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# **Experimental and Numerical Study of Spur Gears With Lightening Holes**

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# HIGHLIGHTS

- To loosen the gear, it was found that in the case of perforating it, 5 holes are optimal in terms of vibration
- The perforation diameter must be equal to 0.1085 \*Dp
- Using c35 gears through design calculations
- Working at different speeds from 0 to 2000 revolutions per minute
- Calculations of stress concentration

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# 1. Introduction

# ABSTRACT

The gear is a very important component of power transmission in a flying vehicle and rotating mechanisms etc... In this paper, the dynamic effect and stresses generated as a result of the mash gear will be studied by designing light-weight gears by making lightening holes of multiple diameters (14, 19, 24 and 29) mm at serval speeds (500,1000,1500,2000) that represent the variables parameter with a fixed dimension between the center of the holes and the center of rotation of the gear 43 mm and the number of 5 holes for gears with reduction ratios of gear mass ranging from (3.6%-15%) that represent the constant parameter. The 5-hole gear is better with a hole diameter (0.1085 \* Dp) which is equivalent to 19 mm in this research with respect to the gear diameter of 175 mm. Where the maximum value of stress concentration in the root of the tooth reached 0.655 M.N/m^2.

The objective Study of vibration in a system containing gear with lightening holes. The study of the open gear system operation depends on the vibrations and noise resulting from exposing the system to several different factors. Knowing the effect of the diameter's holes, and the distance from the center of the gear on the vibration. Finding several relationships in the system, such as the diameter of the holes with array radius. Calculation of process noise in the system. Reaching some points which may improve the gear system, after discovering the critical points that should not be reached during normal operation.

Chen and Shao with the development of mechanical transmission systems, high-speed and high-power gear transmissions will have higher requirements for vibration reduction and service life [1]. Nidal H. and Mohammad A, established a pilot model is established and used it as a reference model to predict the Owen Meese tension in the root flat of the non-perforated gear, utilizing the ABAQUS package using finite element modeling. Then, the forecast and pressure obtained were compared with the AGMA analytics solution. The authors explored the effect of different knot parameters to make holes in the gear body and face profile using the first model and the second model, respectively. In this case, the hardness of the gear was greatly affected [2].

Morrish et al presented an investigation by using ANSYS software on an asymmetric gear tooth with circular holes as stress relieving features for the structural analysis under some loading value, and with modeling the gear tooth as per two modification parameters for holes. The investigation found that CAD modeling and FEA simulation can help generate huge test data with different textures of stress-relieving features in minimum time with minimum effort and with maximum accuracy [3]. S. Mahendran et al, designed a super gear through a design software used and studied the weight loss and stress distribution for cast steel and composite materials, which were analyzed utilizing gearboxes used in automobiles. The analysis found that the tensile value of composite materials is lower than that of cast steel super gear. Moreover, it was concluded that the stress, tension and weight generated by the super gear composite are less than that of cast steel super gear [4].

Emre Ozturk and Zeynep Sonmez, mentioned that lightweight perforations are frequently employed in gears to save weight. The geometric position and radius of lightweight holes and the radius of the gear shaft utilized in a spur gear in aircraft engines were optimized in this study. These values serve as design variables in optimization, with the weight of the gear serving as the objective function and the maximum von Mises stress on the gear serving as the design constraint. ISIGHT software was used to address the optimization challenge. The geometric position of the lightning hole, the radius of the lightning holes, and the radius of the gear shaft utilized in aviation engines were all optimized in this study. ISIGHT 2017 was used to address the optimization problem. The following conclusions were taken from the obtained DOE and optimization results: First, the Latin Hypercube approach produced the best results after a series of experimental design studies using various methods. Second The diameter of the lightning hole by 80 percent is the most critical parameter impacting the weight, according to the experimental design results. The second useful parameter in lowering the gear weight is the square power of the lightning hole diameter, which is 12.5 percent, and the diameter of the gear shaft hole, which is 6.5 percent. Third When the parameters affecting the stress are analyzed, the square power of the gear shaft hole diameter is found to be the most influencing element (25%), and the square power of the gear shaft hole diameter is found to be the least influencing component (5%) [5]. Dhananjay Dolas and Sarfraz Ali Quadr, provided an overview of the overall optimization of super gears under static load. Because gears have many advantages, they also have limitations, such as weight and size. Gear weight makes gears difficult to use for compact and lightweight applications. In this study, some characteristic geometric shapes were incorporated into motion tools to improve the weight and see its effect on stress development. The super gear finite element model was then considered for the study and analysis of static pressure using ANSYS. This specifies the method of analysis of a finite element for the super gear teeth using ANSYS 14.5. Studying the gear blocks with the inclusion of different geometries was continued. The results showed the effect of different geometries on the resulting pressure, and give the best geometry which gives safe results with better super gear mass [6].

