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Ranking Fuzzy Numbers by Geometric Average Method and its Application to Fuzzy Linear Fractional Programming Problems

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Abstract

In this paper, we consider a fuzzy linear fractional programming (FLFP) problem under the condition that the objective function is represented by triangular and trapezoidal fuzzy numbers, while the values of the right-hand side and left-hand side constraints are represented by real numbers. And defined a new ranking function for convert fuzzy linear fractional programming problem into crisp linear fractional programming problem. This proposed approach is based on a crisp linear programming and has a simple structure. Comparing the proposed method to the exiting methods for solving FLFP problems we see it is simple to apply and acceptable. Finally, numerical illustrations are used to demonstrate the suggested methods.

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1. Introduction

The objective of linear fractional programming (LFP) is to find the optimal (maximum or minimum) value of a linear fractional objective function subject to linear constraints on the given variables. The constraints may be either equality or inequality constraints. From the point of view of real world applications, LFP possesses as many nice and extremely useful features, as linear programming (LP). If we have a problem formulated as an LP, we can reformulate this problem as LFP by replacing an original linear objective function with a ratio (fraction) of two linear functions (Bajalinov, 2003).

The fractional programming problems are particularly useful in the solution of economic problems in which various activities use certain resources in various proportions, while the objective is to optimize a certain indicator (Nawkhass and Sulaiman, 2022). Usually the most favorable return on allocation ratio subject to the constraint imposed on the availability of goods. Examples of such situations are financial and corporate planning, production planning (Stancu-Minasian, 1992). Many proposed methods were presented to get a solution for fuzzy programming FP issue such as: in (Charness and Cooper, 1962) showed that by a simple transformation the original LFP problem can be reduced to an (LP) problem that can therefore be solved using a regular simplex method for a linear programming. In (Sapan and Tarni, 2017) a proposed method with ranking function is presented. In (Malathi and Umadevi, 2018), a new technique for solving special type of fuzzy programming is suggested. (Deb and De 2015), introduced a ranking function for solving fully fuzzy linear fractional programming problem with objective function and constraints are trapezoidal fuzzy numbers. Also, (Rasha, 2021) solved FLFP problem using α-cut method. Furthermore, (Deepak at el. 2021), suggested a new ranking function of trapezoidal fuzzy number, for solving fully fuzzy linear fractional programming problem with the objective function and constraints are trapezoidal fuzzy numbers. A new method to find a fuzzy optimal solution of FLFP problems with inequality constraints (Sapan and Tarni, 2017). The objective of this paper is to propose an algorithm on a new ranking function to solve FLFP problem using triangular and trapezoidal fuzzy numbers. This paper contains five sections: in section two we review some concepts of fuzzy set theory, in section three a suggested ranking function was presented for triangular and trapezoidal fuzzy numbers, and study some properties, in section four a new algorithm for solving this problem was applied, in section five different numerical examples are applied and compared with some ranking functions Material and methods

2. Preliminaries

In this section, we will give some basic concepts of fuzzy sets and fuzzy numbers.

Definition (1) (Nalla et al., 2020): Let X be universe of discourse. A fuzzy set $\tilde{A}$ in X can be defined as a set of ordered pairs:

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)), x \in X\},$$

where $\mu_{\tilde{A}}(x): X \to [0,1]$ and $\mu_{\tilde{A}}(x)$ is called membership function.

Definition (2) (Hari and Jayakumar, 2014): A fuzzy set $\tilde{A}$, which is both convex and normal, $\tilde{A}$ is called fuzzy number.

Definition (3) (Al Thabhawi 2019): A fuzzy number $\tilde{A} = (r_1, r_2, r_3)$, $r_1 \le r_2 \le r_3$ with $(r_1, r_2, r_3 \ge 0)$ is called a triangular fuzzy number (TFN) if membership function $\mu_{\tilde{A}}(x)$ is describe as:

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{(x - r_1)}{(r_2 - r_1)}, & r_1 \le x \le r_2 \\ 1, & x = r_2 \\ \dfrac{(r_3 - x)}{(r_3 - r_2)}, & r_2 \le x \le r_3 \\ 0, & \text{Otherwise} . \end{cases}$$

Definition (4) (Rasha, 2016): A fuzzy number $\tilde{A} = (r_1, r_2, r_3, r_4)$, $r_1 \le r_2 \le r_3 \le r_4$ with $(r_1, r_2, r_3, r_4 \ge 0)$ is called a trapezoidal fuzzy number (TrFN) if membership function $\mu_{\tilde{A}}(x)$ is describe as:

