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Maximizing the Value of Decay Rate for a Vibrated Beam

Abtract-*This paper presents design controllers by using control strategies and (LMI) in order to attenuate the vibrations of a beam. An Aluminum beam with fixed-fixed configuration was chosen as an application example of (AVC) of structure. Six conditions had been taken for (AVC) of the beam. In each condition the beam was the same, the changes was in the actuator and in the disturbance according to the location and the kind of the force applied. The attenuation in vibrations of this beam is in maximizing the decay rate (increasing the damping) and limiting the amplitude of the Beam. In this study Pzt actuator had be used, this Pzt have some constraints in the maximum voltage that can be applied, so the input signal must be bounded and limited to some value. In the result there are four requirements for (AVC), stability, input peak bound, output peak bound, and maximizing the decay rate. These requirements had been formulated in the form of (LMI). These (LMI) can be solved by using The Method of Centers For Minimizing The Eigen values. Once the problem solved, the response of the system in the time domain and in frequency domain had been plotted with controller and without controller. The percentage of reduction in the settling time for condition one was (75.9%), while for condition four was (94.6%) and for condition five was (88.32%). The settling time for conditions two and three had increased which means these two conditions are not useful for active vibration control*

Keywords- *LMI, Active Vibration Control, smart structures,Pzt.*

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1. Introduction

Light structures are usually lightly damped, which can cause large amplitude of vibration. Any disturbances in these systems can degrade the demand performance of mechanical systems sand lead to a number of undesirable circumstances. Structural or mechanical failure can often result from sustained vibration (e.g. cracks in airplane wings). Electronic components used in airplanes, automobiles, machines, and so on, may also fail because of vibration, shock, and/or sustained vibration input. Therefore, the elimination of structural vibration has been attracting the attention of engineers. An attractive methodology for attenuation structural vibration is to use Active Vibration Control (AVC) [1].

A linear matrix inequality (LMI) has the standard form [2]

$$F(p) = F_0 + \sum_{i=1}^{m} p_i F_i > 0 \qquad (1)$$

Where $p \in \mathbf{R}^m$ is the variable and the symmetric matrices $F_i = F_i^T \in \mathbf{R}^{n\times n}$, i = 0, 1, 2…, , are given. The inequality symbol in equ.1 means that (p) is positive definite.

In the last decade LMI has been used to solve many roblems, not necessarily active vibration control, which until then were not possible using other methodologies [2,3]. Once formulated in terms of an LMI, a problem can be solved efficiently using convex optimization algorithms, for example, interior-point methods [3]. [4] used LMI to design an output feedback controller to increase the damping in some modes of a cantilever beam. However, the resulting matrix inequalities involved bilinear matrix inequalities (BMI) in unknown variables, and hence it became a no convex optimization problem. Because of this, the BMI could not be solved directly using standard convex optimization software, and it was necessary to use an iterative method, as cone complementary linearization algorithm, described in [4]. The article [5] describes the application of LMI to design an active control system. The positioning of the actuators, the design of a robust state feedback controller and the design of an observer are all achieved using LMI. Active vibration control of a flat plate is chosen as an application example. The simulation system results demonstrate the efficacy of the approach, and show that the control system increases the damping in some modes. [6]Uses a cantilever beam with bounded PZT actuators and sensor. Three control methods that have been successfully

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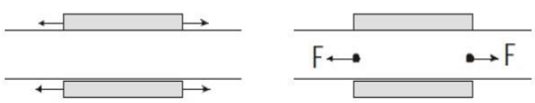
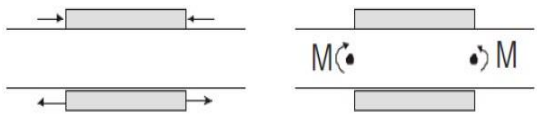
implemented to suppress the vibration of the beam in this study. The first method of active control uses derivative controller. The second method of active control uses the Proportional and Derivative (PD). The third method of active control is the state feedback with a full state observer a linear quadratic regulator (LQR) optimal control method was implemented for vibration suppression in simulation study and real-time control. The LQR state feedback method provided the best vibration suppression compared to the derivative control and PD control. [7] direct output feedback based active vibration control has been implemented on a cantilever beam using Lead Zirconate-Titanate (PZT) sensors and actuators. Three PZT patches were used, one as the sensor, one as the exciter providing the forced vibrations and the third acting as the actuator that provides an equal but opposite phase vibration/force signal to that of sensed to damp out the vibrations. The designed algorithm is implemented on Lab VIEW 2010 on Windows 7 Platform. In this study, a beam will be chosen as an application example of (AVC) of structure. The beam is considered fixed-fixed configuration. The attenuation in vibrations of this beam will be in maximizing the decay rate (increasing the damping) and limiting the amplitude of the beam. The Pzt actuator will be used, the Pzt have some constraints in the maximum voltage that can be applied, and so the input signal must be bounded and limited to some value. The purpose of this study is to illustrate the design procedure of an Active Vibration Control system using LMI. A procedure that was first proposed by [5], is used. A controller is to be designed to satisfy the requirements of stability, input peak bound, output peak bound, and maximizing the decay rate, i.e. increasing the damping. The beam is aluminum with surface bonded PZT (Lead-Zirconate-Titanate) patches. The surface bonded piezoelectric parches are used as actuators and a source of disturbances. If two Pzt elements are fixed on the both asides of the beam element, the longitudinal and lateral motion can be configured for the beam by these two Pzt. If the voltage applied on these elements are in-phase, then a longitudinal motion is obtained. If the voltages applied on the Pzt elements are out-of-phase, then it will generate moments at the end of the Pzt elements, generation a lateral motions [9]. As showed at Fig.1.



