

# Experimental and Theoretical Studies of Last Ply Failure Analysis on the Laminate Polymer Composite Materials with Different Orientation and Stacking Sequence of Fibers Based on Classical Laminate Theory

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## Abstract:

Unidirectional composite materials are among the most important composite materials and have many applications in daily life. The angle of the fibers of the lamina is an important factor in controlling properties of the composite materials as well as the sequence of layers. The aim of this study is to obtain the best laminate with good mechanical properties (strength and stiffness) with the selection of the appropriate failure theory. This study was conducted to carbon /epoxy laminate with different angles (0°,25°,30°,50°,60°,75°,90°) by three theories of failure (maximum stress theory, Tsai-Hill theory and Hashin theory). The strength ratio depending on the last ply failure load was found by using MATLAB program of five layers of laminates and by applied the classical laminate theory. Seventy-two combinations of laminates were obtained with not repeating the layers that have the same angles. The theoretical results showed a large deviation between the theoretical and practical results of the laminate [0/30/60/90/50] for the three theories by (24%).While, there was convergent deviation of the laminate [30/90/060/25] by (11.9% and 9.7%).As for the laminate [60/30/0/90/75],the deviation between the theoretical and practical

results was according to the Hashin and Tsai Hill theories by (9.7%). As for the deviation in relation to the maximum stress theory was by (24.3%).

**Key words:** carbon/epoxy, fibers orientation angles, classical laminate theory, last ply failure load, strength ratio, maximum stress theory, Hashin failure theory, Tsai-Hill failure theory

## 1. Introduction

The Composite materials are produced by mixing two or more materials, one of which is called the reinforcement and the other is called the matrix. They differ greatly in physical and chemical properties, which transfer their properties to the composite material. The laminated composite materials are used in modern applications such as aircraft, automobiles and engineering applications due to its high stiffness and strength relative to weight's ratio and resistance to corrosion and chemical [1]. Single-layer failure analysis is easier than laminate failure analysis, which requires knowledge of the strength of each layer by evaluating the stresses on the principle axis for each layer and choosing the appropriate failure theory. There are many factors that determine the strength of the laminate, the most important of which are the strength of layer, stiffness, the coefficient of thermal and moisture expansion, the stacking sequence, the manufacturing process and orientation angles of lamina [2]. It is known that failure in composite materials is a deterioration of the properties of the components, and the failure may be catastrophic, so it is important to know the failure mechanisms of the composite when designing. There are ways to predict the failure of the first layer in the structured composite. However, this does not mean a catastrophic failure of the composite, but rather leads to the observation of the failure of the first layer, as it is possible for the composite to function properly as a result of transferring the load to other layers in the laminate, so choosing the stacking sequence leads to the selection of the maximum acceptable applied load [3]. The failure theories were studied for the unidirectional layer (carbon/epoxy) and the appropriate failure theory was found, where the Tsai Hill theory was suitable for the single-layer [4]. A numerical study was also presented for a unidirectional composite material (carbon/epoxy) with different fiber angles and included four angles using the Tsai-Hill failure theory to find the appropriate sequence that has good mechanical properties [5]. The failure of the first ply load for glass/epoxy and graphite/ epoxy composites was studied under three different boundary conditions fixed-fixed, fixed-free and simple - supported to find out optimum hybrid composite beam with less deflection, cost and weight [6]. The first layer failure load has been analyzed by using a finite element arithmetic procedure based on the theory of shear deformation in the first place and on (the criteria of maximum stress, maximum strain, Tsai-Hill, Hoffman and Tsai-Wu) as special cases [7]. There was also a study of composite (epoxy /carbon)

for five layers with different angles fibers ( $0^\circ, 45^\circ, -45^\circ, 90^\circ$ ) by finite element analysis method to obtain 1027 combination of stacking sequence and best stacking that have good strength and bending load. In this study, Tsai- Hill's theory was used [8].

In this study, a numerical analysis used by the MATLAB program was dedicated to calculate the maximum strength of the laminate with different fibers orientation angles ( $0^\circ, 25^\circ, 30^\circ, 50^\circ, 60^\circ, 75^\circ, 90^\circ$ ) based on the last ply failure load. The strength ratio of three different failure theories (Maximum stress failure, Tsai-Hill failure and Hashin failure) was calculated. Also, and application the classical laminate theory, for 72 combinations of stacking sequence without repeating layers having the same angles, and with a comparison of the theoretical and experimental result of the laminate [60/30/0/90/75] that obtained the greatest allowable stress.

## 2. Failures theories

Based on the parameters of the strength and macroscopic stresses of the lamina along the principle axes and assuming the linear elastic behavior of the lamina, on this basis, failure theories are applied to the composite materials. The failure theories of composite materials are divided into three groups; (1) non –interactive or limit theories such as (maximum stress and maximum strain theories) , (2)interactive theories such as (Tsai-Hill and Tsai-Wu theories ) , and (3) partially interactive such as ( Hashin , Hashin-Rotem and puck theories) [9].

### 2.1 The maximum stress failure theory (two dimensions)

This theory assumes that failure occurs in the lamina when any stress on the principle axes of the lamina is exceeded the corresponding strength. In other words, the failure of the two-dimensional lamina will occur if one of the equations below is proven [10].

$$X_t < \sigma_1 < X_c \quad (1)$$

$$Y_t < \sigma_2 < Y_c \quad (2)$$

$$|\tau_{12}| < S \quad (3)$$

### 2.2 Tsai-Hil failure theory

Several researchers, such as von Mises, Henky and Nadai, have proposed the deviatoric and distorted energy of two-dimensional isotropic ductile metals. Accordingly, Hill provided anisotropic criterion for ductile metals [11]. And, then Tsai and Azzi suggested the criterion for the orthotropic composite materials, such as unidirectional lamina [12.]. Based on the foregoing. Tsai-Hill theory is presented a single interactive criterion that combines all the components of stress.

$$\frac{\sigma_1^2}{X_t^2} - \frac{\sigma_1\sigma_2}{X_t^2} + \frac{\sigma_2^2}{Y_t^2} + \frac{\tau_{12}^2}{S^2} = 1 \quad (4)$$

