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# Design of Speed-Controller for Brushless DC-Motor Based on Grey Predictor-PID Controller

***Abstract-*** *Brushless direct current (BLDC) motor is a permanent-magnet synchronous motor but the commutator and the brushes are replaced by electronic commutation system. This motor is characterized by many advantages compared with other motors and it being used in a wide range of applications. In BLDC motor, speed control plays an essential role; the speed-controller has to respond rapidly to command changes and to offer enough robustness against the variations of the mechanical load. In this paper, the design of Grey-PID controller is proposed as speed controller of BLDC motor where a new adaptation algorithm is suggested to update the parameters of the adaptive PID controller. The proposed algorithm takes advantages of both of Grey model GM(1,1) and gradient-descent method. The Grey-PID speed control system is simulated by using Matlab\Simulink environment. The simulation results confirm that the proposed controller performs better than the traditional PID controller. By using Grey-PID controller, starting the motor or sudden change of speed can be performed with short rise and settling times, no overshoot and no steady-state error. The proposed controller, also, has a great ability to restrain the fluctuation of speed caused by load disturbance.*

***Keywords***- *Grey predictor, PID controller, Gradient-descent, BLCD motor, Matlab/Simulink.*

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## 1. Introduction

A brushless DC motor is an upgraded version of the conventional DC motor, in this motor the commutator and the brushes have been replaced by electronic commutation system. The construction of the BLDC motor is similar to the permanent magnet synchronous motor while the principle of operation is similar to the DC motor [1]. In order to operate and control BLDC motor a hall-effect sensor that is embedded to the stator, suitable control system, together with 3-phase inverter must be included in the drive system [2]. BLDC motor is characterized by many advantages compared with other motors such as high efficiency; high reliability, nearly maintenance free, high power density, high power factor and low electromagnetic interface and noise levels. BLDC motors are rapidly gaining popularity and it being used in a wide range of applications such as industry, household, computers, automotive, etc. [1,3]. In electrical drive applications, speed control plays an essential role; the speed-controller has to respond rapidly to command changes and to offer enough robustness against the variations of the mechanical load [1,4]. Conventional PID control is widely used as a speed controller for BLDC motor, where this controller is characterized by its robustness, simplicity and reliability. Many mathematical methods (e.g. gradient-descent) and intelligent methods (e.g. fuzzy and neural networks) have been introduced to propose efficient version of adaptive PID controller. In brief, the operation of the adaptive PID controller is based on on-line tuning of the parameters of PID controller which plays a significant role to improve the system performance and expand the area of applications [2,4,5]. Reference [5] added grey model GM(1,2) into the self tuning Neuro-PID control based on radial basis function (RBF) method. Grey-RBF-PID is applied on a small DC servomotor and it performed better than RBF-PID algorithm. Reference [6] proposed a kind of PID control algorithm based on grey theory. This controller can effectively predict and compensate the disturbances and uncertainties in DC servomotor system, thus improving the accuracy of controller. Reference [7] proposed Grey-PID controller based on grey model GM(0,N), the controller is applied into BLDC motor. The motor model is divided into certainty section and uncertainty section. Grey predictor can estimate the uncertainty section. By the proposed strategy, excellent flexibility and adaptability are obtained as well as good robustness and high precision. In

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this paper, a new algorithm is applied to tune the adaptive PID-speed controller; where grey model GM(1,1) is exploited to predict the speed of the motor. Adaptation of the PID parameters is performed based on the predicted speed and by using gradient-descent, which is first-order-iterative optimization algorithm.

## 2. BLDC-Motor Modelling and Control

The BLDC motors have a permanent-magnet rotor and the stator windings are wounded such that the back emf's are trapezoidal. Most BLDC motors have three-phase windings connected in star. The circuit diagram of the BLDC motor supplied by three-phase full-bridge inverter is shown in Figure 1. The power switches T1 to T6 are used to conduct the currents of the windings according to the six gate signals. These signals are determined by the pulse width modulation (PWM) generator and position detector [1,3]. Modeling of the brushless DC motor is based on the following assumptions [1]:

1.Stator resistances are equal.
2.Mutual and self-inductances are constant.
3.The motor does not reach the saturation state.
4.Switching devices of the inverter are ideal.
5.Iron and windage losses are nil.

