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# Discontinuous Control and Stability Analysis of Step-Down DC-DC Voltage Converters

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KEYWORDS

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ABSTRACT

*Step down DC-DC converters are power electronic circuits, which mainly used to convert voltage from a level to a lower level. In this paper, a discontinuous controller is proposed as a control method in order to control Step-Down DC-DC converters. A Lyapunov stability criterion is used to mathematically prove the ability of the proposed controller to give the desired voltage. Simulationsl1 are performedl1 in MATLABl1 software. The simulationl1 resultsl1 are presentedl1 for changesl1 in referencel1 voltagel1 and inputl1 voltagel1 as well as stepl1 loadl1 variations. The resultsl1 showl1 the goodl1 performancel1 of the proposedl1 discontinuousl1 controller.*

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## 1. Introduction

Modern electronic systems require high-quality, small, lightweight, reliable, and efficient power supplies. Therefore, the DC/DC converters are widely used in many industrial and electrical systems. The most familiar are switching power supplies, DC drives, and photovoltaic systems. The stability is an important aspect in the design of switch mode power supplies; a feedback control is used to achieve the required performance [1-3].

Different control techniques are applied to regulate the DC-DC converters, especially buck converters, in order to obtain a robust output voltage [4,5].

As DC/DC converters are nonlinear and time-variant systems, the application of linear control techniques for the control of these converters is not suitable. In order to design linear control system using classical linear control techniques, the small signal model is derived by the linearization around a precise operating point from the state space average model [4]. The controllers based on these techniques are simple to implement; however, it is difficult to account the variation of system parameters, because of the dependence of small signal model parameters on the converter operating point [6].

Sliding mode control is a well-known discontinuous feedback control technique which has been exhaustively explored in many books and journal articles. The technique is naturally suited for the regulation of switched controlled systems, such as power electronics devices, in general, and DC/DC power converters, in particular [2]. Many sliding mode controllers have been proposed and used for DC/DC converters [6–9]. These controllers are direct [6] or indirect control method [6, 7]. The direct method is proposed in [6].

In [7], the output capacitor current of DC/DC converter is used to control the output voltage. The differences of the DC/DC output voltage and the reference voltage enter the proportional-Integrator (PI) type controller, and then the output capacitor current of DC/DC converter is decreased from the output of controller [8]. The output voltage and inductor current are used to control of DC/DC converter in [9]. These references [6–9] have not completely investigated the load and line as well as reference regulations.

In this paper, the application of discontinuous control for step down DC-DC voltage converters has been analyzed. The operation and mathematical model for the converter have been described in section 2. In section 3, the analysis for the control is made. Simulations are performed in section 4 and some conclusions are made in section 5.

## 2. Circuit Description

The topology of a step-down converter is shown in Figure 1. When the switch is on position 1 the circuit is connected to the dc input source resulting an output voltage across the load resistor. If the switch changes its position to position 0, the inductor current will discharge through the load. Controlling switch position the output voltage can be maintained at a desired level lower than the input source voltage [10].

A practical realization for the circuit of Figure 1 is shown in Figure 2.

For a negligible resistance offered by DC-DC converter inductor, the depicted DC-DC step-down converter (Figure 1) can be described by the following equation:

$$L\frac{dI}{dt} = uE - v_o \qquad (1)$$

$I$ is the converter current (Amp), $L$ and $R$ are the inductance (Henry) and load resistance (Ohm) respectively, $E$ and $v_o$ are input source and output voltages (Volts) respectively, and $u$ is the control input, or a switching function, which can acquire two discrete values – either 0 or 1.

For control purposes, the model of Eq. (1) can be written in terms of input voltage $E$ and output voltage $v_o$ only as follows: By Applying Ohm's law, we get $I = \frac{v_o}{R}$ which results in $\frac{dI}{dt} = \frac{\dot{v}_o}{R}$ Substitute for $\frac{dI}{dt}$ in (1) one can get

$$\frac{L}{R}\dot{v}_o = uE - v_o \qquad (2)$$

Rearranging the above equation one can get

$$\dot{v}_o = -\frac{R}{L}v_o + \frac{R}{L}uE \qquad (3)$$

The above equation represents the circuit dynamics of Figure 1 and which will be used in control analysis.

