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Study of Multi-Cracked Cantilever Composite Beams Subjected to External Moving Load

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PAPER INFO

Paper history:
Received ....
Received in revised form ....
Accepted ....

Keywords:
Periodic Load, Cantilever Composite Beams, Multiple Cracked Beams, Moving Load

ABSTRACT

The behaviour of multiple cracked cantilever composite beams is studied when subjected to moving periodic force. In this investigation a new model of multiple cracked composite beams under periodic moving load is solved. Three cracks are considered at different position of the beam for numerical solution. The results from experimental work compared to numerical solution. The multiple cracks are identified easily from the deflection graphs at different force speed. Influences of crack depth at different load speed are investigated.

1. Introduction

The composite beams are being used more frequently in the field of civil and mechanical engineering because of high strength, low weight impact resistance, and high fatigue resistance. The dynamic behaviour of the composite cracked beams under the moving loads is one of the most important issues for researchers. chouiyakh et al. [1] developed the numerical method to study nonlinear response of multi-cracked beams subjected to moving load. The numerical method based on the differential quadrature was used to crack identification. Karimi et al.[2] presented two types of artificial neutral network for crack detection. Parhi et al.[3] investigated crack CATIA U5.A. crack identification procedures were based on the change of natural frequencies and mode shape. Rambabu et al. [4] studied the dynamic behaviour of composite beam with a non-propagating one-edge open crack. The effects of position of crack, depth of crack and volume fraction of fibers upon the bending natural frequencies of the composite beam were presented. Doguscan [5] proposed a formulation to study of the influence of crack position and magnitude on the natural frequencies and mode shapes of the graded beams. Validity of the formulation was established in comparison with the previously published results. kumar and Kumar [6] investigated vibration response of multi-cracked composite beams made of aluminium-reinforced GFRP (glass fiber reinforcement polymer) and aluminium reinforced Nylon. The effects of cracks on the natural frequencies of GFRP and Nylon with the cracks were decreased when they bonded with aluminium. Apate and More [7] reviewed various cost effective reliable, solution method and experimental technique developed by researchers for dynamic behaviour of cracked beams. Kurt et al. [8] investigated dynamic response of cracked beams under a moving load using Finite Element Methods. The effect of moving load speed , crack depth and location upon the response shape presented. Ramadas et al. [9] attempted to use Lamb wave and vibration based technique in an artificial neural network environment for effective detection of transverse crack position and magnitude. Kisa [10], studied vibration behaviour of a composite beam with multiple cracks. Assumptions concerning nonlinearity at the interface of substructure and the fracture mechanics integrated to develop a new method for the modelling of the freo vibration of a cantilever composite beam with multiple cracks. Sarvestan et al. [11] developed a spectral finite element model for dynamic behaviour of cracked viscoelastic Euler-Bernoulli beam under a moving load. The higher accuracy with using less number of elements was observed in comparison with other methods. In this research a new model of multiple cracked composite beam under dynamic moving force with differential equation involves complicated terms is solved. The behaviour of multi crack cantilever composite beams is investigated. The effect of load speed with different crack depth on the dynamic deflection of the cracked cantilever composite beams is studied.

2. Theoretical Model

Fig. 1 shows An Euler-Bernoulli beam with multiple open crack located at position $x_1, x_2, .. x_{i-1}, x_i, x_{i+1}, ... x_N$. A harmonic load with constant speed v is moved on the beam. The beam divided into N segments with length of L_1 and $L_2, .. L_i, L_{i+1}, ... L_N$., [12].