Özek, F, investigated the effect of various geometric holes on the spur gear pressure to reduce their weight. It was found that the model with the circular hole and the stepped gear is the model that best fits the tension value [7]. Ali R. H. et al, in addition to ANSYS, used a 2D finite element model to investigate the spur gears' natural frequency and dynamic response, analyze the gear teeth at various operating speeds, and use transient superposition to displace. And, the horizontal and vertical components of the dynamic stress were determined. The finite element analysis program used in the proposed model of the refinement of the natural frequency by the Ranchos block method and the displacement and dynamic stress by the superposition method of the transient mode showed that bending occurred as the rotational speed of the gear was increased. The stress of dynamic stress also increased, and for the moving loads, the coefficient of maximum dynamic bending root stress gradually increased as the gear rotational speed was increased [8].

Bai, X.M., et al, various shock absorbers and damping rings were designed to reduce the vibration of the gear. In the vibration reduction analysis of the gear system, active vibration reduction must increase processing accuracy. Meanwhile, the parameters of the gear transmission system require redesigning, which has many limitations such as sizeable computational complexity and high processing cost. A more efficient method is called particle damping [9].

#### 2. Model of Rotational System

The specifications of the gear were chosen after it was designed by studying the loads and speeds as shown in Table 1 gear parameters after computational design.

Parameter	Pinon	Gear
Thickness	7.4 <i>mm</i>	6.6 mm
Width	40 mm	40 mm
Root radius	1.4 <i>mm</i>	1.4 <i>mm</i>
Pitch Circle radius	87.5 mm	175 mm
Module	3.5	3.5
No. of teeth	25	50
Pressure angle	20	20

 Table 1: Gears parameters

A metal of its quality (CK35) was used for the gears, and chemical analysis of the metal was taken in specialized laboratories to ensure practical results extracted from this mineral. In the figure shows the position and diameter of holes in each gear which represents 4 cases with the normal state and each case have a serial name as shown in Figure 1.

And system parts (Rig) as shown in Figure 2 (Base, Bearing, Gear, flywheel, motor, etc.)

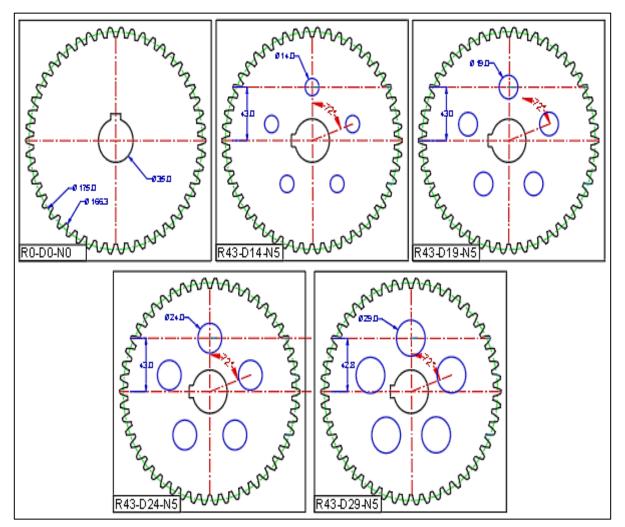


Figure 1: Position and diameter of holes in each gear

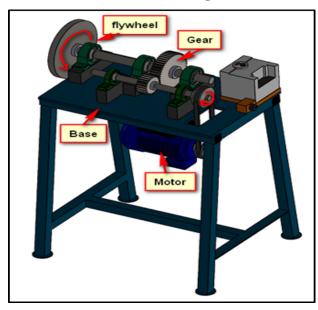




Figure 2: Rotational system in Solidwork program & real

# 3. Analytical Method

To simplify the model that contains 8 DOF and turn it into a simple model with 3 DOF, it was assumed that the gear with the tooth is one rigid part, and the two axes (Shafts) were considered as springs in the direction of rotation the ideal model taken for a sensible gear drive system is shown in Figure 3.

