

An Index of Superiority for the Most Famous International Football Teams

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ABSTRACT

In this research, we propose a sports indicator to measure the historical performance of any sports team. By applying one of the cases of Markov chains through the state of equilibrium to the results of the historical matches that the team contributed to (win, draw, loss), and then applying this indicator to a selection of the most famous international football teams.

It is noted from this study that Brazil historically ranked first in football, followed by Spain, then England, then Argentina, then Italy, then Germany, then the Netherlands, then Portugal, then Denmark, then Belgium.

As a general summary of this study, it is a unique study at the level of the university and Iraq, in which we touched on the role of mathematics and its applications in the field of football. Role in the development of this effectiveness, and stand on the treatment of problems and decision-making.

Keywords : Index, Superiority, International Football Teams.

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الملخص

في هذا البحث نقترح مؤشر رياضي لقياس الأداء التاريخي لأي فريق رياضي. من خلال تطبيق إحدى حالات سلاسل ماركوف عبر حالة التوازن على نتائج المباريات التاريخية التي ساهم بها الفريق (فوز ، تعادل ، خسارة) ، ومن ثم تطبيق هذا المؤشر على مجموعة مختارة من أشهر فرق كرة القدم العالمية.

ويلاحظ من هذه الدراسة أن البرازيل احتلت المرتبة الأولى تاريخياً في كرة القدم ، تليها إسبانيا ، ثم إنجلترا ، ثم الأرجنتين ، ثم إيطاليا ، ثم ألمانيا ، ثم هولندا ، ثم البرتغال ، ثم الدنمارك ، ثم بلجيكا. وكخلاصة عامة لهذه الدراسة فإنه تعد دراسة فريدة على مستوى الجامعة والعراق تطرقنا بها عن دور الرياضيات وتطبيقاتها في مجال كرة القدم فإن كل النتائج التي حصلنا عليها تعطي إشارة عامة

وواضحة، قريبة جدا من الواقع وتعتبر عن أهمية تعدين البيانات في مجال كرة القدم واكتشاف المعرفة لما لها دور في تطوير هذه الفعالية، والوقوف على معالجة المشاكل واتخاذ القرار.

الكلمات المفتاحية : مؤشر ، تفوق ، فرق كرة قدم دولي

1- introducing:

1-1 Introduction and importance of research:

In today's modern world, there is wide interest by the competent authorities and the public in data and information related to sports activities, especially in the results of matches.

With the available data, it may be interesting to turn them into sports models in order to study and analyze them in order to reach a realistic and sound vision of the reality of each sports team, as well as to benefit from them in sports betting.

There is a need to deal with data and information to understand the historical behavior of each team in order to study its development over time as well as to benefit from it in the field of sports betting around the world. Through mathematical modeling and machine learning, we can predict the sports trends of each team.

In probability theory, a stochastic process is a mathematical object usually defined as a family of random variables indexed by an index like time. Stochastic processes are the dynamics part of probability theory, and Markov chains are a special kind of stochastic processes.

Mathematical modeling has witnessed great and wide developments in recent years in various applied fields, and it has become one of the great titles that attracted the attention of researchers in various different disciplines, see for example Sandeep and Vadhera (2020) [12].

An application of the stochastic process in the sport's analysis is not new. Various research papers that predict and analyze game performances in different times using the Markov chain model, have been published. Bellman (1976) has introduced a Markov Chain model to Baseball match results [2]. Ursin (2014) have developed a Markov model for baseball with applications [9]. Norman (1999) analyzed the possibilities to use stochastic processes for statistical modeling in sports sciences, especially [10]. Clarke and Norman (1998) utilized the stochastic techniques in various decision-making processes in cricket [5]. Lames (1988) applied the idea of assess the performance of individual players in team games [7] such as tennis as well as Lames and Hohmann (1997) analyzed in volleyball [8], T Anuthrika , S Arivalzahan, R Tharshan (2019) introduced a Markov chain stochastic model to classify teams. Comprehensive head-to-head match prediction analysis is given for each cricket team based on steady state odds [1].

Our main goal in this paper is to propose a mathematical model in order to use it to study the historical achievements of a particular team in order to stand on the evolutionary behavior of the team and to be able to predict the future results of the team by measuring the team's performance through the total number of victories, losses and draws in official matches approved.

Our main idea is to use a Markov model to model the results of matches, so that the outcome of each match is one of the specified cases: win or lose or draw.