(a)



(b)

Figure 1: a: Pzt actuators in-phase. b. Pzt actuators out-of-phase

Several conditions will be taken in each condition the structure will be the same, the different will be in the actuator placement, the type of the force that will generated by the actuator, and the disturbance placement. The purpose of taking several conditions is to specify the best condition according to actuator and sensor displacement.

2. Nodal State-Space Model

The standard form of the state-space model is:

$$\begin{aligned} \dot{\boldsymbol{X}} &= A\boldsymbol{X} + B_u\boldsymbol{u} + B_w w \\ y &= C\boldsymbol{X} \end{aligned} \quad (2)$$

Where, $\boldsymbol{X}$ is called state vector, $\boldsymbol{u}$ is the state input vector, w is the state disturbance, y is the output vector, A is the state matrix, B_u is the input vector. B_w is the disturbance vector. C is the output vector. The nodal state-space model of a structure can be formulated by using the second order structural model which can be obtained by the finite element method (FEM), such a model is given by[9]:

$$\begin{aligned} M\ddot{\delta} + D\dot{\delta} + K\delta &= B_{ou}\boldsymbol{u} + B_{ow}w \\ y &= C_{o\delta}\delta \end{aligned} \quad (3)$$

In the above equation δ is the n_d x1 nodal displacement vector, where n_d is the number of degree of freedom(DOF), $\dot{\delta}$ is the (n_dx 1) nodal velocity vector; $\ddot{\delta}$ is the (n_dx 1) nodal acceleration vector; $\boldsymbol{u}$ is the 1 x 1 input vector; w is the 1 x 1 disturbance vector; y is the output vector, r x 1, where r is the number of outputs; M is the mass matrix, n_dxn_d; D is the damping matrix, n_dxn_d;and K is the stiffness matrix, n_dxn_d . The input matrix $B_{ou}$and B_{ow}they are n_dx 1, the output displacement matrix is $C_{o\delta}$ r x n_d. In order to obtain state representation from the nodal model equ.3, the latter equation will be rewritten as follows

$$\begin{aligned} \ddot{\delta} + M^{-1}D\dot{\delta} + M^{-1}K\delta & \\ &= M^{-1}(B_{ou}\boldsymbol{u} + B_{ow}w) \\ y &= C_{o\delta}\delta \end{aligned} \quad (4)$$

The state ector X is defined as a combination of the structural displacements, δ, and velocities$\dot{\delta}$, [9]

$$\boldsymbol{X} = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} \delta \\ \dot{\delta} \end{bmatrix} \quad (5)$$

In this case, equ.4 can be rewritten as follows:

$$\dot{X}_1 = X_2$$
$$\dot{X}_2 = -M^{-1}KX_1 - M^{-1}DX_2 + M^{-1}B_{ou}\boldsymbol{u} + M^{-1}B_{ow}w$$
$$y = C_{o\delta}X_1$$

Combining the above equations, the state equations can be obtained as in the standard form equ.2, with the following state-space representation [10]:

$$A = \begin{bmatrix} 0 & I \\ -M^{-1}K & M^{-1}D \end{bmatrix}, \quad B_u = \begin{bmatrix} 0 \\ M^{-1}B_{ou} \end{bmatrix}, \quad B_w = \begin{bmatrix} 0 \\ M^{-1}B_{ow} \end{bmatrix}, \quad C = [C_{o\delta} \;\; 0 \text{ x } C_{o\delta}] \qquad (6)$$

Use $N = n_d \text{x } 2$, A is N x N , B_u and B_w they are$N$x1, and C is 1 x N.

3. State-feedback Analysis via Miss

The problem to be investigated is the state feedback control, with the control law [5]:

$$\boldsymbol{u} = G_{fb}\,\boldsymbol{X} \qquad (7)$$

$[G_{fb}]$ is the "Gain of the feedback" vector and it have to be found in such a way satisfying the requirement of active vibration control system. The stability of the LDI (2) is studied first, that is, whether all trajectories of system of equ.2 converge to zero as t→∞. By the second method of Liapunov, a ufficient condition for this is the existence of a quadratic function

$(\boldsymbol{X}) = \boldsymbol{X^TPX}$ (8)

$\boldsymbol{P} > 0$ that decreases along every *nonzero trajectory* of (2). If there exists such a $\boldsymbol{P}$, equ.2 is said to be $asymptotically\ stable$. The equ.2 is said to be $Norm-Bounded$ LDI if [10]:

$$w^Tw \leq y^Ty \qquad (9)$$

With $\boldsymbol{u} = G_{fb}\ \boldsymbol{X}$, the NLDI is $asymptotically\ stable$ if there exist

