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# A COMPARISON OF FLEXURAL STRENGTH DESIGN TO WORKING STRESS DESIGN, FOR REINFORCED CONCRETE RECTANGULAR SECTIONS SUBJECT TO SIMPLE BENDING

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# مقارنة استعمال طريقة الحمل الاقصى بطريقة الجهد العامل لتصميم المقاطع الخرسانية المسلحة المستطيلة المعرضة للعزوم البسيطة ٠

بقلم

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**خلاصة المقالة :**

يقدم البحث العلاقة بين الطريقتين المستعملتين لتحديد ابعاد المقاطع المستطيلة من الخرسانة المسلحة والمعرضة للعزوم البسيطة ٠ أى بين استعمال طريقة الحمل الاقصى وبين طريقة الجهد العامل ٠ وسيكون بالامكان ( مع وجود بعض التحديدات ) باستعمال اى من الطريقتين للحصول على اجوبة متطابقة ٠ والطريقة تحدد معاملات الجهد التي يجب استعمالها بطريقة الجهد العامل للحصول على نفس مجموعة معاملات الاثقال المحددة عند استعمال طريقة الحمل الاقصى والعكس بالعكس ٠

ABSTRACT:

The paper investigates the relationship between the two standard methods of design that are used for dimensioning reinforced concrete rectangular sections subject to simple bending, namely, the flexural strength design method, and the working stress design method. It makes it possible (within certain limitations) to use either method to obtain identical results. It establishes the stress factors to be used in working stress design that would give results equivalent to a specified set of load factors that are adopted by the flexural strength design method, and vice versa.

## 1. INTRODUCTION:

The adoption of the Flexural Strength Design (F.S.D), previously known as the Ultimate Strength Design, and the Limit State Design methods for dimensioning of reinforced concrete sections, has created (at least in Iraq) a certain misunderstanding, especially by designers still using the old Working Stress Design (W.S.D), as to the suitability of these new methods of design, for conditions prevailing locally. Although the F.S.D method gives more economical designs, yet the load factors used in the flexural strength design as specified by the various codes, do not seem to satisfy those acquainted with the W.S.D, with the result, that the two schools of thought have yet to find a common ground on which to weigh their designs.

It is the aim of this paper to establish a practical design procedure that makes it possible, for a designer of a reinforced concrete section subject to simple bending using the new F.S.D. method, to obtain identical results as would be obtained by another designer using the old W.S.D method. Also to make it possible (within certain limitations) for a designer using the W.S.D method to design for a known load factor (as with the F.S.D method) and obtain the same answers as would be obtained by the F.S.D method.

It is then possible for all designers (whether those using the old W.S.D method or the new F.S.D method) to have a common basis, represented by the load factors used, to establish a well defined criteria whether the sections designed represent a suitable solution for the problem in question.

## 2. DESIGN METHODS:

Design methods for the F.S.D or the W.S.D are standard textbook problems, and are available in various publications 1, 2, 3, 4, 5 to 15. The equations to be used are given below for the two methods, using the A.C.I notations.

### a. Flexural Strength Design:

For an ultimate moment ($M_u$) acting on the section shown in figure (1), and let:

$$M_u = \psi M_{DL} + \eta M_{LL} \quad \text{.....................(1)}$$

where ( $\psi$ ) and ( $\eta$ ) are the load factors adopted. Then

$$M_u = \phi \rho f_y b d^2 (1 - 0.59 \rho \frac{f_y}{f'_c}) \quad \text{.....................(2)}$$

from which the depth of the section (d) is determined

$$d = k_1 \sqrt{\frac{M_u}{b}} \quad \text{.....................(3)}$$

where $k_1 = \sqrt{\dfrac{1000}{\phi \rho f_y (1 - 0.59 \rho \frac{f_y}{f'_c})}}$ .....................(4)

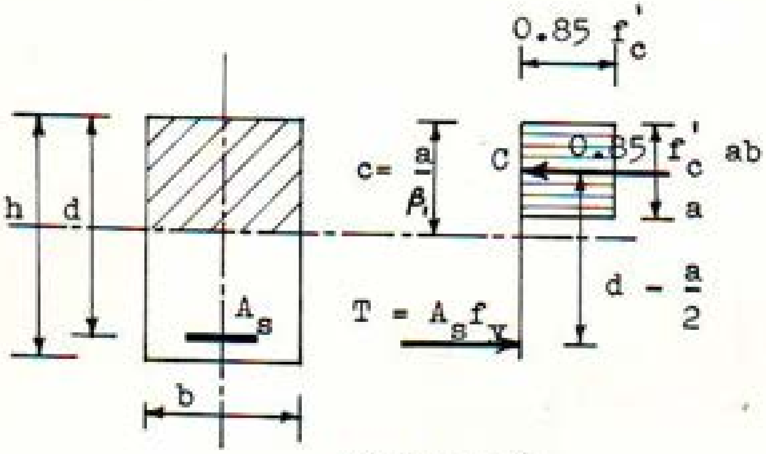



Figure (1)

provide that ($M_u$) is given in (kip inches). Then the area of steel ($A_s$) is given in terms of the steel ratio ($\rho$) from:

$A_s = \rho\ b\ d$ ..........................................(5)

The two equations (3) and (5) above, can be utilized for the design of reinforced concrete sections subject to bending[5] using the charts prepared by the author.

Appendix (2) gives the program prepared by the author using the hand electronic computer (HP 65) for the above method, which can also be used for designing sections with double reinforcement.

b. Working Stress Design:

For a service load moment ($M_e$) acting on the section shown in figure (2), where:

$M_e = M_{DL} + M_{LL}$ ....................................(6)

and if

$$n = \frac{E_s}{E_c} = \frac{29 \times 10^6}{57000 \sqrt{f'_c}} = \frac{508.77}{\sqrt{f'_c}} \qquad \ldots\ldots(7)$$

then from the stress distribution

$$k = \frac{f_c}{f_c + \frac{f_s}{n}} = \frac{1}{1 + \frac{f_s}{n\, f_c}} \qquad \ldots\ldots(8)$$

where $f_c = 0.45\, f'_c$ ......(9)

and $f_s = \gamma f_y$ ......(10)

where ( $\gamma$ ) is the stress factor for steel.

