
e ₁	a ⁶	a ⁶	a ⁶	a ⁴	a ⁴	a ⁴	a ⁴	a ⁶	a ⁴	a ⁴	.	.	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	a ⁶	a ⁴	a ⁴	a ⁴	a ⁴	
e ₂	a ⁴	a ⁶	a ⁶	a ⁶	a ⁴	a ⁴	a ⁴	a ⁴	a ⁶	a ⁴	.	.	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	a ⁶	a ⁴	a ⁴	a ⁴	
e ₃	a ⁴	a ⁴	a ⁶	a ⁶	a ⁶	a ⁴	a ⁴	a ⁴	a ⁴	a ⁶	.	.	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	a ⁶	a ⁴	a ⁴	
e ₄	a ⁴	a ⁴	a ⁴	a ⁶	a ⁶	a ⁶	a ⁴	a ⁴	a ⁴	a ⁴	.	.	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	a ⁶	a ⁴	
e ₅	a ⁴	a ⁴	a ⁴	a ⁴	a ⁶	a ⁶	a ⁶	a ⁴	a ⁴	a ⁴	.	.	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	a ⁶	
e ₆	a ⁶	a ⁴	a ⁴	a ⁴	a ⁴	a ⁶	a ⁶	a ⁶	a ⁴	a ⁴	.	.	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	
e ₇	a ⁴	a ⁶	a ⁴	a ⁴	a ⁴	a ⁴	a ⁶	a ⁶	a ⁶	a ⁴	.	.	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	
e ₈	a ⁴	a ⁴	a ⁶	a ⁴	a ⁴	a ⁴	a ⁴	a ⁶	a ⁶	a ⁶	.	.	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	
e ₉	a ⁴	a ⁴	a ⁴	a ⁶	a ⁴	a ⁴	a ⁴	a ⁴	a ⁶	a ⁶	.	.	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	
e ₁₀	a ⁴	a ⁴	a ⁴	a ⁴	a ⁶	a ⁴	a ⁴	a ⁴	a ⁴	a ⁶	.	.	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	
e ₁₁	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	a ⁶	a ⁴	a ⁴	a ⁴	a ⁴	.	.	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	
e ₁₂	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	a ⁶	a ⁴	a ⁴	a ⁴	.	.	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	
e ₁₃	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	a ⁶	a ⁴	a ⁴	.	.	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	
e ₁₄	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	a ⁶	a ⁴	.	.	a ⁶	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	
