



Velocity Kinematics Analysis and Trajectory Planning of 5 DOF Robotic Arm

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ABSTRACT

Trajectory planning is important in robots to achieve smooth path planning. This paper presents the velocity kinematics analysis and trajectory planning of a 5 DOF robotic arm. The Jacobian matrix is utilized to analyze the velocity kinematics and the third-order polynomial equation is used to determine the path of angle, velocity, and acceleration of the robotic arm. A 5 DOF robotic arm used with revolute joints and the motion of it is performed using the Arduino Mega2560 (microcontroller) which controlling on servo motors. The results of the velocity kinematics indicated the maximum linear velocity occurs with the z-direction and the cubic polynomial equation satisfied a smooth path for angle and velocity but a discontinuous path for acceleration.

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1. Introduction

In recent years, the use of robotics in the manufacturing industries has increased tremendously. Robotics has enabled flexibility in manufacturing processes that enable the production of high-quality goods at relatively low cost [1]. The path planning of the robotic arm is depending on the kinematic analysis, involves both velocity kinematics and displacement kinematics. These kinematics analyses are a linear relationship between joint space and end-effector space and are essential for precise control and motion planning [2,3]. Displacement Kinematics can be divided into two types: Forward kinematics and inverse kinematics [4] as shown in Figure 1. In this work, the forward, inverse, and velocity kinematics of the 5 DOF robotic arm was derived because of its importance in the movement of the arm in the pick and place process.

2. Related Works

Abdulridha and Abaas [5] presented the forward and velocity kinematics for the robotic arm (Lab-Volt 5150) with 5 DOF, the Denavit-Hartenberg (DH) method was used to solve the forward kinematic while the Jacobian method used in velocity kinematic. From results indicated the greater error in position occurred in the z-axis that is because of the weight of the arm and stepper motors.

Dincer and Cevik [6] suggested the combining of cubic polynomial and Bezier curves based on quadratic Bernstein polynomials to obtain the trajectory planning of 2 DOF parallel mechanism, then compared the position, velocity, and acceleration obtained by suggesting method with the cubic polynomial spline method. The results showed the suggested method gave smoother, and the maximum speed and acceleration values are lower than those obtained by the cubic polynomial method. Somasundar and Yedukondalu [1] attempted to use the Jacobian inverse method to determine the joint angles of the KUKA KR5 robot and simulate the path of end-effector for three curves. MATLAB was used to solve the Jacobian inverse method. The results of three cases showed the increase in step size led to reducing the path deviation.

El Haiek et al. [7] presented the using of a genetic algorithm to obtain the trajectory planning of 3 DOF robotic arm. The goals of trajectory were the minimum joint traveling distance and minimum time. The results indicated the efficiency of the genetic algorithm in determined the trajectory length and time. From previous reviews, the researchers focused either the velocity kinematics or path planning to 2 or 3 DOF robotic arm but this work included the velocity kinematics and trajectory planning of 5 DOF robotic arm.

3. Kinematic Modelling

3.1. Forward Kinematics

The forward kinematics shows the transformation from one frame into another one, starting at the base and ending at the end-effector. In this work, A Denavit-Hartenberg (DH) method was used to forward kinematics analysis of the 5 DOF robotic arm, four parameters are required in the DH method (d_i , a_i , θ_i , and α_i) which tell the location of a link-frame of the robot from a previous link-frame. The transformation matrix between two neighbouring frames is expressed as shown in Eq. (1) [5].

$$A_i = R_{z,\theta_i} T_{z,d_i} T_{x,a_i} R_{x,\alpha_i}$$

$$A_i = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & 0 \\ s\theta_i & c\theta_i & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c\alpha_i & -s\alpha_i & 0 \\ 0 & s\alpha_i & c\alpha_i & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_i = \begin{bmatrix} c\theta_i & -s\theta_i c\alpha_i & s\theta_i s\alpha_i & a_i c\theta_i \\ s\theta_i & c\theta_i c\alpha_i & -c\theta_i s\alpha_i & a_i s\theta_i \\ 0 & s\alpha_i & c\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1)$$

Where:

$a_i c\theta_i$: Position of the end-effector in the x-direction (Px).