Also as $j = 1 - \frac{k}{3}$ ......(11)

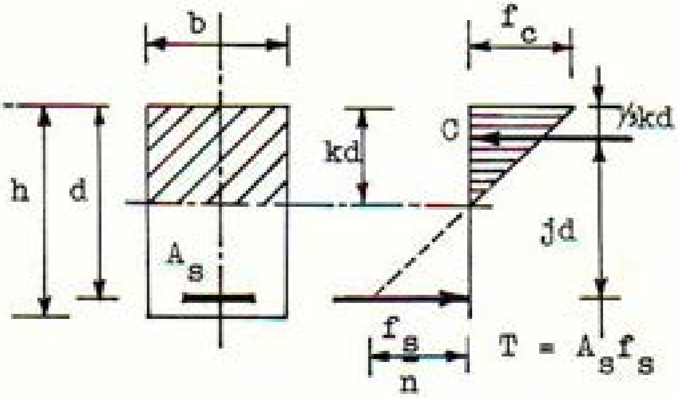



Figure (2)

and if $\rho = \frac{A_s}{b\, d}$ ......(12)

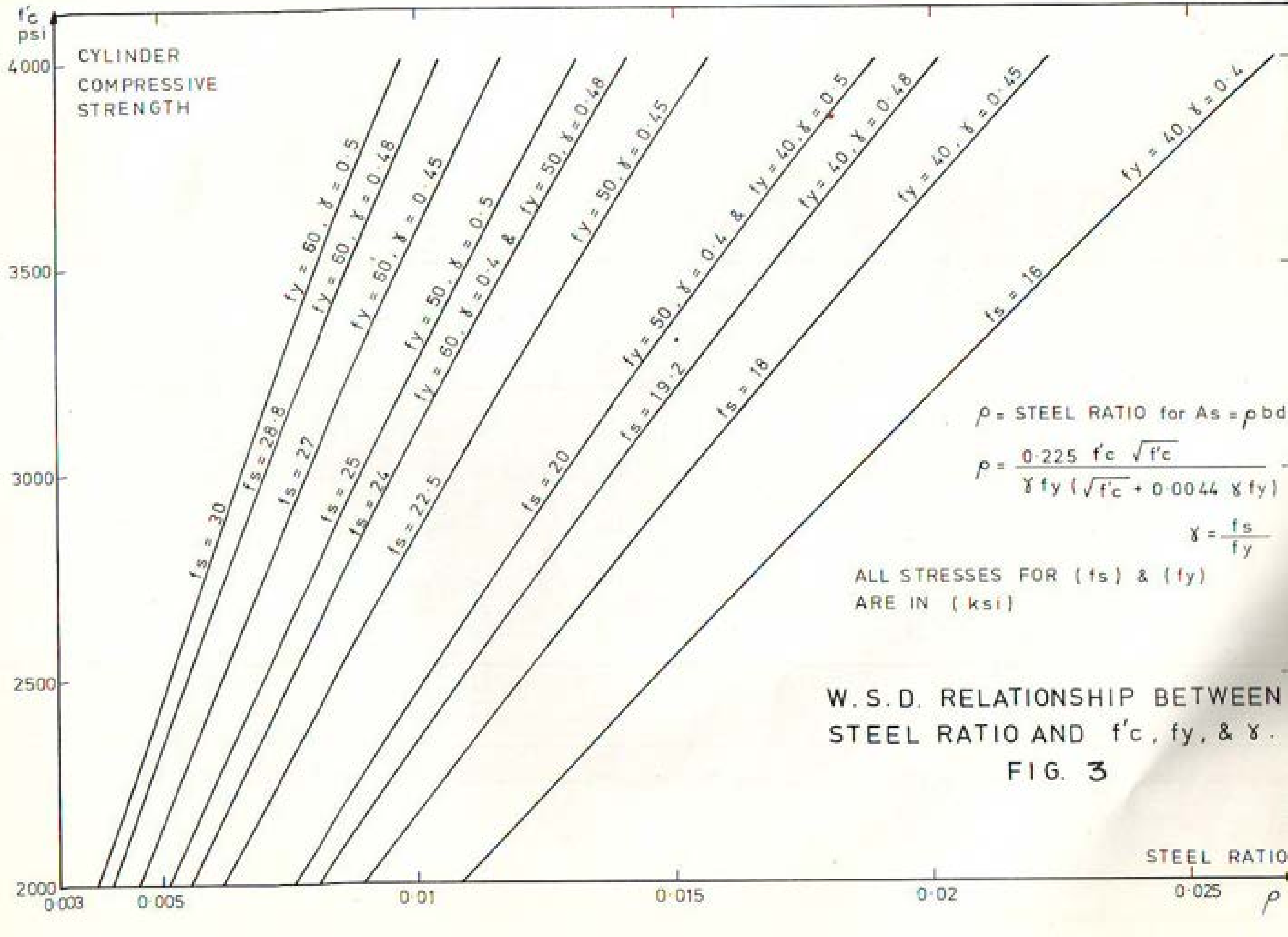
Then the steel ratio ( $\rho$ ) can be determined from equating

C = T

Therefore $\rho = \frac{k}{2} \times \frac{f_c}{f_s}$ ......(13)

The moment ($M_e$) is given by:

$M_e = ½\, f_c\, k\, j\, b\, d^2$ ......(14)




W.S.D. RELATIONSHIP BETWEEN STEEL RATIO AND f'c, fy, & γ.

FIG. 3

from which the depth (d) can be obtained from the equation :

$$d = k_{1w}\sqrt{\frac{M_e}{b}} \quad \text{.........................................(15)}$$

$$\text{where } k_{1w} = \sqrt{\frac{2000}{f_c\ k\ j}} \quad \text{...............................(16)}$$

provided that ($M_e$) is given in (kip inches).

For a balanced design, i.e. for the concrete to reach a full stress of ($f_c = 0.45\ f'_c$), and for the steel to reach a full stress of ($f_s = \gamma\ f_y$), then the value of the steel ratio ( $\rho$ ) can be determined from equation (13) by substituting the values of (n) from (7), (k) from (8), ($f_c$) from (9), and ($f_s$) from (10).

$$\text{Therefore } \rho = \frac{0.225\ f'_c\ \sqrt{f'_c}}{f'_y(\sqrt{f'_c} + 0.0044\ \gamma\ f_y)} \quad \text{............(17)}$$

from which the area of steel ($A_s$) can be determined

$$A_s = \rho\ b\ d \quad \text{.........................................(18)}$$

The curves shown in figure (3), give values of ( $\rho$ ) for various combinations of ($f_y$), ($f_s$), ( $\gamma$ ), and ($f'_c$).

Equations (15) and (16) can be used to determine the depth (d) of the section, while equations (17) and (18) are used to determine the areas of the steel reinforcement for a balanced design.

Appendix (3) gives the program prepared by the author using the hand electronic computer (HP 65) for using the working stress design method for dimensioning a singly reinforced concrete rectangular section subject to simple bending at balanced conditions.