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{(x - r_1)}{(r_2 - r_1)}, & r_1 \le x \le r_2 \\ 1, & r_2 \le x \le r_3 \\ \dfrac{(r_4 - x)}{(r_4 - r_3)}, & r_3 \le x \le r_4 \\ 0, & \text{Otherwise.} \end{cases}$$

3. Ranking Function of Triangular and Trapezoidal Fuzzy Numbers by Geometric Average

Several approaches for the ranking of fuzzy numbers have been proposed in the literatures. An efficient approach for comparing the fuzzy numbers is by the use of a ranking function. We defined the geometric average in descriptive statistics for triangular and trapezoidal fuzzy numbers $\tilde{A}$ as following

Let $\tilde{A} = (r_1, r_2, r_3)$ where $r_1, r_2, r_3 \ge 0$ and $r_1 \le r_2 \le r_3$, be a triangular fuzzy numbers defined the ranking function $GA(\tilde{A})$ as:

$$GA(\tilde{A}) = (\prod_{i=1}^{3}(1 + r_i))^{\frac{1}{3}} - 1 \qquad (1)$$

Let $\tilde{A} = (r_1, r_2, r_3, r_4)$ where $r_1, r_2, r_3, r_4 \ge 0$ and $r_1 \le r_2 \le r_3 \le r_4$, be a trapezoidal fuzzy numbers defined the ranking function GA($\tilde{A}$) as:

$$GA(\tilde{A}) = (\prod_{i=1}^{4}(1 + r_i))^{\frac{1}{4}} - 1 \qquad (2)$$

Let $\tilde{A}$ and $\tilde{B}$ be two arbitrary fuzzy numbers the ranking is:

a) $GA(\tilde{A}) > GA(\tilde{B})$ if and only if $\tilde{A} > \tilde{B}$
b) $GA(\tilde{A}) < GA(\tilde{B})$ if and only if $\tilde{A} < \tilde{B}$
c) $GA(\tilde{A}) = GA(\tilde{B})$ if and only if $\tilde{A} \approx \tilde{B}$

Remark 1: Our ranking function is able to rank the crisp fuzzy numbers, whereas Cheng's Distance method (Cheng, 1998), (Wang et al., 2006), and (Chu and Tsao, 2002) do not.

3.1 Proposition 1

1. If $GA(\tilde{A})$ is a ranking function of $\tilde{A}$ then $GA(\tilde{A})$ belongs to $\tilde{A}$
2. If $\inf \operatorname{supp}(\tilde{A}) > 0$ then $GA(\tilde{A}) > 0$
3. If $\inf \operatorname{supp}(\tilde{A}) > \sup \operatorname{supp}(\tilde{B})$ then $\tilde{A} > \tilde{\tilde{B}}$.

Proof: part (1). Let $\tilde{A} = (r_1, r_2, r_3, r_4)$ be trapezoidal fuzzy number. By definition (4) we have $r_1 \leq r_2 \leq r_3 \leq r_4$ and $r_1, r_2, r_3, r_4 \geq 0$.
Hence, we get $r_1 \leq r_1, r_1 \leq r_2, r_1 \leq r_3$ and $r_1 \leq r_4$, since $r_i \geq 0$ for all i=1,2,…,4, and by the property of inequality, add 1 to both sides of inequalities $r_1 + 1 \leq r_1 + 1, r_1 + 1 \leq r_2 + 1, r_1 + 1 \leq r_3 + 1, r_1 + 1 \leq r_4 + 1$ and $(r_1 + 1).(r_1 + 1).(r_1 + 1).(r_1 + 1) \leq (1 + r_1).(r_2 + 1).(r_3 + 1).(r_4 + 1)$ [by $a \leq c, b \leq d \rightarrow a.b \leq c.d$)], so, $(r_1 + 1)^4 \leq (1 + r_1).(r_2 + 1).(r_3 + 1).(r_4 + 1)$. Take the fourth root ($4^{th}$) for both sides of the inequality.
Get $r_1 + 1 \leq ((1 + r_1).(r_2 + 1).(r_3 + 1).(r_4 + 1))^{\frac{1}{4}}$, subtract both side by (-1), then $r_1 \leq \left((1 + r_1).(r_2 + 1).(r_3 + 1).(r_4 + 1)\right)^{\frac{1}{4}} - 1$ and by equation (2), $r_1 \leq GA(\tilde{A})$. By analogue manner $GA(\tilde{A}) \leq r_4$. Therefore, $r_1 \leq GA(\tilde{A}) \leq r_4$. ■

Example 1: Consider two triangular fuzzy numbers $\tilde{A} = (0.1, 0.4, 1)$ and $\tilde{B} = (0.1, 0.7, 1)$
$GA(\tilde{A}) = ([\prod_{i=1}^{3}(1 + r_i)^{\frac{1}{3}}] - 1) = ((1 + 0.1)(1 + 0.4)(1 + 1))^{\frac{1}{3}} - 1 = 0.4595$
$GA(\tilde{B}) = ([\prod_{i=1}^{3}(1 + r_i)^{\frac{1}{3}}] - 1) = ((1 + 0.1)(1 + 0.7)(1 + 1))^{\frac{1}{3}} - 1 = 0.5522$
Since $GA(\tilde{A}) < GA(\tilde{B})$ therefore $\tilde{A} < \tilde{B}$.