$a_i s\theta_i$: Position of the end-effector in the y-direction (Py).

d_i : Position of the end-effector in the z-direction (Pz).

a_i : Link length.

α_i : Link twist.

d_i : Link offset.

θ_i : Joint angle.

The DH parameters for the 5 DOF robotic arm used are listed in Table 1, where shows rotation about the z-axis, rotation about the x-axis, transition along the z-axis, and transition along the x-axis. By substituting the D-H parameters in Table 1 into Eq (1), we can obtain the individual transformation matrices A_1^0 to A_5^4 , and a global matrix of transformation A_5^0 , as illustrated in Figure 1.

Table 1: DH parameters for the robotic arm

Link	a_i (mm)	α_i (degree)	d_i (mm)	θ_i (degree)
1	0	90	105	θ_1
2	105	0	0	θ_2
3	100	0	0	θ_3
4	0	90	0	θ_4
5	0	0	150	θ_5

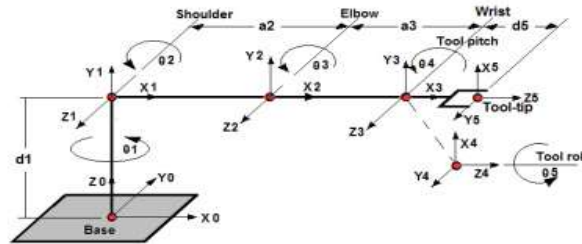


Figure 1: Link coordinate diagram of the robotic arm [5]

$$A_1^0 = R_{z,\theta_1} T_{z,d_1} T_{x,a_1} R_{x,\alpha_1}$$

$$A_1^0 = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & a_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c(90) & -s(90) & 0 \\ 0 & s(90) & c(90) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_1^0 = \begin{bmatrix} c_1 & 0 & s_1 & 0 \\ s_1 & 0 & -c_1 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2)$$

In a similar way will find the A_2^1, A_3^2, A_4^3 , and A_5^4 .

$$A_2^1 = \begin{bmatrix} c_2 & -s_2 & 0 & a_2 c_2 \\ s_2 & c_2 & 0 & a_2 s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (3)$$

$$A_3^2 = \begin{bmatrix} c_3 & -s_3 & 0 & a_3 c_3 \\ s_3 & c_3 & 0 & a_3 s_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (4)$$

$$A_4^3 = \begin{bmatrix} c_4 & 0 & s_4 & 0 \\ s_4 & 0 & -c_4 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (5)$$

$$A_5^4 = \begin{bmatrix} c_5 & -s_5 & 0 & 0 \\ s_5 & c_5 & 0 & 0 \\ 0 & 0 & 1 & d_5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (6)$$

$$A_5^0 = T_1^0 T_2^1 T_3^2 T_4^3 T_5^4$$

$$A_5^0 = \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (7)$$

$$\begin{aligned}
m_{11} &= c_{12345} + s_{15} \\
m_{12} &= -s_5 c_{1234} + s_1 c_5 \\
m_{13} &= c_1 s_{234} \\
m_{14} &= c_1 (d_5 s_{234} + a_3 c_{23} + a_2 c_2) \\
m_{21} &= s_1 c_{2345} - c_1 s_5 \\
m_{22} &= -s_{15} c_{234} - c_{15} \\
m_{23} &= s_1 s_{234} \\
m_{24} &= s_1 (d_5 s_{234} + a_3 c_{23} + a_2 c_2) \\
m_{31} &= c_5 s_{234} \\
m_{32} &= -s_5 s_{234} \\
m_{33} &= -c_{234} \\
m_{34} &= -d_5 c_{234} + a_3 s_{23} + a_2 s_2 + d_1
\end{aligned}$$