e ₁₅	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	a ⁶	.	.	a ⁴	a ⁶	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	
.
.
e ₅₇₁	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	a ⁶	a ⁴	a ⁴	a ⁴	a ⁴	.	.	a ⁴	a ⁴	a ⁶	a ⁴	a ⁴	a ⁶	a ⁶	a ⁴	a ⁴	a ⁴	
e ₅₇₂	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	a ⁶	a ⁴	a ⁴	a ⁴	.	.	a ⁴	a ⁴	a ⁴	a ⁶	a ⁴	a ⁴	a ⁶	a ⁶	a ⁴	a ⁴	
e ₅₇₃	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	a ⁶	a ⁴	a ⁴	.	.	a ⁴	a ⁴	a ⁴	a ⁴	a ⁶	a ⁴	a ⁴	a ⁶	a ⁶	a ⁴	
e ₅₇₄	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	a ⁶	a ⁴	.	.	a ⁶	a ⁴	a ⁴	a ⁴	a ⁴	a ⁶	a ⁴	a ⁴	a ⁶	a ⁶	
e ₅₇₅	a ⁶	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	a ⁶	.	.	a ⁶	a ⁶	a ⁴	a ⁴	a ⁴	a ⁴	a ⁶	a ⁴	a ⁴	a ⁶	
e ₅₇₆	a ⁶	a ⁶	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	.	.	a ⁶	a ⁶	a ⁶	a ⁴	a ⁴	a ⁴	a ⁴	a ⁶	a ⁴	a ⁴	
e ₅₇₇	a ⁴	a ⁶	a ⁶	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	.	.	a ⁴	a ⁶	a ⁶	a ⁶	a ⁴	a ⁴	a ⁴	a ⁴	a ⁶	a ⁴	
e ₅₇₈	a ⁴	a ⁴	a ⁶	a ⁶	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	.	.	a ⁴	a ⁴	a ⁶	a ⁶	a ⁶	a ⁴	a ⁴	a ⁴	a ⁴	a ⁶	
e ₅₇₉	a ⁶	a ⁴	a ⁴	a ⁶	a ⁶	a ⁴	a ⁴	a ⁴	a ⁴	a ⁴	.	.	a ⁴	a ⁴	a ⁴	a ⁶	a ⁶	a ⁶	a ⁴	a ⁴	a ⁴	a ⁴	
e ₅₈₀	a ⁴	a ⁶	a ⁴	a ⁴	a ⁶	a ⁶	a ⁴	a ⁴	a ⁴	a ⁴	.	.	a ⁴	a ⁴	a ⁴	a ⁴	a ⁶	a ⁶	a ⁶	a ⁴	a ⁴	a ⁴	
e ₅₈₁	a ⁴	a ⁴	a ⁶	a ⁴	a ⁴	a ⁶	a ⁶	a ⁴	a ⁴	a ⁴	.	.	a ⁶	a ⁴	a ⁴	a ⁴	a ⁴	a ⁶	a ⁶	a ⁶	a ⁴	a ⁴	
e ₅₈₂	a ⁴	a ⁴	a ⁴	a ⁶	a ⁴	a ⁴	a ⁶	a ⁶	a ⁴	a ⁴	.	.	a ⁴	a ⁶	a ⁴	a ⁴	a ⁴	a ⁴	a ⁶	a ⁶	a ⁶	a ⁴	
e ₅₈₃	a ⁴	a ⁴	a ⁴	a ⁴	a ⁶	a ⁴	a ⁴	a ⁶	a ⁶	a ⁴	.	.	a ⁴	a ⁴	a ⁶	a ⁴	a ⁴	a ⁴	a ⁴	a ⁶	a ⁶	a ⁶	
e ₅₈₄	a ⁶	a ⁴	a ⁴	a ⁴	a ⁴	a ⁶	a ⁴	a ⁴	a ⁶	a ⁶	.	.	a ⁴	a ⁴	a ⁴	a ⁶	a ⁴	a ⁴	a ⁴	a ⁴	a ⁶	a ⁶	
e ₅₈₅	a ⁶	a ⁶	a ⁴	a ⁴	a ⁴	a ⁴	a ⁶	a ⁴	a ⁴	a ⁶	.	.	a ⁴	a ⁴	a ⁴	a ⁴	a ⁶	a ⁴	a ⁴	a ⁴	a ⁴	a ⁶	