Example 2: Consider two trapezoidal fuzzy numbers $\tilde{A} = (1, 3, 4, 5)$ and $\tilde{B} = (2, 2, 2, 2)$
$GA(\tilde{A}) = ([\prod_{i=1}^{4}(1 + r_i)^{\frac{1}{4}}] - 1) = ((1 + 1)(1 + 3)(1 + 4)(1 + 5))^{\frac{1}{4}} - 1 = 2.9359$
$GA(\tilde{B}) = ([\prod_{i=1}^{4}(1 + r_i)^{\frac{1}{4}}] - 1) = ((1 + 2)(1 + 2)(1 + 2)(1 + 2))^{\frac{1}{4}} - 1 = 2$
Since $GA(\tilde{A}) > GA(\tilde{B})$ therefore $\tilde{A} > \tilde{B}$.

4. Algorithm to Solve FLFP Problem using Proposed Ranking Function

The technique is suggested to solve a problem of fuzzy fractional programming utilizing fuzzy programming technique where the coefficients of the objective function are fuzzy numbers. The ranking approach based on geometric average which is used for fuzzy linear fractional programming problem (FLFPP). The technique converts it to a crisp linear fractional programming (CLFP) problem. The following are summarizes of the algorithm. Consider FLFP problem

Maximize $Z(X) = \frac{\tilde{c}X + \alpha}{\tilde{d}X + \beta}$
s.t.
$Ax \leq b,$
$x \geq 0,$
where $A = (A_1, A_2, \ldots, A_n)$ is an m by n matrix, c, d and $x \in R^n$, b, α and β *are scalars.*
The ideas can be summarized as follows:

Step 1: Convert the FLFP problem into the following LFP problem by a new ranking function of fuzzy number

Maximize $Z(X) = \frac{cX+\alpha}{dX+\beta}$
Subject to,
$Ax \leq b,$
$x \geq 0.$
Step 2: Transform the obtained LFP problem into a LP problem by using Charnes-Cooper transformation method
Maximize $Z(X) = cy + \alpha t$
s.t.
$dy + \beta t = 1,$
$Ay - bt \leq 0,$
$y \geq 0, t \geq 0.$
Step 3: Find the optimal solution y in Step 2.
Step 4: Obtain the optimal solution x using the value y in Step 2.
Step 5: Compare the optimal solution with other exiting ranking functions.

5. Numerical Examples

In this section, we illustrate two numerical FLFP problems with triangular and trapezoidal fuzzy numbers, with the help of the recommended ranking functions. The FLFP problem is transformed into a crisp programming problem.

Example 3: Consider the fuzzy linear fractional programming problem

$$\text{Max}\, Z = \frac{(3.3, 4, 5.2)x_1 + (5.3, 6, 7.2)x_2}{(4.3, 5, 6.2)x_1 + (3.3, 4, 5.2)x_2 + (0.3, 1, 2.2)},$$

s.t.
$2x_1 + x_2 \leq 10$
$3x_1 + 4x_2 \leq 26$
$x_1, x_2 \geq 0.$

Apply the proposed algorithm:

Step 1: Convert objective function from fuzzy numbers to crisp value by proposed ranking function as:

$$GA(\tilde{A}) = (\,(1 + r_1)(1 + r_2)(1 + r_3))^{\frac{1}{3}} - 1.$$

The objective function becomes FLP problem (Triangular)

$$\text{Max}\, Z = \frac{4.1083x_1 + 5.8221x_2}{5.1176x_1 + 4.1083x_2 + 1.0263}$$

s.t.
$2x_1 + x_2 \leq 10$
$3x_1 + 4x_2 \leq 26$
$x_1, x_2 \geq 0.$

Step 2: Transformed this LFP problem into LP problem by using transformation of Charnes Cooper, the model programming problem is:

$\text{Max}\, Z = 4.1083y_1 + 5.8221y_2$
s.t.
$5.1176y_1 + 4.1083y_2 + 1.0263t = 1$

$2y_1 + y_2 - 10t \leq 0$
$3y_1 + 4y_2 - 26t \leq 0$
$y_1, y_2, t \geq 0.$

Step 3: The problem is in standard form of programming problem and we can find optimal solution by using simplex method, the optimal solution here is $y_1 = 0, y_2 = 0.2344$ and t = 0.0361.
Step 4: Find the optimal solution x using the value y as: $x_1 = \frac{y_1}{t} = 0$ and $x_2 = \frac{y_2}{t} = 6.4930$. Now, the value of $Z = 1.3647$.