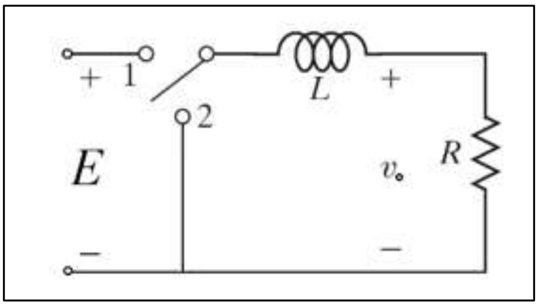

**Figure 1: Schematic of conventional step-down DC-DC voltage converter [10].**

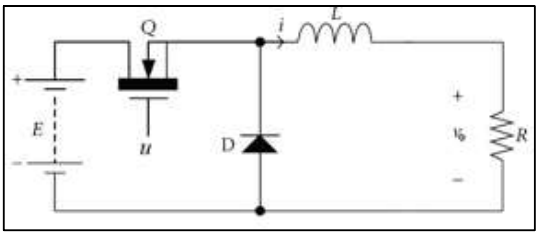

**Figure 2: Practical circuit of step-down DC-DC voltage converter [11].**

## 3. Discontinuous Control Analysis

In this section, the proposed control method is analyzed to show sufficient conditions for the converter to operate correctly.

First, define the voltage error e as

$$e = v_o - v_d \quad (4)$$

Differentiate the above equation

$$\dot{e} = \dot{v}_o - \dot{v}_d \quad (5)$$

Since voltage converters are generally deployed to supply a constant desired voltage, then it's assumed that $\dot{v}_d = 0$. Substitute for $\dot{v}_o$ from Eq. (3) into Eq. (5) results in

$$\dot{e} = -\frac{R}{L}v_o + \frac{R}{L}uE \quad (6)$$

From Eq. (4), $v_o$ can be replaced by $v_o = e + v_d$, then substitute for $v_o$ into Eq. (6) results in

$$\dot{e} = -\frac{R}{L}(e + v_d) + \frac{R}{L}uE \quad (7)$$

The above equation represents the error dynamics of the step-down converter, and the task of the control action *u* is to make the voltage error *e* reach zero value which results in $v_o = v_d$.

In this work, it's assumed that the converter operates in switching mode, that's, the control signal can have only the values 1 (on) and 0 (off), and that control signal is supplied to the transistor Q (Figure 2) which operates as a switch. In this manner, the transistor switch Q controls the circuit current by connecting and disconnecting the circuit to the voltage supply. Based on that, When the output voltage $v_o$ is smaller than the desired $v_d$ $(e < 0)$, then the control action takes the value 1 and the transistor switch Q is on, and when the output voltage $v_o$ is larger than the desired $v_d$ $(e > 0)$, then the control action takes the value 0 and the transistor switch Q is off. Based on that, the control action *u* can be defined as [10]

$$u = \begin{cases} 1\ (on)\ \ if\ \ e < 0\ (v_o < v_d) \\ 0\ (off)\ if\ \ e > 0\ (v_o > v_d) \end{cases}$$

$$= \frac{1}{2}\big(1 - sign(e)\big) \quad (8)$$

Where $sign(e) = \begin{cases} 1 & if\ \ e > 0 \\ -1 & if\ \ e < 0 \end{cases}$

The main contribution of this work is to investigate the operation of the step-down converter under the effect of the proposed control action described by Eq. (8) above and find a suitable value for the supply voltage to operate correctly.

To do so, the Lyapunov stability is applied. First, define a positive definite Lyapunov function as [12]

$$V = \frac{1}{2}e^2 > 0 \quad \forall e \neq 0 \quad (9)$$

Then, for the system of Eq. (7) to be stable, then the time derivative of the Lyapunov function must be negative definite [12]. Thus, differentiating Eq. (9) results in

$$\dot{V} = e\dot{e} < 0 \quad \forall e \neq 0 \tag{10}$$

Substitute for $\dot{e}$ from Eq. (7) results in

$$\dot{V} = e\left[-\frac{R}{L}(e + v_d) + \frac{R}{L}uE\right] < 0 \tag{11}$$

In the case of the output voltage $v_o$ is less than the desired $v_d$ $(v_o < v_d)$, i.e. $(e < 0)$, then the control action $u$ will have the value 1 $(u = 1)$ and Eq. (11) becomes