$p_i = [a^5, a^5, a^5, a^5, \dots, a^5, a^5, a^5, a^5]$

Let $f_i = p_i + \ell_i \quad 1 \leq i \leq 585.$

Now the table of f_i

	1	2	3	4	5	6	7	8	9	10	.	.	576	577	578	579	580	581	582	583	584	585	
f ₁	a	a	a	a ⁵	a ⁵	a ⁵	a ⁵	a	a ⁵	a ⁵	.	.	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a	a ⁵	a ⁵	a ⁵	a ⁵	
f ₂	a ⁵	a	a	a	a ⁵	a ⁵	a ⁵	a ⁵	a	a ⁵	.	.	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a	a ⁵	a ⁵	a ⁵	
f ₃	a ⁵	a ⁵	a	a	a	a ⁵	a ⁵	a ⁵	a ⁵	a	.	.	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a	a ⁵	a ⁵	
f ₄	a ⁵	a ⁵	a ⁵	a	a	a	a ⁵	a ⁵	a ⁵	a ⁵	.	.	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	
f ₅	a ⁵	a ⁵	a ⁵	a ⁵	a	a	a	a ⁵	a ⁵	a ⁵	.	.	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a	
f ₆	a	a ⁵	a ⁵	a ⁵	a ⁵	a	a	a	a ⁵	a ⁵	.	.	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	
f ₇	a ⁵	a	a ⁵	a ⁵	a ⁵	a ⁵	a	a	a	a ⁵	.	.	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	
f ₈	a ⁵	a ⁵	a	a ⁵	a ⁵	a ⁵	a ⁵	a	a	a	.	.	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	
f ₉	a ⁵	a ⁵	a ⁵	a	a ⁵	a ⁵	a ⁵	a ⁵	a	a	.	.	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	
f ₁₀	a ⁵	a ⁵	a ⁵	a ⁵	a	a ⁵	a ⁵	a ⁵	a ⁵	a	.	.	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	
f ₁₁	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a	a ⁵	a ⁵	a ⁵	a ⁵	.	.	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	
f ₁₂	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a	a ⁵	a ⁵	a ⁵	.	.	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	
f ₁₃	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a	a ⁵	a ⁵	.	.	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	
f ₁₄	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a	a ⁵	.	.	a	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	
f ₁₅	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a	.	.	a ⁵	a	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	
.
.
f ₅₇₁	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a	a ⁵	a ⁵	a ⁵	a ⁵	.	.	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a	a	a ⁵	a ⁵	
f ₅₇₂	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a	a ⁵	a ⁵	a ⁵	.	.	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a	a	a ⁵	
f ₅₇₃	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a	a ⁵	a ⁵	.	.	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a	a	a ⁵	
f ₅₇₄	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a	a ⁵	.	.	a	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a	
f ₅₇₅	a	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a	.	.	a	a	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a	
f ₅₇₆	a	a	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	.	.	a	a	a	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	
f ₅₇₇	a ⁵	a	a	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	.	.	a ⁵	a	a	a	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	
f ₅₇₈	a ⁵	a ⁵	a	a	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	.	.	a ⁵	a ⁵	a	a	a	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	
f ₅₇₉	a	a ⁵	a ⁵	a	a	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	.	.	a ⁵	a ⁵	a ⁵	a	a	a	a ⁵	a ⁵	a ⁵	a ⁵	
f ₅₈₀	a ⁵	a	a ⁵	a ⁵	a	a	a ⁵	a ⁵	a ⁵	a ⁵	.	.	a ⁵	a ⁵	a ⁵	a ⁵	a	a	a	a ⁵	a ⁵	a ⁵	
f ₅₈₁	a ⁵	a ⁵	a	a ⁵	a ⁵	a	a	a ⁵	a ⁵	a ⁵	.	.	a	a ⁵	a ⁵	a ⁵	a ⁵	a	a	a	a ⁵	a ⁵	
f ₅₈₂	a ⁵	a ⁵	a ⁵	a	a ⁵	a ⁵	a	a	a ⁵	a ⁵	.	.	a ⁵	a	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a	a	a ⁵	
f ₅₈₃	a ⁵	a ⁵	a ⁵	a ⁵	a	a ⁵	a ⁵	a	a	a ⁵	.	.	a ⁵	a ⁵	a	a ⁵	a ⁵	a ⁵	a ⁵	a	a	a	
f ₅₈₄	a	a ⁵	a ⁵	a ⁵	a ⁵	a	a ⁵	a ⁵	a	a	.	.	a ⁵	a ⁵	a ⁵	a	a ⁵	a ⁵	a ⁵	a ⁵	a ⁵	a	
f ₅₈₅	a	a	a ⁵	a ⁵	a ⁵	a ⁵	a	a ⁵	a ⁵	a	.	.	a ⁵	a ⁵	a ⁵	a ⁵	a	a ⁵	a ⁵	a ⁵	a ⁵	a	