### 2.3 Hashin failure theory

Hashin's theory is a simple theory that predicts the initiation of damage with high accuracy and consists of four separate models of failure. The failure criteria of the 2D lamina are formulated as follows.[13]

Tensile fibers mode  $\sigma_1 > 1$

$$\frac{\sigma_1^2}{X_t^2} + \frac{\tau_{12}^2}{S^2} = 1 \quad (5)$$

Compression fibers mode  $\sigma_1 < 1$

$$-\frac{\sigma_1}{X_c} = 1 \quad (6)$$

Tensile matrix mode  $\sigma_2 > 1$

$$\frac{\sigma_2^2}{Y_t^2} + \frac{\tau_{12}^2}{S^2} = 1 \quad (7)$$

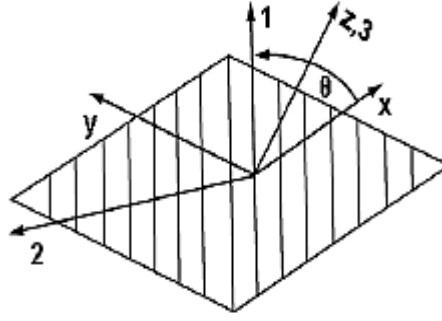
Compression matrix mode  $\sigma_2 < 1$

$$\frac{\tau_{12}^2}{S^2} - \frac{\sigma_2}{Y_c} = 1 \quad (8)$$

### 3. Mathematical Analysis

The laminate consists of several laminas with different angles of fibers in order to obtain strength and high stiffness. The coordinate 1-2 -3 axes are called local axes to represent the angle of the lamina. The direction 1 is aligned with the fibers, the direction 2 is perpendicular to 1 and the directional 3 is perpendicular to axis-1 and axis-2 as the figure (1). The x,y,z is called the global axes .

The stress-strain relationship is presented in the following expressions [14]:



**Figure 1 A lamina principle material coordinate system**

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} \quad (9)$$

Where  $Q_{ij}$  have defined the stiffness matrix

$$Q_{11} = \frac{E_1}{1-\nu_{21}\nu_{12}}, \quad Q_{12} = \frac{E_2\nu_{12}}{1-\nu_{21}\nu_{12}}, \quad Q_{22} = \frac{E_2}{1-\nu_{21}\nu_{12}}, \quad Q_{66} = G_{12} \quad (10)$$

And  $E_{1,2}$  is young's moduli in directions 1 and 2 ,  $\nu_{21}\nu_{12}$  are poison's ratios in directions 1 and 2,  $G_{12}$  is shear modulus in 1-2 plane

The relationship between stress –strain in x-y coordinate is

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \end{bmatrix} \quad (11)$$

Where

$$\begin{aligned} \overline{Q}_{11} &= Q_{11}(\text{COS}\theta)^4 + Q_{22}(\text{SIN}\theta)^4 + 2(Q_{12} + 2Q_{66})(\text{SIN}\theta)^2(\text{COS}\theta)^2 \\ \overline{Q}_{12} &= (Q_{11} + Q_{22} - 4Q_{66})(\text{SIN}\theta)^2(\text{COS}\theta)^2 + Q_{12}((\text{SIN}\theta)^4 + (\text{COS}\theta)^4) \\ \overline{Q}_{22} &= Q_{11}(\text{SIN}\theta)^4 + Q_{22}(\text{COS}\theta)^4 + 2(Q_{12} + 2Q_{66})(\text{SIN}\theta)^2(\text{COS}\theta)^2 \\ \overline{Q}_{16} &= (Q_{11} - Q_{12} - 2Q_{66})(\text{COS}\theta)(\text{SIN}\theta)^3 - (Q_{22} - Q_{12} - 2Q_{66})(\text{COS}\theta)^3(\text{SIN}\theta) \\ \overline{Q}_{26} &= (Q_{11} - Q_{12} - 2Q_{66})(\text{COS}\theta)(\text{SIN}\theta)^3 - (Q_{22} - Q_{12} - 2Q_{66})(\text{COS}\theta)^3(\text{SIN}\theta) \\ \overline{Q}_{66} &= (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66})(\text{SIN}\theta)^2(\text{COS}\theta)^2 + Q_{66}((\text{SIN}\theta)^4 + (\text{COS}\theta)^4) \end{aligned} \quad (12)$$

### 3.2 classical laminate theory

It is a tool capable of analyzing the complex coupling effect of composite materials, as it can predict the strains, curvatures and displacements that formed in the composite as a result of mechanical and thermal loading, As in plate analysis of homogeneous materials, but there are differences in the stress-strain relationship. The classical laminate theory is based on several assumptions.[15]

1. laminate consideration is an orthotropic
2. Linear behavior assumption of each lamina
3. The thickness (h) is greater than the displacements u, v, and w
4. Transverse shear strain and shear stress in out of the plane is negligible,

$$\gamma_{xz} = \gamma_{yz} = \tau_{xz} = \tau_{yz} = 0$$

The stain displacement equation can be written

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} + Z \begin{bmatrix} k_x \\ k_y \\ k_{xy} \end{bmatrix} \quad (13)$$

Where  $\begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix}$  are midplane strains and  $\begin{bmatrix} k_x \\ k_y \\ k_{xy} \end{bmatrix}$  are midplane curvatures

By substitute equation (11) in equation (13)