## 3. STRESS FACTORS AND LOAD FACTORS:

There are basic differences between the two standard methods of design for reinforced concrete sections. These differences are given below :

a. The W.S.D method uses a linear stress distribution with a maximum stress for concrete in compression ($f_c$) which decreases linearly to zero at the neutral axis, while the F.S.D method adopts an equivalent rectangular stress block whose ordinate for concrete stress is ($0.85\ f'_c$) and with a depth (a) which is less than the depth of the neutral axis (c).

b. The W.S.D adopts a working stress for steel ($f_s$), where ($f_s = \gamma f_y$), using a stress factor for steel ( $\gamma$ ). This steel stress factor is always less than (0.5), and varies usually between (0.4 to 0.5). For concrete a constant stress factor of (0.45) is used, and therefore ($f_c$) is always taken as equal to ($0.45\ f'_c$).

c. The F.S.D method uses a load factor ( $\psi$ ) for the dead load, and a load factor ( $\eta$ ) for the live load, together with another factor $\phi = 0.9$. The steel yield stress ($f_y$), and the concrete compressive cylinder strength at (28) days ($f'_c$) being the basis of the analysis.

d. The W.S.D method is related to the working stresses under prevailing service loads, while the F.S.D method is related to the ultimate stresses that occur in the concrete and steel at ultimate load conditions. The relationship between these two sets of conditions is not linear, and varies according to the stress factors, and the load factors used.

e. The range of the load factors used for the F.S.D method can vary according to the prevailing conditions. This is not possible using the standard W.S.D method. It will be shown later that due to the stress factors chosen (0.45 for the concrete, and between 0.4 to 0.5 for the steel), then the low load factors such as (1.4) for the dead load, and (1.7) for the live load cannot be obtained.

## 4. THE RELATIONSHIP BETWEEN W.S.D AND F.S.D:

To find the relationship between the W.S.D and F.S.D methods, equations (3) and (15) for an equal depth (d) to be obtained by both methods:

$$d = k_1 \sqrt{\frac{M_u}{b}} = k_{1w} \sqrt{\frac{M_e}{b}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (19)$$

Then substituting values of ($k_1$) from (4), ($M_u$) from (1), ($k_{1w}$) from (16) and ($M_e$) from (6):

$$\sqrt{\frac{1000}{\phi\, \rho\, f_y\, (1 - 0.59\, \rho\, \frac{f_y}{f'_c})}} \times \sqrt{\frac{\psi M_{DL} + \eta M_{LL}}{b}}$$

$$= \sqrt{\frac{2000}{f_c\, k\, j}} \times \sqrt{\frac{M_{DL} + M_{LL}}{b}} \quad \ldots\ldots\ldots\ldots\ldots (19')$$

Then substituting the value of ( o ) from equation (17), (k) from (8), (9) and (10), (j) from (11) and putting

$$\frac{M_{DL}}{M_{LL}} = \xi \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (20)$$

and let

$$\frac{\psi M_{DL} + \eta M_{LL}}{M_{DL} + M_{LL}} = \frac{\psi \xi + \eta}{\xi + 1} = R \quad \ldots\ldots\ldots (21)$$

the following equation can be obtained:

$$R = \frac{\psi \xi + \eta}{\xi + 1} = \frac{\left(1 - 0.1328 \times \frac{\sqrt{f'_c}}{\gamma(\sqrt{f'_c} + 0.0044\,\gamma f_y)}\right)\left(228.9465\, f'_c + \gamma f_y \sqrt{f'_c}\right)^2}{254.385\, \gamma (\sqrt{f'_c} + 0.0044\, \gamma f_y)\sqrt{f'_c}\,(152.631\, f'_c + \gamma f_y \sqrt{f'_c})} \quad \ldots (22)$$

The value of (R) is, therefore a function of the variables ($f_y$, $f'_c$ and $\gamma$). When the values of (R) are calculated for various combinations of stresses for a range of ( $\gamma$ ) varying between (0.5) to (0.4), then Table (1) is obtained.

Table (1) VALUES OF (R)

| $f_y$ psi | $f'_c$ psi | For values of $\gamma = f_s / f_y$ 0.5 | 0.48 | 0.45 | 0.4 |
|---|---|---|---|---|---|
| 40000 | 2000 | 1.84 | 1.91 | 2.02 | 2.24 |
| | 2500 | 1.84 | 1.91 | 2.03 | 2.24 |
| | 3000 | 1.85 | 1.92 | 2.03 | 2.25 |
| | 3500 | 1.85 | 1.92 | 2.03 | 2.25 |
| | 4000 | 1.85 | 1.92 | 2.03 | 2.25 |
| 50000 | 2000 | 1.83 | 1.90 | 2.02 | 2.24 |
| | 2500 | 1.84 | 1.91 | 2.02 | 2.24 |
| | 3000 | 1.84 | 1.91 | 2.02 | 2.24 |
| | 3500 | 1.84 | 1.91 | 2.02 | 2.24 |
| | 4000 | 1.84 | 1.91 | 2.03 | 2.24 |
| 60000 | 2000 | 1.83 | 1.90 | 2.02 | 2.24 |
| | 2500 | 1.83 | 1.90 | 2.02 | 2.24 |
| | 3000 | 1.83 | 1.90 | 2.02 | 2.24 |
| | 3500 | 1.84 | 1.90 | 2.02 | 2.24 |
| | 4000 | 1.84 | 1.91 | 2.02 | 2.24 |

Table (1) exhibits an interesting characteristic. It can be seen that the values of (R) can be approximated to be a function of ( $\gamma$ ) only, irrespective of the values of ($f_y$) and ($f'_c$) for the ranges studied. Table (2) gives these values :

Table (2) $R = f(\gamma)$

| $\gamma$ | 0.5 | 0.48 | 0.45 | 0.4 |
|---|---|---|---|---|
| R | 1.84 | 1.91 | 2.02 | 2.24 |

If a linear relationship is assumed for $R = f(\gamma)$, then the following straight line formula can be obtained:

$$R = 3.84 - 4.01\gamma \quad \text{.........................(23)}$$

On the other hand, if a parabolic relationship is assumed, then the following parabolic formula is obtained:

$$R = 5.37 - 10.91\gamma + 7.69\gamma^2 \quad \text{..............(24)}$$

This relationship is shown on the curve indicated $R = f(\gamma)$ in figure (4). The same figure is also used to show the relationship between the various load factors ($\psi$) and ($\eta$), the dead to live load ratio ($\xi$) and the value of (R).

Equations (23) and (24) give the relationship between the load factor coefficients that are used in the F.S.D method, to the steel stress coefficients ($\gamma$) which is used in the W.S.D method.

It then become possible to use either the F.S.D method to obtain results identical with the W.S.D method for a given steel stress coefficients, or vice versa (but within certain limitations), to use the W.S.D method and a defined set of load factors to obtain results identical to those that can be obtained by using the F.S.D method.