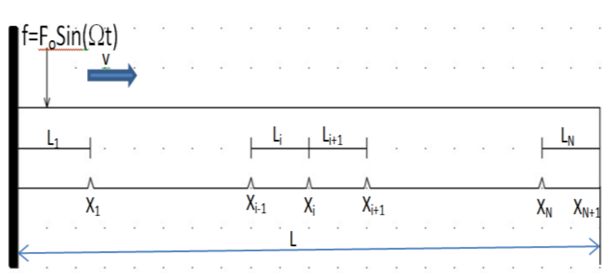

Figure 1. Euler-Bernoulli Beam with Multiple Open Crack Subjected to Moving Load, [12]

The governing equation for the whole beam with uniform cross section is,[12].

$$D\frac{\partial^4 w}{\partial x^4} + I_o\frac{\partial^2 w}{\partial t^2} = f(t)\delta(x-vt)H(\frac{L}{v}-t) \quad (1)$$

Where w=w(x,t) is the deflection of the beam, δ(x) and H(x) are the Dirac and Heaviside function which can be defined as

$$\int_{-\infty}^{\infty}\delta(x)dx = 1, \quad \begin{cases}\delta(x) = 0, & for\ x \neq 0 \\ indefinite & for\ x = 0\end{cases} \quad (2)$$

$$H(x) = \begin{cases}0, & x < 0 \\ 1, & x > 0\end{cases} \quad (3)$$

I_o is the inertia of the beam as

$$I_o = b\int_{-\frac{h}{2}}^{\frac{h}{2}}\rho^{(j)}dz \quad (4)$$

$\rho^{(j)}$ is the material density for the jth layer, h and b is the height and width of the beam, respectively. D is the reducing bending stiffness of the beam which is defined as

$$D = D_{11} - \frac{B_{11}^2}{A_{11}} \quad (5)$$

and

$$\{A_{11}, B_{11}, D_{11}\} = b\int_{-\frac{h}{2}}^{\frac{h}{2}}\bar{Q}_{11}^{(j)}\{1, z, z^2\}dz \quad (6)$$

$\bar{Q}_{11}^{(j)}$ is the stiffness coefficient of the jth layer.

The rotational spring with sectional flexibility model for the crack and the Euler-Bernoulli theory is used for each segment. The governing equation for the free vibration can be written as

$$D\frac{\partial^4 w_i}{\partial {x_i}^4} + I_o\frac{\partial^2 w_i}{\partial t^2} = 0, i = 1,2, \dots, N+1 \quad (7)$$

Method of the separation of variables is used to determine the natural frequencies.

$$w_i(x,t) = \emptyset(x)e^{iwt} \quad (8)$$

By substituting Eq. (8) in Eq. (7)

$$\emptyset_i''''(x) - \beta^4\emptyset_i(x) = 0 \quad x_{i-1} < x < x_i \quad i = 1,2, \dots . N+1 \quad (9)$$

Where

$$\beta^4 = \frac{I_o\omega^2}{D} \quad (10)$$

Equation (9) can be solved for crack location at x_i to find the function of $\emptyset$ for each segment

$$\emptyset_{i(x)} = A_i \sin(\beta(x - x_{i-1})) + B_i \cos(\beta x(x - x_{i-1})) + C_i Sinh(\beta x(x - x_{i-1})) + D_i Cosh\big(\beta x(x - x_{i-1})\big) \quad 0 \quad x_{i-1} < x < x_i \quad i = 1,2, \dots . N + 1 \qquad (11)$$

The boundary conditions for a cantilever beam are

$$w_{(0,t)} = w'_{(0,t)} = w''_{(L,t)} = w'''_{(L,t)} = 0 \qquad (12)$$

The rotational spring model is used at crack location $x=x_i$

$$w_i(x_i, t) = w_{i+1}(x_i, t), w''_i(x_i, t) = w''_{i+1}(x_i, t), w'''_i(x_i, t) = w'''_{i+1}(x_i, t), \ w'_{i+1}(x_i, t) - w'_i(x_i, t) = \vartheta L w''_{i+1}(x_i, t) \qquad (13)$$

Whereϑ sectional flexibility for a single-sided crack beam can be written as

$$\vartheta = 6\pi \bar{d_\iota}^2 f(\bar{d_\iota})(\frac{h}{L}) \qquad (14)$$

$$f(\bar{d_i}) = 0.6348 - 1.035\bar{d_\iota} + 3.7201\bar{d_\iota}^2 - 5.177\bar{d_\iota}^3 + 7.553\bar{d_\iota}^4 - 7.332\bar{d_\iota}^5 + 2.4909\bar{d_\iota}^6 \qquad (15)$$

$$\bar{d_\iota} = \frac{d_i}{h} \qquad (16)$$

Where d is the crack depth.