II. Velocity kinematics

Velocity kinematics is used to change the velocities of joints into Cartesian velocities using the Jacobian matrix. Where this matrix is important to control the robotic arm movement to satisfy the smooth path planning. The relationships between the joint velocity and the linear and angular velocity of the end effector are shown following [5]:

$$\dot{p} = J_p(\theta)\dot{\theta} \quad (8)$$

$$\dot{w} = J_w(\theta)\dot{\theta} \quad (9)$$

By combining the Eqs (8 and 9) will give the Jacobian matrix:

$$J_\theta = \begin{bmatrix} J_p \\ J_w \end{bmatrix} = [6 * n] = \begin{bmatrix} J_{p1} & \dots & J_{pn} \\ J_{w1} & \dots & J_{wn} \end{bmatrix} \quad (10)$$

$$v = J(\theta)\dot{\theta} \quad (11)$$

Where :

θ : Joint angle.

$\dot{\theta}$: joint velocity.

\dot{p} : linear velocity for end-effector.

\dot{w} : angular velocity for end-effector.

The number of rows in the Jacobian matrix equal to the number of DOF in the cartesian coordinate (three linear and three angular) while the number of the columns is equal to the DOF number in the joints [5].

The matrix of Jacobian for revolute joint can be obtained using the following equations:

$$J_{pi} = z_{i-1} \times (o_n - o_{i-1}) \quad (12)$$

$$J_{wi} = z_{i-1} \quad (13)$$

Where:

J_{pi} : Linear Jacobian matrix.

J_{wi} : Angular Jacobian matrix.

In this work, the 5 DOF robotic arm with revolute joints were used, therefore the Jacobian matrix will be obtained as follow:

$$J(\theta) = \begin{bmatrix} z_0 \times (o_5 - o_0) & z_1 \times (o_5 - o_1) & z_2 \times (o_5 - o_2) & \dots \\ z_0 & z_1 & z_2 & \dots \\ \dots & z_3 \times (o_5 - o_3) & z_4 \times (o_5 - o_4) & \dots \\ & z_3 & z_4 & \dots \end{bmatrix} \quad (14)$$

From the forward kinematics can be calculated the linear part of $J(\theta)$, the values of $(o_n - o_{i-1})$ for 5 DOF robotic arm used are:

$$o_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{aligned}
 o_1 &= \begin{bmatrix} 0 \\ 0 \\ d_1 \end{bmatrix} \\
 o_2 &= \begin{bmatrix} a_2 c_1 c_2 \\ a_2 s_1 c_2 \\ a_2 s_2 + d_1 \end{bmatrix} \\
 o_3 &= \begin{bmatrix} a_3 c_1 c_{23} + a_2 c_1 c_2 \\ a_3 s_1 c_{23} + a_2 s_1 c_2 \\ a_3 s_{23} + a_2 s_2 + d_1 \end{bmatrix} \\
 o_4 &= \begin{bmatrix} a_3 c_1 c_{23} + a_2 c_1 c_2 \\ a_3 s_1 c_{23} + a_2 s_1 c_2 \\ a_3 s_{23} + a_2 s_2 + d_1 \end{bmatrix} \\
 o_5 &= \begin{bmatrix} d_5 c_1 s_{234} + a_3 c_1 c_{23} + a_2 c_1 c_2 \\ d_5 s_1 s_{234} + a_3 s_1 c_{23} + a_2 s_1 c_2 \\ -d_5 c_{234} + a_3 s_{23} + a_2 s_2 + d_1 \end{bmatrix}
 \end{aligned}$$