$R = 3 \cdot 37 - 10 \cdot 91\,\gamma + 7 \cdot 69\,\gamma^2$ (PARABOLIC FORMULA)

or $R = 3 \cdot 84 - 4 \cdot 01\,\gamma$ (STRAIGHT LINE FORMULA)

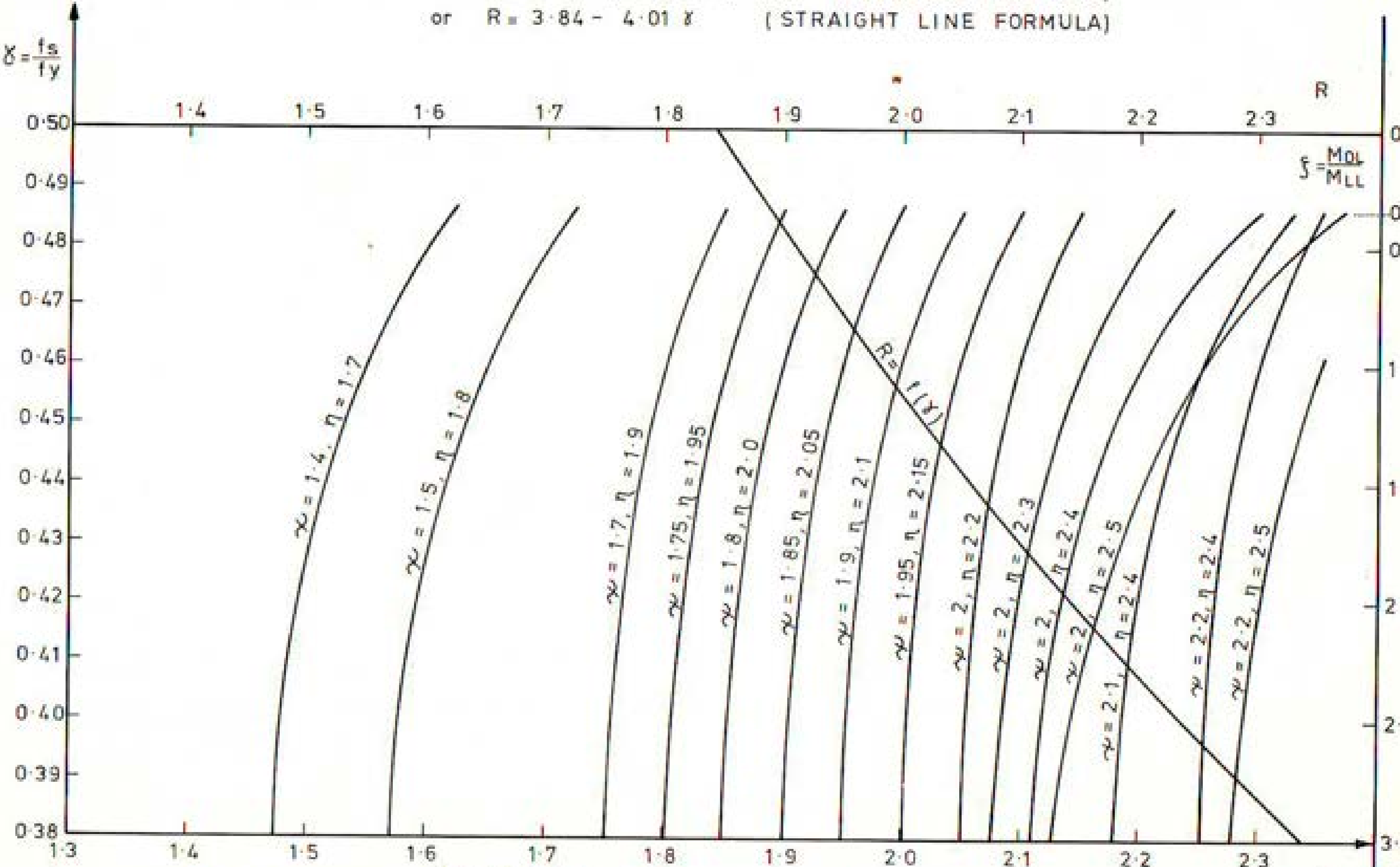



FIG. 4

MOMENTS, STEEL STRESSES, AND LOAD FACTORS RELATIONSHIPS

## 5. PROCEDURE FOR DESIGN:

### a. The Use of F.S.D method to Obtain Results Identical to W.S.D method.

In such a case, it is assumed that the designer using the W.S.D method has established the following quantities : ($f_s$) for a known ($f_y$), ($f'_c$), ($M_{DL}$), ($M_{LL}$) and the breadth of the section (b).

Then the procedure to be used by the designer using the F.S.D method is as follows :

i. Obtain $\xi = \frac{M_{DL}}{M_{LL}}$

ii. Find $\gamma = \frac{f_s}{f_y}$

iii. Using curves shown in figure (4), for the given ( $\gamma$ ), then from curve R = f ( $\gamma$ ), the value of (R) is determined. It is also possible to obtain the value of (R) by substituting value of ( $\gamma$ ) either in equation (23) or (24).

iv. Using also figure (4), for the value of (R) found, and for the given ( $\xi$ ), the load factors ( $\psi$ ) and ( $\eta$ ) can be obtained and directly read from the figure.

v. The value of $M_u = \psi M_{DL} + \eta M_{LL}$ can now be calculated.

vi. Using figure (3), or equation (17), and substituting values of ($f'_c$), ( $\gamma$ ) and ($f_y$), then the value of the steel ratio ( $\rho$ ), representing the balanced steel ratio for the working stress design method, can be obtained.

vii. It is now possible to use any method for F.S.D to obtain the depth of the section (d), and the area of the steel reinforcement ($A_s$). Use can be made by adopting the procedure suggested by the Author[5], where ($k_1$) is determined for the given ($f_y$), ($f'_c$) and ($\rho$), then:

$$d = k_1 \sqrt{\frac{M_u}{b}} \quad \text{and} \quad A_s = \rho\, b\, d$$

A typical example is solved using the above procedure in Appendix (1).

**b. The Use of The W.S.D Method to Obtain Results Identical to F.S.D Method.**

In such a case, it is assumed that the designer using the W.S.D method is given the load factors that are to be used.

There is a certain limitation which confronts the user of the W.S.D method in this respect. This is clear from figure (4), where it is noted that the range of the curve $R = f(\gamma)$ for values of ( $\gamma$ ) between (0.4) and (0.5) lies only in the region of load factors from ($\psi$ =1.8) and ( $\eta$ =2) to ($\psi$ =2.2) and ( $\eta$ =2.5) only, i.e. a designer using the W.S.D method can only use load factors in this range only.