The β and coefficients of A_i, B_i, C_i, D_i, can be find by Substituting equation (12) - (16) in equation (11) as

$$\begin{cases} B_1 + D_1 = 0 \\ A_1 + C_1 = 0 \\ A_i \sin(\beta L_i) + C_i \sinh(\beta L_i) - B_{i+1} - D_{i+1} = 0 \\ -A_i \sin(\beta L_i) + C_i \sinh(\beta L_i) + B_{i+1} - D_{i+1} = 0 \\ -A_i \cos(\beta L_i) + C_i \cosh(\beta L_i) + A_{i+1} - C_{i+1} = 0 \\ -A_i \cos(\beta L_i) + C_i \cosh(\beta L_i) - A_{i+1} - \beta\vartheta L B_{i+1} \\ \quad -C_{i+1} + \beta\vartheta L D_{i+1} = 0 \\ -A_{N+1} \sin(\beta L_{N+1}) - B_{N+1} \cos(\beta L_{N+1}) + C_{N+1} \sinh(\beta L_{N+1}) \\ \quad +D_{N+1} \cosh(\beta L_{N+1}) = 0 \\ -A_{N+1} \sin(\beta L_{N+1}) + B_{N+1} \cos(\beta L_{N+1}) + C_{N+1} \sinh(\beta L_{N+1}) \\ \quad +D_{N+1} \cosh(\beta L_{N+1}) = 0 \end{cases} \qquad (17)$$

Eq. (17) can be written as

$$[S(\beta)]\{A\} = 0 \qquad (18)$$

Where $\{A\} = \{A_i, B_i, C_i, D_i \ (i = 1,2, \dots, N + 1)\}^T$ and $[S(\beta)]$ is 4(N+1)×4(N+1) matrix. For non-trivial solutions of Eq. (18)

$$|S(\beta) = 0| \qquad (19)$$

The nonlinear Eq. (19) is solved to determine Eigen values βj and modes ϕj. The coefficient vector {A} are chosen such that

$$\int_0^L m_p \emptyset_m \emptyset_n dx = \delta_{ij} \begin{cases} \delta_{mn} = 1 \ for \ m = n \\ \delta_{mn} = 0 \ for \ m \neq n \end{cases} \qquad (20)$$

The modal representation is used for a cantilever beam subjected to the moving dynamic force with constant speed

$$w(x, t) = \sum_{j=1}^{\infty} \Phi_j(x) q_{j(t)} \qquad (21)$$

Where Φj(x) is the Eigen function of jth mode

$$\Phi(x) = \sum_{i=1}^{N+1} \emptyset_i(x - x_{i-1})[H(x - x_{i-1}) - H(x - x_i)] \qquad (22)$$

and $q_{j(t)}$ is the jth modal amplitude.

$$\ddot{q}_j(t) + \omega_j^2 q_j(t) = f(t)\Phi_j(vt) \qquad (23)$$

Where f(t) is dynamic force

$$f(t) = F_o \sin(\Omega t) \qquad (24)$$

Where F_o and Ω are force amplitude and frequency, respectively. New mark method is used to solve Eq. (24).

3. Experimental Setup

The experimental method that used for this investigation explained in reference [12]. Three Cracks are created on the surface of cantilever beam with the same dimension and locations used in numerical solution. Fig. 2 shows the System setup which used for experimental work. Laminated composite beams made of epoxy and carbon fiber in twenty layers with fiber angle of zero. A harmonic force from an exciter is applied to the specimen. The exciter held by mounting fixture; move over the surface of the specimen with a controlled constant speed from the fixed to free end of beam. Force and acceleration are measured by atypical load cell and accelerometer, respectively. The dimensions of test specimen are selected as the same dimensions used in the numerical solution which are 900mm×100mm with thickness of 5.5 mm. A Module is used to transfer the data from the accelerometer and load cell in order to analyse. All equipment is Bruel & Kaje.