1- 2 The search problem:

Several years ago, the rating was determined by the collective opinion of writers and journalist coaches. It is clear that these principles of arrangement are unacceptable, because in many cases people's opinions are "biased". For example, a sports analyst may be influenced by a particular team's playing style that would influence its decision, moreover, many of those who vote are considered in ranking polls (especially football coaches) not to see all the matches of each team during the season and are dependent on their personal perceptions or judgments of other professionals. Therefore, this ranking approach can produce "unfair" results.

Accordingly, the problem of the current research is that there is an urgent need to find mathematical models that work on finding a numerical indicator for the purpose of arranging sports teams, especially in the game of football using the historical results of international teams.

1-3 The purpose of the research:

- Finding a numerical index (evidence) as a mathematical model using a stochastic Markov chain to classify teams. A comprehensive analysis is presented to predict the future match based on steady state probabilities.

1- 4 Research areas:

- Human field: the official national teams of each of the following teams (English, Spain, Argentina, Brazil, Portugal, Denmark, Germany, Italy, Belgium, France and the Holland).
- The historical field: the official matches of the international teams that you have participated in since its inception until the year 2020.

1-5 Search terms:

1-5-1 Variables:

The characteristics and advantages that show a difference in their values are called variables, and there are two types of variables:

- Descriptive (Qualitative) variables:

They are the variables that represent specific characteristics and characteristics that are measured descriptively (not including numerical significance) such as color, gender, DNA.

- Quantitative Variables

They are the variables that have a numerical value resulting from a numerical measurement of something, as a specific physical aspect, and these variables are either continuous (continuous) and usually take values in the form of real numbers, such as: height, weight ... etc. or discrete (discrete) and usually It takes values in the form of integers (positive or negative), such as: number of balls, number of goals, number of pitches etc.

- State space:

It is defined as the set of all possible values of the stochastic process called the state space and symbolized by S.

The state space can be finite or infinite depending on the number of its elements. Also, it can be continuous if it takes continuous values, or discrete if it takes separate values.

- Markov processes:

The stochastic process $\{X(t); t \in T\}$ is said to be a Markovian process if the values are

$t_1 < t_2 < \dots < t_n < t$ as follows:

$$\begin{aligned} P\{a \leq X_n \leq b / X(t_1) = X_n, \dots, X(t_n) = X_n\} \\ P\{a \leq X_n \leq b = X(t_n) = X_n\} \end{aligned} \quad (1)$$

It is called a Markov chain if the state space is discrete, which is often in the form of a set of.

It is called a continuous Markov process if the state space is continuous which is in the form

$$-\infty < t < \infty$$

- Markov chains :

Markov chains are a special type of Markov processes that can be represented by the discrete state space and the discontinuous and continuous parameter space, and they are a sequence of random variables that fulfill the Markov property and the systems that have this property are called Markov chains.

3- Research Methodology:

In this section we focus our attention on stochastic processes with discrete state space in discrete time . For convenience, we will symbolize the state space elements, regardless of their names, with the integer $\{1,2,3,\dots\}$ (with some special exceptions).

If the discrete random variable represents the "state" of an observed system at time .

As is common in specialized scientific sources, we will symbolize the stochastic process with a discrete state space with the symbol $\{X_n; n = 0,1,2,\dots\}$ or for short $\{X_n\}$.

Usually, the n index stands for time, because this index is the most commonly used in applications, however n can be any other index, such as the location somewhere. Since the index n represents time, " n " represents the present, " $n-k$ " represents the past before k units of time, and " $n+k$ " represents the future after k units of time. The observed "data" is actually from the stochastic process, so let it be that the stochastic process $\{X_n\}$ is said to have the Markovian property if it $X_0, X_1, \dots, X_n, \dots$ achieves the following:

$$\begin{aligned} P(X_n = j / X_{n-1} = i, X_{n-2} = a, X_{n-3} = b, \dots, X_0 = c) \\ = P(X_n = j / X_{n-1} = i) = p_{ij}, \end{aligned}$$

For all n, i, j, a, b, c . Gagniuc (2017).

The stochastic process has a discrete state space and bears the Markovian property.

The transition possibilities from state i to state j are called transitional probabilities and are symbolized by them . If the transition probabilities do not

depend on time then the Markov chain is described as time-homogeneous, or stationary.

When the state space S is finite, the probability distribution of transitions between states can be represented by a matrix called the Transition Matrix, and is symbolized by it, as the element (i, j) of this matrix represents the transition probability.

$$p_{ij} = P(X_n = j / X_{n-1} = i) \quad (2)$$

A transition matrix is a stochastic matrix that satisfies the following two conditions:

- 1- All of its elements are non-negative (because they are probabilities).
- 2- The sum of each of its rows is 1 (because the sum of the total probabilities is equal to 1).