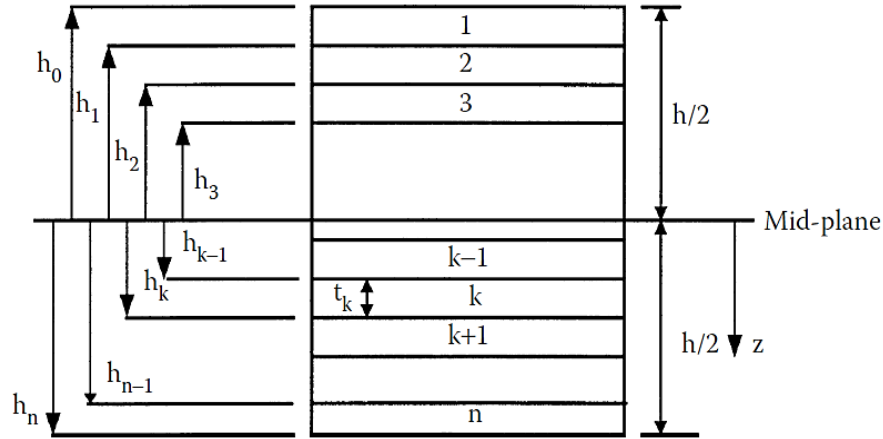
$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} + Z \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix} \begin{bmatrix} k_x \\ k_y \\ k_{xy} \end{bmatrix} \quad (14)$$

The resultant force of the laminate in the x–y plane can be obtained by summing the stresses in each layer through a laminate of thickness [15].

$$\begin{bmatrix} \sigma_1 \\ \sigma_1 \\ \tau_{12} \end{bmatrix} = [T] \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} \quad (15)$$

$$[T] = \begin{bmatrix} c^2 & s^2 & 2sc \\ s^2 & c^2 & -2sc \\ -sc & sc & c^2 - s^2 \end{bmatrix} \quad (16)$$

Where  $[T]$  is the transformation matrix



**Figure 2 Layers location in the laminate[15]**

$$N_x = \int_{-h/2}^{h/2} \sigma_x dz, \quad N_y = \int_{-h/2}^{h/2} \sigma_y dz, \quad N_{xy} = \int_{-h/2}^{h/2} \sigma_{xy} dz \quad (17)$$

$$M_x = \int_{-h/2}^{h/2} \sigma_x Z dz, \quad M_y = \int_{-h/2}^{h/2} \sigma_y Z dz, \quad M_{xy} = \int_{-h/2}^{h/2} \sigma_{xy} Z dz \quad (18)$$

Where  $h/2$  is the half thickness of laminate

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \sum_{k=1}^n \int_{z_{k-1}}^{z_k} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} dz \quad (19)$$

$$\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} = \sum_{k=1}^n \int_{z_{k-1}}^{z_k} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} Z dz \quad (20)$$

$$A_{ij} = \sum_{k=1}^n [\bar{Q}_{ij}]_k (h_k - h_{k-1}), \quad i=1,2,6, \quad j=1,2, \quad (21)$$

$$B_{ij} = \frac{1}{2} \sum_{k=1}^n [\bar{Q}_{ij}]_k (h_k^2 - h_{k-1}^2), \quad i=1,2,6, \quad j=1,2,6, \quad (22)$$

$$D_{ij} = \frac{1}{3} \sum_{k=1}^n [\bar{Q}_{ij}]_k (h_k^3 - h_{k-1}^3), \quad i=1,2,6, \quad j=1,2,6, \quad (23)$$

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} + \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{bmatrix} k_x \\ k_y \\ k_{xy} \end{bmatrix} \quad (24)$$

$$\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} + \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{bmatrix} k_x \\ k_y \\ k_{xy} \end{bmatrix} \quad (25)$$

Where

$A_{ij}$  is extensional stiffness matrix

$B_{ij}$  is coupling stiffness matrix

$D_{ij}$  is bending stiffness matrix

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \varepsilon_{xy}^0 \\ k_x \\ k_y \\ k_{xy} \end{bmatrix} \quad (26)$$

$$\text{Strength ratio} = \frac{\text{Maximum load applied}}{\text{applied load}} = \frac{\text{ultimate strength}}{\text{applied stress}}$$

If  $SR=1$  means the failure load or  $SR>1$  that means the lamina is safe can be increasing the factor of  $SR$  or  $SR<1$  that means the lamina is unsafe, therefore the factor  $SR$  must be reduced.

## 4. Experiential work

### 4.1 Materials

The method used in this research is the method of hand lay-up processes. It is considered one of the oldest and simplest methods in the process of manufacturing composite sheets. Carbon fibers with 0 orientation (Sika Wrap -300 Company, Switzerland)[16] are used as a material for reinforcement in the laminate, with a volume fraction of 30% and its properties in Table (1), and the binder in laminate is the epoxy (type of Quickmast 105) with volume fraction 70%, which is characterized by low creep and can penetrate into cracks easily. It is non-shrinkable and has good chemical resistance and its properties[17] were listed in Table (2). The mold is made of glass and has dimensions of (300x300x5)mm as in the figure(3).



**Table 1 physical and mechanical properties of carbon fibers [16]**

Materials	Density (g/cm <sup>3</sup> )	Tensile strength (Gpa)	Young's modulus(Gpa)	Poisson's ratio
Carbon fibers	1.80	3.9	230	0.2

**Table 2 physical and Mechanical properties of Epoxy [17]**

Materials	Density (g/cm <sup>3</sup> )	Tensile strength (Mpa)	Compressive strength (Mpa)	Young's modulus(Gpa)	Poisson's ratio
Epoxy Quickmast- 105	1.1	≥25	70	4.6	0.3



**Figure 3 Glass mold shape**

#### **4.2 Manufacturing steps**

1. The mold is cleaned and put the transparent nylon in its base to prevent the epoxy from sticking to the mold, and prepare the epoxy material that consists of the hardener and the base and in a proportion 0.33%
2. Putting the first layer of carbon fibers on top of the epoxy and using special rollers for the purpose of wetting the fibers with the epoxy material, then placing the epoxy material and the

second layer of carbon fibers, and so on and according to the conclusions of the Mat lab program to manufacture laminate of five layers with different angles

3. The mold is placed in the vacuum's system (figure 4) for 24 hours to get rid of air bubbles and maintain the volume fraction ratios of the fibers and matrix.

## 5. Tensile tests

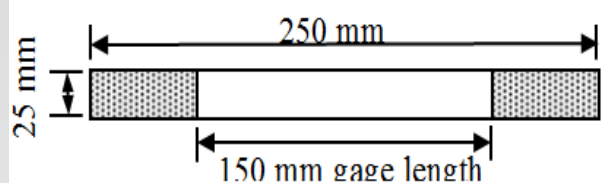
The sheets are cut by a 'special lathe machine (Suda ST1212 CNC router) ' into samples according to the measurements standard as in the figures (5 and 6). Then a tensile test is carried out according to the (ASTM-D3039) standard with a ' tensile machine (LARYEE-50 kN) ' by strain rate (2 mm/min).