$s_i = [a^6, a^6, a^6, \dots, a^6, a^6, a^6]$

Let $g_i = s_i + l_i \quad 1 \leq i \leq 58$

Now the table of g_i

	1	2	3	4	5	6	7	8	9	10	.	.	576	577	578	579	580	581	582	583	584	585	
g ₁	a ⁴	a ⁴	a ⁴	a ⁶	a ⁶	a ⁶	a ⁶	a ⁴	a ⁶	a ⁶	.	.	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁴	a ⁶	a ⁶	a ⁶	a ⁶	
g ₂	a ⁶	a ⁴	a ⁴	a ⁴	a ⁶	a ⁶	a ⁶	a ⁶	a ⁴	a ⁶	.	.	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁴	a ⁶	a ⁶	a ⁶	
g ₃	a ⁶	a ⁶	a ⁴	a ⁴	a ⁴	a ⁶	a ⁶	a ⁶	a ⁶	a ⁴	.	.	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁴	a ⁶	a ⁶	
g ₄	a ⁶	a ⁶	a ⁶	a ⁴	a ⁴	a ⁴	a ⁶	a ⁶	a ⁶	a ⁶	.	.	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁴	a ⁶	
g ₅	a ⁶	a ⁶	a ⁶	a ⁶	a ⁴	a ⁴	a ⁴	a ⁶	a ⁶	a ⁶	.	.	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁴	
g ₆	a ⁴	a ⁶	a ⁶	a ⁶	a ⁶	a ⁴	a ⁴	a ⁴	a ⁶	a ⁶	.	.	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	
g ₇	a ⁶	a ⁴	a ⁶	a ⁶	a ⁶	a ⁶	a ⁴	a ⁴	a ⁴	a ⁶	.	.	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	
g ₈	a ⁶	a ⁶	a ⁴	a ⁶	a ⁶	a ⁶	a ⁶	a ⁴	a ⁴	a ⁴	.	.	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	
g ₉	a ⁶	a ⁶	a ⁶	a ⁴	a ⁶	a ⁶	a ⁶	a ⁶	a ⁴	a ⁴	.	.	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	
g ₁₀	a ⁶	a ⁶	a ⁶	a ⁶	a ⁴	a ⁶	a ⁶	a ⁶	a ⁶	a ⁴	.	.	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	
g ₁₁	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁴	a ⁶	a ⁶	a ⁶	a ⁶	.	.	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	
g ₁₂	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁴	a ⁶	a ⁶	a ⁶	.	.	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	
g ₁₃	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁴	a ⁶	a ⁶	.	.	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	
g ₁₄	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁴	a ⁶	.	.	a ⁴	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	
g ₁₅	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁴	.	.	a ⁶	a ⁴	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	
.
.
g ₅₇₁	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁴	a ⁶	a ⁶	a ⁶	a ⁶	.	.	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁴	a ⁴	a ⁶	a ⁶	a ⁶	
g ₅₇₂	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁴	a ⁶	a ⁶	a ⁶	.	.	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁴	a ⁴	a ⁶	a ⁶	
g ₅₇₃	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁴	a ⁶	a ⁶	.	.	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁴	a ⁴	a ⁶	
g ₅₇₄	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁴	a ⁶	.	.	a ⁴	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁴	a ⁴	
g ₅₇₅	a ⁴	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁴	.	.	a ⁴	a ⁴	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁴	
g ₅₇₆	a ⁴	a ⁴	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	.	.	a ⁴	a ⁴	a ⁴	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	
g ₅₇₇	a ⁶	a ⁴	a ⁴	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	.	.	a ⁶	a ⁴	a ⁴	a ⁴	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	
g ₅₇₈	a ⁶	a ⁶	a ⁴	a ⁴	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	.	.	a ⁶	a ⁶	a ⁴	a ⁴	a ⁴	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	
g ₅₇₉	a ⁴	a ⁶	a ⁶	a ⁴	a ⁴	a ⁶	a ⁶	a ⁶	a ⁶	a ⁶	.	.	a ⁶	a ⁶	a ⁶	a ⁴	a ⁴	a ⁴	a ⁶	a ⁶	a ⁶	a ⁶	
g ₅₈₀	a ⁶	a ⁴	a ⁶	a ⁶	a ⁴	a ⁴	a ⁶	a ⁶	a ⁶	a ⁶	.	.	a ⁶	a ⁶	a ⁶	a ⁶	a ⁴	a ⁴	a ⁴	a ⁶	a ⁶	a ⁶	
g ₅₈₁	a ⁶	a ⁶	a ⁴	a ⁶	a ⁶	a ⁴	a ⁴	a ⁶	a ⁶	a ⁶	.	.	a ⁴	a ⁶	a ⁶	a ⁶	a ⁶	a ⁴	a ⁴	a ⁴	a ⁶	a ⁶	
g ₅₈₂	a ⁶	a ⁶	a ⁶	a ⁴	a ⁶	a ⁶	a ⁴	a ⁴	a ⁶	a ⁶	.	.	a ⁶	a ⁴	a ⁶	a ⁶	a ⁶	a ⁶	a ⁴	a ⁴	a ⁴	a ⁶	
g ₅₈₃	a ⁶	a ⁶	a ⁶	a ⁶	a ⁴	a ⁶	a ⁶	a ⁴	a ⁴	a ⁶	.	.	a ⁶	a ⁶	a ⁴	a ⁶	a ⁶	a ⁶	a ⁶	a ⁴	a ⁴	a ⁴	
g ₅₈₄	a ⁴	a ⁶	a ⁶	a ⁶	a ⁶	a ⁴	a ⁶	a ⁶	a ⁴	a ⁴	.	.	a ⁶	a ⁶	a ⁶	a ⁴	a ⁶	a ⁶	a ⁶	a ⁶	a ⁴	a ⁴	
g ₅₈₅	a ⁴	a ⁴	a ⁶	a ⁶	a ⁶	a ⁶	a ⁴	a ⁶	a ⁶	a ⁴	.	.	a ⁶	a ⁶	a ⁶	a ⁶	a ⁴	a ⁶	a ⁶	a ⁶	a ⁶	a ⁴	