$Q > 0$

Such that

$$\begin{bmatrix} \begin{pmatrix} QA^T + Y^TB_u^T + AQ \\ +B_uY + B_wB_w^T \end{pmatrix} & QC^T \\ CQ & -I \end{bmatrix} < 0 \qquad (10)$$

Where$Y = G_{fb}Q$. The applicability of the second method of Liapunov to control theory is not limited to stability analysis. It can be applied to the study of the transient response [10] (Decay rate) behavior of linear and nonlinear systems. The formulation that impose the decay rate on the close loop system is as follows:

$$\dot{V}(X) \leq -2 \propto V(X) \qquad (11)$$

Where α is the decay rate. Then $V(X(t)) \leq V(X(0))e^{-2\alpha t}$,[2]. So the constraint of the decay rate can be expressed as follows:

$$\begin{pmatrix} \lambda Q - (AQ \\ +QA^T + B_uY + Y^TB_u^T) \end{pmatrix} \geq 0 \qquad (12)$$

Just as quadratic tability, it can also be interpreted in terms of invariant ellipsoids [5], it can interpret quadratic stabilizability in terms of hold able ellipsoids. The ellipsoid

$$\varepsilon_Q = \{x \in R^n | X^TQ^{-1}X < 1\} \qquad (13)$$

is hold able for the system of equ.2 if there exists a state-feedback gain G_{fb} such that is invariant for the system of equ.2 with $\boldsymbol{u} = G_{fb}\boldsymbol{X}$. If LMI equ.10 holds, then the *Liapunov function* $V(X) = X^TQ^{-1}X$ satisfies $\dot{V}(X) < 0$,this means that

$$X(0) \in \varepsilon_Q \rightarrow \forall t > 0, X(t) \in \varepsilon_Q \qquad (14)$$

The Pzt actuator have some constraints in the maximum voltage that can be applied, and this, also, can be treated as LMI. So a new constraint is added on the input peak, to ensure that: ‖(t)‖ < μ , ∀t ≥ 0.

Where μ is the input peak bounder. This can be formulated in LMI form

$$\begin{bmatrix} Q & Y \\ Y^T & \mu^2 I \end{bmatrix} > 0 \qquad (15)$$

In this case, for each initial condition $X(0)$. Use $\mu = 10$ [5].

Equ.7 is a property that can be used to bound the output signals. This means that for every initial condition started inside the ellipsoid, all states will stay inside the ellipsoid [5].

$$\max_{x(t)\in\varepsilon_Q} y(t)^Ty(t) \leq \beta^2 \qquad (16)$$

Where β is the maximum value of the output peak in Euclidian norm. With $y = CX$. Equ.16 can be formulated as a constraint on Q.

$$\beta^2 I - CQC^T > 0 \qquad (17)$$

By using **Shur complements** [3], the condition of equ.14 can be formulated as LMI

$$\begin{bmatrix} 1 & X(0)^T \\ X(0) & Q \end{bmatrix} \geq 0 \qquad (18)$$

Consequently, if $Q > 0$ exists, that satisfies equ.10 and equ.17, then, for every initial condition (0) , where equ.18 holds, the output peak signal is restricted by β.

The input command $\boldsymbol{u}$ and the output y ensure [9]

$$\forall t \geq 0, \quad \begin{cases} \|\boldsymbol{u}(t)\| < \rho e^{-\alpha t} \\ \|y(t)\| < \beta e^{-\alpha t} \end{cases} \qquad (19)$$

In summary, the controller design ng the result of the following LMI problem [1]:

$$\min \lambda \quad \text{subject to} \tag{20}$$

$$\begin{bmatrix} \begin{pmatrix} -QA^T - Y^T B_u^T - \\ AQ - B_u Y - B_w B_w^T \end{pmatrix} & -QC^T \\ -CQ & I \end{bmatrix} > 0 \tag{21}$$

$$\beta^2 I - CQC^T > 0 \tag{22}$$

$$\begin{bmatrix} 1 & X(0)^T \\ X(0) & Q \end{bmatrix} \geq 0 \tag{23}$$

$$\begin{bmatrix} Q & Y \\ Y^T & \rho^2 I \end{bmatrix} > 0 \tag{24}$$

$$Q > 0 \tag{25}$$

$$\begin{pmatrix} \lambda Q - (AQ + QA^T \\ +B_u Y + Y^T B_u^T) \end{pmatrix} \geq 0 \tag{26}$$

Where $Y = G_{fb}Q$. The problem above solved by *thod of Centers For Minimizing the Generalized Eigenvalue Problems*. [11].
In this paper, the problem has been solved by using Matlab toolbox. The commands for solving the problem is described in[12].