If the state space is in the form of $S = \{1, 2, \dots, N\}$, then the general form of a one-step transition matrix is as follows:

$$P = \begin{matrix} & \begin{matrix} X_n = 1 & 2 & \dots & N \end{matrix} \\ \begin{matrix} X_{n-1} = 1 \\ 2 \\ \vdots \\ N \end{matrix} & \begin{pmatrix} p_{11} & p_{12} & \dots & p_{1N} \\ p_{21} & p_{22} & \dots & p_{2N} \\ \cdot & \cdot & \dots & \cdot \\ p_{N1} & p_{N2} & \dots & p_{NN} \end{pmatrix} \end{matrix} \quad (3)$$

The matrix $P = (P_{ij})$ is the string transition matrix Thanoon (2011).

As we mentioned, the transition possibilities from state i to state j , are called transitional probabilities, meaning that the transition probability is the conditional probability $p_{ij} = P(X_1 = j / X_0 = i)$. Or more precisely, the probability of moving from state i to state j is one-step. The probability of moving from state i to state j in n steps is denoted by it $p_{ij}^{(n)}$, and it is clear that:

$$p_{ij}^{(n)} = P(X_n = j / X_0 = i); n = 1, 2, \dots$$

It is noted that $p_{ij}^{(1)} = p_{ij} = P(X_1 = j / X_0 = i)$, and

$$\begin{aligned} p_{ij}^{(2)} &= P(X_2 = j / X_0 = i) = \sum_{r \in S} P(X_2 = j, X_1 = r / X_0 = i) \\ &= \sum_{r \in S} P(X_2 = j / X_1 = r, X_0 = i) P(X_1 = r / X_0 = i) \\ &= \sum_{r \in S} P(X_2 = j / X_1 = r) P(X_1 = r / X_0 = i) \end{aligned}$$

According to the Markovian property

$$\begin{aligned} \therefore p_{ij}^{(2)} &= \sum_{r \in S} P(X_1 = r / X_0 = i) P(X_2 = j / X_1 = r) \\ &= \sum_{r \in S} p_{ir} p_{rj}. \end{aligned}$$

As for the transition matrix with k steps, $P^{(k)}$, this matrix represents the probability of moving from state i to state j in k steps, that is, the elements of this matrix are.

$$p_{ij}^{(k)} = P(X_n = j / X_{n-k} = i) \quad (4)$$

Axiom : Thanoon (2011)

For any $0 < k < n$, the transitional probability of n steps from state i to state j satisfies the following equation:

$$p_{ij}^{(n)} = \sum_{r \in S} p_{ir}^{(k)} p_{rj}^{(n-k)}.$$

the proof:

$$\begin{aligned} p_{ij}^{(n)} &= P(X_n = j / X_0 = i) \\ &= \sum_{r \in S} P(X_n = j, X_k = r / X_0 = i) \end{aligned}$$

Because the string must be in some state, say r , at time k .

$$\therefore p_{ij}^{(n)} = \sum_{r \in S} P(X_n = j / X_k = r) P(X_k = r / X_0 = i)$$

Sines $P(A \cap B / C) = P(A / B \cap C) P(B / C)$.

That is:

$$p_{ij}^{(n)} = \sum_{r \in S} p_{ir}^{(k)} p_{rj}^{(n-k)}.$$

The series must have a steady state (or steady state) distribution if there is a vector like this given the transition probability matrix P :

$$\pi = \pi p \quad (5)$$

If the finite Markov chain is irreducible, then:

$$\lim_{n \rightarrow \infty} p^n = \pi = \begin{bmatrix} \pi_1 & \cdots & \pi_n \\ \vdots & \ddots & \vdots \\ \pi_1 & \cdots & \pi_n \end{bmatrix} \quad (6)$$

where $\pi = (\pi_1, \pi_2, \dots, \pi_n)$ with $0 < \pi_j < 1$ and $\sum \pi_j = 1$

This constant probability vector can be viewed as the only long-run distribution of a random variable.