Also, the angular part of $J(\theta)$ can be obtained using the forward kinematics, the values of z_i are:

$$\begin{aligned}
 z_0 &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \\
 z_1 &= \begin{bmatrix} s_1 \\ -c_1 \\ 0 \end{bmatrix} \\
 z_2 &= \begin{bmatrix} s_1 \\ -c_1 \\ 0 \end{bmatrix} \\
 z_3 &= \begin{bmatrix} s_1 \\ -c_1 \\ 0 \end{bmatrix} \\
 z_4 &= \begin{bmatrix} c_1 & s_{234} \\ s_1 & s_{234} \\ -c_{234} \end{bmatrix}
 \end{aligned}$$

Then

$$z_0 \times (o_5 - o_0) = \begin{bmatrix} -d_5 s_1 s_{234} - a_3 s_1 c_{23} - a_2 s_1 c_2 \\ d_5 c_1 s_{234} + a_3 c_1 c_{23} + a_2 c_1 c_2 \\ 0 \end{bmatrix} = J_{11} \quad (15)$$

$$z_1 \times (o_5 - o_1) = \begin{bmatrix} d_5 c_1 c_{234} - a_3 c_1 s_{23} - a_2 s_2 c_1 \\ d_5 s_1 c_{234} - a_3 s_1 s_{23} - a_2 s_2 s_1 \\ d_5 s_{234} + a_3 c_{23} + a_2 c_2 \end{bmatrix} = J_{12} \quad (16)$$

$$\begin{aligned}
 z_2 \times (o_5 - o_2) &= \\
 \begin{bmatrix} d_5 c_1 c_{234} - a_3 c_1 s_{23} \\ d_5 s_1 c_{234} - a_3 s_1 s_{23} \\ d_5 s_{234} + a_3 c_{23} \end{bmatrix} &= J_{13} \quad (17)
 \end{aligned}$$

$$z_3 \times (o_5 - o_3) = \begin{bmatrix} d_5 c_1 c_{234} \\ d_5 s_1 c_{234} \\ d_5 s_{234} \end{bmatrix} = J_{14} \quad (18)$$

$$z_4 \times (o_5 - o_4) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = J_{15} \quad (19)$$

The Jacobian matrix will become:

$$J(\theta) = \begin{bmatrix} J_{11} & J_{12} & J_{13} & J_{14} & J_{15} \\ 0 & s_1 & s_1 & s_1 & c_1 s_{234} \\ 0 & -c_1 & -c_1 & -c_1 & s_1 s_{234} \\ 1 & 0 & 0 & 0 & -c_{234} \end{bmatrix}$$

4. Trajectory Planning

The trajectory planning is meant to find the path from initial to the final configuration of the end-effector of the robotic arm or for each joint, that is, the path with time to satisfying the smooth variation of position, velocity, and acceleration in the robotic arm [8]. In this work, a cubic polynomial method is using in determining the trajectory planning with four constraints, thus to determine the cubic trajectory for angular displacement as follow:

$$q(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 \quad (21)$$

And for velocity and acceleration as

$$\dot{q}(t) = a_1 + 2a_2 t + 3a_3 t^2 \quad (22)$$

$$\ddot{q}(t) = 2a_2 + 6a_3 t \quad (23)$$

Where:

t: The time (sec).

q: The angular displacement (deg).

\dot{q} : The angular velocity (deg/sec).

\ddot{q} : The angular acceleration (deg/sec²).

$a_0, a_1, a_2, \text{ and } a_3$: Independent coefficients.