The mathematical-model of the BLDC motor can be described by the following matrix equation [2]:

$$\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \begin{bmatrix} R & 0 & 0 \\ 0 & R & 0 \\ 0 & 0 & R \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \begin{bmatrix} L_i & 0 & 0 \\ 0 & L_i & 0 \\ 0 & 0 & L_i \end{bmatrix} \begin{bmatrix} \frac{di_a}{dt} \\ \frac{di_b}{dt} \\ \frac{di_c}{dt} \end{bmatrix} + \begin{bmatrix} e_a \\ e_b \\ e_c \end{bmatrix}$$

(1)

Where$v_a, v_b$ and $v_c$ are the applied phase voltages, $i_a$, $i_b$ and $i_c$ are the phase currents, $L_i$ is the total inductance ($L_i$=L-M), L is the self-inductance and M is the mutual-inductance, R is the resistance of the stator winding. $e_a$,$e_b$ and $e_c$ are the trapezoidal emf. The motion equation has the form:

$$T_e - T_L = J\frac{d\Omega}{dt} + B\Omega$$

(2)

Where $T_e$ is the electromagnetic torque (N.m), $T_L$ is the torque of load (N.m), J is the moment of inertia (kg.m$^2$), B is the friction coefficient (N.m.sec) and Ω is the speed (red/sec).The drive system of the BLDC motor, commonly, consists of dual-closed-loop controller. The outer loop is for speed control, while the inner loop is for current or torque control (outside the scope of this paper) [1]. Figure 2 shows the general schematic diagram of BLDC motor control system. The input of the speed control is called speed error that equals the difference between the reference speed and the actual motor speed, the output of the speed controller represents the reference current. The reference current signal, with the measured currents, are processed by the current controller, the current controller determines the duty-cycle of the PWM generator. The required operating speed of the BLDC motor within the rated speed may be achieved by variation of the armature voltage through the duty-cycle of the PWM. If the rotation speed is higher or lower than the reference speed, the speed controller contributes on decreasing or increasing the duty-cycle in order to achieve the desired operating speed [1,2].

### 3. PID Speed-Controller of BLDC Motors

The standard Proportional-Integral-Derivative (PID) speed controller is deal with the error e(t) between the reference speed and the actual speed. Then, the plant is controlled by the signal u(t) which includes a combination of proportion-integration-derivative terms. The typical structure of this controller is shown in Figure 3, the corresponding control function can be expressed as[4,5]:

$$u(t) = K_p e(t) + K_i \int_0^t e(t)dt + K_d \frac{de(t)}{dt}$$

(3)

Where $K_p, K_i$ , and $K_d$ are the coefficients for PID terms, respectively.Once the configuration of the PID speed controller is determined, the parameters $K_p$, $K_i$ and $K_d$ need to be specified. Fixed parameters may not attain a reasonable performance because the drive system of the BLDC motor is multivariable and nonlinear; the defect appears clearly at transient conditions [6]. The target of the PID-speed controller is to achieve the following features [7]:

1. Fast response.
2. No steady-state error for a wide range of speeds.
3. Minimal overshoot.

Since the dynamic characteristics of the BLDC motor are complex, it is required to use an adaptive PID speed-controller with an efficient auto-tuning technique to achieve the above three features [5].

