Unviersty of Anbar

Anbar Journal Of Engineering Science©
journal homepage: http:// http://www.uoanbar.edu.iq/Evaluate/

Fuzzy Reliability-Vulnerability for Evaluation of Water Supply System Performance.

S. A. Mutlag [a*] and A. H. Kassam [b]

[a] Department Mechanical Engineering, Al-Anbar University, Ramadi-Iraq,
[b] Department of Production Engineering and Metallurgy, University of Technology, Baghdad-Iraq

P A P E R I N F O

Paper history:
Received … … …
Received in revised form … … …
Accepted … … …

Keywords:
Reliability, Fuzzy set, Fuzzy reliability, Performance Indexes, Water Supply System.

A B S T R A C T

The reliability of water supply system is a critical factor in the development and the ongoing capability to succeed in life and people's health. Determining of its, with high certainty, for performance of water supply system is developed to ensure the sustainability of system. Reliability (Re) plays a great role in evaluation of system sustainability. The probability approaches have been used to evaluate the reliability problems of systems. The probability approach is failed to address the problems of reliability evaluation that comes by subjectivity, human inputs and lack of history data. This research proposed two models; I) traditional model: fuzzy reliability measure suggested by Duckstein and Shresthaand then developed by El-Baroudy; and II) developed model: fuzzy reliability-vulnerability model. The two models implemented and evaluation of water supply system by using two hypothetical systems (G and H). System (G) consists of a single pump and System (H) consists of a two parallel pumps. Triangular and trapezoidal membership functions (MFs) are used to investigate of the reliability measure to the form of the membership function. The results agree with expectations that the reliability of parallel component system {Re_H (0.53)} is higher than the reliability of single component system {Re_G (0.47)}. Moreover, the result by using fuzzy set reduces the effect of subjectively in process of decision-making (DM). The fuzzy reliability vulnerability is able to handle different fuzzy representations and different operation environment of system.

1. Introduction

When developed of the modern technology in the 1960s, the reliability tools are becoming a vital technics in the engineering analyses. Therefore, the reliability in probabilistic approaches has extended to achieve the evaluation of systems performance reliability. The probabilistic reliability approach has become very popular tool to addresses with uncertainty problems [1].

The fuzzy set theory has been addresses to analyze the Fuzzy Reliability (FR) that becomes a proper tool to handle the systems performance reliability with the uncertainty variables. Some of researches have been work by researchers for establishing the Reliability Analysis (RA) based on the fuzzy set theory [2].

* Corresponding author. Tel.: +964 07812818819
E-mail address: satmutt1961@yahoo.com

One of the major goals of systems design is to confirm that the systems perform satisfactorily under many fluctuations the possible of future environment. This premise is particularly exact of large and complex systems (such as Water Supply Systems WSS). The WSS contain conveyance facilities (i.e. pumps and pipes) and reservoirs. These elements are interconnected in complicated networks. Each element is vulnerable in service caused by natural hazards or human error [3].

1.1. Problem definition

The problem of uncertainty that comes with the lack of certainty, human input, lack of history and incomplete record keeping , which impede the decision making process[3].

In another hand, how to obtain failure probabilities when events do not have probability distributions function (pdf) of their lifetime to failures [4].

Because uses of the complicated arithmetic operations to analyze system reliability that arithmetic operations of the complicated extended algebraic operations is using in probabilistic approach [2].

There is a real need to convert to new approaches that can compensate for uncertainty of human perception. A new perspective and methodology is model the behavior of the components as well as the system using the theory and methods for fuzzy sets [5].

1.2. Research Objectives

To develop a model to evaluate the reliability based on performance data, not on failure data as used in the traditional reliability. And developed a model for evaluate reliability by using fuzzy reliability performance through fuzzy set operations without the need to failures events or pdf.

The study proposed two models for fuzzy reliability performance indexes: First model: the traditional model; fuzzy reliability measure suggested by Duckstein and Shrestha (1998) then developed by El-Baroudy; Second model: developed model ;fuzzy reliability-vulnerability model and comparing between them.

1.3. Research Focused on

The methodology of the research consist of three parts: the first part presents the many approaches used to deal with the problems of system performance SP reliability. Second part illustrated the calculation to development of methodology for fuzzy reliability indexes suggested by EL-Baroudy: (1) the reliability-vulnerability index and (2) robustness index. The illustration is proposed by detailed and discussed of the fuzzy performance models. The third Part examines the utility of the two models using two hypothetical cases study (two systems G and H).

1.4. Probability of Traditional Reliability Model

The problem of systems reliability has attracted attention from statisticians and probabilities researchers. The probability reliability analysis (RA) has been used to deal with the problems of uncertainty in system performances (SP) [4].

A Prior knowledge of Probability Density Functions (pdf) for both load L and resistance R and their overlapping (PF) of pdfs are a necessity in the probability approach as in fig (1). Practically, the data of failure is insufficient to provide the information for pdf of distribution. Even if the data of failure is available for estimating the distribution, approximation approach is almost used necessary to calculate reliability of system [5].

$$P_F = \int_0^{\infty} \left[\int_0^{l} f_{L,R}(l,r)\, dr \right] dl$$

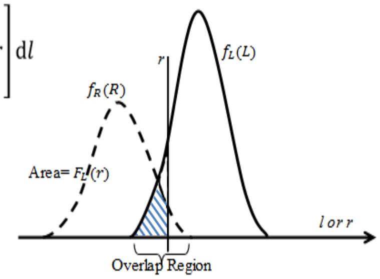

Figure 1. show the probability of failure (PF)[5]

The subjective judgment of the decision makers is used to estimate the pdf of the failure. The subjective pdf is another approach to addresses the data insufficiency. In the subjective probability approach, the degree of exactness is dependent the estimation the judgment of decision makers [6]. Always, only the probability approaches were used for systems reliability analysis (RA). However, the probability approaches are failed to deal with the problems of uncertainty that drives along with subjectivity human inputs and lack of history. There are needs to investigate a developed approach that can handle the problems uncertainty [7, 8].