**Figure 4 Vacuum's system**



**Figure 5 Dimension specimen**



**Figure 6 Specimens before test**

## 6. Matlab application steps

- Enter the stiffness matrix as in equation 10, the thickness of the laminate ( $h$ ) and the midplane distance for 5 layers ( $h_0, h_1, h_2, h_3, h_4$ )
- Enter the angles of the layers of the laminate, where (I) represents the angles of the first layer, (J) represents the angles of the second layer, (K) represents the angles of the third

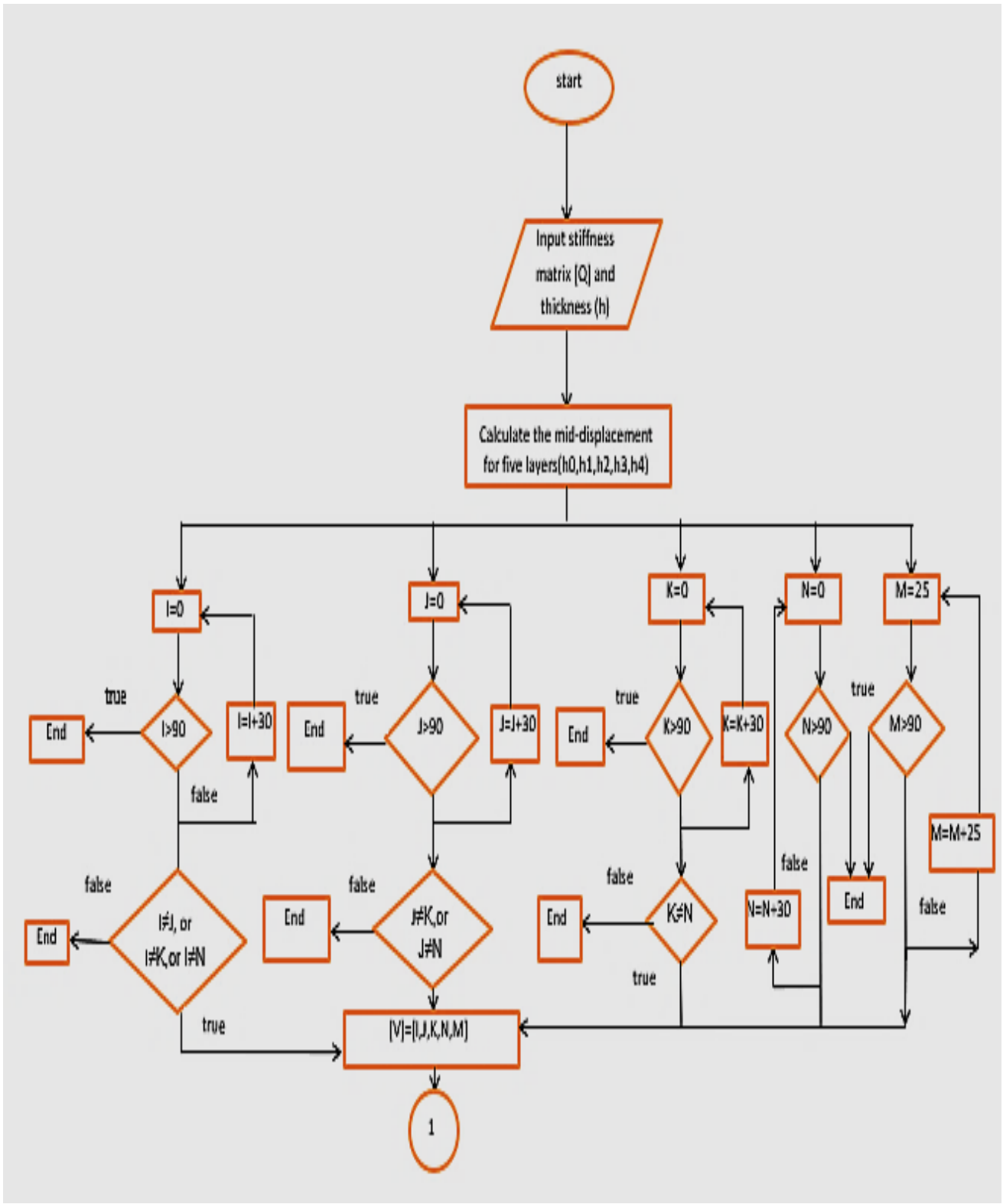
layer, (N) represents the corners of the fourth layer, and (M) is represents the angles of the fifth layer.