Find the shortest distance between two different code words in the matrices above, so that the shortest distance is 73 and the largest distance is 585. $m_i, v_i, h_i, k_i, o_i, w_i, p_i, s_i, \ell_i, a_i, c_i, b_i, n_i, e_i, f_i, g_i$, where $i = 1, \dots, 585$

$d(m_i, \ell_i) = 73$	$d(c_i, a_i) = 585$	$d(g_i, e_i) = 439$
$d(v_i, \ell_i) = 73$	$d(c_i, b_i) = 585$	$d(g_i, n_i) = 585$
$d(h_i, \ell_i) = 585$	$d(c_i, n_i) = 439$	$d(g_i, f_i) = 585$

$d(k_i, \ell_i) = 585$	$d(c_i, e_i) = 585$	$d(h_i, m_i) = 585$
$d(o_i, \ell_i) = 585$	$d(c_i, f_i) = 585$	$d(h_i, v_i) = 585$
$d(w_i, \ell_i) = 585$	$d(c_i, g_i) = 585$	$d(h_i, k_i) = 585$
$d(p_i, \ell_i) = 585$	$d(n_i, a_i) = 585$	$d(h_i, o_i) = 585$
$d(s_i, \ell_i) = 585$	$d(n_i, b_i) = 585$	$d(h_i, w_i) = 585$
$d(\ell_i, a_i) = 585$	$d(n_i, c_i) = 439$	$d(h_i, p_i) = 585$
$d(\ell_i, b_i) = 585$	$d(n_i, e_i) = 585$	$d(h_i, s_i) = 585$
$d(\ell_i, c_i) = 585$	$d(n_i, f_i) = 585$	$d(o_i, m_i) = 585$
$d(\ell_i, n_i) = 585$	$d(n_i, g_i) = 585$	$d(o_i, v_i) = 585$
$d(\ell_i, e_i) = 585$	$d(e_i, a_i) = 585$	$d(o_i, k_i) = 585$
$d(\ell_i, f_i) = 585$	$d(e_i, b_i) = 585$	$d(o_i, w_i) = 585$
$d(\ell_i, g_i) = 585$	$d(e_i, c_i) = 585$	$d(o_i, p_i) = 585$
$d(a_i, b_i) = 585$	$d(e_i, n_i) = 585$	$d(o_i, s_i) = 585$
$d(a_i, c_i) = 585$	$d(e_i, f_i) = 585$	$d(v_i, m_i) = 585$
$d(a_i, n_i) = 585$	$d(e_i, g_i) = 439$	$d(v_i, v_i) = 585$
$d(a_i, e_i) = 585$	$d(f_i, a_i) = 585$	$d(v_i, k_i) = 585$
$d(a_i, f_i) = 585$	$d(f_i, b_i) = 439$	$d(v_i, o_i) = 585$
$d(a_i, g_i) = 585$	$d(f_i, c_i) = 585$	$d(v_i, w_i) = 585$
$d(b_i, a_i) = 439$	$d(f_i, e_i) = 585$	$d(v_i, p_i) = 585$
$d(b_i, c_i) = 585$	$d(f_i, n_i) = 585$	$d(v_i, s_i) = 585$
$d(b_i, n_i) = 585$	$d(f_i, g_i) = 585$	$d(m_i, a_i) = 73$
$d(b_i, e_i) = 585$	$d(g_i, a_i) = 585$	$d(m_i, b_i) = 585$
$d(b_i, f_i) = 585$	$d(g_i, b_i) = 439$	$d(m_i, c_i) = 585$
$d(b_i, g_i) = 585$	$d(g, c_i) = 585$	$d(m_i, n_i) = 585$