$$\dot{V} = e\left[-\frac{R}{L}(e + v_d) + \frac{R}{L}E\right] < 0 \tag{12}$$

Since $e < 0$, then the other term enclosed in brackets must be positive to ensure that $\dot{V} < 0$, then

$$-\frac{R}{L}(e + v_d) + \frac{R}{L}E > 0 \tag{13}$$

Multiply both sides by $\frac{L}{R}$ gives

$$-e - v_d + E > 0 \tag{14}$$

Since $e < 0$, then $-e > 0$ already, which leaves

$$-v_d + E > 0 \tag{15}$$

And finally

$$E > v_d \tag{16}$$

On the other hand, if the output voltage $v_o$ is greater than the desired $v_d(v_o > v_d)$, i.e. $(e > 0)$, then the control action u will have the value 0 $(u = 0)$, and Eq. (11) becomes

$$\dot{V} = e\left[-\frac{R}{L}(e + v_d)\right] < 0 \tag{17}$$

Since $e > 0$, then the other term enclosed in brackets must be negative to ensure that $\dot{V} < 0$, then

$$-\frac{R}{L}(e + v_d) < 0 \tag{18}$$

Since $e > 0$ and $v_d$ assumed to be positive as mentioned earlier, $R$ and $L$ are positive, then the above inequality is proved.

The above results of Eq.'s (16) and (18) shows that in case that the desired voltage $v_d$ is positive, then the supply voltage $E$ must always be greater than the desired voltage $v_d$, in addition, when the output voltage is greater than the desired, then the control action $u$=0 and the output voltage will decay to the desired. A block diagram of the overall system is shown in Figure 3.

A final note can be made for the equivalent control action. Back to Eq. (7), when the voltage error $e$ reaches zero, i.e. $= 0$ , which also leads to $\dot{e} = 0$, then Eq. (7) becomes

$$0 = -\frac{R}{L}(v_d) + \frac{R}{L}uE \tag{19}$$

Which leads to

$$u_{eq} = \frac{v_d}{E} \tag{20}$$

What Eq. (20) means is that, despite that the controller action $u$ has a discontinuous nature which alternate between values 0 and 1 only, the average of that signal is equal to the ratio between the desired voltage $v_d$ and supply voltage $E$.

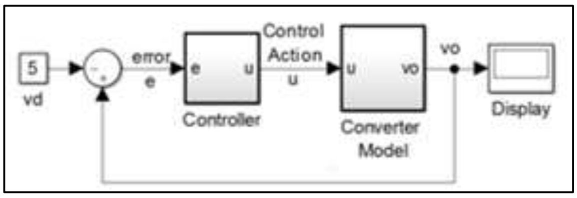

**Figure 3: Block diagram of step-down DC-DC voltage converter**

## 4. Simulation and Results

In order to verify that the operation of the converter and controller, a numerical simulation using MATLAB is performed. For all simulations, the sampling time assumed to be 0.01msec and the inductance $L = 0.2\ H$. the next simulations show the converter operation under different operation conditions which include the variety of the supply voltage $E$ and the load resistance $R$.

### *I. Constant supply voltage*

In the first simulation, the converter is assumed to supply a constant value desired voltage. In this simulation, the desired voltage is $v_d = 5$ volts. The converter is assumed to be supplied by $E = 12$ volts source and load resistance $R = 10\, ohm$. The simulation results are shown in Figures 4-9. Figure 4 shows the output voltage $v_o$, Figure 5shows the load current $I$, the voltage error $e$ is shown in Figure 6, the Lyapunov function is shown in Figure 7 and control action $u$ and its equivalent value are shown in Figures 8 and 9 respectively.

Figure 8 below shows the control action described by Eq. (8). It can be seen how the control action oscillates between values 0 and 1 in order to keep the output voltage $v_o$ at the desired level. However, to get more insight into the operation of control action, the control signal can be filtered to get rid of the high-frequency components and the averaged control signal is obtained. In this work, a second-order filter is applied with the time constant of 1ms sec is used and the equivalent control action is shown in Figure 9. The equivalent control action here is obtained only for explanation purpose and do not play rule in performance. It's worth noting that the final equivalent control value should be compatible with Eq. (20), which in this case $u_{eq} = \frac{v_d}{E} = \frac{5}{12} = 0.417$.