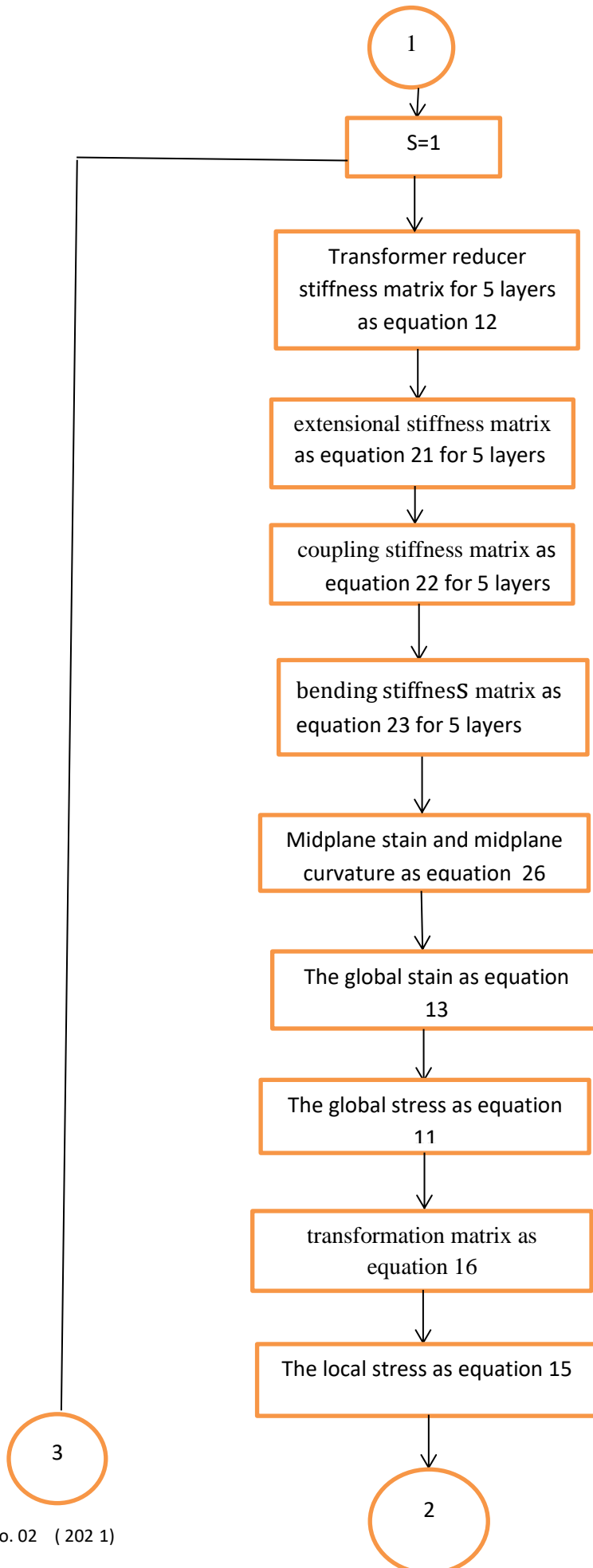
- Set the conditions for non-repetition of the angle of the fibers in the laminate.
- Set a counter (s) for the five layers.
- Calculation of the transformed reduced stiffness matrix for the five layers according to Equation 12.
- Calculation of the extensional stiffness matrix for five layers according to equation 21.
- Calculation of the coupling stiffness matrix for five layers according to equation 22.
- Calculation of the bending stiffness matrix for five layers according to equation 23.
- Calculation of the midplane strain and midplane curvature for five layers according to equation 26 with applied load ( $N_x=1\text{N/m}$ ) in x-direction only
- Calculation of the global strain for five layers according to equation 13.
- Calculation of the global stress for five layers according to equation 11.
- Calculation transformation matrix for five layers according to equation 16.
- Calculation of the local stress for five layers according to equation 15.
- Calculation of the stress ratio for five layers according to mode failure theories

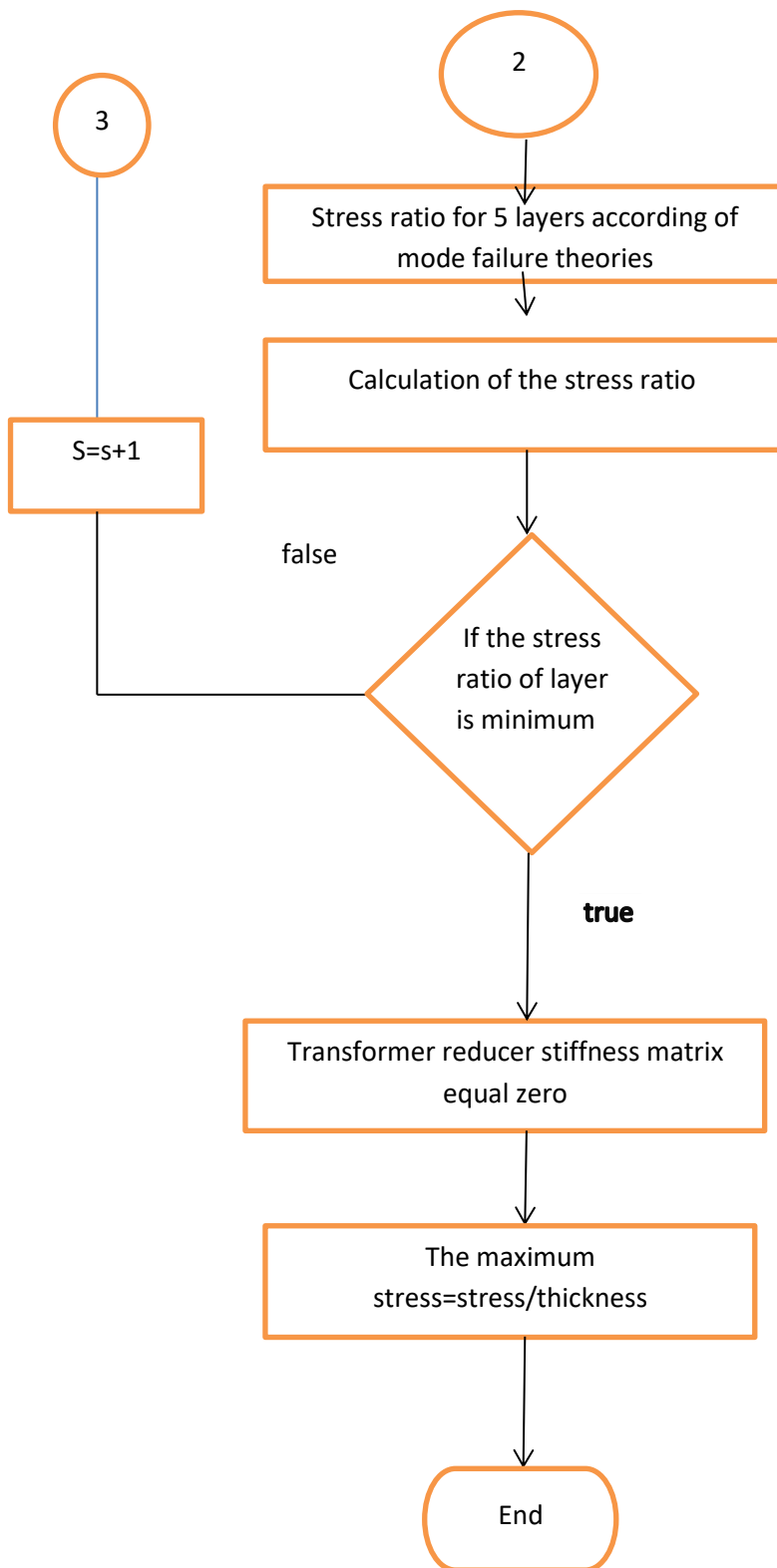
For example, the stress ratio is according to the mode of maximum stress failure as below

$$\text{Minimum Stress ratio} = \left[ \frac{X_t}{\sigma_1}, \frac{Y_t}{\sigma_2}, \frac{s_{12}}{\tau_{12}} \right].$$