It is assumed that the values of ( $\psi$ ) and ( $\eta$ ), ($f_y$), ($f'_c$), ($M_{DL}$), ($M_{LL}$) and (b) are given. The procedure is as follows :

i. Obtain $\xi = \frac{M_{DL}}{M_{LL}}$

ii. For the value of ( $\xi$ ) obtained, and the given values of ( $\psi$ ) and ( $\eta$ ), the value of (R) is either obtained from figure (4), or calculated from equation (21):

$$R = \frac{\psi \xi + \eta}{\xi + 1}$$

iii. From figure (4), for the value of (R) obtained and from curve R = f( $\gamma$ ), the value of ( $\gamma$ ) can be obtained It is also possible to obtain ( $\gamma$ ) from either equation (23) or from equation (24) for the value of (R) as calculated in (ii) above.

iv. The value of ($f_s$) to be used in the W.S.D method is now obtained from $f_s = \gamma f_y$

v. The section can then be designed as a balanced section for the given ($M_{DL}$), ($M_{LL}$), ($f_s$), ($f'_c$) and (b) by any of the standard methods of design using the W.S.D method.

vi. When comparison is made to the F.S.D method, it is necessary that the designer using the F.S.D method to use a steel ratio ( $\rho$ ) as would be obtained from equation (17), or from the curves shown in figure (3)

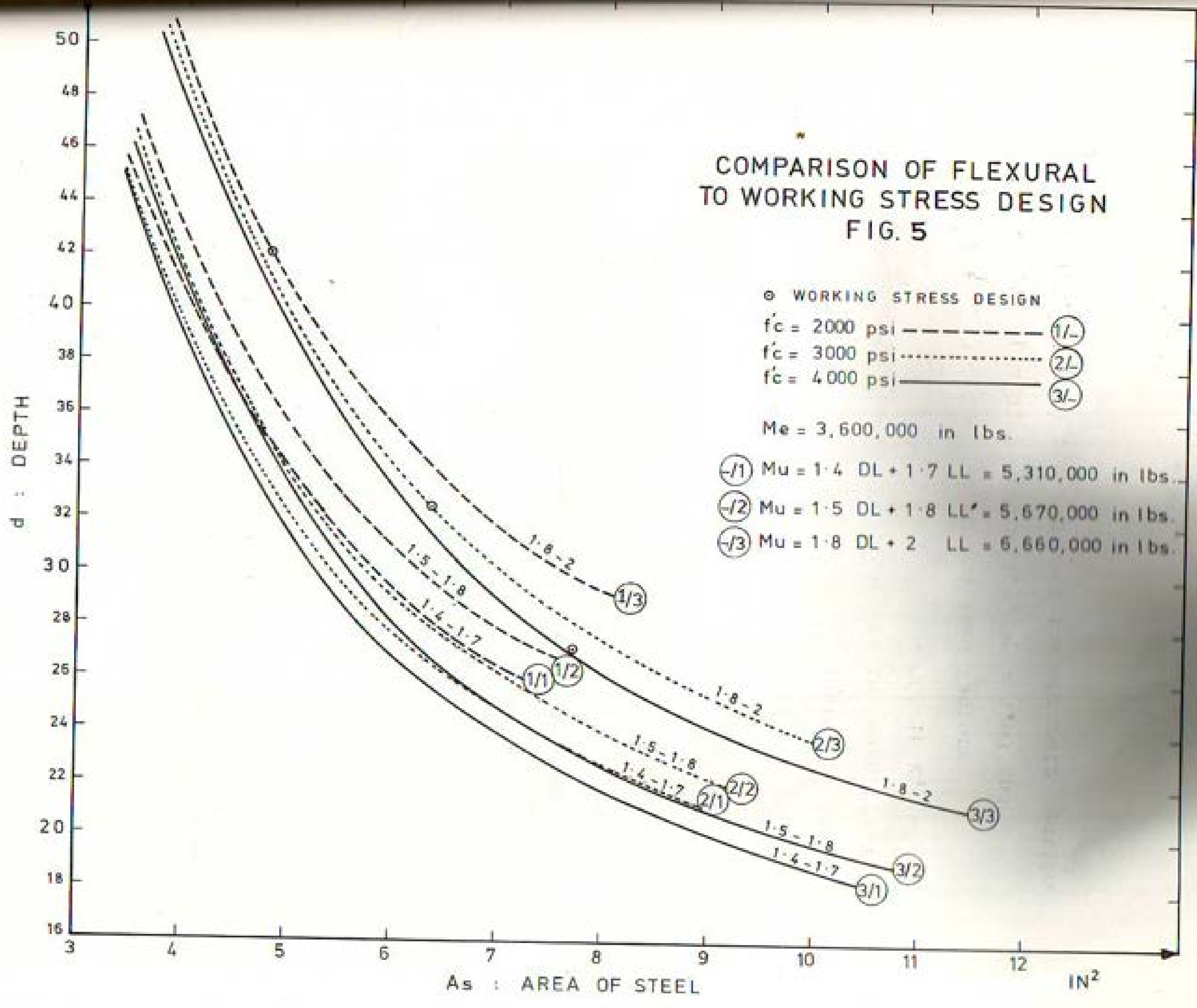
A typical example using the procedure given above is given in Appendix (1).

## 6. COMPARISON OF F.S.D. TO W.S.D METHODS :

a. The F.S.D method gives a more realistic explanation of the capacity of reinforced concrete sections to the loads to which they may be subjected to.

b. It is possible to vary the load factors in the F.S.D method in such a range, that it can cover all the various conditions of the concrete construction industry. e.g. it can take into consideration the high quality control of concrete production, by decreasing the load factors used to the minimum values that are specified internationally. This is not possible by using the W.S.D method which has a minimum possible load factor of only ($\psi$ =1.8 and $\eta$ = 2) for a maximum ( $\gamma$ ) = (0.5).

c. The Author has prepared a very large range of comparative designs by using the two methods. Figure (5) shows a typical comparison. It considers the case for : $f_y$ = 40 ksi, $M_{DL}$ = 2700 in k, $M_{LL}$ = 900 in k, i.e. ( $\xi$ = 3). Then using the load factors ($\psi$ = 1.4, $\eta$ = 1.7), ($\psi$ = 1.5, $\eta$ = 1.8) and ($\psi$ = 1.8, $\eta$ = 2) the depth of the concrete section


COMPARISON OF FLEXURAL
TO WORKING STRESS DESIGN
FIG. 5
WORKING STRESS DESIGN
f'c = 2000 psi (1/–)
f'c = 3000 psi (2/–)
f'c = 4000 psi (3/–)
Me = 3,600,000 in lbs.
(–/1) Mu = 1·4 DL + 1·7 LL = 5,310,000 in lbs.
(–/2) Mu = 1·5 DL + 1·8 LL' = 5,670,000 in lbs.
(–/3) Mu = 1·8 DL + 2 LL = 6,660,000 in lbs.
1·8 – 2
1·5 – 1·8
1·4 – 1·7
1/1
1/2
1/3
2/1
2/2
2/3
3/1
3/2
3/3
d : DEPTH
As : AREA OF STEEL
IN²
16
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22
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and the area of the steel are calculated for a breadth ($b = 15"$) for various steel ratios ( $\rho$ ) by the F.S.D method as given by the A.C.I code, for the three cases of $f'_c$ = 2000, 3000 and 4000 psi. It is only when ( $\psi = 1.8$, $\eta = 2$) that the W.S.D method at balanced conditions can give identical results with the F.S.D. This is shown in figure (5).