Furthermore, the steady-state probabilities of π_j are obtained as

$$\lim_{n \rightarrow \infty} p_{ij}^{(n)} \forall i, j \in S \quad \text{Thanoon (2011)}.$$

3-1 Building a Markov Chain Model:

If we give a sequence of observed cases (verified observations), the first thing to note is that these observations may be in the form of symbols consisting of two or more letters, or of integers, usually in the form of 1.2....N. To facilitate computer processing, it is preferable to convert literal symbols into integers. If we assume that the achieved observations are:

$$\{A D B A A C B D D A D A C D D B A D C D D B D A D C C A B\}$$

These symbols can be encoded into integers as follows: A=1, B=2, C=3 and D=4. It is possible to benefit from the following program, which encodes any set of letters or symbols into integers, and this program is based on the previous verified observations, which represent the results obtained by the teams since its inception until 2020, which number (11) and which were obtained through

the website (Wikipedia, the free encyclopedia), the numbers of transitions from each state to another were obtained and the transition probabilities were estimated using the maximum likelihood criterion.

The following table shows the total number of matches played by each team n , the number of wins, the number of draws, and the number of losses for each team included in this study.

Table (1) : Number of official matches played by each of the selected world's most prominent teams since its inception until 2020, with cases of wins, draws and losses for each.

	Team	Total number of matches played	Number of Wins	Number of Draws	Number of Losses
1	England	1007	573	242	192
2	Spain	724	421	169	134
4	Brazil	1007	643	206	158
5	Portugal	645	313	151	181
6	Denmark	880	412	179	289
7	Germany	978	562	205	211
8	Italia	848	449	236	163
9	Belgium	809	362	170	277
10	France	886	440	185	261
11	Holland	624	337	127	160

To build a Markov chain model, X_n is the team's performance on the last day and is named as

State 1: The team wins the match (W).

State 2: The team is tied in the match (D).

State 3: The team loses the match (L).

So, X_n is a random process that has a value from 1 to 3 in states. Furthermore, consider the team's performance in any sequence of letters chosen from the combination (W,D,L). Suppose $n_1 . n_2 . n_3$ denotes the number of letters "W", "D", "L" respectively in the sequence. Frequencies Matrix for transitions from one state to another

The three cases are here and this sequence of results is recorded in a

$[3 \times 3]$ matrix.

$W \quad D \quad L$

$$F = \begin{matrix} W \\ D \\ L \end{matrix} \begin{bmatrix} n_{ij} & n_{ij} & n_{ij} \\ n_{ij} & n_{ij} & n_{ij} \\ n_{ij} & n_{ij} & n_{ij} \end{bmatrix} \quad (7)$$

The transition probability matrix P was constructed by dividing each element by the corresponding row total.

$$P = \begin{matrix} W \\ D \\ L \end{matrix} \begin{bmatrix} P_{ij} & P_{ij} & P_{ij} \\ P_{ij} & P_{ij} & P_{ij} \\ P_{ij} & P_{ij} & P_{ij} \end{bmatrix} \quad (8)$$

And also $\sum_j P_{ij} = 1$

This transition probability matrix is called the $[3 \times 3]$ matrix of the transition probability matrix of the first step of the Markov chain. Moreover, each row of P is the probability distribution related to the transition from state i to state j .

Further, the steady-state distribution is examined and steady-state probabilities are obtained

$$\pi_j = \lim_{n \rightarrow \infty} p_{ij}^n \quad \forall i, j \in S$$

In light of this interpretation of the stationary distribution, we propose the following mathematical index for historical sports excellence; HSE; as the final summary of the historical results of any sports team that played matches and the results of which were one of the three cases: win, draw or loss:

$$I_{HSE} = (2\pi_W + \pi_T - 2\pi_L)100\% \quad (9)$$

4- Applying Markovian Modeling:

We consider the result of any sporting match as states of a 3- stats Markov chain with states: Win (W), a Draw (D), or a Loss (L).

The basic assumption in the Markovian model is that the result of the current match depends on the result of the previous match, and that the result of that that preceded it depends on the outcome of its previous match, and the result of that previous match depends on the outcome of the match that preceded it and so on; see e.g.

If $\{X_1, X_2, \dots\}$ represents the results of consecutive matches for a specific team, then the probabilistic model expressing the Markovian property can be formulated mathematically as follows; see e.g. Gagniuc (2017):

$$P(X_n | X_{n-1}, X_{n-2}, \dots) = P(X_n | X_{n-1}); n = 1, 2, \dots \quad (10)$$

Table (2) shows the frequencies had observed when moving from one state to another in successive matches; for example, WD stands for the transition from a win state to a draw state in two consecutive matches, while DW stands for the transition from a draw state to a win state in two consecutive matches.