Step 5: Using ranking function (Rasha, 2021), and (Iden and Anfal, 2015) for comparison with the proposed method, from Table 1

Table 1: Comparison proposed method with existing triangular ranking methods

Ranking Method	**Rasha Method**	**Iden and Anfal Method**	**Proposed Method**
Optimal Solution	$Z = 1.3434$	$Z = 1.3369$	$Z = 1.3647$

Example 4: Consider the FLFP problem (Trapezoid)

$$\text{Max}\, Z = \frac{(5,6,7,8)x_1 + (3,5,6,7)x_2}{(1,2,3,4)x_1 + (5.5,7,8.5,9)}$$

s.t
$2x_1 + 3x_2 \leq 4$
$3x_1 + 3x_2 \leq 6$
$x_1, x_2 \geq 0.$

Apply the proposed algorithm:
Step 1: Convert objective function from fuzzy numbers to crisp value by proposed ranking function as:

$$GA(\tilde{A}) = ((1+r_1)(1+r_2)(1+r_3)(1+r_4))^{\frac{1}{4}} - 1.$$

The objective function becomes LFP problem

$$Max\, Z = \frac{6.4155x_1 + 5.0548x_2}{2.3097x_1 + 7.3836}$$

s.t.
$2x_1 + 3x_2 \leq 4$
$3x_1 + 3x_2 \leq 6$
$x_1, x_2 \geq 0.$

Step 2: Transformed this LFP problem into linear programming problem by using transformation of Charnes-Cooper, the model programming problem as:

$$Max\, Z = 6.4155y_1 + 5.0548y_2$$

s.t,
$2.3097y_1 + 7.3836t = 1$
$2y_1 + 3y_2 - 4t \leq 0$
$3y_1 + 3y_2 - 6t \leq 0$
$y_1, y_2, t \geq 0.$

Step 3: the problem is standard form of linear programming problem and we can find optimal solution by using simplex method, optimal solution here is $y_1 = 0.1666, y_2 = 0$ and $t = 0.0833$.

Step 4: find the optimal solution x using the value y as: $x_1 = \frac{y_1}{t} = 2$ and $x_2 = \frac{y_2}{t} = 0$. Now, Z= 1.069.

Step 5: Using ranking function (Yager, 1981), and (Deepak and Priyank 2021). Compare with proposed method.

Table 2: Comparison proposed method with existing trapezoidal ranking methods

Ranking Method	Yager Method	Deepak at. el Method	Proposed Method
Optimal Solution	$Z = 1.0400$	$Z = 1.0277$	Z= 1.0690.

6. Conclusion

In this paper, we presented a new algorithm to convert the FLFP problem to crisp FLP problem and solving crisp FLP problems by steps of the proposed algorithm. Also, we introduced a new ranking function method to covert the objective function of FLFP problem to crisp FLP problem with only the objective function is fuzzy numbers. The advantage of the ranking method is for using triangular and trapezoidal fuzzy numbers. Finally, the numerical examples and their result show clearly the usefulness of the proposed method.

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ترتيب الأعداد الضبابية بطريقة المتوسط الهندسي وتطبيقها على مسائل البرمجة الخطية الكسرية الضبابية

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الخلاصة

في هذا البحث ، نأخذ في الاعتبار مشكلة البرمجة الكسرية الخطية الضبابية (FLFP) بحيث ان دالة الهدف تمثل باعداد ضبابية مثلثة وشبه المنحرف ، بينما يتم تمثيل قيم الجانب الأيمن وقيود الجانب الأيسر بواسطة اعداد حقيقية. الطريقة المقترحة تعتمد على البرمجة الخطية الاعتيادية التي لها تركيبة بسيطة. بمقارنة الطريقة المقترحة بالطرق الموجودة لحل مشكلات FLFP، نرى أنها سهلة التطبيق ومقبولة. أخيرًا ، تم استخدام توضيحات عددية للطرق المقترحة.

الكلمات الدالة: مشكلة البرمجة الجزئية الخطية الضبابية (FLFP) ، دالة الهدف ، دالة الترتيب ، المتوسط الهندسي.