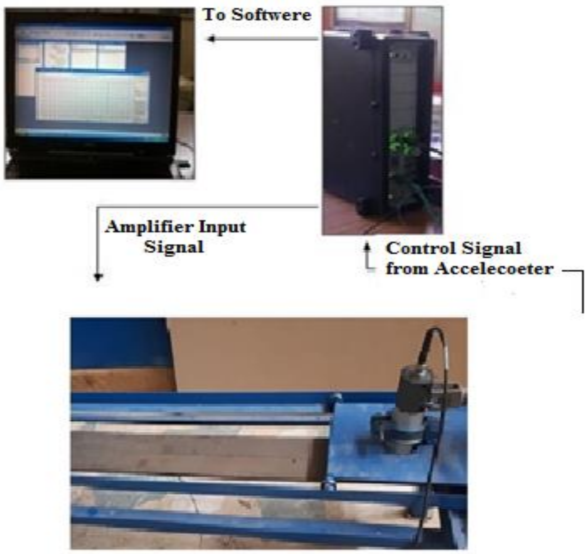

Figure 2. Vibration Measurement Scheme

4. Numerical Example and Discussion

For numerical solution and validation of the present model consider a cantilever beam with three different cracks at locations of x_1=200mm, x_2=450mm, and x_3=700mm. The composite beam made from twenty layer carbon fiber with epoxy matrix. The inertia and reduced matrix are I_o=0.8756 and D= 231.44, respectively. The beam dimensions are length L=900mm, width b=100mm and height h=5.5 mm.

The equations in part two can be solved for triple cracked beam and 16 equations are obtained from Eq. 17.

$$\begin{cases} B_1 + D_1 = 0 \\ A_1 + C_1 = 0 \\ A_1 \sin(\beta L_1) + C_1 \sinh(\beta L_1) - B_2 - D_2 = 0 \\ A_2 \sin(\beta L_2) + C_2 \sinh(\beta L_2) - B_3 - D_3 = 0 \\ A_3 \sin(\beta L_3) + C_3 \sinh(\beta L_3) - B_4 - D_4 = 0 \\ -A_1 \sin(\beta L_1) + C_1 \sinh(\beta L_1) + B_2 - D_2 = 0 \\ -A_2 \sin(\beta L_2) + C_2 \sinh(\beta L_2) + B_3 - D_3 = 0 \\ -A_3 \sin(\beta L_3) + C_3 \sinh(\beta L_3) + B_4 - D_4 = 0 \\ -A_1 \cos(\beta L_1) + C_1 \cosh(\beta L_1) + A_2 - C_2 = 0 \\ -A_2 \cos(\beta L_2) + C_2 \cosh(\beta L_2) + A_3 - C_3 = 0 \\ -A_3 \cos(\beta L_3) + C_3 \cosh(\beta L_3) + A_4 - C_4 = 0 \\ -A_1 \cos(\beta L_1) + C_1 \cosh(\beta L_1) - A_2 - \beta\vartheta_1 L B_2 \\ \quad -C_2 + \beta\vartheta_1 L D_2 = 0 \\ -A_2 \cos(\beta L_2) + C_2 \cosh(\beta L_2) - A_3 - \beta\vartheta_2 L B_3 \\ \quad -C_3 + \beta\vartheta_2 L D_3 = 0 \\ -A_3 \cos(\beta L_3) + C_3 \cosh(\beta L_3) - A_4 - \beta\vartheta_3 L B_4 \\ \quad -C_4 + \beta\vartheta_3 L D_4 = 0 \\ -A_4 \sin(\beta L_4) - B_4 \cos(\beta L_4) + C_4 \sinh(\beta L_4) \\ \quad +D_4 \cosh(\beta L_4) = 0 \\ 4 \sin(\beta L_4) + B_4 \cos(\beta L_4) + C_4 \sinh(\beta L_4) \\ \quad +D_4 \cosh(\beta L_4) = 0 \end{cases} \quad (25)$$

The values of β and ϕ can be obtained from non-trivial solutions of Eq. (25). The values of $\boldsymbol{\vartheta_1}$, $\boldsymbol{\vartheta_2}$, and $\boldsymbol{\vartheta_3}$ are determined from Eq. (14) for different crack depth. The MATLAB and MAPLE program are used in numerical solution.