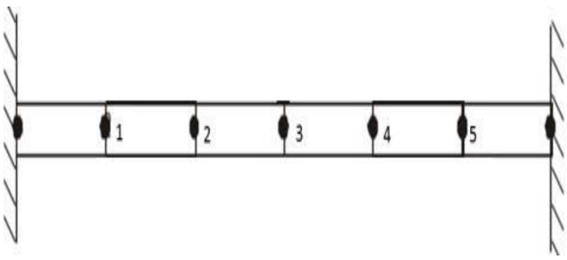
4. Application Example

In this study, a beam is chosen as an application example of (AVC) of structure. This beam is considered to be fixed-fixed configuration, the properties of the beam is described in Table.1.

Table.1 Beam properties and dimensions.

Property	Value
Length	600 mm
Width	30 mm
Thickness	5 mm
Young's modulus	70 Gpa
Density	2710 kg.m^{-3}

By the (FEM), the beam is discretized mathematically into a finite number of rectangular elements with equally dimensions as shown in Figure 2, each element has 2_ nodes each node has three degree of freedom: horizontal displacement(u), vertical displacement(v), and in plane rotation,θ.



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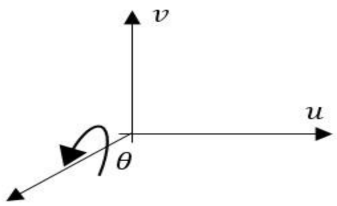

(b)

Figure 2.a: Schematic diagram of the discretized beam b. The local coordinate of each node.

The discretized beam consisting of a finite number of lumped elements had been described by ordinary differential equations equ.3 in which time is the independent variable. Several conditions had been taken for AVC as follows:

Condition1 In this condition, one Pzt actuator, one Pzt disturbance, and one sensor is used as described in Figure 3. The actuator is driven out-of-phase. It generate pair of moments one at node 1, and the other at node 2. The Pzt-Disturbance will be driven out-of-phase. It will generate pair of moments one at node 4 and the other at node 5. The sensor measure the vertical displacement of the node 3. The vectors input, disturbance and the output will be as follows:

$$B_{ou}{}^T = [0\ 0\ -1\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0] \tag{27}$$

$$B_{ow}{}^T = [0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ -1\ 0\ 0\ 1] \tag{28}$$

$$C_{o\delta} = [0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0] \tag{29}$$

$$\boldsymbol{X}(\boldsymbol{0}) = [0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0 - 1\ 0\ 0\ 1] \tag{30}$$

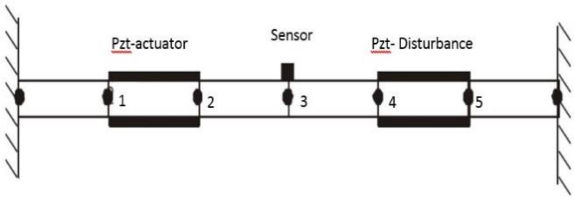

Figure 3 :Schematic picture showing the system of condition1

The impulse response in the time domain with controller off is shown in Figure 4, with the controller on for condition1 is shown in Figure 5, and in the frequency domain is shown in Figire 6.

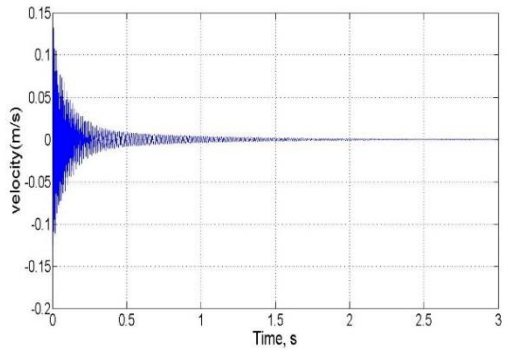

Figure 4: The impulse response in time domain for the condition1 without controller.

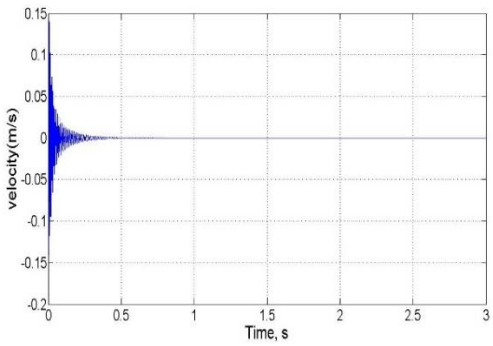

Figure 5: The impulse response in the time domain for condition1 with controller on.

Condition2 In this condition, one actuator driven in-phase is used, it generates longitudinal forces at nodes 1 and 2, and an impulse hammer as a source of disturbance is used. The description of this condition is shown in Figure 7.
The vectors input, disturbance, and the output will be as follows

$$B_{ou}{}^{T} = [-1\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0] \quad (31)$$

$$B_{ow}{}^{T} = [0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0] \quad (32)$$

$$C_{o\delta} = [0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0] \quad (33)$$

$$X(0) = [0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0] \quad (34)$$

The impulse response in the time domain with controller off is shown in Figure 8, with the controller on is shown in Figure 9, and in the frequency domain is shown in Figure 10.