The four constraints are the initial time (t_0), final time (t_f), initial velocity ($\dot{q}(t_0)$), and final velocity ($\dot{q}(t_f)$), where these constraints were applied in Eqs (21 and 22) as follow:

$$q(t_0) = a_0 + a_1 t_0 + a_2 t_0^2 + a_3 t_0^3 = q_0 \quad (24)$$

$$q(t_f) = a_0 + a_1 t_f + a_2 t_f^2 + a_3 t_f^3 = q_f \quad (25)$$

$$\dot{q}(t_0) = a_1 + 2a_2 t_0 + 3a_3 t_0^2 = v_0 \quad (26)$$

$$\dot{q}(t_f) = a_1 + 2a_2 t_f + 3a_3 t_f^2 = v_f \quad (27)$$

The v_0 and v_f are equal to zero and the $t_0 = 0$, these constraints are applied in the above equations, we get:

$$q_0 = a_0 + a_1 t_0 + a_2 t_0^2 + a_3 t_0^3 \quad (28)$$

$$q_0 = a_0 \quad (29)$$

$$q_f = a_0 + a_1 t_f + a_2 t_f^2 + a_3 t_f^3$$

$$v_0 = a_1 + 2a_2 t_0 + 3a_3 t_0^2 \quad (30)$$

$$a_1 = 0$$

$$v_f = a_1 + 2a_2 t_f + 3a_3 t_f^2 \quad (31)$$

$$a_1 + 2a_2 t_f + 3a_3 t_f^2 = 0$$

Solving the previous equations, yields:

$$a_0 = q_0$$

$$a_1 = 0$$

$$a_2 = \frac{3(q_f - q_0)}{t_f^2}$$

$$a_3 = \frac{-2(q_f - q_0)}{t_f^3}$$

After applying these independent coefficients in Eqs (21, 22, and 23), we get:

$$q(t) = q_0 + \frac{3(q_f - q_0)}{t_f^2} t^2 - \frac{2(q_f - q_0)}{t_f^3} t^3 \quad (32)$$

$$\dot{q}(t) = \frac{6(q_f - q_0)}{t_f^2} t - \frac{6(q_f - q_0)}{t_f^3} t^2 \quad (33)$$

$$\ddot{q}(t) = \frac{6(q_f - q_0)}{t_f^2} - \frac{12(q_f - q_0)}{t_f^3} t$$

5. Methodology

The 5 DOF robotic arm was used in this work as shown in Figure 2. Four cases were performed for testing the robotic system in sorting objects from the conveyor belt into the discharge station as shown in Figure 3. The Jacobian method was used to calculate the velocity of joints. Also, the cubic polynomial equation was used to determine the trajectory planning.

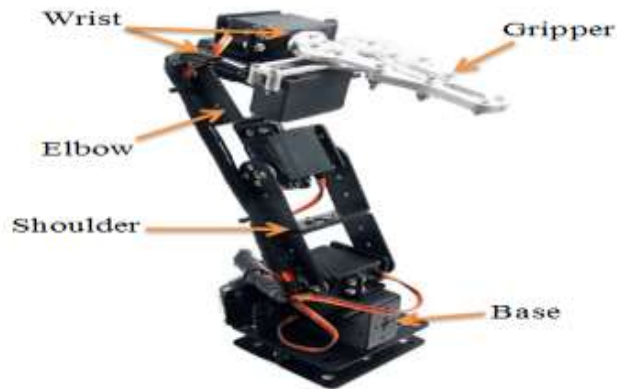


Figure 2: The robotic arm.