The four fist natural frequencies and mode shapes for three cracks depth ratio of d/h =0.2are obtained and compared with the experimental values in order to validation of the present method.

The lowest four natural frequencies that obtained from numerical solutions are: f1=11.2 Hz, f2= 70.19 Hz, f3= 196.49 Hz, and f4= 383.56 Hz. In other hand the experiment results gave the four first natural frequencies as: f1=10 Hz, f2= 74 Hz, f3= 204 Hz, and f4= 376 Hz. It is resulted that the numerical solution is in good agreement compared to the experiment.

The lowest four first modes of the beam are used in modal expansion to obtain response of the system. The normalized modes for four lowest modes of cantilever cracked beam are shown in Fig 3.

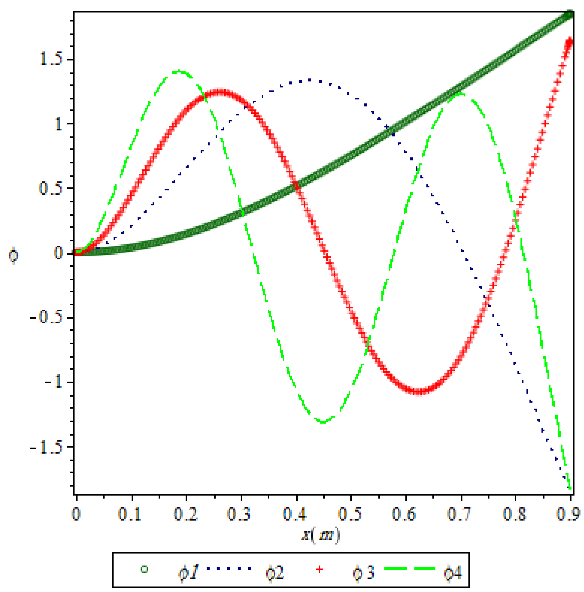

Figure 3. Four Lowest Modes of a Cantilever Cracked Beam

Figs. 4 and 5 show the variation of dynamic deflection with velocity ratio for a cracked cantilever composite beam with three cracks at positions x_1=200mm, x_2=450mm, x_3=700mm with crack depth of d/h=0.2.

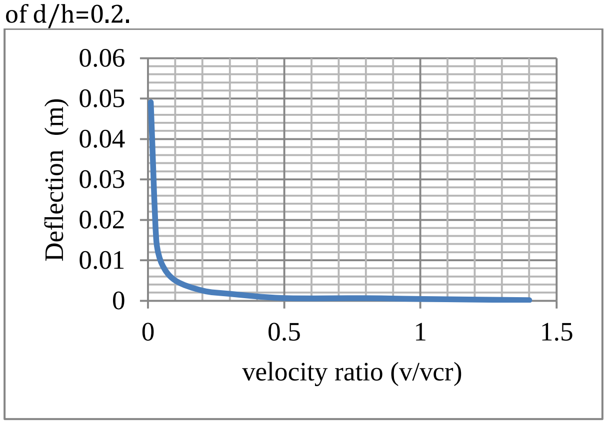

Figure 4. ForcedDeflection at Different Velocity Ratio (f=11.15 Hz , x1=200mm, x2=450mm, and x3=700mmd1/h1=d2/h2=d3/h3=0.2, F_o=2N)

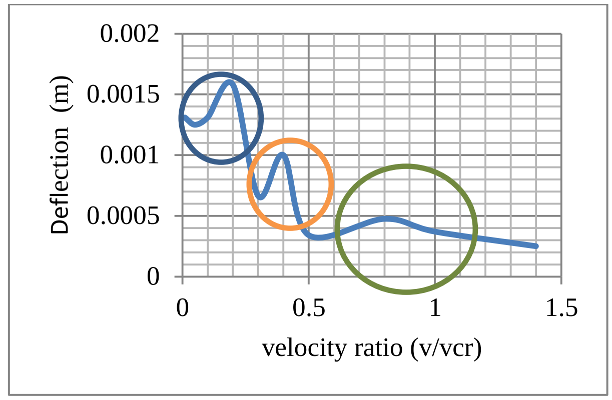

Figure 5. ForcedDeflection at Different Velocity Ratio (f=15.92 Hz , x1=200mm, x2=450mm, and x3=700mm, d1/h1=d2/h2=d3/h3=0.2, F_o=2N)