2. Letreture Review

Some of the approaches are introduced to solve reliability evaluation problems. The following sections, present a review of related researches of the evolving conversion from initially binary state then multi-state finally fuzzy reliability (FR) measures. Julwan, (2014) [5] addressed fuzzy set to evaluate fuzzy reliability model without dependence on historical failure data through qualitative data treatment. He used fuzzy binary state instead of probability theory and discussed that the probability theory does not fully capture the real behavior of most systems, especially systems of several performance levels. Practically, the data of failure is insufficient to provide the information for pdf of distribution. Even if the data of failure is available for estimating the distribution, approximation approach is almost used necessary to calculate reliability of system [5].

Huang, (2014)[9]addressed the multi-state for system to evaluate reliability of system performance. He treated the system as multiple-state of a system through multiple-state of components. The results gave more realistic of system performance that can be used in evaluation reliability and decision making (DM) processes. The probability approaches are failed to deal with the problems of uncertainty that drives along with subjectivity human inputs and lack of history.

Shujie and Zire.(2015)[8] proposed a model for the evaluation of fuzzy reliability. They noted that the single value threshold is not easy to determine. The developed model used the system states as fuzzy event variable. The results showed that the fuzzy model was feasible and effective. They recommended that there are needs to investigate a developed approach that can handle the problems uncertainty.

From the results above it becomes commonly identified that the binary state of fuzzy reliability (FR) modeling of systems or components is very simple and not fully capturing the real behavior of several systems, that have different performance levels.

Kapur, et al. (2011)[6] proposes a methodology to develop two models to evaluate reliability of the components and systems using the fuzzy theory. They noted that fuzzy binary state modeling for reliability systems do not capture the most performance of systems, which has many performance levels. They demonstrated how fuzzy multi-state system modeling of the system or component captured most realistic system performance for its assess plus processes of decision making.

Guijie, et al. (2015)[2] proposed a fuzzy reliability evaluation model with fuzzy variables. They infer that the fuzzy theory expands the traditional concept of a probability. Fuzzy set theory has been used to complement probability theory to evaluate the system reliability when statistical data or knowledge is insufficient for probabilistic analysis.

3. Methodology

3.1. Fuzzy Reliability Measure (traditional model)

The Duckstein and Shrestha(1998)[11] first to propose a fuzzy reliability measure that can be used in the case of both, Load *L* and Resistance *R*, being fuzzy. They proposed uses safety margin S as measure for the system failure, that is:

$$S = R - L \quad (1)$$

The membership function MF of safety margin S from Equ.(1), is considered the system failure. Failure is defined as the condition when *L* larger than *R* and accordingly S < 0. From Fig. (2), the define of fuzzy reliability index, (FR) as :

$$FR = \frac{\int_{S>0} \mu_S(S) d(S)}{\int_S \mu_S(S) d(S)} \quad (2)$$

Where: $\mu_S(S)$: MF of the safety margin.

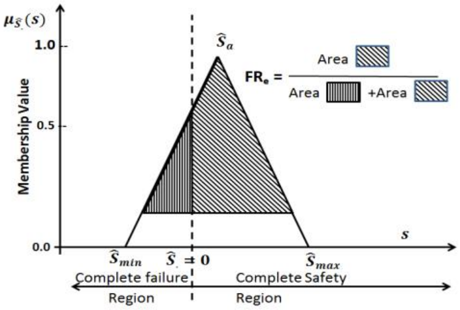

Figure 2. fuzzy reliability measure proposed by Duckstein and Shrestha[8].

The proposed fuzzy reliability measure treats the triangle MF of safety margin as a probability density function (pdf). Figure 2 shows, the zone under the μ(S) MF (shaded zone), represented by $\int_{S>0} \mu_S(S) d(S)$ as the failure zone where total zone is $\int \mu_S(S) d(S)$. By the definition of fuzzy set, a fuzzy failure is represented by a fuzzy set that involve several failure events, according to their membership functions MFs. By using the fuzzy system state $\hat{S}$ as a failure where the failure has some level of evented. Therefore, the system reliability is measure based on the comparison of two fuzzy sets: one representing the system state (safety margin) and the other fuzzy set representing the level of performance levels [12].

3.2. Developed model

3.2.1.Definition of Fuzzy Failure

Guijie, et al. (2015)[2] addressed the case of both fuzzy Resistance $(\hat{R})$ and fuzzy Load $(\hat{L})$. He proposed measure to used fuzzy Safety margin $(\hat{S})$, as a measure for the system failure or as system state, that is:

$$\hat{S} = \hat{R} - \hat{L} \quad (3)$$

Where: $\hat{S}$: fuzzy safety margin; and $\hat{R}$ *and* $\hat{L}$: fuzzy Resistance and fuzzy Load.

Ahmad, (2010)[13] and Guijie, et al.,(2015)[2] explain fuzzy sets, by defining the boundaries of the partial failure zone are ambiguous and varies from one judgment of decision maker to the other on the personal perception of failure. Therefore, these boundaries could not be determined precisely.