To clarify in an understandable way, Table No. (2) was prepared according to the frequency matrix, which was referred to in the equation (7).

Table (2) The observed frequencies of states for the selected teams.

Team	WW	WD	WL	DW	DD	DL	LW	LD	LL
Argentina	243	75	79	75	31	27	80	26	30
Belgium	173	83	106	76	40	53	113	46	118
Brazil	411	130	102	134	45	27	98	30	29
Denmark	191	83	137	83	36	60	138	60	91
England	320	140	113	143	53	45	109	49	34
France	246	87	107	92	41	51	101	57	103
Germany	322	124	116	119	42	44	121	39	50
Holland	182	70	84	70	25	32	85	31	44
Italia	244	127	77	114	70	52	91	38	34
Portugal	164	72	77	74	34	43	74	45	61
Spain	253	91	76	103	37	29	65	40	29

The elements of the frequency matrices of the teams have been transformed into transition matrices of probabilities; each with a dimension 3×3 .

After obtaining the matrix of frequencies, we calculate the transition matrix, this program is called MCestimation Thanoon (2011), as this program finds both the matrix of frequencies as well as the transition matrix for any observations made from a Markovian chain consisting of S from the possible cases, and according to the definition of the relative frequency of the probability, that We form the transition matrix by dividing each element in the frequency matrix by the sum of the row in which it is located. That is, the transitional probabilities are calculated as follows:

Table (3) Elements of transition probability matrices for the selected teams.

	Team	WW	DW	LW	WD	DD	LD	WL	DL	DD
1	Argentina	0.6106	0.5682	0.5809	0.1884	0.2348	0.1985	0.2010	0.1970	0.2206
2	Belgium	0.4647	0.5289	0.4205	0.2193	0.2314	0.1692	0.3160	0.2397	0.4103
3	Brazil	0.6526	0.5923	0.5926	0.1715	0.2231	0.1778	0.1759	0.1846	0.2296
4	Denmark	0.5036	0.4876	0.4333	0.2029	0.2149	0.2167	0.2935	0.2975	0.3500
5	England	0.5594	0.5785	0.5885	0.2500	0.2190	0.2344	0.1906	0.2025	0.1771
6	France	0.5603	0.4030	0.4456	0.2069	0.1613	0.1606	0.2328	0.2984	0.3938
7	Germany	0.5519	0.6183	0.5808	0.1975	0.1985	0.1617	0.2506	0.1832	0.2575
8	Holland	0.5401	0.5556	0.5250	0.2077	0.1984	0.2000	0.2522	0.2460	0.2750
9	Italia	0.5476	0.5723	0.4803	0.2381	0.2516	0.3071	0.2143	0.1761	0.2126
10	Portugal	0.5385	0.5089	0.4848	0.1885	0.2857	0.2348	0.2731	0.2054	0.2803
11	Spain	0.6361	0.5667	0.5688	0.1833	0.2583	0.2110	0.1806	0.1750	0.2202

In Table No. (4), the possibility of the transitional matrix was clarified for each team to facilitate the discussion process and according to the conditions of the transitional matrix in equation (8).

4-1 The Proposed Index:

In this index we have given to the winning state a weight equivalent to two draws, and for the losing state a weight equivalent to winning, but with a negative sign.

Some properties of this proposed mathematical index can easily be inferred:

- 1- Theoretically, its highest value is 100% when all the team's results are a win without any draw or losing.
- 2- The sign of the index is a clear indication of the positive or negative results of the team.
- 3- When comparing historically between a group of teams, the team that has the highest value from this index is the best as an outcome of its historical results.

Note that, when calculating this index for a group of teams, it is preferable to normalize it by dividing the value of each team's index by the value of the largest index of them and then multiplying the result by 100%.

4-2 Results and Discussion:

Using the transition probability matrices for the selected teams shown in Table (3), the Stationary probability distribution of wins, draws and losses for each team were obtained and shown in Table (4).

Table (4) Stationary probability distribution of wins, draws and losses for each team.

	Team	π_W	π_D	π_L
1	Argentina	0.5961	0.1997	0.2042
2	Belgium	0.4632	0.2052	0.3316
3	Brazil	0.6304	0.1820	0.1876
4	Denmark	0.4783	0.2097	0.3120
5	England	0.5695	0.2396	0.1909
6	France	0.5231	0.1850	0.2919
7	Germany	0.5714	0.1891	0.2395
8	Holland	0.5394	0.2038	0.2568
9	Italia	0.5402	0.2556	0.2042
10	Portugal	0.5159	0.2222	0.2619
11	Spain	0.6093	0.2038	0.1869

From the last table, values of the mathematical index for historical sports excellence index; HSEI; were calculated and shown in the next table together with the corresponding normalized values.