It is clear from figure (5) that for the same depth for a concrete section, then the design by the F.S.D method would give steel areas about (25 per cent) less than by using the W.S.D method. This is the case when taking the load factors ( $\psi = 1.4$, and $\eta = 1.7$) and comparing it to a balanced working stress design using a steel stress factor ( $\gamma = 0.5$), which corresponds to load factors of about ($\psi = 1.8$ and $\eta = 2.0$).

e. The working stress design was based originally only on the steel and concrete stress factors, irrespective of the load factors. Therefore, the W.S.D method was unable, except by using the procedure suggested by the Author, to correlate with the actual load factors at which the section can reach its failure strength.

## 7. CONCLUSIONS :

a. It is possible to use the flexural strength design method to obtain identical results as would be obtained by using the working stress design method, and vice versa, it is possible (except for low load factors) to use the working stress design method to obtain identical results as would be obtained by using the flexural strength design method.

b. The load factors, the ratio of the dead to the live loads, can all be directly related to the steel stress factor ( $\gamma$ ) which is used in the working stress design method.

c. The use of the flexural strength design method give more economical and realistic designs.

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## REFRENCES :

1. G. Winter, & A.H. Nilson, "Design of Concrete Structures", 8th Edition 1973, McGraw-Hill.

2. P.M. Ferguson, "Reinforced Concrete Fundamentals", 3rd Edition 1973, Wiley.

3. A.C.I Building Code (318 - 71).

4. Reinforced Concrete Design Handbook - Working Stress Method, A.C.I SP - 3 , 1965.

5. El-Katib, M.T., "Flexural Strength Design Charts for Reinforced Concrete Beams Subject to Bending or to an Axial Force With Large Eccentricity With or Without Comparison Reinforcement" Al-Muhandis - Iraqi Society of Engineers, Serial 59, July 1976.

6. A.C.I Design Handbook, SP - 17 , (1973).

7. Unesco, "Reinforced Concrete - An International Manual" Butterworths, 1971.

8. B.S.I., "The Structural Use of Concrete" CP 110, Parts: 1, 2 & 3, (1972).

9. C.E. Reynolds, & J.C. Steedman "Reinforced Concrete Designer's Handbook", 8th Edition 1974, Cement and Concrete Association.

10. P.E. Regan & C.W. YU, "Limit State Design of Structural Concrete", 1973, Chatto & Windus.

11. R.P. Johnson, "Structural Concrete", 1967, McGraw-Hill.

12. F.K. Kong, & R.H. Evans "Reinforced and Prestressed Concrete", 1975 Nelson.

13. A.L.L. Baker, "Limit - State Design of Reinforced Concrete", 1970 C. & C.A.

14. R. Park & T. Paulay, "Reinforced Concrete Structures", 1975, Wiley.

15. B.P. Hughes, "Limit State Theory for Reinforced Concrete", 1971, PITMAN.

NOTATIONS:

$a$ = The depth of the rectangular stress block.

$A_s$ = The area of the tensile steel reinforcement.

$b$ = The breadth of the section.

$c$ = The depth of the neutral axis.

$d$ = The distance from the centroid of the tensile steel reinforcement to the top compressive fibre. The effective depth of section.

$f_c$ = $f'_c$ x 0.45 psi, concrete stress at service load.

$f_c$ = Concrete cylinder 12" by 6", compressive strength at 28 days in (psi).

$f_s$ = Steel stress at service load = $\gamma f_y$ psi.

$fy$ = Yield stress of steel psi.

$h$ = The total depth of the concrete section.

$j$ = Ratio of distance (jd) between resultants of compressive and tensile stresses to effective depth.

$k_1$ = A constant to determine the depth, using the flexural strength design.

$k_{1w}$ = A constant to determine the depth, using the working stress design.

$k$ = Ratio of distance (k d) between extreme fiber and neutral axis to effective depth.

$M_e$ = Moment at service loads.

$M_u$ = Ultimate moment.

$M_{DL}$ = Moment due to dead load.

$M_{LL}$ = Moment due to live load.

R = A ratio = $\frac{\psi \xi + \eta}{\xi + 1} = \frac{\psi M_{DL} + \eta M_{LL}}{M_{DL} + M_{LL}}$

$E_c$ = Modulus of Elasticity of Concrete = 57000 $f'_c$ psi.

$E_s$ = Modulus of Elasticity of Steel = 29 $10^6$ psi.

$\psi$ = Load factor for dead load.

$\eta$ = Load factor for live load.

$\xi$ = Ratio of dead to live loads.

$\gamma$ = Stress factor for steel = $\frac{f_s}{f_y}$.

$\rho$ = The ratio of the area of the tensile reinforcement to effective area of concrete.

$\phi$ = Moment capacity factor, = 0.9 for simple bending.

C = Resultant of compressive stresses in flexural computations.

T = Resultants of tensile stresses.

## APPENDIX: 1.

### Example (1):

A designer uses the Working Stress Method for designing a reinforced concrete section. If the stresses, moments, and section characteristics used are as follows :

$f_s$ = 27000 psi for $f_y$ = 60000 psi, $f'_c$ = 3000 psi,

$M_{DL}$ = 942 in k. $M_{LL}$ = 554 in k. b = 12 in.

It is required to design this section by the flexural design method to give the same results, as will be obtained by the use of the working stress design method, and also to find the load factors that would result in such a case.

Solution :

1. Obtain $\xi = \frac{M_{DL}}{M_{LL}} = \frac{942}{554} = 1.7$

2. Find $\gamma = \frac{f_s}{f_y} = \frac{27000}{60000} = 0.45$

3. From figure (4), for $\gamma = 0.45$, from curve $R = f(\gamma)$

   find R = 2.02

Note: The value of (R) can also be obtained by substituting in the straight line formula (equation : 23) or in the parabolic formula (equation : 24).

4. From figure (4), at $\xi = 1.7$, and R = 2.02, the load factors $\psi = 1.95$, and $\eta = 2.15$ are obtained.

These load factors correspond to the actual load factors corresponding to the working stress design.

The value of $(M_u)$ is calculated from $M_u = \psi M_{DL} + \eta M_{LL}$

$$M_u = 1.95 \times 942 + 2.15 \times 554 = 3028 \text{ in.k}$$

From figure (3), for $f_y = 60$ ksi, $\gamma = 0.45$, i.e. $f_s = 27$ ksi, and $f'_c = 3$ ksi, the value of $(\rho)$ is obtained $\rho = 0.0079$. As an alternative, the value of $(\rho)$ can be determined by substituting in equation (17).