Partial failure is more realistic in the performance evaluation process of water supply systems. The performance level (*PL*) represented as a fuzzy membership function (MF) in the following equation [1,3]:

$$\hat{S}(s) = \begin{cases} 0, & if\, s \le s_1 \\ \varphi(s), & if\, s \in [s_1, s_2] \\ 1, & if\, s \ge s_2 \end{cases} \quad (4)$$

Where $\hat{S}$ the fuzzy MF of safety margin; $\varphi(s)$ is functional for representing the subjective view of

PL; s_2, s_1 upper and lower bounds of the partial performance Zone, respectively.

Figure (3) is represented of the definition presented in Equ.(4). The lower s_1 and upper s_2 bounds of the partial failure Zone are presented in Equ.(4). The value of the safety margin $\hat{S}(s)$ less than s_1 is definitely complete failure. The value of the safety margin $\hat{S}(s)$ higher s_2 is definitely complete safety (reliability) and therefore fit in the performance zone. The reliability of performance level (PL) quantified as the following [13]:

$$PL = \frac{s_2 * s_1}{s_2 - s_1} \quad (5)$$

Where: PL is the reliability measure of the performance level. The partial failure is alternate definition of failure for a select between the upper and lower bounds, and performance level. The developed model provides a comprehensive tool for performance [12].

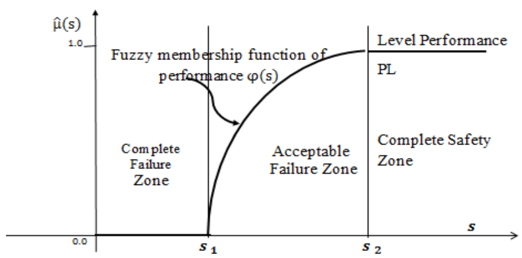

Figure 3. shown representation of partial failure zone[9]

3.2.2. Compatibility Measure

The aims of comparing between the two fuzzy membership functions MFs is to determine the two fuzzy sets match. The reliability evaluation, presented in this research, involves a comparative analysis *Cm* for system state and the predefined performance level (PL). As shows in Fig (4) and Equ(2) The fuzzy compatibility measure is [1]:

$$Cm = \frac{\text{weighting overlap zone between PL and system state } \hat{S}}{\text{weighting intire zone of system state } \hat{S}} = \frac{WOA}{WA} \quad (6)$$

Where: Cm: Compatibility measure between the system state $\hat{S}$ MF and the performance level *(PL);* **WOA** : The weighting of overlap zone between the system state $(\hat{S})$ and the performance level (PL);and $WA_{\hat{S}}$: The weighting zone of the system-state $\hat{S}$.

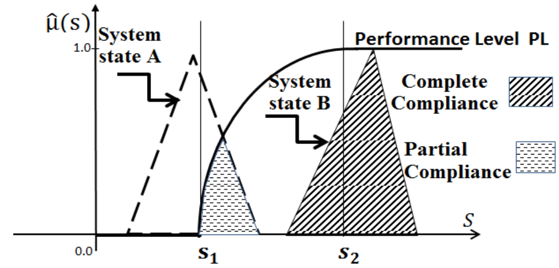

Figure 4.: two compliance between system state and PL[2]

3.2.3. Combined Fuzzy Reliability and Fuzzy Vulnerability

Reliability and vulnerability were used to provide a complete description of system performance SP in case of system failure and to determine the magnitude of the failure event. In order to calculate system reliability at many performance levels must be defined these levels to reflect the several perceptions of the decision makers [3].

A comparison between the fuzzy system state and the fuzzy performance level provide perfect information for about both system reliability and system vulnerability in same time, as in Fig (5). The fuzzy reliability-vulnerability R_v index is formulated as equation follows[12]:

$$R_v = \frac{\max_{i \in K} \{Cm_1, Cm_2, \ldots .. Cm_i\} * PL_{max}}{\max_{i \in K} \{PL_1, PL_2, \ldots .. PL_i\}} \quad (7)$$

Where: R_v is index of fuzzy reliability-vulnerability, PL_{max} is the reliability measure of performance level corresponding to the system state with maximum compatibility $max\{Cm\}$ value; PL_i is the reliability measure of the i^th performance level; Cm_i is the compatibility measure for system state with the i^th performance level; and K is the total number of the performance level.

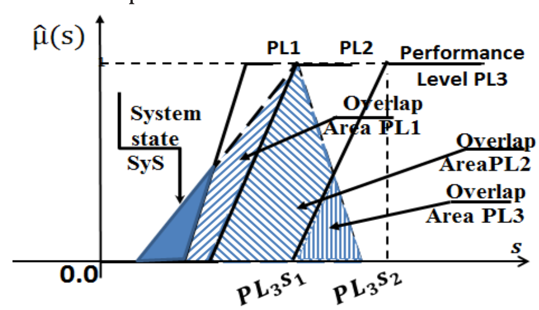

Figure 5. overlap area between system state and three PLs [7]

3.2.3. Fuzzy Robustness indexes

Hashimoto et al., (1982)[4] define the Robustness index (R_o) is the capability of system to adjust to fluctuation of possible future load in fuzzy performance environment, the fuzzy represented of fluctuation changing in future environments obtained by a definition of the performance level and a changing in the system state MF. As a result, the robustness of system is defined as changing in the compatibility measure:

$$R_O = \frac{1}{Cm_1 - Cm_2} \quad (8)$$

Where R_O is the fuzzy robustness index, Cm_1, Cm_2 are the compatibility measure before and after changing in environment, respectively. Equation.(8) reflects that if the higher changing in compatibility the fuzzy robustness of performance is lower value. Therefore the high robustness values allow the system to better performance adapt to new environment.