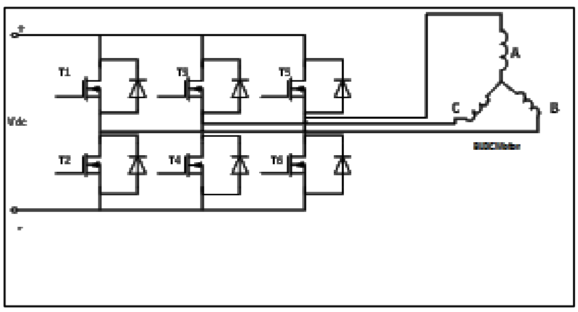

**Figure 1: The electrical circuit of the BLDC motor drive [1,3]**

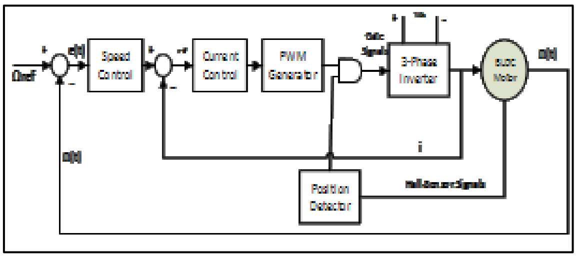


**Figure 2: General block diagram of BLDC motor drive system [1,2]**

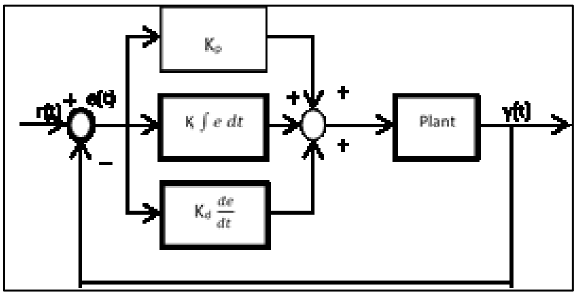


**Figure 3: PID control system [4]**

## 4. Theory of Grey-Predictor

Grey system is a powerful tool for prediction analysis. Actually, this predictor utilizes just small amount of previous information to obtain an accurate prediction without excessive computations. The idea of grey predictor is based on what called Accumulated-Generating-Operation (AGO) on the information sequence to produce a new series. This series is used to formulate a difference equation, and the coefficients of the difference equation can be obtained by using least-squares algorithm. The solution of grey difference equation is then derived, according to this solution. The predicted value is found by using Inverse-Accumulating-Generation-Operation (IAGO). Grey theory introduces several grey models GM(m,N), where m is the order of the difference equation while N is the number of input values. Grey model GM(1,1) is the first-order with one input variable, it is the most widely used for prediction problems [6-8].In this work, GM(1,1) model is used to predict the variation of the motor speed. The procedure of formulating grey model and predicting the motor speed is described in the following:

A- Establish the initial information ( speed values) sequence

$$\Omega^{(0)} = (\Omega^{(0)}(1), \Omega^{(0)}(2), , , , \Omega^{(0)}(n)) \quad (4)$$

where n is the size of the data. One can create a sequence to describe the variation of the information by using fewer (at least four) data points [8,9]. Reference [10] choose n to be 5, where more computations will be required if n is greater than that. The next equal dimension information sequence is obtained by eliminating the most initial data and adding the latest data. At the next sampling time, $\Omega^{(0)}$ will be: $\Omega^{(0)} = (\Omega^{(0)}(2), \Omega^{(0)}(3), , , , \Omega^{(0)}(n+1))$ and so on [5,8].

B- Obtain the accumulated-generating-operation (AGO) of information sequence [8,9]:

$$\Omega^{(1)} = AGO(\Omega^{(0)}) = (\Omega^{(1)}(1), \Omega^{(1)}(2), , , , \Omega^{(1)}(n)) \quad (5)$$

where:

$$\Omega^{(1)}(n) = \sum_{i=1}^{n} \Omega^{(0)}(i) \quad (6)$$

The mean value of two successive data is:

$$\Omega^{(1)}_{Mean}(i) = \frac{\Omega^{(1)}(i-1)+\Omega^{(1)}(i)}{2} \quad (7)$$

where $i = 1, 2, \ldots\ldots, n$

C- Grey difference equation of GM(1,1) has the follow form:

$$\Omega^{(0)}(i) + a\Omega^{(1)}(i) = b \quad (8)$$

and the corresponding first order differential equation is:

$$\frac{d\Omega^{(1)}(t)}{dt} + a\Omega^{(1)}(t) = b \quad (9)$$

D- The coefficients of Eq. (9) are calculated by least-square method as follows [7,9]:

$$\begin{bmatrix} a \\ b \end{bmatrix} = (B^T B)^{-1} B^T w \quad (10)$$

where the data matrix $B = \begin{bmatrix} -\Omega^{(1)}_{Mean}(2) & 1 \\ -\Omega^{(1)}_{Mean}(3) & 1 \\ \vdots & \vdots \\ -\Omega^{(1)}_{Mean}(n) & 1 \end{bmatrix}$

and the data vector $w = \begin{bmatrix} \Omega^{(0)}(1) \\ \Omega^{(0)}(2) \\ \vdots \\ \Omega^{(0)}(n) \end{bmatrix}$

The solution of Eq. (9) at time (i+1) [8,10]:

$$\Omega^{(1)}(i+1) = \left[\Omega^{(0)}(1) - \frac{b}{a}\right] e^{-ai} + \frac{b}{a} \quad (11)$$

By using inverse-accumulating-generation-operation (IAGO), the predicted speed at time (i+1) is obtained as follows [8,10]:

$$\widehat{\Omega}(i+1) = \Omega^{(0)}(i+1) = \left[\Omega^{(0)}(1) - \frac{b}{a}\right] e^{-ai}(1 - e^{a}) \quad (12)$$

## 5. Proposed Design of the Speed Controller

A new speed controller is proposed in this paper, the structure of this controller includes the following stages:

1-Estimation of the motor speed: based on the last five values of the speed, Grey model GM(1,1) is formulated to predict the next state of speed. Grey system is an effective predictor, it needs little information and calculation convenient but it gives an estimated speed $\widehat{\Omega}(i+1)$ with reasonable accuracy [11].