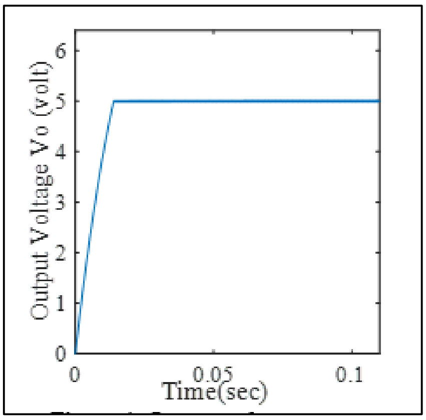

**Figure 4: Output voltage $v_o$.**

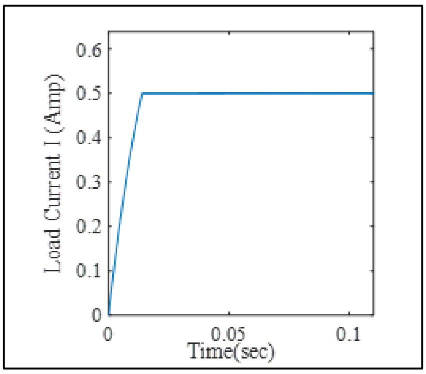

**Figure 5: Load current.**

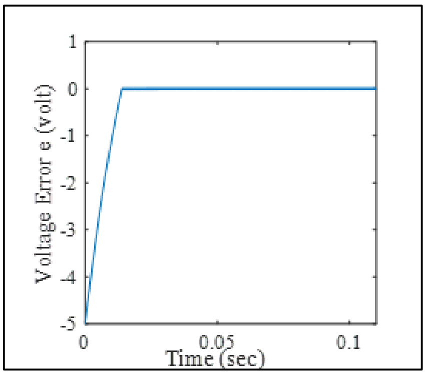

**Figure 6: Voltage error e.**

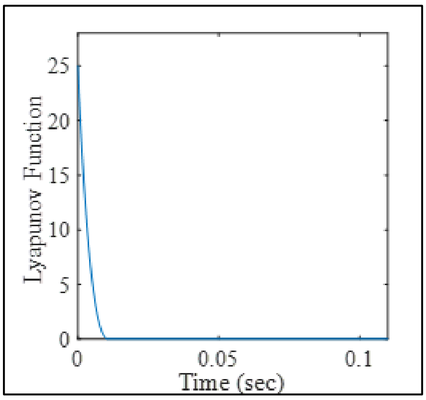

**Figure 7: Lyapunov function V.**

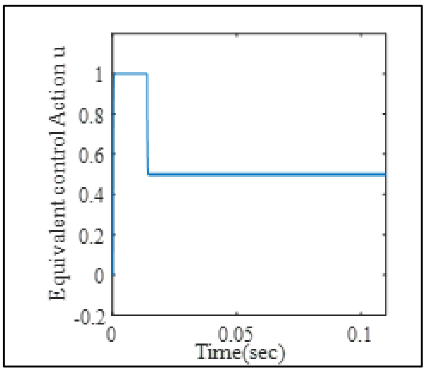

**Figure 8: Control action u.**

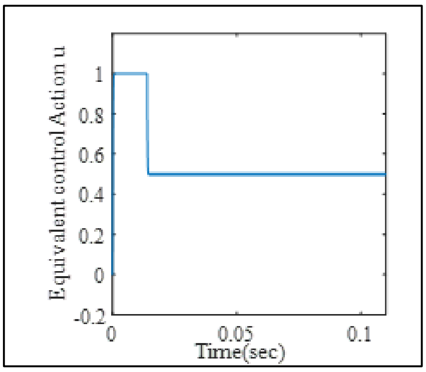

**Figure 9: Equivalent control action $u_{eq}$.**

*II. Variable supply voltage*

In this simulation, the supply voltage source assumed to be variable, in this case, the supply voltage has the form $E(t) = 12 + 3\sin(1000t)$. Figure 10 below shows the waveform of the supply voltage. The other parameters of the circuit are the same as the previous simulation.

