AN OPTIMUM EXPANDED MODEL OF $\overline{X}$ CONTROL CHARTS WHEN THERE ARE SEVERAL ASSIGNABLE CAUSES

by

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نموذج امثل موسع للوحات ضبط المتوسط
عند تعدد المسببات غير العشوائيـــــــــــة

بقلـــم
الدكتور تركي ابراهيم سلطان
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خلاصة المقالـــــة :ــ

في بحث سابق[٧] تمت دراسة اسلوب التصميم الامثل للوحات ضبط المتوسط عندما يكون هناك مسببا واحدا غير عشوائيا يحدث بطريقة عشوائية• والبحث الحالـي مخصص لدراسة الحالة التي يكون فيها عدد من المسببات العشوائيـــــــــــة • وقد ادخل في النموذج ١٤ متغيـرا مستقلا تؤثر على التصميم الامثـــــــل • ومن ذلك النموذج يمكن ايجاد القيم المثلى لحجم العينة والزمن بين كل عينه واخرى• علاوة على المسافة بين حدى الضبط •

ABSTRACT:

In a previous work[7], the optimum design of $\overline{X}$ charts, when there is a single assignable cause occuring randomly was studied. The present article extends the study to allow for the occurrence of several assignable causes.
14 independent variables are encountered in the model.
The optimum values of the size and frequency of sampling, and the position of control limits are to be seeked.

1. INTRODUCTION :

Considering the maximum income criterion of $\overline{X}$ control charts, several authers (1-8) derived a formula giving the long run average net income per unit time as a function of the parameters, sample size n and its frequency h and multiple k of sigma determining the control limits, when there is a single assignable cause occuring randomly with known effect. This model includes 14 independent parameters that may affect the net income function.

In the presented study, we build a model to allow for the occurance of multiple assignable causes.

2. FEATURES OF THE MODEL :

The main features of the model are (2,7) :-

a. Samples of n items are taken every h hours of operations.
The quality characteristic X of each item is determined, and a sample mean $\overline{X}$ is plotted on an $\overline{X}$ chart with control limits at $\pm k\sigma$.

b. The process is at any time in one of the two states.
Either it is in control or it has been disturbed by the occurrence of an assignable cause A_j which produces a shift in the process mean of $\delta_j \sigma$ where σ is the standard deviation of X. This standard

deviation is assumed to remain invarient with the shift in the mean.

c. It is assumed that when the process has been disturbed by a given assignable cause, it is free from the occurrence of other assignable causes.

d. When the process is in control, the occurrence times of the various assignable causes are assumed to be independently exponentialy distributed with mean times $1/\lambda_j$ according to waiting time analysis (3) The time at which the process goes out of control is distributed with mean $1/\lambda$ where

$$\lambda = \sum \lambda_j \text{ ....(1)}$$

e. When the process goes out of control owing to the occurrence of cause A_j, the average length of time before the presence of A_j is detected, is made up of several parts.

Let P_j be the probability that a point falls outside the control limits after the occurrence of cause A_j. Since we assume that the distribution of $\overline{X}$ is normal,

$$P_j = \int_{-\infty}^{-k-\delta_j\sqrt{n}} \frac{e^{-z^2/2}}{\sqrt{2\pi}}\,dz + \int_{k-\delta_j\sqrt{n}}^{\infty} \frac{e^{-z^2/2}}{\sqrt{2\pi}}\,dz \text{ ....(2)}$$

Then the average number of samples that will be taken following the occurrence of cause A_j will be $(1/P_j)$. If the shift occurred immediately following the taking of a sample, the average time before a later sample would fall outside the limits would be $(1/P_j)h$.

f. Let α be the probability of a point falling outside the control limits, when the process is in the state of control. Then

$$\alpha = 2\int_{k}^{\infty}\frac{e^{-z^2/2}}{\sqrt{2\pi}}\,dz \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

It is assumed that the rate of production is sufficiently high, such that the probability of a change in the process average during the taking of a sample may be neglected.

g. The average time the process will be out of control before a sample point falls outside the control limits is:

$$\left(\frac{1}{P_j} - \frac{1}{2} + \frac{\lambda_j h}{12}\right) h$$

h. Considering the time required in taking and inspecting a sample and computing the results, the delay in plotting a point, will be given by (en+1), where e is the time per sampling and charting which is directly related to the sample size n, and l is a constant term independent of the sample size.

i. D_j, is the average time taken to look for an assignable cause A_j after a point has been found to fall outside the control limits.

j. G_j, is the average time required for adjustment and repair of the process to bring it back to a state of control, when a sample point falls outside the control limits.