2-Calculation of the estimated speed error, where:

$$\hat{e}(i+1) = \Omega_{ref} - \widehat{\Omega}(i+1) \quad (13)$$

3-Update the parameters of the adaptive PID:

*I. Evaluation of incremental values ($\Delta k_p$, $\Delta k_i$ and $\Delta k_d$):*

The performance function is defined as [5]:

$$E(k) = \frac{1}{2}\hat{e}^2(i+1) \quad (14)$$

By discretization of Eq. (3), the control law of the adaptive PID in discrete form is obtained as [1,12]:

$$u(i) = K_p e_p(i+1) + K_i e_i(i+1) + K_d e_d(i+1) \quad (15)$$

The three error functions ($e_p$, $e_i$ and $e_d$) are defined as [6]:

Proportional error function:

$$e_p(i+1) = \hat{e}(i+1) \quad (16)$$

Integration error function:

$$e_i(i+1) = \hat{e}(i+1) - 2\hat{e}(i) + \hat{e}(i-1) \quad (17)$$

Derivative error function:

$$e_d(i+1) = \hat{e}(i+1) - \hat{e}(i) \quad (18)$$

A regulating adaptation rule of the PID parameters is derived by using the gradient-descent algorithm accompanied with the chain rule, the objective is to minimize the performance function (Eq. (14)) [5]:

$$\Delta k_p = -\psi\frac{\partial E}{\partial k_p} = -\psi\frac{\partial E}{\partial \widehat{\Omega}}\frac{\partial \widehat{\Omega}}{\partial u}\frac{\partial u}{\partial k_p} \quad (19)$$

$$\Delta k_i = -\psi\frac{\partial E}{\partial k_i} = -\psi\frac{\partial E}{\partial \widehat{\Omega}}\frac{\partial \widehat{\Omega}}{\partial u}\frac{\partial u}{\partial k_i} \quad (20)$$

$$\Delta k_d = -\psi\frac{\partial E}{\partial k_d} = -\psi\frac{\partial E}{\partial \widehat{\Omega}}\frac{\partial \widehat{\Omega}}{\partial u}\frac{\partial u}{\partial k_p} \quad (21)$$

Where $\psi$ is the learning coefficient for the update algorithm, $\frac{\partial \widehat{\Omega}}{\partial u}$ is a Jacobian, which denote the sensitive relation between the plant output (motor speed) and the output signal of the PID controller. Substituting equations (13), (14) and (15) into equations (19) , (20) and (21) gives [5]:

$$\Delta k_p = \psi\hat{e}(i+1)\frac{\partial \widehat{\Omega}}{\partial u}e_p(i+1) \quad (22)$$

$$\Delta k_i = \psi\hat{e}(i+1)\frac{\partial \widehat{\Omega}}{\partial u}e_i(i+1) \quad (23)$$

$$\Delta k_d = \psi\hat{e}(i+1)\frac{\partial \widehat{\Omega}}{\partial u}e_d(i+1) \quad (24)$$

*II. Determination of the new values of Kp, Ki and Kd [12,13]:*

$$K_p^{new} = K_p + \Delta k_p$$
$$K_i^{new} = K_i + \Delta k_i$$
$$K_d^{new} = K_d + \Delta k_d$$

The essential idea of this speed controller is that the update procedure is based on the predicted speed not the actual speed. Since the grey predictor provides an accurate prediction without excessive computations, the PID control will be already adapted to deal with any sudden or gradual, large or small change in speed. The proposed Grey-PID speed controller of the BLDC motor is simulated by using Matlab-Simulink environment. Figure 4 shows the block-diagram of the proposed speed controller.