- The transformed reduced stiffness matrix of the layer that has minimum stress ratio is zero.
- The steps are repeated by deleting the layer that has the minimum stress ratio.
- Finally, the stress ratio is divided by the thickness of the laminate, and choose the maximum permissible stress.







**Figure 7 Flow chart of MATLAB program**

## 7. Results and discussion

Micromechanical analysis of the constituents of the unidirectional lamina is used to find ultimate strengths, shear moduli and Poisson's ratio [18]. While  $E_1$  and  $E_2$  calculate empirically for the purpose of calculating the reduced stiffness matrix.

$$X_t = 478.2 \text{ Mpa}, X_c = 63.4 \text{ Mpa}, Y_t = 12.22 \text{ Mpa}, Y_c = 26.74 \text{ Mpa}, S = 29.47 \text{ Mpa}$$

$$E_1 = 14.15 \text{ Gpa}, E_2 = 15.4 \text{ Mpa}, \nu_{12} = 0.305$$

The strength ratio of five layers of carbon/epoxy with different angles ( $0^\circ, 25^\circ, 30^\circ, 50^\circ, 60^\circ, 75^\circ, 90^\circ$ ) is calculated based on the last layer failure load for three different failure theories (maximum stress failure, Tsai-Hill failure and Hashin failure) with a load of 1N/m applied in the x-direction only ( $N_x$ ) and comparing them by using MATLAB program by applying classical laminate theory. Where 72 possibilities were obtained for the stacking sequence of a laminate with not repetition of the same lamina in laminate. The results were as follows

1. Theoretical results in Table (3), The difference in the sequence of layers as well as the angles of the fibers leads to the difference in the loading of the laminate.
2. According to the mechanical properties of the lamina, there is a very good match in the calculation of the allowable stresses for three theories (Hashin's theory, Tsai-Hill theory and Maximum stress theory), as is clear from the results in the Table (3). While the maximum stress theory has a slight increase in the strength value of some laminate
3. Three laminates with different stress values (minimum, medium, and maximum stress) were selected for the purpose of comparison as Table (4), it was noted that the practical results of the laminate  $[0/30/60/90/50]$  and laminate  $[30/90/0/60/25]$  have a higher tensile strength than the theoretical results, while the laminate  $[60/30/0/90/75]$  was on the contrary, and The reason may be the presence of bubbles between the layers that have not been eliminated by the vacuum's system.
4. A convergent deviation was also observed between the theoretical and practical results of the three theories of laminate  $[30/90/0/60/25]$  by (11.9% and 9.7%), while the laminate  $[0/30/60/90/50]$ , There was a high amount of deviation between the theoretical and practical results of the three theories by 24%. For the laminate  $[60/30/0/90/75]$  there was a difference in the amount of deviation between maximum stress theory (24.32%) and both of theories Hashin and Tsai-Hill (9.7%) as in the Table (4).

5. Figure (8) shows the stress-strain relationship for the three laminates, where the relationship was linear elastic.

6. Figure (9) shows the shape of the fracture after conducting a tensile test, as the fracture occurs towards the angles of the fibers .

**Table 3 Maximum stress of five layers with different angles for three different failure theories by using the MTLAB program**