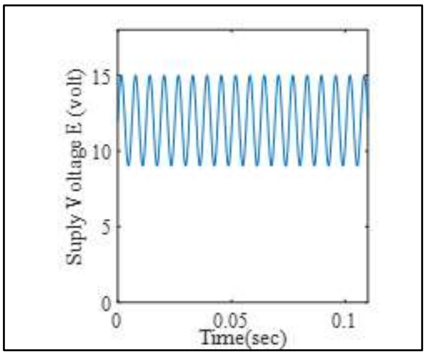

**Figure 10: Supply voltage E.**

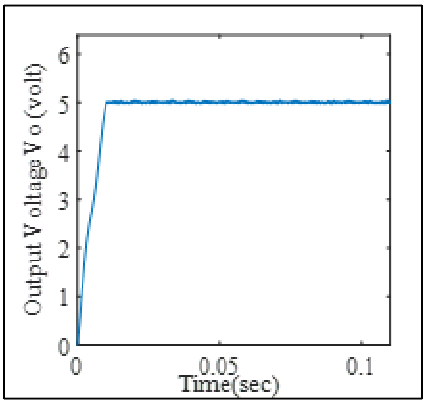

**Figure 11: Output voltage $v_o$.**

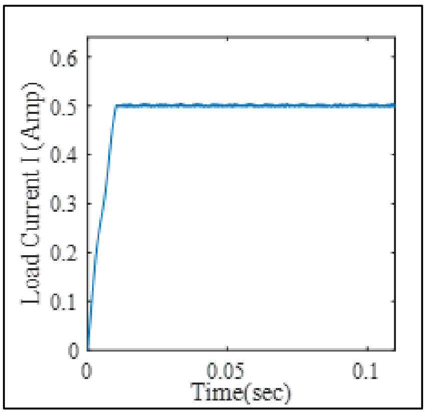

**Figure 12: Load current.**

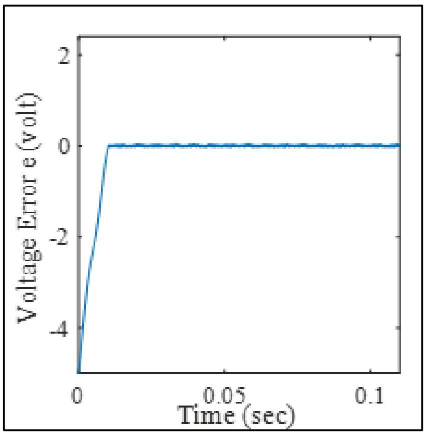

**Figure 13: Voltage error e.**

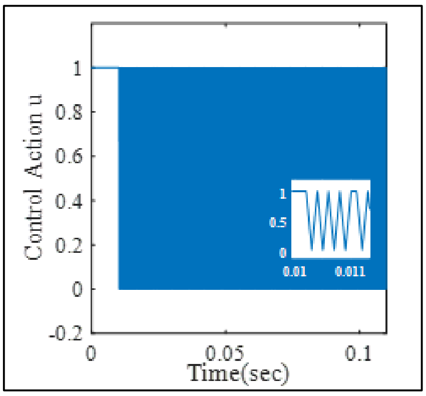

**Figure 14: Control action u.**

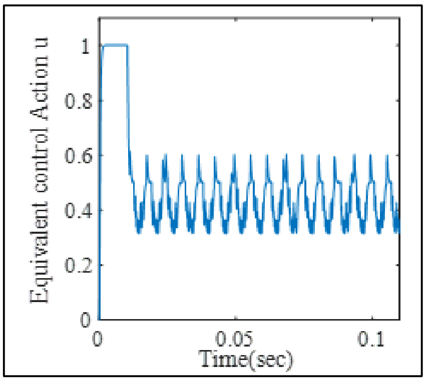

**Figure 15: Equivalent control action $u_{eq}$.**

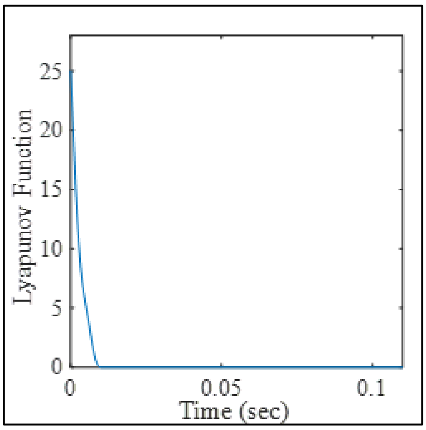

**Figure 16: Lyapunov function V.**

*III. Load variation*

In this last simulation, the case of the load resistance value is considered. The supply Voltage E is considered to be constant as in the first simulation. The load assumed to change its value from 10 ohms to 5 ohms at time instant t=0.03sec. Figures 17-23 below show the converter performance under load variation.