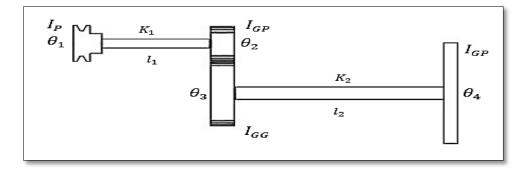


Figure 3: Gear Model for Rotational system

"Steel Structure System", which provides a "stable speed" load, a rigid model, and the simplest dynamic loading model of the gear system. To represent the static moment in Figure 3, the "input torque" creates a velocity angle that can be calculated using the "low moment of inertia" [10].

$$I_{read} = I_P + I_{GP} + \left[ \left( I_{GG} + I_F \right) \left( \frac{1}{i} \right)^2 \right]$$
(1)

Then the angular accelerations are:

$$\ddot{\theta}_1 = \ddot{\theta}_2 = \frac{M_T}{I_{read}} , \ \ddot{\theta}_4 = \ddot{\theta}_3 = \ddot{\theta}_1(\frac{1}{i})$$
(2)

The two gearbox shafts have different torque values, which are measured using the free-body diagram:

$$\mathbf{M}_1 = (I_{read} - I_P)\ddot{\theta}_1 \tag{3}$$

$$M_4 = \ddot{\theta}_4 I_4 \tag{4}$$

When applying the assumptions in the equations to simplify the system to find its natural frequency, as through these assumptions, it turns from 8DOF to 3DOF. Figure 4 shows the lowering oscillator chain of the drive gear system with an arbitrary standard model compared to Figure 3.

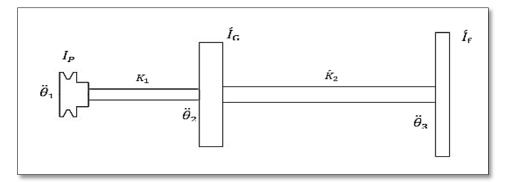


Figure 4: Equivalent Gear Model for Rotational system

 $I_P \ddot{\theta}_1 + K_1 \theta_1 - K_1 \theta_2 = 0 \tag{5}$ 

$$\hat{I}_{G}\ddot{\theta}_{2} - K_{1}\theta_{1} + (K_{1} + \dot{K}_{2})\theta_{2} - \dot{K}_{2}\theta_{3} = 0$$
(6)

$$\hat{I}_{f}\ddot{\theta}_{3} - \hat{K}_{2}\theta_{2} + \hat{K}_{2}\theta_{3} = 0$$
<sup>(7)</sup>

Matrix will be:

$$\begin{bmatrix} I_P & 0 & 0 \\ 0 & \hat{I}_G & 0 \\ 0 & 0 & \hat{I}_f \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \\ \ddot{\theta}_3 \end{bmatrix} + \begin{bmatrix} K_1 & -K_1 & 0 \\ -K_1 & (K_1 + \dot{K}_2) & -\dot{K}_2 \\ 0 & -\dot{K}_2 & \dot{K}_2 \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix} = 0$$

To find natural frequency by solving (EOM), Natural frequency is determined from this Equation:

$$-\omega_n^2 \left[ \omega_n^4 - \omega_n^2 \left( K_1 * \frac{I_P + \hat{I}_G}{I_P \, \hat{I}_G} + \hat{K}_2 \, \frac{\hat{I}_G + \hat{I}_f}{\hat{I}_G \, \hat{I}_f} \right) + K_1 \, \hat{K}_2 \, \frac{I_P + \hat{I}_G + \hat{I}_f}{I_P \, \hat{I}_G \, \hat{I}_f} \right] = 0 \tag{8}$$