The BLDC motor used in the simulation has the following specifications [7]:

Rated power (Watt): 1000
Rated torque (N.m): 3
Rated voltage (Volt): 500
Rated speed (rpm): 3000
Number of phases: 3
Number of poles: 4
Stator phase resistance R (ohm): 2.875
Stator phase inductance L (mH): 8.5
Moment of Inertia (kg.m$^2$): $0.8\times10^{-3}$
Friction coefficient (N.m.sec): $1\times10^{-3}$

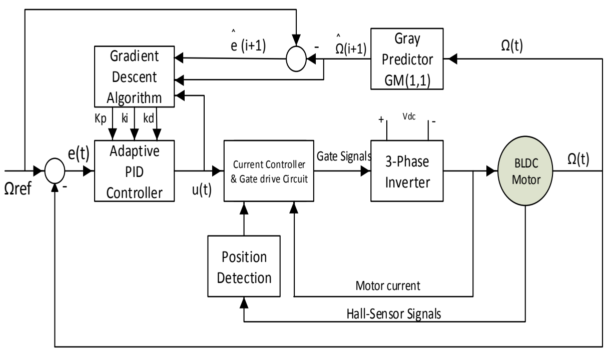


**Figure 4: Grey-PID speed controller of the BLDC motor**

## 6. Simulation Results

In order to investigate and evaluate the performance of the Grey-PID as a speed controller of BLDC motor, simulations have been conducted in accordance with the proposed system shown in Figure 4. Figure 5 shows the controller signal u(t) at different operating states. In this figure, the motor is started with reference speed of 500 rpm and no-load. At time instant 0.2 second, the reference speed is increased to 1000 rpm and then load torque of 2 N.m is applied to motor at time 0.4 second. Results of the proposed controller were compared with the traditional PID controller under the same operating conditions. Figure 6 shows the speed response at starting the BLDC motor with reference speed of 3000 rpm (rated speed) and load torque of 0.5 N.m. It can be observed that the Grey-PID controller performs better speed response than the PID controller where the rising time is much shorter, no overshoot and no steady-state error. Result of another starting test is shown in Figure 7, reference speed, here, is 1000 rpm and the load torque is 3 N.m (rated value). Here, also, the Grey-PID control shows excellent performance and it is extremely superior to the traditional PID controller. The behavior of the motor at sudden change of reference speed value is shown in Figure (8). In this test, the reference speed is reduced from 3000 to 500 rpm at time instant 0.25 second while the motor is operating at no load, the performance of Grey-PID controller, again, is much better than the traditional PID controller. The effect of sudden variations of the load on the speed of motor is appeared in Figure 9 and Figure 10. The motor is subjected to load removal at time instant 0.3 second i.e. the torque is suddenly reduced from full-load (3N.m) to no-load, in this case Grey-PID controller exhibits a great ability to maintain the motor speed without any obvious change. On the contrary, PID controller allows an increase of speed up to about 3.75% and it requires 0.115 second to retrain the speed of the motor to the reference speed and that is shown in Figure 9.The motor is subjected to sudden load at time instant 0.3 second i.e. the torque is raised from no-load to full-load (3N.m), in this case Grey-PID controller exhibits a great ability to maintain the motor speed without any obvious reduction. On the contrary, PID controller allows a drop in speed up to about 3.5% and it requires 0.105 second to retrain the motor speed to the reference speed and that shown in Figure 10.