Condition3 In this condition, two actuators driven in-phase is used, it generates longitudinal forces at nodes 1,2,4, and 5, and an impulse hammer is used as a source of disturbance. The description of this condition is shown in Figure 11. The vectors input, disturbance, and the output will be as follows:

$$B_{ou}{}^{T} = [-1\ 0\ 0\ \ 1\ 0\ 0\ 0\ 0\ 0 - 1\ 0\ 0\ 1\ 0\ 0] \quad (35)$$

$$B_{ow}{}^{T} = [0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0] \quad (36)$$

$$C_{o\delta} = [0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0] \quad (37)$$

$$\boldsymbol{X}(\boldsymbol{0}) = [0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0] \quad (38)$$

The impulse response in the time domain with controller off is shown in Figure 8, with the controller on is shown in Figure 12, and in the frequency domain is shown in Figure 13.

Condition4 In this condition, two actuators driven out-of-phase is used, it generates moments at the end of the Pzt elements at nodes 1,2,4, and 5, and an impulse hammer is used as a source of disturbance. The description of this condition is shown in Fig.11.The vectors input, disturbance, and the output will be as follows:

$$B_{ou}{}^{T} = [0\ 0 - 1\ \ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0 - 1\ 0\ 0\ 1] \quad (39)$$

$$B_{ow}{}^{T} = [0\ 0\ 0\ \ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 00\ 0\ 0\ 0] \quad (40)$$

$$C_{o\delta} = [0\ 0\ 0\ 0\ 0\ 0\ 0\ 10\ 0\ 0\ 0\ 0\ 0\ 0] \quad (41)$$

$$\boldsymbol{X}(\boldsymbol{0}) = [0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0] \quad (42)$$

The impulse response in the time domain with controller off is shown in Fig.8, with the controller on is shown in Fig.14, and in the frequency domain is shown in Fig.15.

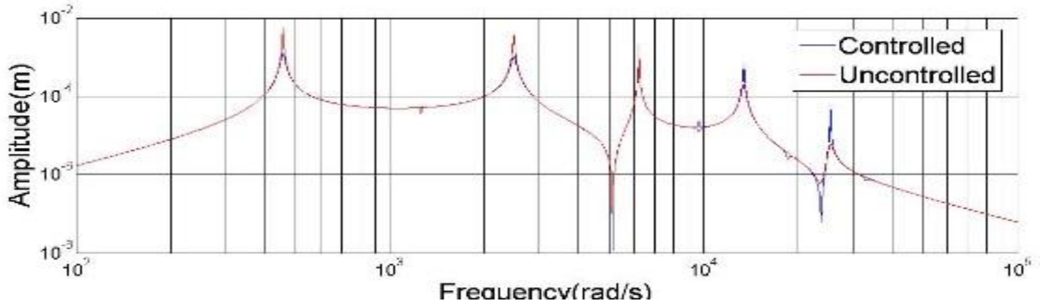

Figure 6: The impulse response in frequency domain for the condition1 with controller and without controller.

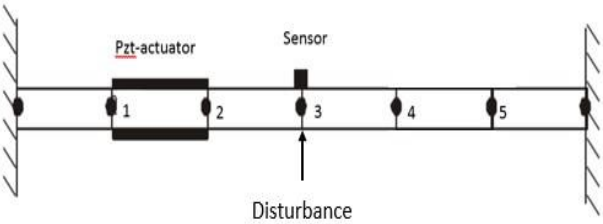

Figure 7: Schematic picture for the condition2 showing the actuator, sensor and the disturbance location.

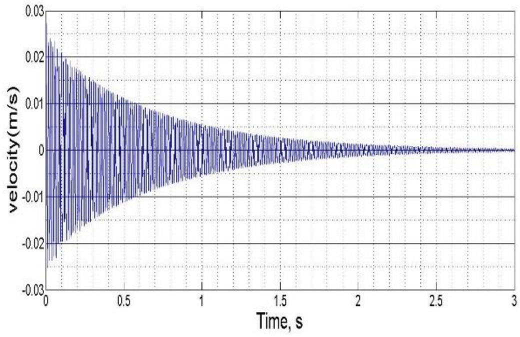

Figure 8: The impulse response in time domain for the condition2, 3,4,5,6 without controller.

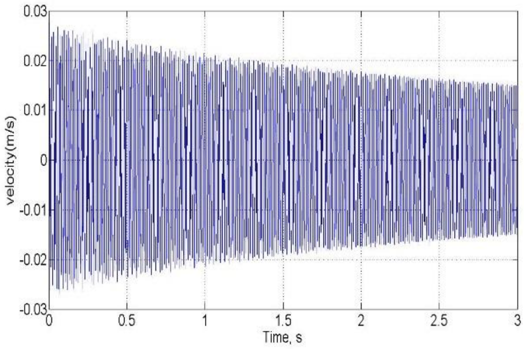

Figure 9: The impulse response in time domain for the condition2 controlled.