An applied dynamic Absorber will be

$$\omega_{n\,2,3}^{2} = \frac{1}{2} \left( K_{1} * \frac{I_{P} + \hat{I}_{G}}{I_{P} \,\hat{I}_{G}} + \hat{K}_{2} \, \frac{\hat{I}_{G} + \hat{I}_{f}}{\hat{I}_{G} \,\hat{I}_{f}} \right) \mp \sqrt{\frac{1}{4} \left( K_{1} * \frac{I_{P} + \hat{I}_{G}}{I_{P} \,\hat{I}_{G}} + \hat{K}_{2} \, \frac{\hat{I}_{G} + \hat{I}_{f}}{\hat{I}_{G} \,\hat{I}_{f}} \right)^{2} - K_{1} \, \hat{K}_{2} \, \frac{I_{P} + \hat{I}_{G} + \hat{I}_{f}}{I_{P} \,\hat{I}_{G} \, \hat{I}_{f}}} \quad (9)$$

When  $\omega_1 = 33.4$ 

From the above equation, the first, second and third natural frequencies of the system can be found, respectively, with the values of Table 2, frequency. Their mass should be calculated after pre-calculation, as shown in Table 3 [11].

$$\hat{I}_{GG} = I_{GG} - I_{hole} \tag{10}$$

$$I_{hole} = \frac{m r^2}{2} + m d^2 \tag{11}$$

After using Equation (10), (11) to find a moment of inertia for each

Putting all the values of the constants and the values of the variables in Equation (9), produces the values of the first, second and third natural frequencies of the system for each gear.

Table 2: The moment of inertia for gears

Name	Í <sub>GG</sub>	Units
R0-D0-N0	0.026077800	$kg.m^2$
R43-D14-N5	0.025783679	$kg.m^2$
R43-D19-N5	0.025587599	$kg.m^2$
R43-D24-N5	0.025391518	$kg.m^2$
R43-D29-N5	0.025589514	$kg.m^2$

Table 3: Theoretical natural frequency

Name	Natural Frequency Hz			
	$\omega_{n1}$	$\omega_{n2}$	$\omega_{n3}$	
R0-D0-N0	33.4	163.678	545.742	
R43-D14-N5	33.4	164.463	546.428	
R43-D19-N5	33.4	164.990	546.893	
R43-D24-N5	33.4	165.849	547.659	
R43-D29-N5	33.4	167.132	548.820	

To compare actual and theoretical results, it is necessary to calculate the frequency of the gear, which represents the true frequency in all cases. This is consistent because it depends on the speed of rotation and the number of teeth in the gear. Each gear produces frequencies (gear mesh frequencies) associated with the gear and gear teeth running speed [12].

$$\omega_{mesh} = \frac{nN}{60} \tag{12}$$

As the frequency is calculated for different speeds 500, 1000, 1500 and 2000 as shown in Table 4.

Table 4: Gear Mesh Frequency for Rotational system

Speed rpm	Frequency Hz	
500	33.1740	
1000	66.3481	
1500	99.5222	
2000	132.6963	

# 4. Results and Discussion

#### **4.1** Practical Vibration

As the operating frequency was calculated for the different speeds, which are 500, 1000, 1500 and 2000 at room temperature with the use of dampers through the (acceleration) sensor MPU 6050, then the data was processed by the Sigveiw software using the FFT transformation to find the frequencies as shown in the Tables 5 below.

Table 5:	Practical Fre	equency for	different speed
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Name	Pra	Practical Frequency Hz	
	Near Gears		
	Х	Y	Z
	500 rpm	l	
R0-D0-N0	30.542	23.95	31.934
R43-D14-N5	36.328	24.609	26.147
R43-D19-N5	24.465	31.541	36.475
R43-D24-N5	32.739	28.711	30.908
R43-D29-N5	33.471	30.298	30.566
	1000 rpr	n	
R0-D0-N0	57.593	54.348	60.669
R43-D14-N5	71.948	78.613	57.764
R43-D19-N5	35.034	69.214	76.563
R43-D24-N5	53.662	32.471	69.043
R43-D29-N5	52.352	25.179	71.834
	1500 rpr	n	
R0-D0-N0	55.908	101.32	113.77
R43-D14-N5	63.477	105.93	84.717
R43-D19-N5	100.83	99.365	71.045
R43-D24-N5	88.135	74.951	91.064
R43-D29-N5	98.877	66.161	83.495
	2000 rpr	n	
R0-D0-N0	142.19	99.375	47.188
R43-D14-N5	94.588	118.13	152.81
R43-D19-N5	157.5	110.31	115.31
R43-D24-N5	97.5	113.13	144.09
R43-D29-N5	82.603	117.715	176.391

To clarify how the values were extracted from the LabVIEW program and analyzed by the sigveiw program X-axis is Frequency (Hz) and Y-axis is Amplitude  $(mm^{-6})$  is shown in Figure 5.