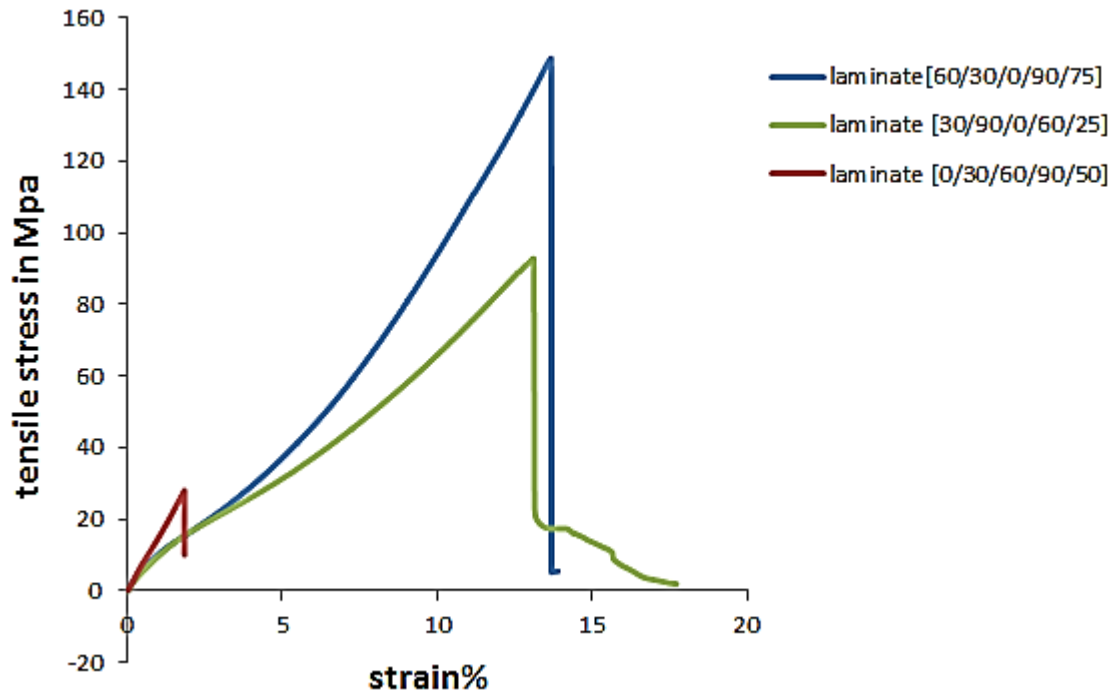
No	Code of laminate	Hashin failure theory	Tsai-HILL failure theory	Maximum stress failure
		Maximum stress(pa)	Maximum stress(pa)	Maximum stress(pa)
1	0/30/60/90/25	3.5857e+07	3.5857e+07	3.6275e+07
2	0/30/60/90/50	1.9684e+07	1.9677e+07	1.9706e+07
3	0/30/60/90/75	2.2954e+07	2.2954e+07	2.7700e+07
4	0/30/90/60/25	3.2931e+07	3.2922e+07	3.3246e+07
5	0/30/90/60/50	2.5659e+07	2.5586e+07	2.5750e+07
6	0/30/90/60/75	2.3210e+07	2.3209e+07	3.1020e+07
7	0/60/30/90/25	3.9862e+07	3.9860e+07	4.0241e+07
8	0/60/30/90/50	2.7490e+07	2.7485e+07	2.7553e+07
9	0/60/30/90/75	2.7543e+07	2.7541e+07	2.7983e+07
10	0/60/90/30/25	4.3433e+07	4.3413e+07	4.3704e+07
11	0/60/90/30/50	2.2054e+07	2.2050e+07	2.2181e+07
12	0/60/90/30/75	2.1624e+07	2.1622e+07	2.1904e+07
13	0/90/30/60/25	2.9097e+07	2.9087e+07	2.9221e+07
14	0/90/30/60/50	1.9307e+07	1.9305e+07	1.9419e+07
15	0/90/30/60/75	2.3745e+07	2.3744e+07	2.4007e+07
16	0/90/60/30/25	3.4170e+07	3.4103e+07	3.4309e+07
17	0/90/60/30/50	1.4767e+07	1.4762e+07	1.4818e+07
18	0/90/60/30/75	1.9822e+07	1.9820e+07	2.0027e+07
19	30/0/60/90/25	4.5688e+07	4.5686e+07	4.6408e+07
20	30/0/60/90/50	2.3369e+07	2.3363e+07	2.3441e+07
21	30/0/60/90/75	3.1623e+07	3.1624e+07	3.1654e+07
22	30/0/90/60/25	3.6426e+07	3.6417e+07	3.6767e+07
23	30/0/90/60/50	2.1556e+07	2.1404e+07	2.1811e+07
24	30/0/90/60/75	1.5879e+07	1.5865e+07	1.5894e+07
25	30/60/0/90/25	5.4985e+07	5.4967e+07	5.5360e+07
26	30/60/0/90/50	1.0361e+08	1.0361e+08	1.2882e+08
27	30/60/0/90/75	9.0783e+07	9.0760e+07	1.2006e+08
28	30/60/90/0/25	3.0224e+07	3.0208e+07	3.0412e+07
29	30/60/90/0/50	3.2703e+07	3.2696e+07	3.2879e+07
30	30/60/90/0/75	3.5616e+07	3.5610e+07	3.5764e+07
31	30/90/0/60/25	8.1530e+07	8.1537e+07	8.3660e+07
32	30/90/0/60/50	8.1576e+07	8.1578e+07	1.2851e+08
33	30/90/0/60/75	1.1419e+08	1.1420e+08	1.3566e+08
34	30/90/60/0/25	3.6644e+07	3.6641e+07	3.6989e+07
35	30/90/60/0/50	4.1112e+07	4.1109e+07	4.1503e+07
36	30/90/60/0/75	4.2373e+07	4.2370e+07	4.2665e+07
37	60/0/30/90/25	6.1797e+07	6.1794e+07	6.2533e+07
38	60/0/30/90/50	3.3777e+07	3.3763e+07	3.3802e+07
39	60/0/30/90/75	3.4762e+07	3.4760e+07	3.5254e+07
40	60/0/90/30/25	6.2696e+07	6.2684e+07	6.3049e+07



41	60/0/90/30/50	2.7627e+07	2.7623e+07	2.7790e+07
42	60/0/90/30/75	2.7797e+07	2.7796e+07	2.8179e+07
43	60/30/0/90/25	4.4966e+07	4.4960e+07	4.8685e+07
44	60/30/0/90/50	1.0585e+08	1.0584e+08	1.3895e+08
45	60/30/0/90/75	1.6310e+08	1.6311e+08	1.8483e+08
46	60/30/90/0/25	2.2932e+07	2.2929e+07	2.3100e+07
47	60/30/90/0/50	2.3562e+07	2.3559e+07	2.3743e+07
48	60/30/90/0/75	2.6335e+07	2.6332e+07	2.6554e+07
49	60/90/0/30/25	5.5608e+07	5.5584e+07	5.5681e+07
50	60/90/0/30/50	8.2832e+07	8.2838e+07	8.5625e+07
51	60/90/0/30/75	7.6435e+07	7.6441e+07	7.7440e+07
52	60/90/30/0/25	2.5211e+07	2.5210e+07	2.5541e+07
53	60/90/30/0/50	2.5096e+07	2.5095e+07	2.5466e+07
54	60/90/30/0/75	2.5460e+07	2.5459e+07	2.5837e+07
55	90/0/30/60/25	5.7373e+07	5.7338e+07	7.2462e+07
56	90/0/30/60/50	2.9477e+07	2.9470e+07	3.2292e+07
57	90/0/30/60/75	3.0868e+07	3.0865e+07	3.1008e+07
58	90/0/60/30/25	4.4796e+07	4.4764e+07	4.4843e+07
59	90/0/60/30/50	2.0030e+07	2.0023e+07	2.0099e+07
60	90/0/60/30/75	3.1623e+07	3.1624e+07	3.1654e+07
61	90/30/0/60/25	8.0360e+07	8.0366e+07	8.3718e+07
62	90/30/0/60/50	6.7012e+07	6.7012e+07	7.3048e+07
63	90/30/0/60/75	1.0475e+08	1.0475e+08	1.3177e+08
64	90/30/60/0/25	3.3211e+07	3.3210e+07	3.3803e+07
65	90/30/60/0/50	3.5108e+07	3.5107e+07	3.5751e+07
66	90/30/60/0/75	3.5762e+07	3.5761e+07	3.6390e+07
67	90/60/0/30/25	8.1987e+07	8.1991e+07	9.1611e+07
68	90/60/0/30/50	8.3989e+07	8.3987e+07	8.4963e+07
69	90/60/0/30/75	7.8084e+07	7.8090e+07	7.8466e+07
60	90/60/30/0/25	3.0492e+07	3.0490e+07	3.1205e+07
71	90/60/30/0/50	2.8419e+07	2.8418e+07	2.9043e+07
72	90/60/30/0/75	2.6172e+07	2.6171e+07	2.6731e+07