The critical velocity v_{cr} of a cantilever composite beam is calculated with considering the first mode

$$v_{cr} = \frac{1.8751}{L}\sqrt{\frac{D}{I_o}} \qquad (26)$$

For the resonance frequency the forced deflections are decreased with increasing the speed ratio, with the same behavior of single cracked beam [12]. But for another frequency such as 15.92 the different curve profile is obtained as shown in Fig. 5. This behavior is related to the existence of three cracks instead of one, and three parts is recognized from the graph (blue, orange, and green) which are in a different manner with a single crack [12]. The results are showed that the forced deflection of multi cracked beam strongly depended to the speed ratio and the existence of multi cracks can be easily identified.

Figs6 and 7 show the forced deflection for the different crack depth at various crack position. Firstly, three cracks are created with depth ratio of 0.2 then for study the effect of different crack position and depth ratio, the first crack depth ratio incrased to 0.4 then the second and finally the third increased to 0.4. The same procedure was done for the crack depth ratio of 0.6.

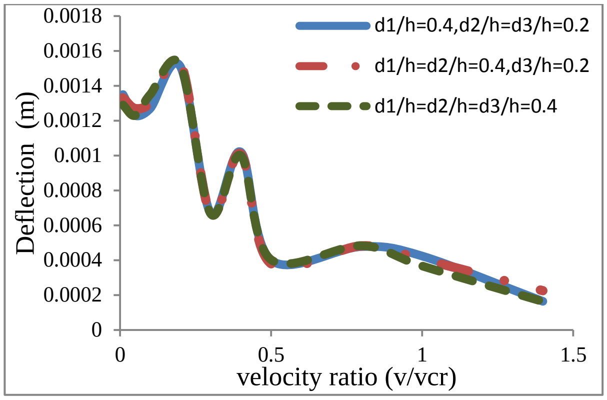

Figure 6. ForcedDeflection at Different Velocity Ratio (f=15.92 Hz , x1=200mm, x2=450mm, and x3=700mm, F_o=2N, and Different Crack Depth Ratio 0.2 & 0.4)

There is very low difference between the curves, because of the forced deflection is highly depended to the speed ratio and number of crack. When the depth of cracks increased step by step from the fixed end to free side there is not important change in dynamic deflection of the cracked beam

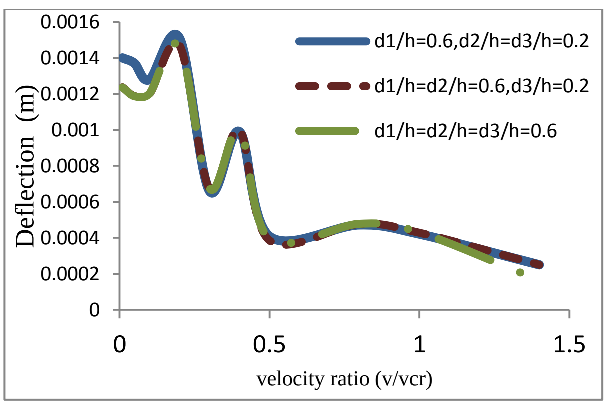

Figure 7. ForcedDeflection at Different Velocity Ratio (f=15.92 Hz , x1=200mm, x2=450mm, and x3=700mm, F_o=2N, and Different Crack Depth Ratio 0.2 & 0.6)

Fig 8 represented the forced deflection when the cracks have the same crack depth ratio for each case. Three cases are considered; firstly d1/h =d2/h=d3/h=0.2, secondly d1/h=d2/h=d3/h=0.4 and finally d1/h=d2/h=d3/h=0.6. It can be observed that the smaller value of deflection is related to more deep cracks for lower velocity but for higher value of the speed ratio, the forced deflection is increased by increasing the depth ratio.