$d(m_i, e_i) = 585$	$d(m_i, a_i) = 73$	$d(w_i, a_i) = 585$
$d(m_i, f_i) = 585$	$d(m_i, b_i) = 585$	$d(w_i, b_i) = 585$
$d(m_i, g_i) = 585$	$d(m_i, c_i) = 585$	$d(w_i, c_i) = 585$
$d(k_i, m_i) = 585$	$d(m_i, n_i) = 585$	$d(w_i, n_i) = 585$
$d(k_i, v_i) = 585$	$d(m_i, e_i) = 585$	$d(w_i, e_i) = 73$
$d(k_i, k_i) = 585$	$d(m_i, f_i) = 585$	$d(w_i, f_i) = 585$
$d(k_i, o_i) = 585$	$d(m_i, g_i) = 585$	$d(w_i, g_i) = 512$

$d(k_i, w_i) = 585$	$d(k_i, a_i) = 585$	$d(s_i, a_i) = 585$
$d(k_i, p_i) = 585$	$d(k_i, b_i) = 585$	$d(s_i, b_i) = 585$
$d(k_i, s_i) = 585$	$d(k_i, c_i) = 73$	$d(s_i, c_i) = 585$
$d(w_i, m_i) = 585$	$d(k_i, n_i) = 512$	$d(s_i, n_i) = 585$
$d(w_i, v_i) = 585$	$d(k_i, e_i) = 585$	$d(s_i, e_i) = 512$
$d(w_i, h_i) = 585$	$d(k_i, f_i) = 585$	$d(s_i, f_i) = 585$
$d(w_i, k_i) = 585$	$d(k_i, g_i) = 585$	$d(s_i, g_i) = 73$
$d(w_i, o_i) = 585$	$d(o_i, a_i) = 585$	$d(p_i, m_i) = 585$
$d(w_i, p_i) = 585$	$d(o_i, b_i) = 585$	$d(p_i, v_i) = 585$
$d(w_i, s_i) = 585$	$d(o_i, c_i) = 512$	$d(p_i, h_i) = 585$
$d(s_i, v_i) = 585$	$d(o_i, n_i) = 73$	$d(p_i, k_i) = 585$
$d(s_i, h_i) = 585$	$d(o_i, e_i) = 585$	$d(p_i, w_i) = 585$
$d(s_i, k_i) = 585$	$d(o_i, f_i) = 585$	$d(p_i, o_i) = 585$
$d(s_i, o_i) = 585$	$d(o_i, g_i) = 585$	$d(p_i, s_i) = 585$
$d(s_i, w_i) = 585$	$d(p_i, a_i) = 585$	$d(m_i, e_i) = 585$
$d(s_i, p_i) = 585$	$d(p_i, b_i) = 585$	$d(m_i, f_i) = 585$
$d(s_i, m_i) = 585$	$d(p_i, c_i) = 512$	$d(m_i, g_i) = 585$
$d(v_i, a_i) = 73$	$d(p_i, n_i) = 585$	$d(v_i, n_i) = 585$
$d(v_i, b_i) = 585$	$d(p_i, e_i) = 585$	$d(v_i, e_i) = 585$
$d(v_i, c_i) = 585$	$d(p_i, f_i) = 585$	$d(v_i, f_i) = 585$
$d(v_i, g_i) = 585$	$d(p_i, g_i) = 585$	