. The design of the section by the flexural strength design method can then be carried out for : $f_y$ 60 ksi, $f'_c = 3$ ksi, $M_u = 3028$ in.k., $\rho = 0.0079$, $b = 12"$. Using charts prepared by the Author(5), use chart No. (13), $k_1 = 1.6088$

$$d = k_1\sqrt{\frac{M_u}{b}} = 1.6088 \times \sqrt{\frac{3028}{12}} = 25.56 \text{ inches.}$$

$$A_s = 0.0079 \times 12 \times 25.56 = 2.42 \text{ sq. in.}$$

8. The corresponding calculations by the designer using the working stress method for a balanced design would be as follows :

$$M_e = 942 + 554 = 1496 \text{ in.k.}$$

$$n = \frac{508.77}{\sqrt{f'_c}} = \frac{508.77}{\sqrt{3000}} = 9.289$$

at $f_c = 0.45 f'_c$

then $k = \dfrac{1}{1 + \dfrac{f_s}{n\, f_c}} = \dfrac{1}{1 + \dfrac{27000}{9.289 \times 0.45 \times 3000}}$

Therefore k = 0.3171

The value of $j = 1 - \frac{1}{3}k = 1 - \frac{1}{3} \times 0.3171 = 0.8943$

$\rho = \dfrac{k\, f_s}{2\, f_s} = \dfrac{0.3171 \times 0.45 \times 3000}{2 \times 27000} = 0.0079$ as before.

$K = f_s\, \rho\, j = 27000 \times 0.0079 \times 0.8943 = 191.44$

$d = \sqrt{\dfrac{M_e}{Kb}} = \sqrt{\dfrac{1496000}{191.44 \times 12}} = 25.52$ inches.

$A_s = 0.0079 \times 12 \times 25.52 = 2.42$ inches sq.

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Example (2):

A designer using the working stress design method, wishes to design a section that should have a load factor of (2) for the dead load, and a factor (2.3) for the live load. It is required to determine the value of ($f_s$) that must be used to accomplish this. A comparison of the design by the W.S.D and F.S.D methods is also required.

Given : $f_y = 40$ ksi, $f'_c = 2000$ psi, $M_{DL} = 746$ in.k

$M_{LL} = 538$ in.k, b = 12 inches.

Solution:

1. Obtain $\xi = \frac{M_{DL}}{M_{LL}} = \frac{764}{538} = 1.42$

2. Obtain $R = \frac{\psi\xi + \eta}{\xi + 1} = \frac{2 \times 1.42 + 2.3}{1.42 + 1} = 2.124$

   Note: It is possible to read directly from figure (4) the value of (R), by reading ( $\xi$ = 1.42), then from the curve of load factors ($\psi$ = 2, $\eta$ = 2.3), (R) is read to be (2.124).

3. From curve R = f ( $\gamma$ ), in figure (4), and for (R=2.124) the value of ( $\gamma$ ) is read to be (0.424). Alternatively, it can be obtained from the straight line formula $R = 3.84 - 4.01\gamma$ . Therefore $2.124 = 3.84 - 4.01\gamma$ from which $\gamma = 0.428$, Alternatively the parabolic equation $R = 5.37 - 10.91\gamma + 7.69\gamma^2$ gives, by solving this quadratic equation ( $\gamma$ = 0.424). Therefore $f_s = 0.424 \times 40000 = 16960$ psi.

4. The section can now be designed by the working stress design method. At $f_s = 16960$ psi, $f'_c = 2000$ psi, $f_c = 0.45 f'_c$. Therefore $f_c = 0.45 \times 2000 = 900$ psi, b = 12 in. $M_e = 764 + 1302$ in. k.

   $n = \frac{508.77}{\sqrt{3000}} = 11.376$

   (k) from equation (8) $= \dfrac{1}{1 + \dfrac{16960}{11.376 \times 900}} = 0.3764$

   (j) from equation (11) $= 1 - \frac{1}{3} \times 0.3764 = 0.8745$

   ( $\rho$ ) from equation (13) $= \frac{0.3764 \times 900}{2 \times 16960} = 0.01$

   $K = f_s \rho j = 16960 \times 0.01 \times 0.8745 = 148.14$

$$d = \sqrt{\frac{M_e}{K\ b}} = \sqrt{\frac{1302000}{148.14 \times 12}} = 27.06 \text{ inches.}$$

$$A_s = \rho\ b\ d = 0.01 \times 12 \times 27.06 = 3.24 \text{ sq. in.}$$

5. Comparing the above solution, with that carried out by the F.S.D method:

$$M_u = \psi M_{DL} + \eta M_{LL} = 2 \times 764 + 2.3 \times 538 = 2765 \text{ in.k}$$

at $\xi = \frac{764}{538} = 1.42$

from figure (4), at (R) = 2.125 for ( $\xi$ = 1.42) and the load factors curve for ( $\psi$ = 2, and $\eta$ = 2.3).

For R = 2.125, and from curve R = f( $\gamma$ ), the value of ( $\gamma$ ) is read to be (0.424). Substituting in equation (17), the value of ( $\rho$ ) = 0.0099, alternatively from figure (3), $\rho$ = 0.01 by interpolation.

Using chart No. 2 prepared by the author[5],

$k_1 = 1.7791$

$$d = k_1 \sqrt{\frac{M_u}{b}} = 1.7791 \times \sqrt{\frac{2765}{12}} = 27.006 \text{ inches.}$$

$$A_s = \rho\ b\ d = 0.01 \times 12 \times 27 = 3.22 \text{ sq. in.}$$

i.e. identical results.

## APPENDIX: 2.

Program for designing reinforced concrete sections subject to bending, with single or double reinforcement.

Use hand electronic computer HP 65.

Equation used:

$$k_1 = \sqrt{\frac{1000}{\phi\,\rho\,f_y\{(1-0.75\alpha)\,[1-0.59\rho\,(1-075\alpha)\,\frac{f_y}{f'_c}] + 0.675\}}}$$

$$d = k_1 \sqrt{\frac{M_u}{b}} \quad , \quad A_s = \rho\, b\, d \quad , \quad A'_s = \alpha\,\rho\, b\, d$$

Program:

| | | | | |
|---|---|---|---|---|
| f | RCL 4 | ENTER | 1/x | RTN |
| CLX | x | RCL 4 | 1 | LBL |
| LBL | - | x | 0 | D |
| A | RCL 1 | - | 0 | RCL 6 |
| DSP | x | x | 0 | ENTER |
| . | RCL 2 | . | x | RCL 8 |
| 4 | ÷ | 6 | f | x |
| STO 1 | RCL 3 | 7 | √x | RCL 3 |
| STO 2 | x | 5 | STO 7 | x |
| STO 3 | . | ENTER | RTN | R/S |
| STO 4 | 5 | RCL 4 | LBL | RCL 4 |
| STO 5 | 9 | x | C | x |
| STO 6 | x | + | RCL 5 | R/S |
| RTN | CHS | RCL 1 | ENTER | |
| LBL | 1 | x | RCL 6 | |
| B | + | RCL 3 | ÷ | |
| 1 | 1 | x | f | |
| ENTER | ENTER | . | √x | |
| . | . | 9 | RCL 7 | |
| 7 | 7 | x | x | |
| 5 | 5 | g | STO 8 | |
| ENTER | | | | |

$$d = \sqrt{\frac{M_e}{K\ b}} = \sqrt{\frac{1302000}{148.14 \times 12}} = 27.06 \text{ inches.}$$

$A_s = \rho\ b\ d = 0.01 \times 12 \times 27.06 = 3.24$ sq. in.