Table (5) Values of I_{HSE} for the selected football teams and the corresponding normalized values.

	Team	I_{HSEI}	Normalized
1	Argentina	0.9835	92.1225
2	Belgium	0.4684	43.8741
3	Brazil	1.0676	100.0000
4	Denmark	0.5999	56.1915
5	England	0.9968	93.3683
6	France	0.6474	60.6407
7	Germany	0.8529	79.8895
8	Holland	0.7690	72.0307
9	Italia	0.9276	86.8865
10	Portugal	0.7302	68.3964
11	Spain	1.0486	98.2203

It is clear from the obtained results of this proposed historical index, that the Brazilian team comes in the lead, followed by the teams It is noted from this study that Brazil ranked first in football historically, followed by Spain, then Spain, then England, then Argentina, then Italy, then Germany, then the Holland, then Portugal, then Denmark, and Belgium at the bottom of these ten selected teams, see Figure (1).

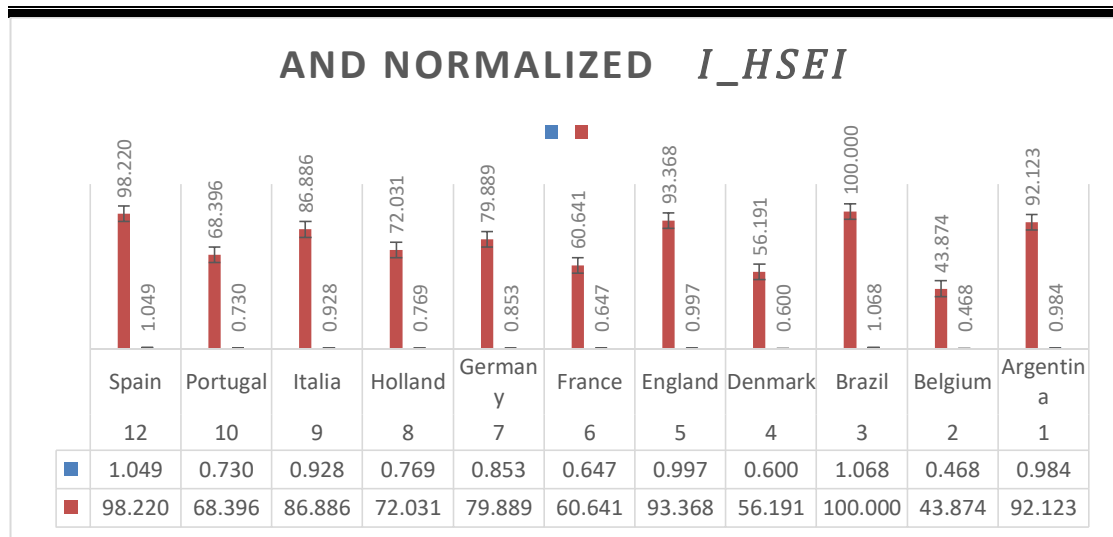


Figure (1) : Classification of the selected teams according to the proposed mathematical index for historical sports excellence.

5- Conclusions and recommendations

1- We have compared 11 teams according to their potential as rankings by giving an indicator that reflects the current strength of the team. Our analysis shows that the Markov chain models that gave the advantage of not ignoring the historical activity of the teams, which is the basis of the team's superiority through the match result in terms of win, tie or loss, and that the best models are those that determine the least number of parameters for the teams.

2- In this paper, we have found an easy and very important way to get to the top ranked teams using Markov chains of official historical matches of football teams around the world, and how it can be exploited by scheduling the matches played by each team. Since its inception in a smart way. We've set goals for teams to move up the rankings, and we've shown through our experiences how opportunities can be taken to move up the table. And the chances of advancement in the table in terms of attendant possibilities.

3- Our proposed methods can be used to improve the team ranking by football teams, which may lead to a better ranking of these teams.

4- We also benefited from betting odds as a tool of high value in treating the available information and predicting sporting events. The betting odds themselves are a measure of the expectation of success in the next match and using our approach we can relate these market expectations directly to the quantitative rating of each team, i.e., a measure of team quality.

6- We can also use our suggestion for all team and individual games like basketball, volleyball, handball.... etc, among the games followed by people, analysts, etc.

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