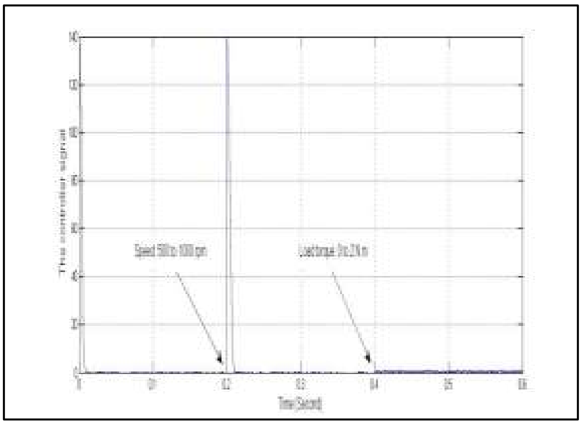

**Figure 5: The signal of Grey-PID controller**

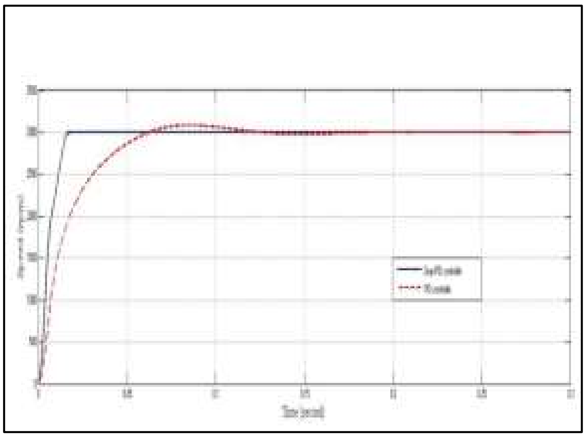

**Figure 6: Starting the motor with reference speed of 3000 rpm and T=0.5N.m**

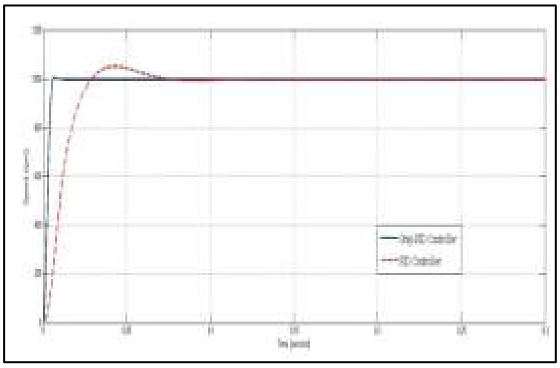

**Figure 7: Starting the motor with reference speed of 1000 rpm and T=3N.m**

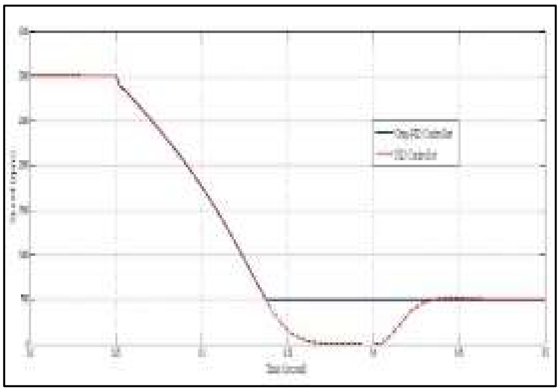

**Figure 8: Speed of the motor at changing the reference speed**

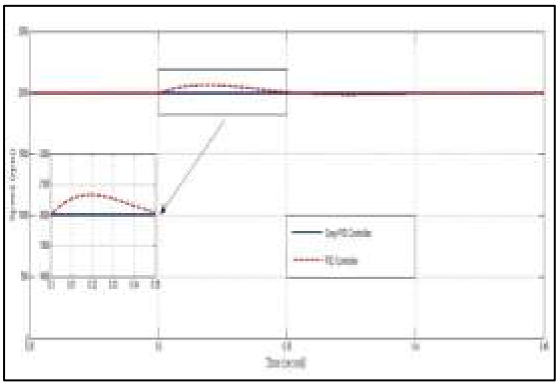

**Figure 9: Speed of motor at decreasing the load**

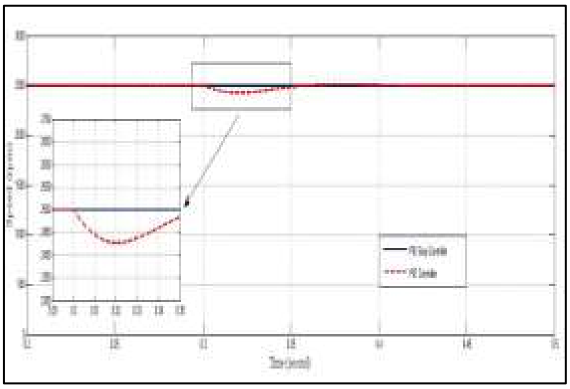

**Figure 10: Speed of motor at increasing the load**

## 7. Conclusion

In this paper, an adaptation algorithm is suggested to update the parameters of the adaptive PID controller. The proposed algorithm takes advantages of both of grey model GM(1,1) and gradient-descent method. The Grey-PID control is designed as a BLDC motor speed controller. The idea of this speed controller is that the update procedure is based on the predicted speed. Since the grey predictor provides an accurate prediction, the PID control will be already adapted to deal with any sudden change in speed. The performance of the proposed controller is compared with the traditional PID controller under the same operating conditions. The simulation results showed that the Grey-PID controller is superior to the PID controller. The proposed controller has a great ability to restrain the fluctuation of speed caused by load disturbance. In addition, at starting the motor or sudden change of speed, the controller improves the performance of the BLDC motor drive system where the speed response has short rise and settling times, no overshoot and no steady-state error.

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