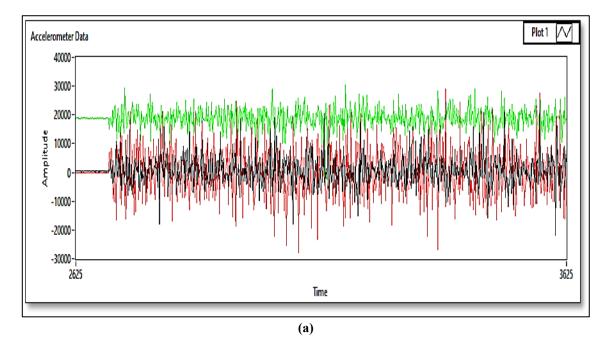


Figure 5: (a) LabVIEW software (b)Time Domain (Sigview) (c) Frequency Domain (Sigview)

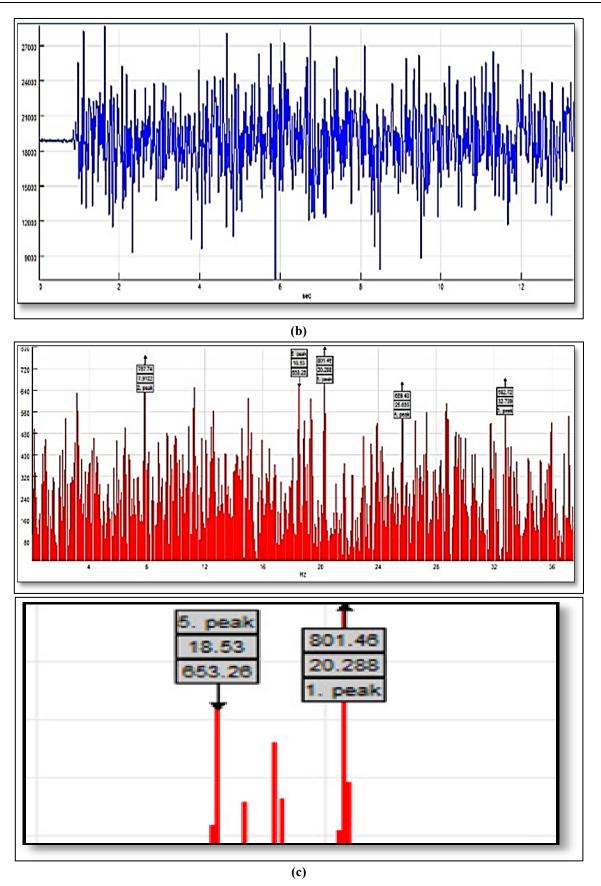


Figure 5: Continued

When applying the practical case with the theoretical case using the Ansys software, it was found that the value of the natural frequency is higher than the hypothetical case due to the presence of dampers and bearings, where the value of the frequency is 229 Hz, while that the default value is 164 Hz. Figure 6 shows the mod 1 shape of the system and the center point of vibration, and below are some properties for mash in Ansys software.

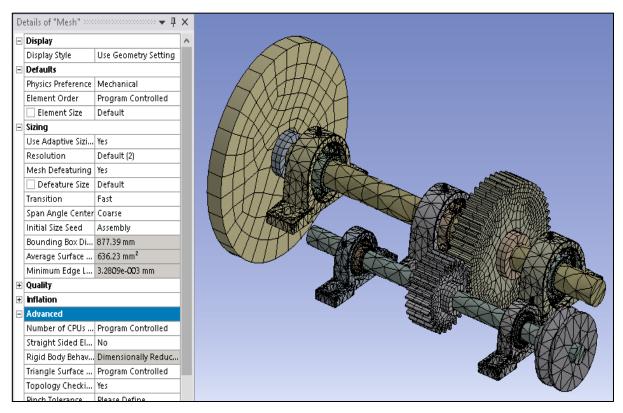


Figure 6: Mash (assort) properties for Rotational System in Ansys program in normal state

The modal continence a damped Coriolis effect must be neglected because is not firmly connected to the ground, and applied full load with full speed 209 rad/s CW by belt drive when it is transient the rotational speed from the motor to a second pully.