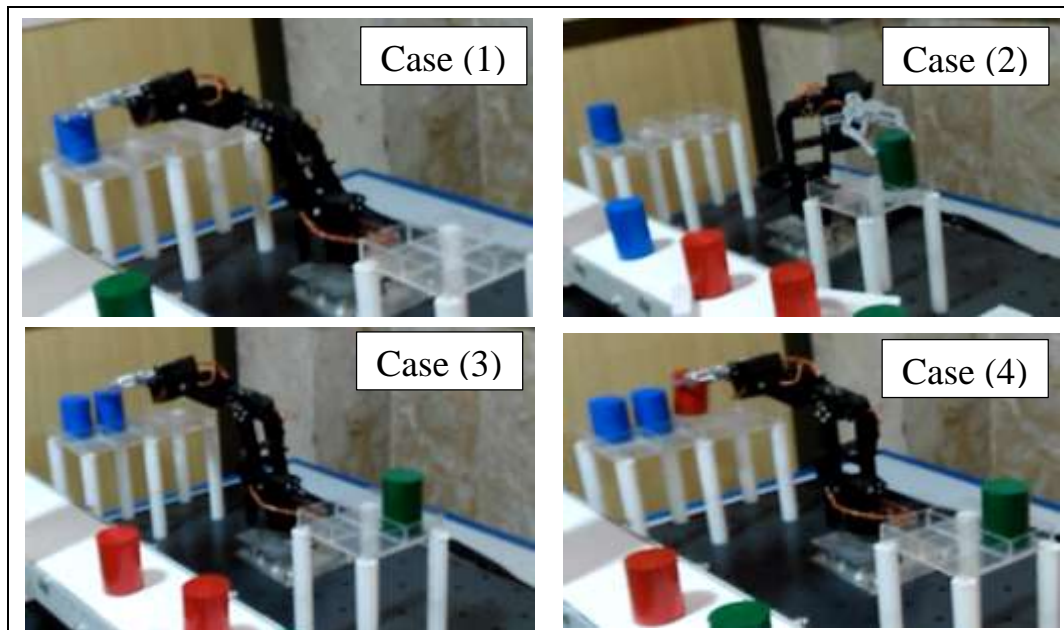


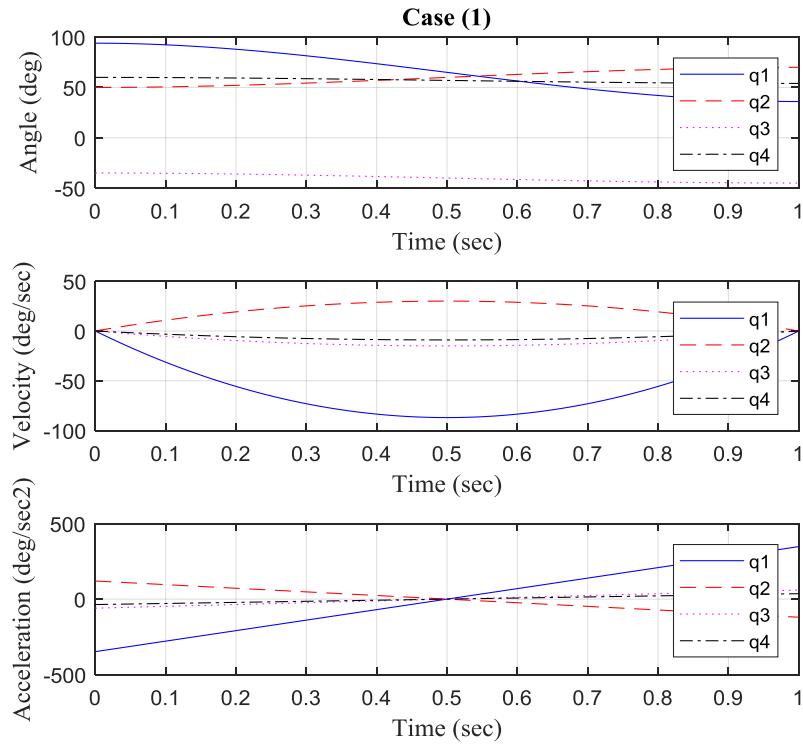
Figure 3: The four cases.