k. The proportion of time, β , the process will be in control is :

$$\beta = \left(\frac{1}{\Sigma \lambda_j}\right) \Big/ \left(\frac{1}{\Sigma \lambda_j} + \Sigma B_j\right) \quad \ldots\ldots\ldots\ldots\ldots \text{[illegible]}$$

where,

$$B_j = a_j h + (en+l) + D_j + G_j \quad \text{and,}$$

$$a_j = \left(\frac{1}{P_j} - \frac{1}{2} + \frac{\lambda_j h}{12}\right)$$

Hence the time proportion during which the process is out of control, equals

$$\gamma = 1 - \beta \quad \ldots\ldots\ldots\ldots\ldots \text{[illegible]}$$

l. The average expected number of false alarms per unit time is :

$$\frac{\beta \alpha}{h}$$

m. Considering T to be the cost of looking for an assignable cause when none exists, the expected loss per unit time of operations due to false alarms will be :

$$\frac{\beta \alpha T}{h}$$

n. The average number of times per unit time the process actually goes out of control is:

$$\mathcal{E}_j = \frac{1}{(1/\Sigma \lambda_j) + \Sigma B_j}$$

o. If W_j is the average cost for finding an assignable cause A_j when it occures, the average cost per unit time will be $\mathcal{E}_j W_j$.

p. If the average cost of adjustment and repair of an assignable cause A_j is F_j, the average cost per unit time will be $\mathcal{E}_j F_j$.

q. When the process mean shifts from $\bar{\bar{X}}$ to $\bar{\bar{X}} \pm \delta_j \sigma$ due to occurrence of an assignable cause A_j, it will be assumed that the proportion of defective items produced will be increased. V_0 be the average income per unit time occurring from operation of the process at the standard level $\bar{\bar{X}}$, V_{1j} be the average income per unit time occurring from operation at the new level $\bar{\bar{X}} + \delta_j \sigma$, and V_{2j} be that income per unit time for the other new level $\bar{\bar{X}} - \delta_j \sigma$.

r. In the long run, the probability that a sample point will fall outside the upper control limit is assumed to equal the probability that a sample point will fall under the lower control limit.

s. The average income per unit time from the occurrence moment of an assignable cause A_j untill noticing a sample point falling outside the upper control limit is :

$$\frac{V_{1j}(ah + en + 1)}{2\left(\frac{1}{\lambda_j} + B_j\right)}$$

and the income per unit time untill a sample point is noticed fall under the lower control limit is :

$$\frac{V_{2j}\ (ah + en + 1)}{2\left(\frac{1}{\lambda_j} + B_j\right)}$$

t. The cost part per unit time for keeping the control chart, is assumed to be given by the simple linear function (b/h) + (cn/h), where b is the cost of sampling and charting per sample which is independent of the sample size, and c is the cost of measuring an item and of other control chart operations directly related to the size of the sample.

$$S = \frac{V_0 \sum \lambda_j U_j + \sum \lambda_j (M_{1j} + M_{2j})(u_j/2) + (\alpha T/h)}{1 + \sum \lambda_j B_j}$$

$$+ \frac{\sum \lambda_j (W_j + F_j)}{1 + \sum \lambda_j B_j} + \frac{b + cn}{h} \quad \ldots\ldots\ldots\ldots\ldots (7)$$

The average net income I will be a maximum when S is a minimum for certain values of n, h and k, since V_0 is independent of these three variables. Hence the optimum solution will be obtained by minimizing S.

For certain values of the 14 independent parameters (δ_j, λ_j, V_0, V_{1j}, V_{2j}, e, l, D_j, G_j, T, W_j, F_j, b and c), the model can be solved by the aid of the electronic computer[7].

REFERENCES :

1. Duncan, A.J., The economic design of $\overline{X}$ charts used to maintain current control of process", Journal of the American Statistical Association, Vol. 51, 1956, pp 228 - 42.

2. Duncan, A.J., "The economic design of $\overline{X}$ charts when there is a multiplicity of assignable causes", Journal of the American Statistical Association, Vol. 66, 1971, pp 107 - 21.

3. Feller, W., "An introduction to probability theory and its application", Vol. 1, John Wiley & Sons, 1950.

4. Goel, A.L., Jain, S.C., and Wu, S.M., "An algorithm for the determination of the economic design of $\overline{X}$ charts based on Duncan's model", Journal of the American Statistical Association, Vol. 63, 1968, pp 304 - 20.

5. Goel, A.L., and Wu, S.M., "Economically optimum design of cusum charts", Management Science, Vol. 19, 1973, pp 1271 - 82.

6. Sultan, T.I., and Roshdy, T.H., "An optimum expanded design of cumulative sum control charts", The Eleventh Annual Conference of Statistics and Scientific Computations, Special issue, Vol. 11, 1975, pp 1 - 12.

7. Sultan, T.I., and Roshdy, T.H., "An optimum expanded design of $\overline{X}$ control charts", The Second Annual Operations Research Conference, Special issue, Vol. 2, 1975, pp 212 - 24.

8. Taylor, H.M., "The economic design of cumulative sum control charts", Technometrics, Vol. 10, 1968, PP 479-88.