By ANSYS software it finds the mode shape from 1 to 7 the area where deformation occurs and is dangerous is located near the flywheel, because of the torque coming from it. the representation of the practical values extracted on the graph is promised as shown in Figure 7.

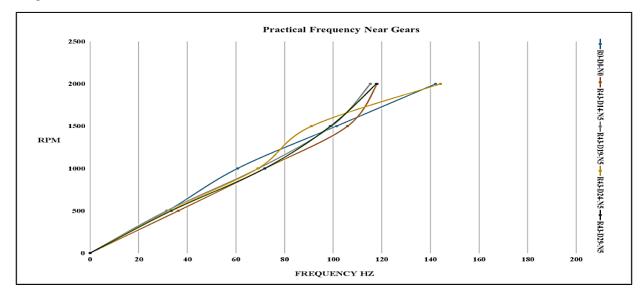


Figure 7: Speed & Frequency chart

The graph where shows the relationship between speed and frequency as a function of the size of the holes, where it is clear that in the absence of holes in the gears, the relationship is close to the curved relationship. For the presence of holes, we note that when the diameter of the hole is 19 mm in gear, the shape of the relationship is semi-curved, and by extracting the Equations from the curves and using the graphics software, the equations can be found and choosing the most suitable, it turns out that:

$$speed(rpm) = 13.8224 * x^{1.0291}$$
 (13)

x: is the value of frequency

Where the value of x represents the practical frequency of the system that contains the gear with 5 holes and a dimeter 19 mm.

It was found that in the case of 5 holes in the gear with a diameter of 19 mm, the frequency is somewhat stable and better than the gears with 14, 19 and 29 mm.

Through the study of the previous research, most of the research did not discuss the effect of the holes on the vibrations and the vibration they generate that harm the system in general and the gears in particular. The effect of vibrations on the system was discussed in this research, which distinguishes it from the rest of the research

Circular holes and how to increase the impact of these holes on the machine and especially its efficiency. Ideal conditions for systems with groove super gears are found by considering vibration and noise values as factors. Two main results findings in the system's performance were that the 5-hole gears with a diameter of 19 mm were superior to the gears of 14, 24, and 29 in diameter in terms of frequency and noise.

#### 4.2 Stress Concentration in Gear

The highest value of the torque in the system is taken, which is produced at a speed of 2000 rpm, and by extracting the tangential force  $(W_t)$  and the axial force  $(W_n)$  of the gear, the

$$W_t = \frac{2T}{D_p} \tag{14}$$

where T = 7.265 N.m and  $D_p = 175mm$ 

$$W_n = W_t \tan\theta \tag{15}$$

After solving Equations (14) and (15), apply these values in the ANSYS program to get the maximum stress and strain.  $W_t = 81.892 N$  and  $W_n = 29.806 N$ .

Using Ansys to simulate the real case and find the stress concentration for each gear, whether it is with holes or without holes. We note in Figure 8 that the distribution of stresses in gears varies between each case and the optimum case was clarified in the gear (R43-D19-N5) and (R43-D29-N5) respectively. The values of the stresses are shown in Figure 9 the highest values of stress concentration.