If we substitute the values of $n = 585, d = 73, e = 36$ in inequality of Theorem 3.1 we get $M = 8^{581}$ Hence C is a $(585, 8^{581}, 73)$ - code.

$$M \left\{ \binom{n}{0} + \binom{n}{1}(q-1) + \dots + \binom{n}{e}(q-1)^e \right\} \leq q^n$$

$$8^{581} \left\{ \binom{585}{0} + \binom{585}{1}(8-1) + \binom{585}{2}(8-1)^2 + \binom{585}{3}(8-1)^3 + \binom{585}{4}(8-1)^4 + \binom{585}{5}(8-1)^5 + \binom{585}{6}(8-1)^6 + \binom{585}{7}(8-1)^7 + \binom{585}{8}(8-1)^8 + \binom{585}{9}(8-1)^9 + \binom{585}{10}(8-1)^{10} + \binom{585}{11}(8-1)^{11} + \binom{585}{12}(8-1)^{12} + \binom{585}{13}(8-1)^{13} + \binom{585}{14}(8-1)^{14} + \binom{585}{15}(8-1)^{15} + \binom{585}{16}(8-1)^{16} + \binom{585}{17}(8-1)^{17} + \binom{585}{18}(8-1)^{18} + \binom{585}{19}(8-1)^{19} + \binom{585}{20}(8-1)^{20} + \binom{585}{21}(8-1)^{21} + \binom{585}{22}(8-1)^{22} + \binom{585}{23}(8-1)^{23} + \binom{585}{24}(8-1)^{24} + \binom{585}{25}(8-1)^{25} + \binom{585}{26}(8-1)^{26} + \binom{585}{27}(8-1)^{27} + \binom{585}{28}(8-1)^{28} + \binom{585}{29}(8-1)^{29} + \binom{585}{30}(8-1)^{30} + \binom{585}{31}(8-1)^{31} + \binom{585}{32}(8-1)^{32} + \binom{585}{33}(8-1)^{33} + \binom{585}{34}(8-1)^{34} + \binom{585}{35}(8-1)^{35} + \binom{585}{36}(8-1)^{36} \right\} < 8^{585}$$

C is not perfect

CONCLUSIONS

We have constructed the Projective linear codes with the parameters $n, k,$ and d depending on the order of Galois Field $F_8,$ and have also studied the relationship between the finite Projective space and coding theory such that the columns of the generator matrix of any linear code are considered the points in the Projective space.

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تطبيقات الفضاء الإسقاطي $PG(3,8)$ في نظرية الترميز

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الخلاصة

الهدف الرئيسي من هذا البحث هو تقديم العلاقة بين موضوع نظرية الترميز والفضاء الإسقاط الثلاثي الأبعاد في الحقل الثامن . نجد النقاط والخطوط والمستويات لحقل كالمو من الرتبة 8 وذلك باستخدام المعادلات الجبرية , ومن ثم نكون مصفوفة إسقاطية ذا نظام ثنائي صفر وواحد . نجمع عناصر حقل كالمو من الرتبة 8 مع المصفوفة الإسقاطية يتكون عندنا سبعة مصفوفات إسقاطية ونجد أقصر مسافة بين نقطتين مختلفتين من المصفوفات إذ كانت أعلى مسافة التي حصلنا عليها 585 وأقصر مسافة هي 73 . واختبر الكود . ومن ثم تم إنشاء القيمة القصوى لحجم الشفرة على مجال محدود من الرتبة الثامنة ومصفوفة حدود مع المعلمة n (طول الشفرة) , d (الحد الأدنى للشفرة) , e (تصحيح الخطأ في الشفرة) . نختبر الكود في نظرية الترميز حيث ان طول الشفرة 581 والحد الأدنى للشفرة 73 وتصحيح خطأ في الشفرة 36 . نطبق نظرية الترميز للتحقق من كونها تامة او غير تامة.