Condition5 In this condition, one actuator driven out-of-phase is used, it generates moments at the end of the Pzt elements at nodes 1 and 2, and an impulse hammer is used as a source of disturbance. The description of this condition is shown in Figure 7. The vectors input, disturbance, and the output will be as follows:

$$B_{ou}{}^{T} = [0\ 0 - 1\ \ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ ] \qquad (43)$$

$$B_{ow}{}^{T} = [0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ ] \qquad (44)$$

$$C_{o\delta} = [0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0] \qquad (45)$$

$$\boldsymbol{X(0)} = [0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ ] \qquad (46)$$

The impulse response in the time domain with controller off is shown in Figure 8, with the controller on is shown in Figure 16, and in the frequency domain is shown in Figure 17.

Condition6 In this condition, two actuators will be used, one of them is driven out-of phase, it generates moments at the end of the Pzt elements at nodes 4 and 5, the other is driven in-phase, it generates longitudinal forces at the nodes 1 and 2. An impulse hammer is used as a source of disturbance. The description of this condition is shown in Fig.11.The vectors input, disturbance, and the output will be as follows:

$$B_{ou}{}^{T} = [-1\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0 - 1\ \ 0\ \ 0\ 1\ ] \qquad (46)$$

$$B_{ow}{}^{T} = [0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ ] \qquad (48)$$

$$C_{o\delta} = [0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0] \qquad (49)$$

$$\boldsymbol{X(0)} = [0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ ] \qquad (50)$$

The impulse response in the time domain with controller off is shown in Fig.8, with the controller on is shown in Fig.18, and in the frequency domain is shown in Fig.19.

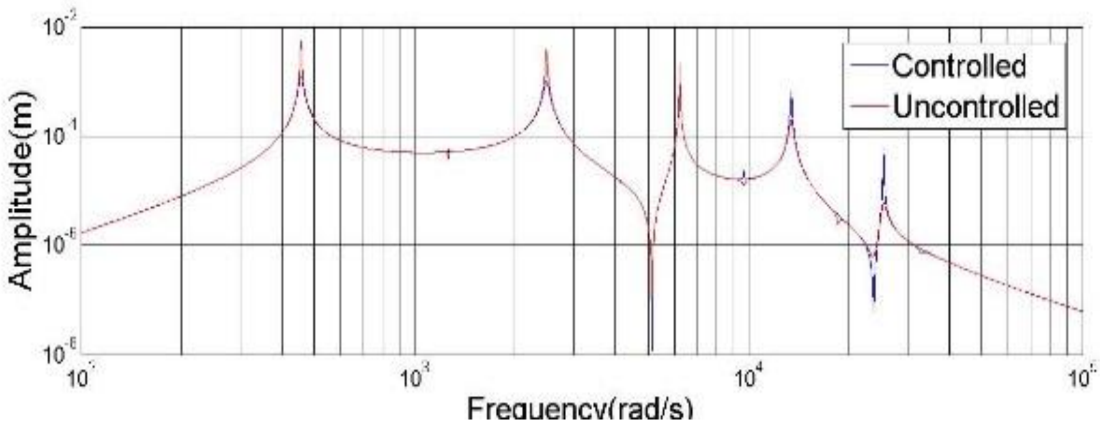

Figure 10: The impulse response in frequency domain for the condition2 with controller and without controller

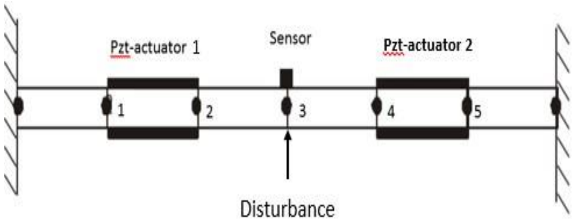

Figure 11: Schematic picture showing the actuator, sensor and the disturbance location.

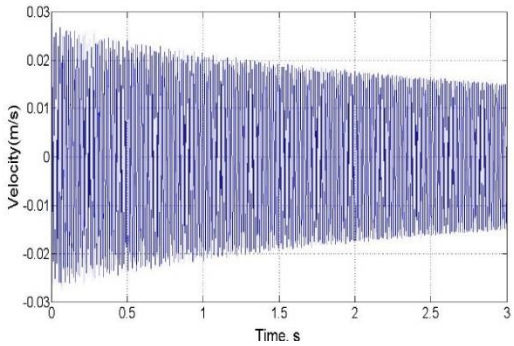

Figure 12: The impulse response in time domain for the condition3 controlled.

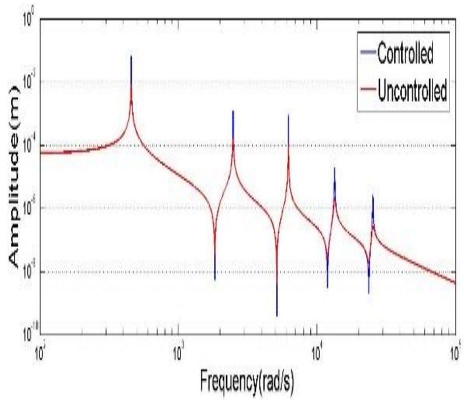

Figure 13: The impulse response in frequency domain for the condition3 with controller and without controller

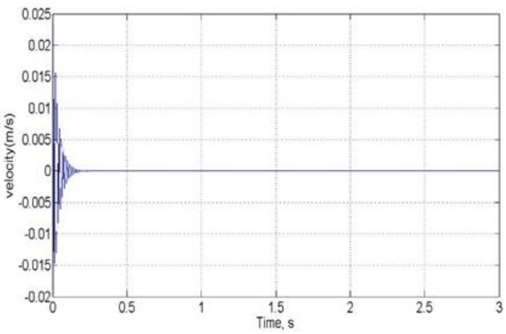

Figure 14: The impulse response in time domain for the condition4 controlled.