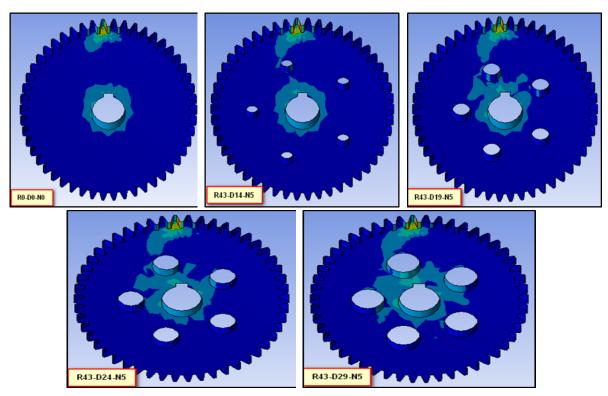


Figure 8: Stress concentration for each case

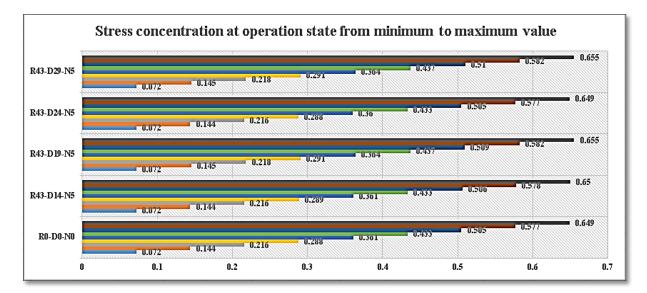


Figure 9: Stress concentration at operation state from minimum to the maximum value

When comparing the values for the stress concentration in the cases of gears in Table 6, it was found that the defined spur gears are in which the stresses are concentrated on the tooth root of the spur gear, which leads to tooth damage, as shown in the case (R0-D0-N0) where the stresses are concentrated in the root zone.

In the case of lightning hole gears, the stresses are distributed over many areas in the spur gear, and this is due to the presence of holes that absorb the stresses as much as possible. The spur gears (R43-D19-N5) and (R43-D29-N5) have a maximum stress concentration value that is 91% higher than other gears.

# **5.** Conclusion

- Lightweight gear manufacturing through the use of perforated gear blanks is widespread in the industry. Helping engineers meet their mechanical and environmental needs, which primarily include reducing mass and increasing operating speed. However, structural integrity and durability must be maintained.
- The current study conducted a comprehensive study on gears and how to increase their effectiveness by controlling their weight with circular holes and studying the effect of these holes and their diameter on performance in general and in particular.
- The ideal condition of the perforated gear system is found by calculating the vibration and noise values of two components. The 5-hole gear is better with a hole diameter (0.1085 \* Dp) which is equivalent to 19 mm in this research with respect to the gear diameter of 175 mm, as it was found to be better than the holes with 14, 24, and 29 mm diameters than 5-hole gears In terms of frequency and weight with parameters of durability and distribution of stress concentration, which are two important factors in the performance of the system, where the maximum value of stress concentration in the root of the tooth reached  $0.655 \ M.N/m^2$

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# **Author Contributions**

Methodology Ali Raad Hassan; Almurtadha Ahmed; Software Almurtadha Ahmed; Formal Analysis Almurtadha& Ali.; Writing-Original Draft Preparation, Ali; Writing-Review & Editing, Almurtadha Ahmed; Ali Raad Hassan. "All authors have read and agreed to the published version of the manuscript."

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# Data availability statement

The data that support the findings of this study are available on request from the corresponding author.

# **Conflicts of Interest**

"The authors declare no conflict of interest."

# **Parameters and Symbol**

Symbol	Parameter	Unite
$I_P$	Moment of Inertia for pully	$kg.m^2$
$I_{GP}$	Moment of Inertia for pinon	$kg.m^2$
$I_f$	Moment of Inertia for flywheel	$kg.m^2$
$I_{GG}$	Moment of Inertia for Gear	$kg.m^2$
Í <sub>G</sub>	Equivalent Mass Moment of Inertia for Gears Meshing	$kg.m^2$
$l_1$	Length of small shaft	m
$l_2$	Length of a big shaft	т
$\overline{K_1}$	Stiffness of small shaft	N/m
<i>K</i> <sub>2</sub>	Stiffness of the big shaft	N/m
Κ́2	Equivalent Stiffness of big shaft	N/m
$\omega_n$	Natural Frequency	Hz
$\omega_{mesh}$	Rotation Frequency of the Gear Shaft	Hz
$n_G$	The Rotational Speed of Gear	rpm
$n_P$	The Rotational Speed of the Pinion	rpm
N2	Number of the Gear Teeth	-
N1	Number of the Pinion Teeth	-
EOM	Equation of motion	-

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