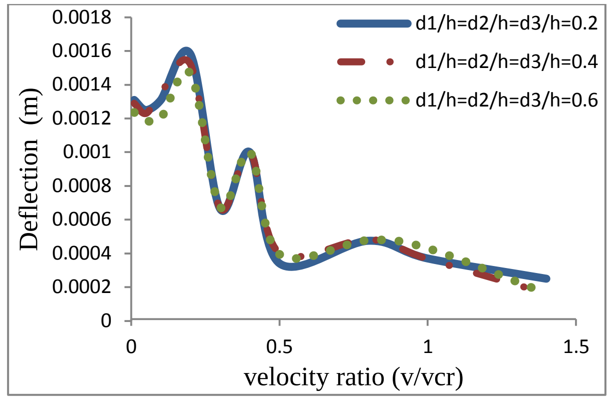

Figure 8. ForcedDeflection at Different Velocity Ratio (f=15.92 Hz , x1=200mm, x2=450mm, and x3=700mm, F_o=2N, and Different Crack Depth Ratio)

This phenomenon can be explained by the effect of the velocity ratio on forced deflection. Fig 9 represented the 3-D graph of forced deflection for the case of crack depth ratio 0.2 and speed ratio of 0.01.

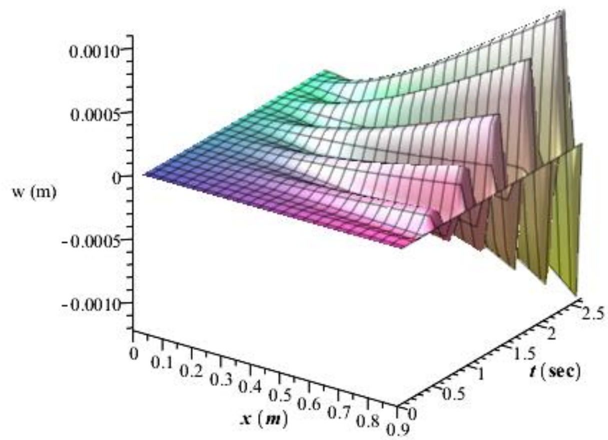

Figure 9. 3-DForced Deflection at Velocity Ratio of 0.01 (f=15.92 Hz , x1=200mm, x2=450mm, and x3=700mm d1/h1=d2/h2=d3/h3=0.2, F_o=2N)

The variation of dynamic deflection for the free end can be shown in a graph as Fig 10. It is obvious from the Figs 9 and 10, show moving dynamic force provide reasonable condition for increasing deflection smoothly in a periodic form.

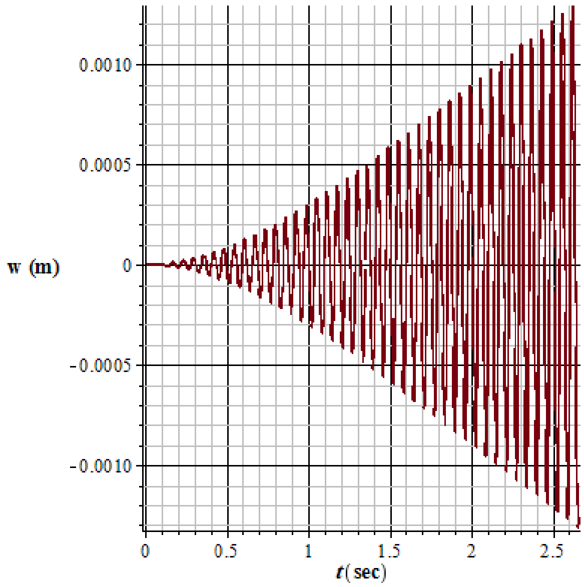

Figure 10. Forced Deflection at Free End for Velocity Ratio of 0.01 (f=15.92 Hz , x1=200mm, x2=450mm, and x3=700mm d1/h1=d2/h2=d3/h3=0.2, F_o=2N)