3.3. Multi-Element Systems

The networks of multi-element systems are (a) serial, (b) parallel, and (c) combined as in Fig (6). Every element has a MF, representative the state of element.

The networks of multi-element systems are (a) serial, (b) parallel, and (c) combined as in Fig (6). Every element has a MF, representative the state of element. The system failure for parallel elements occurs if all the elements of system will fail. In this case, the maximum fuzzy operator is used to determine the system state $\hat{S}_P$ of parallel system as follow[13].

$$\hat{S}_P = max(\hat{S}_1, \hat{S}_2, \dots . \hat{S}_m) \quad (9)$$

Where, m : total number of parallel elements of system.

For a serial element system, the minimum fuzzy operator is used and failure of the system occurs if any of its elements fails. The system state $\hat{S}_S$ serial sys

tem as follow [12]:

$$\hat{S}_S = min(\hat{S}_1, \hat{S}_2, \dots . \hat{S}_n) \quad (10)$$

Where, n : total number of serial elements.

Other combinations of system configurations are deal as different combinations of serial or parallel subsystems. Using either Equs (9) or (10), independently and the overall system reliability index S_{Sys} is obtained, as follow [12].

$$\hat{S}_{SyS} = min\,max\,(\hat{S}_1, \hat{S}_2, \dots . \hat{S}_{n,m}) \quad (11)$$

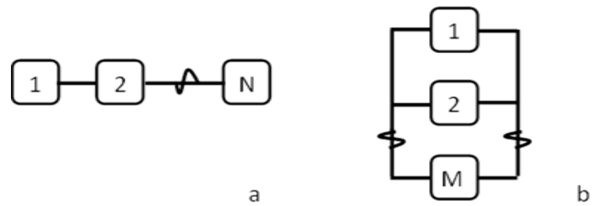

Figure 6. shows network of serial and parallel system.
(a) the network of serial system of N elements.
(b)the network of parallel system of M of elements.

4. Cases Study

The suggested fuzzy reliability FR measures and index of the fuzzy reliability of performance used to evaluate two cases. Such as shown in Fig.(7), system G consists of a single pump, single pipeline and tank, while system H consists of two parallel pumps, two pipelines and tank. In expect, the two parallel pumps in system H increases system reliability as a resulting in higher than system G reliability. Therefore, the value system reliability reflects the variance between two systems (G and H). Notice that the both G and H are under the same load (D) requirement and have the same supply (C) (capacity).

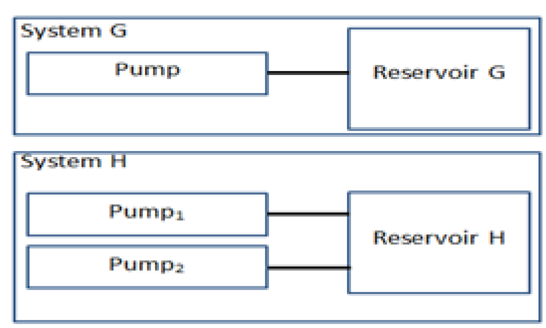

Figure 7. Schematic representation of the hypothetical case studies

The major aims of the cases study are; (I) examine the reliability measure of system H have a greater than reliability measure of system G by suggested that the system H greater than parallel element from system G. (II) Illustrate computational the performance indexes R_v andR_o . (III) Assess the sensitivity of the indexes to many form of MF i.e., triangular (Tg MF) and trapezoidal (Tp MF) to choice form of the MF.

Supposing that the pumps reliability for two systems (G and H) are control the entire reliability of system, two dissimilar scenarios implied to system H: (1) two pumps have the equal C and (2) single of the pumps has a double C greater than the another pump. The totality of the C of the both pumps, in two scenarios (1,2), is equal to C of pump in the system G. Table 1 summary the four scenarios of two systems(G and H) as follow:

Scenario 1, fuzzy triangular membership (Tg MF) represents that a fuzzy demand (Tg D MF) and fuzzy capacity (Tg C MF) for two systems. The Tg D MF and Tg C MF of system divided into the two pumps in system H under the ratio 1:1 (equal value).

Scenario 2, The Tg D MF and Tg C MF of system divided into the two pumps in system H under the ratio 1:2 (nonequivalent value).

Scenario 3, fuzzy trapezoidal membership (Tp MF) represents that a (Tp D MF) and Tp C MF) for two systems. The Tp D MF and Tp C MF of system divided into the two pumps in system H under the ratio 1:1 (equal value).

Finally, **Scenario 4**, The Tp D MF and Tp C MF of system divided into the two pumps in system H under the ratio 1:2 (nonequivalent value).

4.1. Application of Fuzzy Reliability

The usefulness of two models the fuzzy reliability FR measure suggested by Duckstein and Sherstha and the fuzzy reliability indexes model suggested by El-baroudy the both using for performance systems are evaluated by the compare of the results for different levels.

4.1.1.First model

To determine a system state of system is used safety margin concept in Equ.(3). For example; for system H scenario2: from Table (1) the three values of fuzzy capacity (C) and fuzzy demand (D) are nonequivalent values. To determine a Combination of two parallel elements nonequivalent elements for this scenario (2) by using Equ.(10). The results this scenario Combination and system state for four scenarios are listed in Table (1) in two last right columns. Then determine system state of every scenario by using Equ.(3). The result listed in Table (2) used for two models to represent the system states of four scenarios.