**Table 4 comparison between Experimental and theoretical strengths for laminate of sample [60/30/0/90/75], [30/90/0/60/25] and [0/30/60/90//50]**

Laminate	Theoretical Strength (Mpa)		Experimental Strength (Mpa)	Deviation % With Hashin and Tsai –Hill and Experimental	Deviation % With Mximum stress theory and Experimental
	Hashin theory and Tsai-Hill theory	Maximum stress theory			
60/30/0/90/75	163.1	184.8	148.64	9.7%	24.32%
30/90/0/60/25	81.57	83.66	92.68	11.9%	9.7%
0/30/60/90/50	19.68	19.7	26.211	24.9%	24.8%



**Figure 8** Experimental stress-strain diagram for laminate [60/30/0/90/75],[30/90/0/60/25]and [0/30/60/90/50]



**Figure 9** specimens after test

## 8. Conclusion

The maximum allowable stress of the laminate with different fiber orientation angles was calculated by determining the strength ratio for three failure theories based on the last ply failure load. The maximum stress theory represented the limited criterion, Tsai-Hill theory represented the interactive criterion, and Hashin's theory represented the partial criterion by using MATLAB program for laminate consisting of five layers with different angles. Seventy-two possibilities for stacking sequence were obtained. The theoretical results in calculating the strength value of the plates showed a good convergence of the three failure theories, while there was a slight difference in the strength value of some laminates in theory of maximum stress. Three laminates (minimum, medium, and maximum stress value) of theoretical results were selected for the purpose of tensile

testing and comparison. The practical results of the laminate [0/30/60/90/50] and the [30/90/0/60/25] were higher than the theoretical results, while the laminate [60/30/0/90/75] was on the contrary. The deviation between the theoretical and practical results was different. The laminate [30/90/0/60/25] showed the least deviation in the three theories.

## 9. Future work

Conducting a fatigue test for the three laminates ([0/30/60/90/50], [30/90/0/60/25] [60/30/0/90/75]) and to know its behavior when applying a dynamic load.

## Nomenclature

$X_t$	ultimate tensile strength (in fibers direction)
$X_C$	ultimate compressive strength (in fibers direction),
$Y_t$	ultimate tensile strength (in perpendicular fibers direction),
$Y_C$	ultimate compressive strength (in perpendicular fibers direction),
<b>S</b>	ultimate shear strength (in plane 1-2)
$\sigma_1$	longitudinal strength
$\sigma_2$	transverse strength
$\tau_{12}$	shear strength in plane (1-2)
Z,h	distance from neutral axis, total thickness of laminate.
$\sigma_x$	stress in x-direction
$\sigma_y$	stress in y- direction
$\tau_{xy}$	shear stress in x-y plane
$N_X$	normal force per unit length in x-direction
$N_Y$	normal force per unit length in y-direction
$N_{XY}$	shear force per unit length in x-y plane
$\epsilon_x$	strain in x- direction
$\epsilon_y$	strain in y-direction
$\epsilon_{xy}$	shear strain in x-y plane
$M_X$	bending moment per unit length in x-direction
$M_y$	bending moment per unit length in y-direction
$M_{xy}$	twisting moment per unit length

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### الدراسات التجريبية والنظرية لتحليل فشل الطبقة الأخيرة على مواد البوليمر المركبة ذات التوجهات المختلفة وتسلسل التراص للألياف بناءً على نظرية التصفيح الكلاسيكية

**الخلاصة:** تعتبر المواد المركبة أحادية الاتجاه من بين أهم المواد المركبة ولها تطبيقات عديدة في الحياة اليومية. تعد زاوية ألياف الصفيحة عاملاً مهماً في التحكم في خصائص المواد المركبة بالإضافة إلى تسلسل الطبقات. الهدف من هذه الدراسة هو الحصول على أفضل صفائح ذات خصائص ميكانيكية جيدة (قوة وصلابة) مع اختيار نظرية الفشل المناسبة. أجريت هذه الدراسة على صفائح الكربون / الأيبوكسي بزوايا مختلفة (0 درجة ، 25 درجة ، 30 درجة ، 50 درجة ، 60 درجة ، 75 درجة ، 90 درجة) بثلاث نظريات للفشل (نظرية الإجهاد الأقصى ، نظرية تساي هيل وهاشين) تم العثور على نسبة القوة اعتماداً على حمل فشل الطبقة الأخير باستخدام برنامج MATLAB المكون من خمس طبقات من الشرائح وتطبيق نظرية التصفيح الكلاسيكية. تم الحصول على اثنين وسبعين توليفة من الشرائح بدون تكرار الطبقات التي لها نفس الزوايا. أظهرت النتائج النظرية انحرافاً كبيراً بين النتائج النظرية والعملية للصفائح [50/90/60/30/0] للنظريات الثلاث بنسبة (24٪)، بينما كان هناك انحراف متقارب للصفائح [25/060 / 90/30] بنسبة (11.9٪) و (9.7٪). أما بالنسبة للرفائق [75/90/0/30/60] فكان الانحراف بين النتائج النظرية والعملية بحسب نظريتي هاشين وتساوي هيل بنسبة (9.7%) أما الانحراف بالنسبة لنظرية الضغط الأقصى فكان بنسبة (24.3٪).