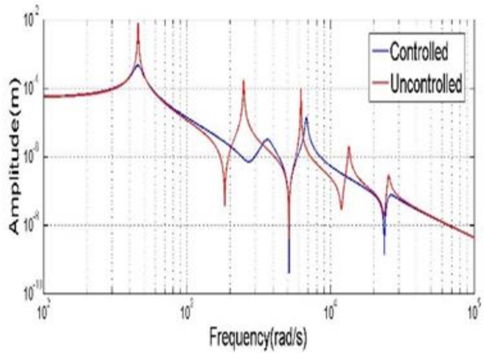

Figure 15: The impulse response in frequency domain for the condition4 with controller and without controller

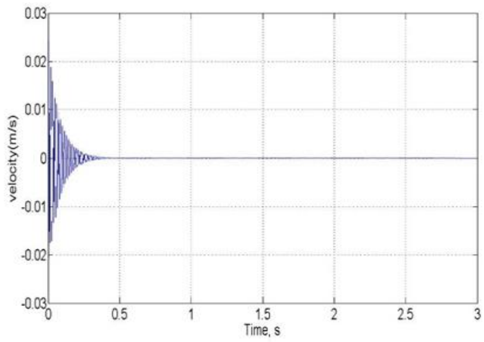

Figure 16: The impulse response in time domain for the condition5 controlled.

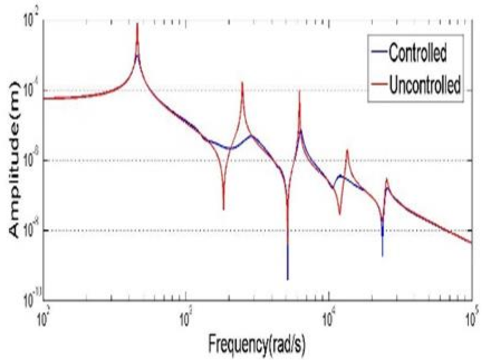

Figure 17: The impulse response in frequency domain for the condition5 with controller and without controller

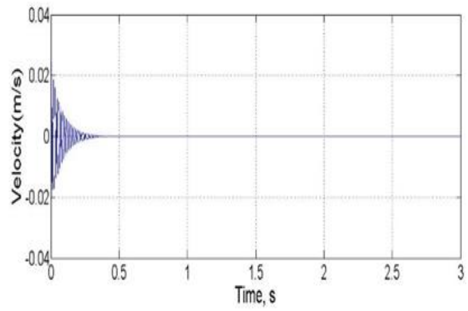

Figure 18: The impulse response in time domain for the condition6 controlled.

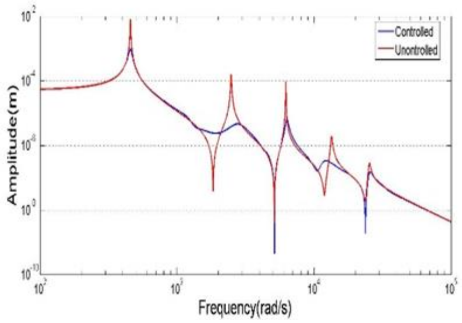

Figure 19: The impulse response in frequency domain for the condition6 with controller and without controller