5. Comparing the above solution, with that carried out by the F.S.D method:

$M_u = \psi M_{DL} + \eta M_{LL} = 2 \times 764 + 2.3 \times 538 = 2765$ in.k

at $\xi = \frac{764}{538} = 1.42$

from figure (4), at (R) = 2.125 for ( $\xi$ = 1.42) and the load factors curve for ( $\psi$ = 2, and $\eta$ = 2.3).

For R = 2.125, and from curve R = f( $\gamma$ ), the value of ( $\gamma$ ) is read to be (0.424). Substituting in equation (17), the value of ( $\rho$ ) = 0.0099, alternatively from figure (3), $\rho$ = 0.01 by interpolation.

Using chart No. 2 prepared by the Author(5),

$k_1 = 1.7791$

$$d = k_1 \sqrt{\frac{M_u}{b}} = 1.7791 \times \sqrt{\frac{2765}{12}} = 27.006 \text{ inches.}$$

$A_s = \rho\ b\ d = 0.01 \times 12 \times 27 = 3.22$ sq. in.

i.e. identical results.

## APPENDIX: 2.

**Program for designing reinforced concrete sections subject to bending, with single or double reinforcement.**

**Use hand electronic computer HP 65.**

**Equation used:**

$$k_1 = \sqrt{\frac{1000}{\phi \rho f_y \{(1-0.75\alpha)\left[1-0.59\rho\ (1-075\alpha)\ \frac{f_y}{f'_c}\right] + 0.675\}}}$$

$d = k_1 \sqrt{\frac{M_u}{b}}$ , $A_s = \rho\ b\ d$ , $A'_s = \alpha \rho\ b\ d$

<u>Program:</u>

| | | | | |
|---|---|---|---|---|
| f | RCL 4 | ENTER | 1/x | RTN |
| CLX | x | RCL 4 | 1 | LBL |
| LBL | - | x | 0 | D |
| A | RCL 1 | - | 0 | RCL 6 |
| DSP | x | x | 0 | ENTER |
| . | RCL 2 | . | x | RCL 8 |
| 4 | ÷ | 6 | f | x |
| STO 1 | RCL 3 | 7 | $\sqrt{x}$ | RCL 3 |
| STO 2 | x | 5 | STO 7 | x |
| STO 3 | . | ENTER | RTN | R/S |
| STO 4 | 5 | RCL 4 | LBL | RCL 4 |
| STO 5 | 9 | x | C | x |
| STO 6 | x | + | RCL 5 | R/S |
| RTN | CHS | RCL 1 | ENTER | |
| LBL | 1 | x | RCL 6 | |
| B | + | RCL 3 | ÷ | |
| 1 | 1 | x | f | |
| ENTER | ENTER | . | $\sqrt{x}$ | |
| . | . | 9 | RCL 7 | |
| 7 | 7 | x | x | |
| 5 | 5 | g | STO 8 | |
| ENTER | | | | |

Procedure :

Insert Program.

Press: A

Write value of ($f_y$ psi) and press STO 1.

Write value of ($f_c'$ psi) and press STO 2.

Write value of ( $\rho$ ) and press STO 3.

Write value of ( $\alpha$ ) and press STO 4. ( Note: If no compression reinforcement, then leave as zero).

Write value of ($M_u$ in. k.) and press STO 5.

Write value of (b in.) and press STO 6.

Press: B, gives $k_1$

Press: C, gives d

Press: D, gives $A_s$

Press: R/S, gives $A_s'$ (for case of compression reinforcement).

Note: The above program can be used in metric system, with the stresses given (kg/cm$^2$), the moment given (t.cm), and the dimensions given (cm).

Appendix: 3.

Program for designing a reinforced concrete section by working stress method subject to simple bending with single reinforcement.

Use hand electronic computer HP 65.

Equations used:

$$n = \frac{E_s}{E_c} = \frac{508.77}{\sqrt{f_c'}} \; , \quad k = \frac{1}{1 + \frac{f_s}{n \; 0.45 \; f_c'}} \; , \quad j = 1 - \tfrac{1}{3}k$$

$$\rho = \frac{0.225 \; f_c' \; k}{f_s} \; , \quad K = f_s \, \rho \, j \; , \quad d = \sqrt{\frac{M_e}{K \, b}} \; , \quad A_s = \rho \, b \, d$$

Program:

| | | | | |
|---|---|---|---|---|
| f | RCL | STO 6 | ÷ | ÷ |
| CLX | ENTER | R/S | RCL 6 | RCL |
| LBL | f | 1 | x | 9 |
| A | √x | ENTER | STO 8 | − |
| DSP | ÷ | 3 | R/S | f |
| . | STO 5 | ÷ | LBL | √x |
| 4 | R/S | RCL 6 | C | STO 2 |
| STO 1 | RCL 1 | x | RCL 1 | RTN |
| STO 2 | ENTER | CHS | ENTER | LBL |
| STO 3 | RCL 2 | 1 | RCL 7 | E |
| STO 4 | ÷ | + | x | RCL 4 |
| RTN | . | STO 7 | RCL 8 | ENTER |
| LBL | 4 | R/S | x | RCL 2 |
| B | 5 | RCL 2 | STO | x |
| 5 | ÷ | ENTER | 9 | RCL 8 |
| 0 | RCL 5 | . | RTN | x |
| 8 | ÷ | 2 | LBL | RTN |
| . | 1 | 2 | D | |
| 7 | + | 5 | RCL 3 | |
| 7 | 6 | x | ENTER | |
| ENTER | 1/x | RCL 1 | RCL 4 | |

Procedure :

Insert Program.

Press: A

Write value of ($f_s$ psi) and press STO 1.

Write value of ($f_c'$ psi) and press STO 2.

Write value of ($M_e$ in. lbs.) and press STO 3.

Write value of ( b in. ) and press STO 4.

Press: B, gives n

Press: R/S, gives k

Press: R/S gives j

Press: R/S gives $\rho$

Press: C, gives K

Press: D, gives d

Press: E, gives $A_s$

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