Table 1. Summary the four scenarios of two systems (G and H):

No.	Scenario description	System G		pump	System H		Aggregation of system H	
		Capacity C m^3/day	Demand D m3/day		Capacity C m^3/day	Demand D m^3/day	Capacity C m^3/day	Demand D m^3/day
1	Tg MF with equal values distribution between C and D of pumps in system H	(0, 3, 6)	(1, 2, 4)	1	(0, 1.5, 3)	(0.5, 1, 2)	(0, 1.5, 3)	(0.5, 1, 2)
				2	(0, 1.5, 3)	(0.5, 1, 2)		
2	Tg MF with non-equal values distribution between C and D of pumps in system H	(0, 3, 6)	(1, 2, 4)	1	(0, 1, 2)	(0.3, 0.7, 1.3)	(0, 2, 4)	(0.7, 1.3, 2.7)
				2	(0, 2, 4)	(0.7, 1.3, 2.7)		
3	Tp MF with equal values distribution between C and D of pumps in system H	(0,1, 5, 6)	(1,2,3,4)	1	(0, 0.5, 2.5, 3)	(0.5, 1, 1.5, 2)	(0, 0.5, 2.5, 3)	(0.5, 1, 1.5, 2)
				2	(0, 0.5, 2.5, 3)	(0.5, 1, 1.5, 2)		
4	Tp MF with non-equal values distribution between C and D of pumps in system H	(0,1, 5, 6)	(1,2,3,4)	1	(0, 0.3, 1.7, 2)	(0.3, 0.7, 1, 1.3)	(0, 0.7, 3.3, 4)	(0.7, 1.3, 2, 2.7)
				2	(0, 0.7, 3.3, 4)	(0.7, 1.3, 2, 2.7)		

Table 2. Summary details of system state of two systems (G & H).

No. scenario	First Model	
	System state G	System stat H
1	(0,1,5)	(0,0.5,2.5)
2	(0,1,5)	(0,0.7,3.3)
3	(0,0,3,5)	(0,0,1.5,2.5)
4	(0,0,3,5)	(0,0,2,3.3)

By using GeoGebra program to draw and determine the Success Area and Failure area of two systems (G and H) as Fig (8) and By using a Equ.(2) to determine the fuzzy reliability measure, the results are listed in Table (3).

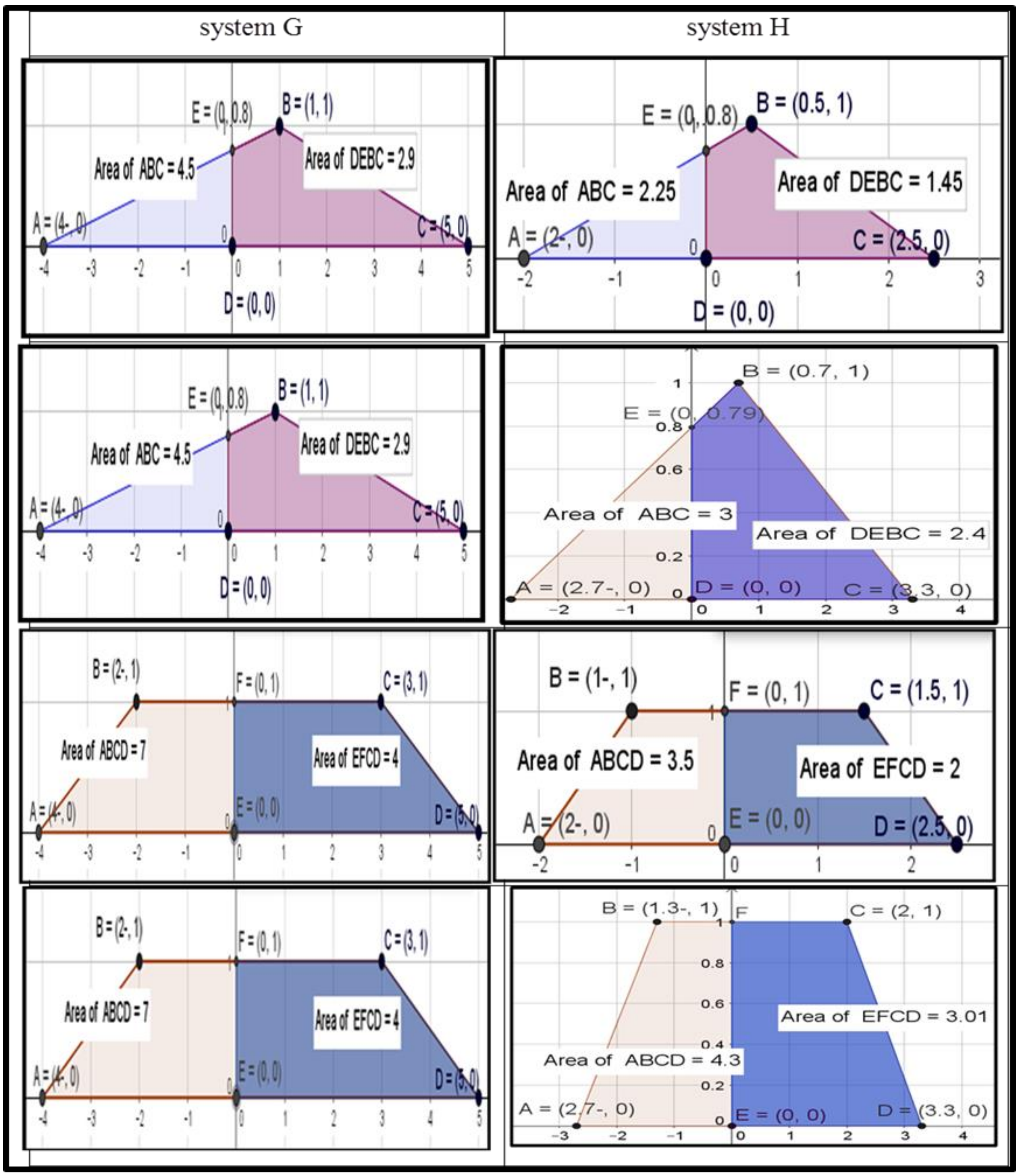

Figure 8. shown the Success Area and Failure area of two systems (G and H).