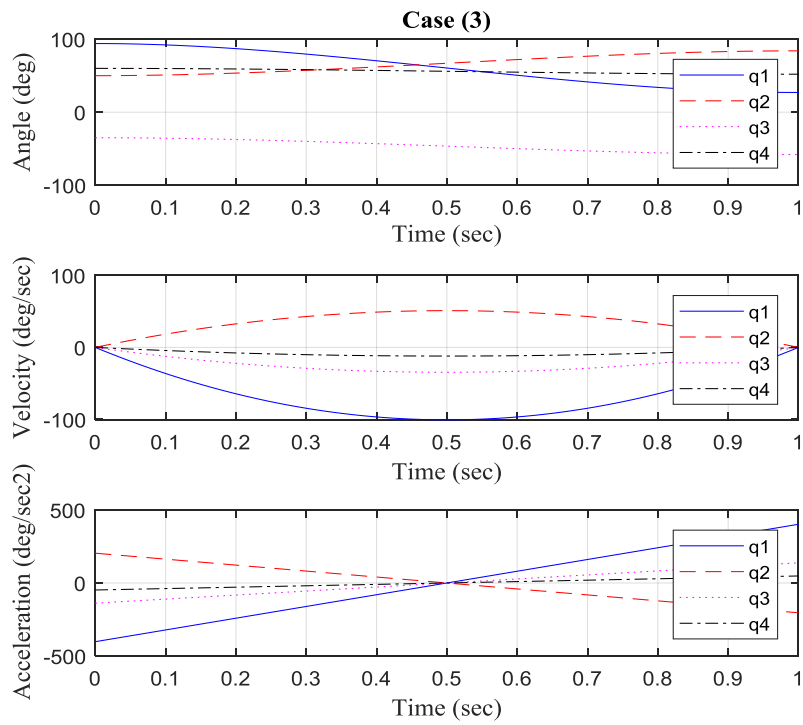
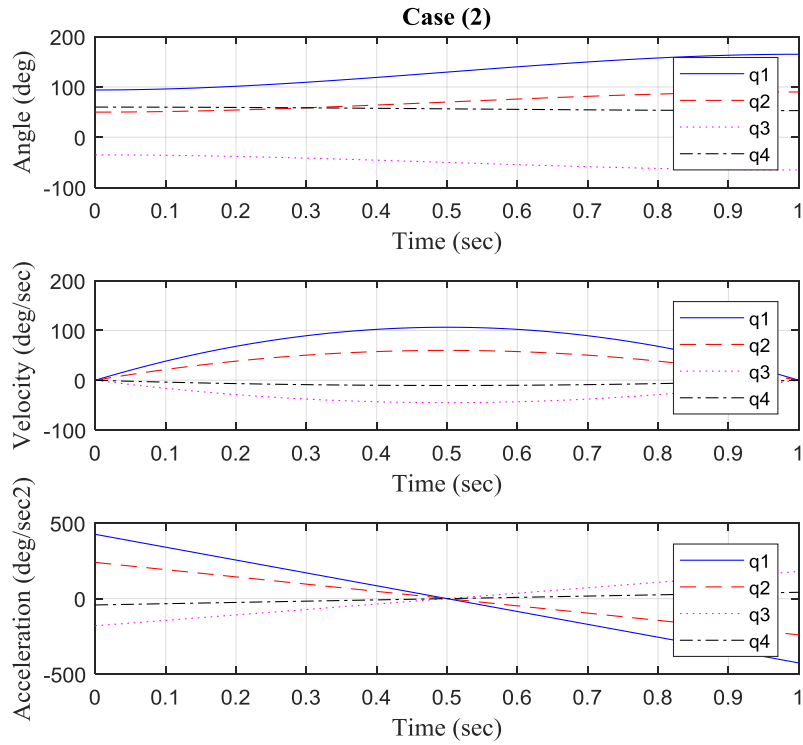
6. Results and Discussion

The velocities of the end-effector of the robotic arm for four cases shown in Figure 3 were calculated using Eq (11) with joint velocity (20 deg/sec) and listed in Table 2. Cubic polynomial trajectories were applied in this work, and the Eqs (32, 33, and 34) were used to determine the path of angle, velocity, and acceleration respectively. A MATLAB program was used to plot these results as shown in Figure 4.