The behavior of the beam at high speed ratio is considered as another case for this study. The best case is critical speed for comparison with result of low velocity ratio. Fig 11 show 3-D graphs are obtained from the results of forced deflection when the crack depth ratio is 0.2 and the force moved with critical speed

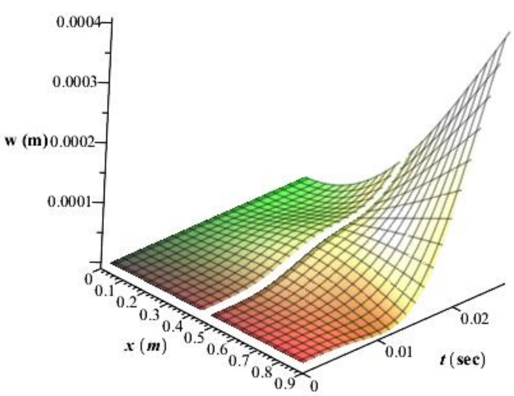

Figure 11. 3-DForced Deflection at Critical Velocity (f=15.92 Hz , x1=200mm, x2=450mm, and x3=700mm d1/h1=d2/h2=d3/h3=0.2, F_o=2N)=2N)

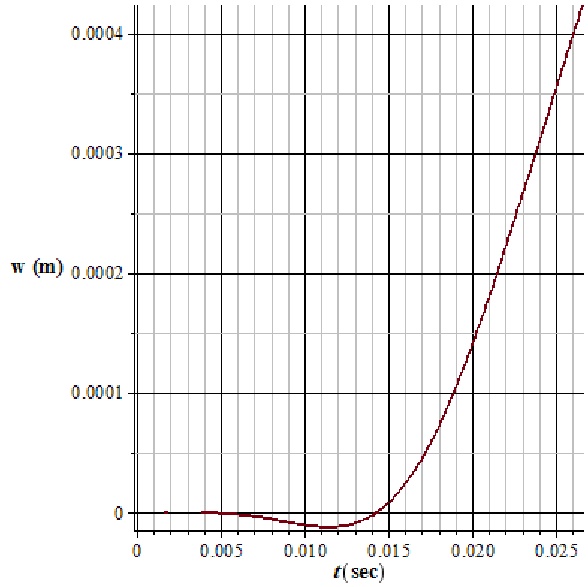

Figure 12. Forced Deflection at Free End for Critical Velocity (f=15.92 Hz , x1=200mm, x2=450mm, and x3=700mm d1/h1=d2/h2=d3/h3=0.2, F_o=2N)

Fig 12 shows the dynamic deflection at the free end when the force moving with the critical speed on the cracked cantilever beam. Figs 11 and 12 illustrate a great increasing in beam deflection and do not show any periodic deflection, because of high speed of the force. There is not enough time to dynamically deflection of the beam and this is reason for unusual behavior. Thus, it can be result that the behavior of the beam under the moving load is hardly depended to the speed of the load.

5. Conclusions

The dynamic of the multiple cracked composite beams when subjected to a periodic moving load is studied. From investigation of multiple crack cantilever composite beams with, the following conculusions are obtained

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1- Good agreements are obtained from comparison between the numerical solution and experimental results.

2- The dynamic deflections in the resonance state are decreased with increasing the speed ratio.

3- The multi cracked state can be distinguished easily, with plotting the graph of dynamic deflection versus speed ratio.

4- For the multiple cracks there is not important change in the graphs if depth ratio increased from the fixed end to free end by the same value.

5- The dynamic deflection show decreasing in low speed of moving periodic load and increasing with higher velocity of dynamic load, when crack depth increased with the same value for all cracks.

6- The speed of moving load has the major effect on change of periodic deflection of beam to non-periodic behavior.

Nomenclature

D	reducing bending stiffness
I_o	inertia of the beam
b	beam section width
h	beam section height
w	beam deflection
$\delta(x)$	Dirac function
$H(x)$	Heaviside function
Q_{11}	stiffness coefficient
ϑ	sectional flexibility for a single-sided crack beam
d	crack depth
F_o	force amplitude
Ω	force frequency
v_{cr}	critical force speed
L	beam length

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