In Table 3 the results of the fuzzy reliability (FR) for two systems (G and H) by using the first model of fuzzy reliability measure of Duckstein and Sherstha. For example, scenario 1 and 3, yield the same reliability values of (FR_G 0.64, FR_H 0.64) and (FR_G 0.57, FR_H 0.57), respectively, for both systems G and H. This reliability measure does not indicate the difference in reliability values between the two systems. This consideration was misleading and

Table 3. Success and Failure areas and fuzzy reliability FR measure of two systems (G & H)

No. scenario		Single pump			Parallel pumps		
		System G			System H		
		Success Area	Failure Area	FR measure	Success Area	Failure Area	FR measure
Tg	1	2.9	4.5	0.64	1.45	2.25	0.64
	2	2.9	4.5	0.64	3	2.4	0.80
Tp	3	7.0	4.0	0.57	3.5	2.0	0.57
	4	7.0	4.0	0.57	4.3	3.01	0.70

resulted in the inappropriate use of the system state MF and the aggregation rules for multicomponent systems.

4.1.2. Second model

A comparison between the fuzzy system state MF and the level of performance MF provides information about the both system reliability and system vulnerability at the same time, as show in Fig 5. The system reliability is based on the proximity of the system state to the level of performance of system. The measure of proximity is expressed by the compatibility measure suggested in Equ (7).

1- Performance Levels PL

To evaluate performance levels PL is by reflecting a decision maker's perception. To reflect the subjectivity of the decision maker through different of the performance are assigned to different levels.

The usefulness of the performance indexes (developed model) evaluated using three performance levels. The levels; (1) high reliability level PL_1, (2) reliable level PL_2 and (3) unreliable level PL_3 were defined subjectively by experts. These levels are represented by three trapezoidal fuzzy sets, (0.5, 0.8, 15, 15), (0.7, 1.0, 15, 15) and (0.8, 1.2, 15, 15) respectively as in Fig.(9). The reliability measures of these performance levels (PL) by using Equ(5) are 2.40, 2.33, and 1.33 respectively.

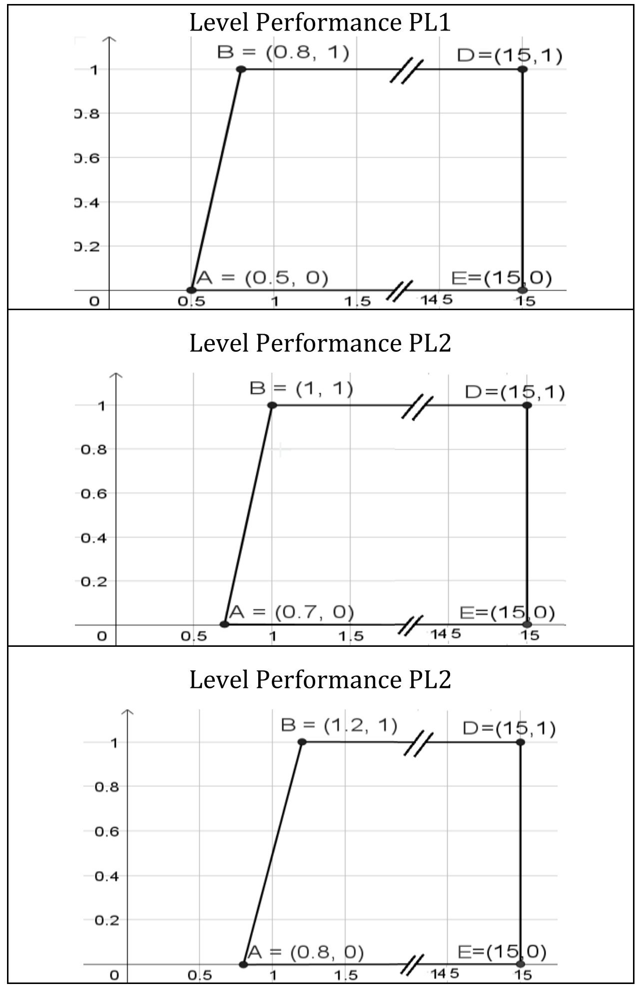

Figure 9. represented three trapezoidal fuzzy sets of PLs

2-Determine Success Area (A) and Failure Area

The three values of system state of systems in Table (4) are used with GeoGebra program to draw and determine the total Area and Overlap area of two systems (G and H). By used Equs. (6,7 and 8) to determine the fuzzy reliability-vulnerability index, and fuzzy robustness index. Fig (10) shows the triangle system state (Tg MF) and trapezoidal system state (Tp MF), respectively, used for four scenarios with three performance levels MF. The results are listed in Table (5).