4. Discussion

A state feedback controller designed to satisfy some requirements. The requirements are stability, input peak bound, output peak bound, and maximizing the decay rate, i.e. increasing the damping. These requirements have all been achieved using LMI. A set of LMIs are formulated. Then they had been solved by the Matlab program, the commands for using Matlab is described in [12]. In this program the feedback state vector had been found. This vector had been substituted in the state space representation described in equ.7 and equ.3. After that, the impulse response in the time domain and in frequency domain had been plotted. The plot of impulse response in the time domain gives an indication of the decay rate. Where the settling time will be found in this plot. As the settling time decreased, the decay rate is increased. In condition1, the settling time is reduced from (1.19s) to (0.286s) as shown in Fig.4 and Fig.5 so the ratio of reduction is (1.19/0.286=4.1608), this means that the damping had increased. The impulse response in the time domain gives also an indicator of stability by observing the response whether it converge to its equilibrium state or diverge to infinity. It is clear that in condition1 the system is stable as shown in Fig.5, i.e. the response in the time domain converge to zero. In the impulse response in frequency domain shown in Fig.6, some of the peaks at the resonance frequencies are reduced for example in mode1 the amplitude is reduced from (5.493mm) to (1.272mm), and in mode3 the amplitude is attenuated by (2.907mm). The attenuation in other modes are described in Table 2. Nevertheless, some peaks have increased, for example, the amplitude at modes 7 and 9 increase by 0.4427 and 0.0462, respectively. This occurred, because of phenomena called $control spillover$. System vibrates in composite mode of vibration, when the system degree of freedom is reduced to specific No. of degree of freedom to be controlled, the effect of other mode of vibration is predominate and the $control spillover$ phenomena appears. Therefore, the actuator placement is very important to choose which mode have to be controlled. For a small number of actuators a typical solution to the location problem is found through a search procedure. For large numbers of locations there is a procedure for actuator placement as described in [10]. In this study several locations of actuator placement have been taken and described as conditions. In condition2 and condition3, the settling time is increased, which means at the conditions the controller have not satisfying the requirement, i.e. control spillover phenomena have been occurred. So this kind of actuator at this placement is not useful for (AVC) for the beam. In condition4, the settling time is reduced from (2.63s) to (0.142s) so the ratio of reduction is (2.63/0.142=18.52), this means that the damping have increased more than those in condition1, because in condition 4 two actuators have been used while in condition 1 one actuator have been used. In the impulse response in the frequency domain, all of the peaks at the resonance frequencies are reduced for example in mode1 the amplitude are reduced from (7.968mm) to (0.8002mm), and in mode3 the amplitude are reduced from (0.1562) to (0.004701). The attenuation in other modes are described in Table 3. In this condition there is no peak increased which means there no control spillover effect.

Table 2: Control performance for the condition1

Mode	Uncon. Amp. (mm)	Con. Amp. (mm)	Attenuation
1	5.493	1.272	-4.221
2	0.03	0.05	0.02
3	3.937	1.03	-2.907
4	0.0156	0.0135	-2.09e-3
5	2.013	1.116	-0.897
6	0.0124	0.0228	0.01038
7	0.204	0.6467	0.4427
8	0.0024	0.0038	1.47e-3
9	0.0060	0.0523	0.0462

In condition5, the settling time are reduced from (2.63s) to (0.307s) so the ratio of reduction is (2.63/0.307=8.566), this means that the damping have increased relative to the uncontrolled condition. In the impulse response in the frequency domain, all of the peaks at the resonance frequencies are reduced, the attenuation is shown in Table 4. In condition6, the settling time is reduced from (2.63s) to (0.314s) so the ratio of reduction is (2.63/0.314=8.375), this means that the damping have been increased relative to the uncontrolled condition. In the impulse response in the frequency domain, all of the peaks at the resonance frequencies are reduced, the attenuation is shown in Table 5.

Table 3: Control performance for the condition4

Mode	Uncon. Amp. (mm)	Con. Amp. (mm)	Attenuation
1	7.968	0.8002	-7.1678
2	0.0053	0.00537	0.000
3	0.1562	0.00470	-0.151499
4	0.87e-3	0.87e-3	0.000
5	9.40e-3	0.41e-3	-8.99e-3
6	0.27e-3	0.273e-3	0.000
7	1.90e-3	0.277e-3	-1.626e-3
8	0.11e-3	0.113e-3	0.000
9	0.27e-3	0.155e-3	-0.120e-3

Table 4: Control performance for the condition5

2	0.0053	0.0053	0.000
3	0.1562	0.0031	-0.153033
4	0.87e-3	0.87e-3	0.000
5	9.40e-3	0.48e-3	-8.9264e-3
6	0.27e-3	0.27e-3	0.000
7	1.90e-3	0.27e-3	-1.628e-3
8	0.11e-3	0.11e-3	0.000
9	0.27e-3	0.15e-3	-0.1262e-3

Table 5: Control performance for the condition6

Mode	Uncon. Amp. (mm)	Con. Amp. (mm)	Attenuation
1	7.968	0.8014	-7.1666
2	0.0053	0.0053	0.000
3	0.1562	4.67e-3	-0.151524
4	0.87e-3	0.87e-3	0.000
5	9.40e-3	4.10e-3	-5.303e-3
6	0.27e-3	0.27e-3	0.000
7	1.90e-3	0.27e-3	-1.6234e-3
8	0.11e-3	0.11e-3	0.000
9	0.27e-3	0.15e-3	-0.1218e-3

5. Conclusion

Table 6 shows the ratio of the settling time, this ratio is determined by dividing the settling time of uncontrolled system over the settling time of controlled system,. This table gives the best condition for maximizing the decay rate. It can be noted that the best of them is condition4, because in this condition two actuators have been used each of them gives two moments at the ends of it. From the impulse response in the frequency of this condition, it can be noted that all modes have been controlled otherwise for the condition1 where the first five modes have been controlled. This gives an implantation that if it is required to control the first five modes, then it will convenient to use conditio1 for control. In addition, if it is required to control on all modes, then it will be convenient to use condition4.

Table 6: Settling time ratio for each condition

Condition	Settling time ratio	Percentage of decreasing the settling time
1	4.1608	75%
4	18.52	94%
5	8.566	88.32%
6	8.3757	88.06%

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