Table 2: The linear and angular velocity of end-effector

Case No.	Linear Velocity *10 ³ (mm/sec)			Angular Velocity *10 ³ (deg/sec)		
	v _x	v _y	v _z	w _x	w _y	w _z
1	-4.793	3.286	13.178	0.051	-0.037	0.016
2	0.625	-5.082	12.429	-0.003	0.063	0.016
3	-4.004	3.517	12.618	0.045	-0.045	0.016
4	-3.131	3.89	12.252	0.035	-0.053	0.016





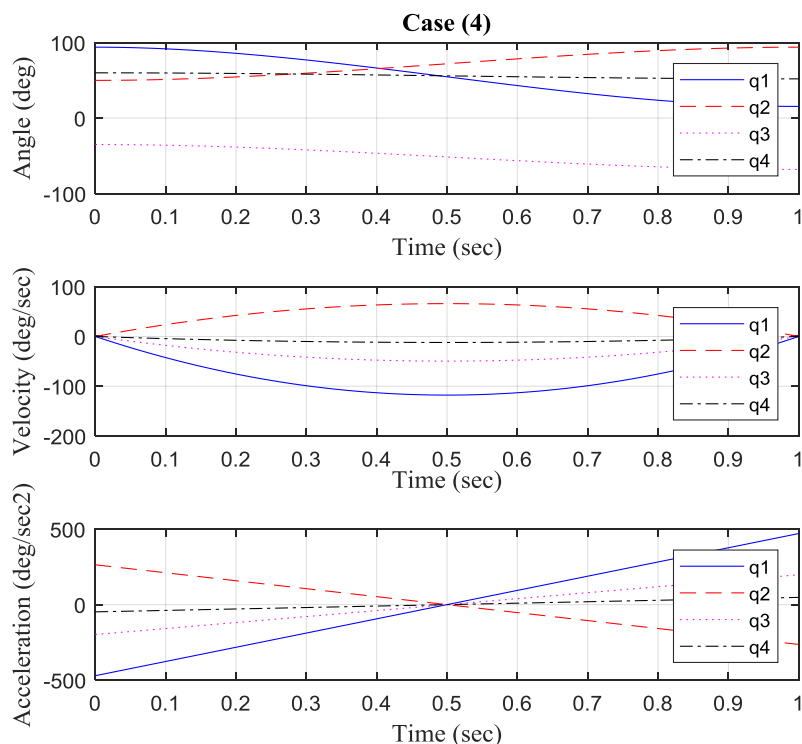


Figure 4: Cubic polynomial trajectory of four cases.

From Table 2, found that the highest values of linear velocity were with the z-direction where the maximum value (13.178×10^3 mm/sec) and in an angular velocity the velocity with the y-direction was had the highest values where the maximum value was (0.063×10^3 deg/sec). These results indicated the linear velocity more effect on the end-effector velocity from an angular velocity. The negative sign of the velocity values means that the direction of the movement axis of the end-effector was opposite the direction of the axis at the base of the robotic arm.

The cubic polynomial trajectory (angle, velocity, and acceleration) of cases showed in Figure 3, it was concluded that the cubic polynomial gives a continuous angle and velocity while discontinuous in acceleration because the second derivative of position with time was a linear equation. The discontinuous in acceleration leads to vibration and reduces the accuracy of the trajectory and to avoid this problem, a high order polynomial (fifth-order) must be used to determine the angle, velocity, and acceleration.

The initial and final value of the fifth joint of the robotic arm was constant (90°) in all cases, therefore when applied in Eqs (33 and 34) given zero value to velocity and acceleration.

7. Conclusions

From the velocity kinematics analysis of the robotic arm, the results indicated the maximum linear velocity occurs with the z-direction and the velocity of the end-effector of the robotic arm used does not depend on the length of the first link and the linear velocity do not depend on the fifth joint angle. Also, the angular velocity does not depend on the length of the second, third, and fifth links. While in trajectory planning, a cubic polynomial trajectory achieves a smooth path for angle and velocity but a discontinuous path for acceleration.

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