Table 4. Summary details of (G and H) systems state

No. scenario		Second Model	
		System state G	System stat H
1		(0,1,5)	(0,0,2.5)
2		(0,1,5)	(0,0,3.3)
3		(0,0,3,5)	(0,0,1.5,2.5)
4		(0,0,3,5)	(0,0,2,3.3)

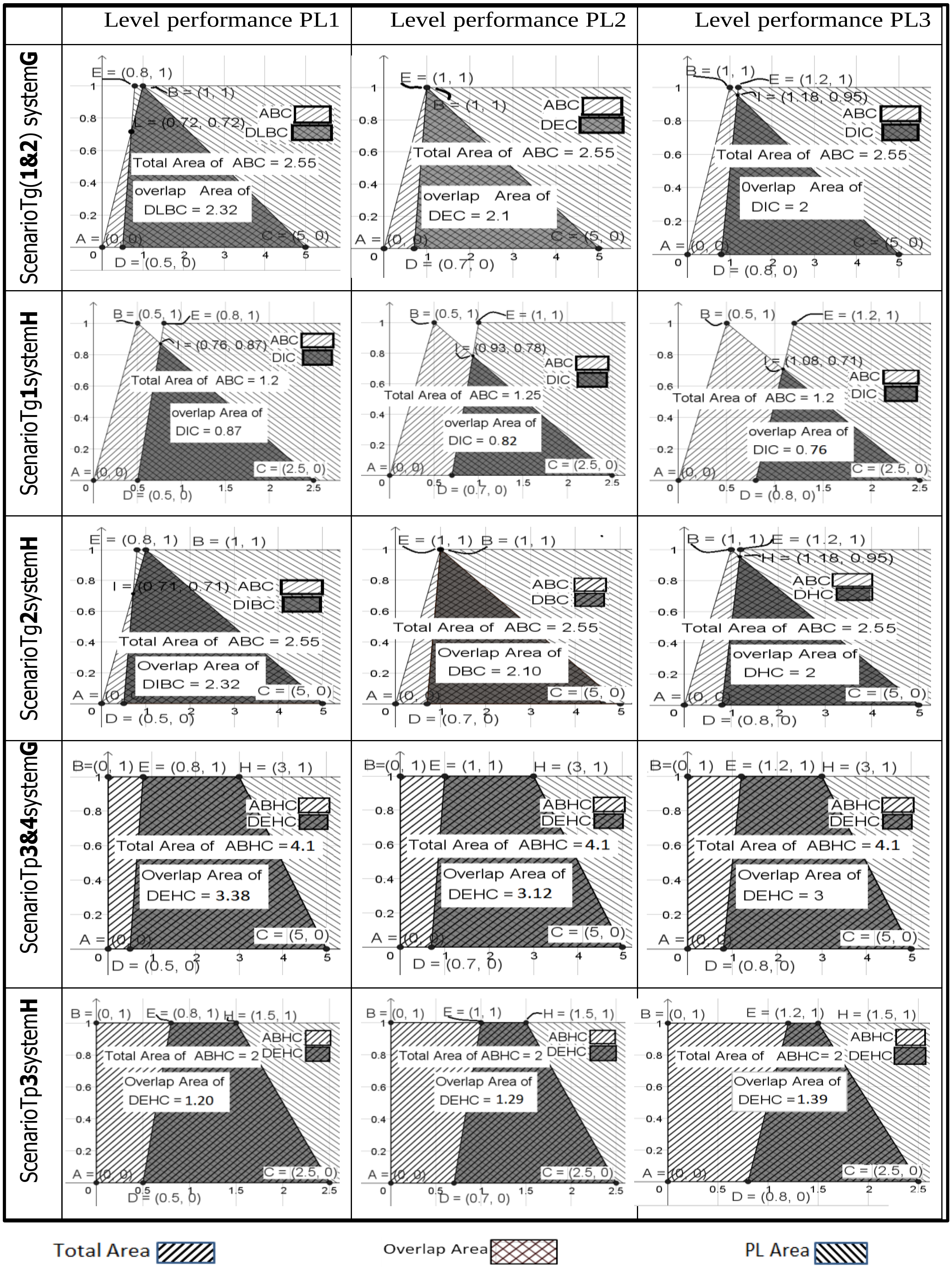

Figure 10. Shows the Success Area and Failure Area (Tg and Tp MFs) for four scenarios with three PLs.

Table (5): results of determine *Rv* and *Ro* indexes for Tg and Tp MF for four scenarios with three PLs MF

scenarios		Tg for system G									Tg for system H								
		Total Area			Overlape Area				Indexes		Total Area			Overlape Area				Indexes	
		A	CG	WA	OA	CG	WOA	Cm	Rv	Ro	A	CG	WA	OA	CG	WOA	Cm	Rv	Ro
Tg	1	2.55	2.00	5.10	2.32	2.10	4.87	0.955	0.531	17.35	1.20	1.00	1.20	0.87	1.36	1.19	0.989	0.550	115.17
					2.10	2.18	4.58	0.898						0.82	1.43	1.18	0.981		
					2.00	2.25	4.50	0.882						0.76	1.46	1.11	0.925		
Tg	2	2.55	2.00	5.10	2.32	2.10	4.87	0.955	0.531	17.35	1.60	1.29	2.06	1.34	1.53	2.04	0.991	0.550	125.06
					2.10	2.18	4.58	0.898						1.21	1.68	2.03	0.983		
					2.00	2.25	4.50	0.882						1.14	1.74	1.98	0.961		
Tp	3	4.10	2.23	9.14	3.38	2.34	7.91	0.865	0.481	16.75	2.00	1.00	2.00	1.39	1.38	1.92	0.959	0.533	33.00
					3.12	2.36	7.36	0.805						1.29	1.44	1.86	0.929		
					3.00	2.50	7.50	0.820						1.20	1.50	1.80	0.900		
Tp	4	4.10	2.23	9.14	3.38	2.34	7.91	0.865	0.481	16.75	2.60	1.33	3.45	2.00	1.65	3.30	0.958	0.532	32.97
					3.12	2.36	7.36	0.805						1.83	1.75	3.20	0.928		
					3.00	2.50	7.50	0.820						1.65	1.83	3.01	0.874		

From Table(5) the fuzzy reliability-vulnerability Rv of systems G and H(0.531, 0.55) in scenarios (1) and (2), respectively is higher than the fuzzy reliability-vulnerability of systems G and H(0.48 to 0.53) in scenario 3 and 4, respectively. These results are support the fact of hypothesis that the larger system reliability result from greater parallel. The form of MF effect on the conclusion associated with system reliability.

To determine the robustness Ro of system by the performance level MF was changed from the reliable level PL2 to the high reliable level PL1. To reach this level by using two parallel pumps (as in Table 5) increased the robustness Ro of system. The Ro of system increased as the assessment of the fuzzy robustness index. In scenario (1), the index of Ro increased from (17.35 to 115.17).

In the case of the trapezoidal membership function (Tg MF) is increase three times than in the case of triangular the membership function (Tp MF). Therefore, the amount of system robustness Ro depends on the form of MF. The load L distribution between the two parallel pumps affects the robustness Ro of the system. The amount of Ro increases from (115.17 to 125.6) between scenario (1 and 2). No significant change observed between scenario (3 and 4) because the load not equal.

5. Conclusion

The probability approach fails to address the problems of uncertainty by subjectivity judgment, human error, and the lack of system performance data. A fuzzy reliability-vulnerability index quantifies fuzzy reliability and fuzzy vulnerability of multi-element systems, reflecting differences in connections of systems at fuzzy environment. The methodology analysis based on performance data (i.e. design Capacity and average Demand) without using failure data (as in traditional reliability measure). The models suggested in this study demonstrate performance consistent with expectations. Where the parallel system H is (TpRvH 0.55) more reliable than of the single system G (TpRvG 0.48). The selected subjectively three PL performance levels will assist the decision maker to select better performance level.

Nomenclature

L	load
R	resistance
$\hat{L}$	fuzzy Load
$\hat{R}$	fuzzy Resistance
G	single System
H	parallel System
MFs	membership functions
pdf	probability distributions function
FR	Fuzzy Reliability
RA	reliability analysis
Tg MF	fuzzy triangular membership
Tp MF	fuzzy trapezoidal membership
WSS	Water Supply Systems
Rv	fuzzy reliability-vulnerability index
R_O	fuzzy robustness index

References:

[1] Ahmad SS, Simonovic SP. A New Method for Spatial Analysis of Risk in Water Resources Engineering Management. *Open Civil Engineering Journal*. 2007; 1:1-2.

[2] Guijie LG, Lu Z, Xu J. A Fuzzy Reliability Approach for Structures Based on the Probability Perspective. Elsevier *Structural Safety*. 2015 May 31;54:10-8.

[3] Dejan V. Water Resource Decision Making Under Uncertainty. Ph.D. Thesis, Department of

Civil and Environmental Engineering, University of Western Ontario London, Canada, 2011.

[4] Hashimoto T, Stedinger JR, Loucks DP. Reliability, Resiliency, and Vulnerability Criteria For Water Resource System Performance Evaluation. *J Water resources research*. 1982 Fep 1;18(1):14-20.

[5] Julwan PH. Fuzzy probability on reliability study of nuclear power plant probabilistic safety assessment: Elsevier *a review. Progress in Nuclear Energy*. 2014 Sep 30;76:73-80.

[6] Kapur KC, Li Z. Models and measures for fuzzy reliability and relationship to multi-state reliability. *International Journal of Performability Engineering*. 2011 May;7(3):241.

[7] Julwan PH, Lu J, Zhang G, Pedrycz W. A fuzzy reliability assessment of basic events of fault trees through qualitative data processing. Elsevier *Fuzzy Sets and Systems*. 2014 May 16;243:50-69.

[8] Shujie S, Zhang H, Li C, Lu H, Hu Y. Fuzzy reliability estimation for cutting tools. Elsevier *Procedia CIRP*. 2014 Jan 1;15:62-7.

[9] Huang HZ, Liu Y, Levitin G. Reliability and performance assessment for fuzzy multi-state elements. Proceedings of the Institution of Mechanical Engineers, Part O: Research Geat *Journal of Risk and Reliability.* 2008 Dec 1;222(4):675-86.

[10] Pandey D, Tyagi SK, Kumar V. Reliability analysis of a series and parallel network using triangular intuitionistic fuzzy sets. Applicat. *Appl. Math.-An Int. J.* 2011;6(11):1845-55.

[11] Duckstein L, Sherstha B. A Fuzzy Reliability Measure for Engineering Applications. ch7 *Uncertainty Modelling and Analysis in civil Engineering CRC* Press LLC;1998.

[12] El-Baroudy I, Simonovic SP. Fuzzy Criteria for the Evaluation of Water Resource Systems Performance. *J Water resources research*. 2004 Oct 1;40(10).

[13] Ahmad, S. Shohan,"Spatial and Temporal Risk Assessment for Water Resources Decision Making" Ph.D. Thesis, Department of Civil and Environmental Engineering , University of Western Ontario London, Ontario, Canada, 2010.