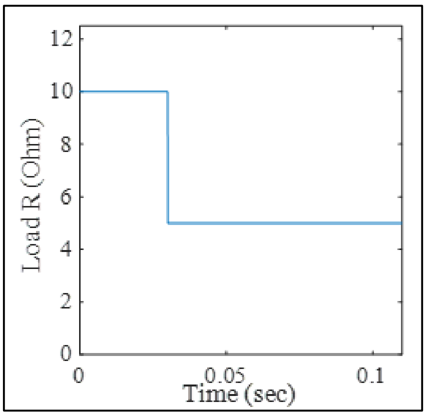

**Figure 17: Load value R.**

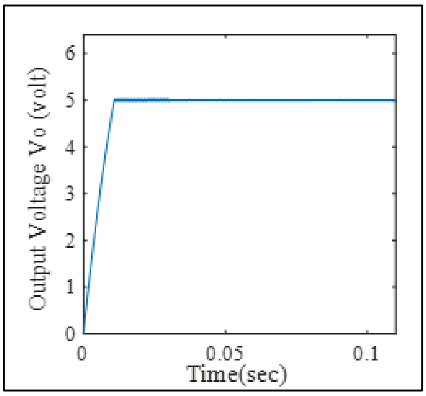

**Figure 18: Output voltage $v_o$.**

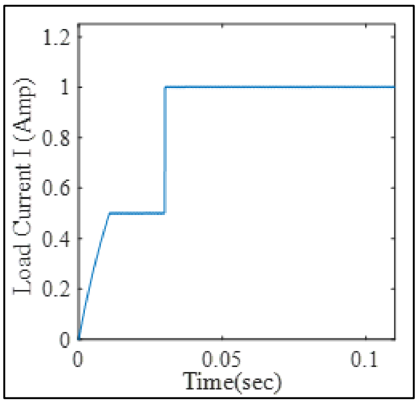

**Figure 19: Load current.**

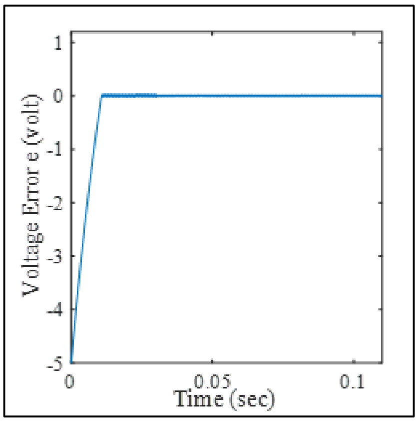

**Figure 20: Voltage error e.**

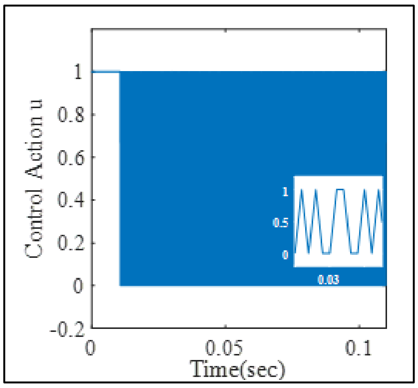

**Figure 21: Control action u.**

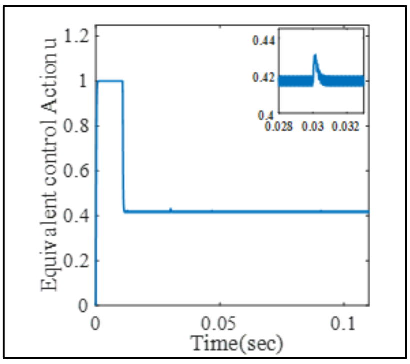

**Figure 22: Equivalent control action $u_{eq}$.**

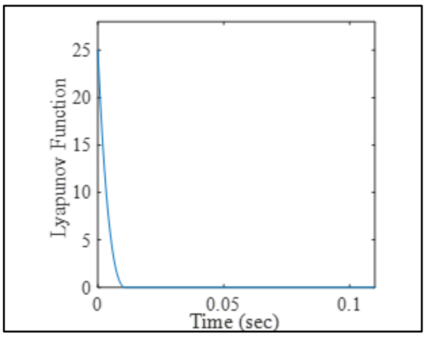

**Figure 23: Lyapunov function V.**

## 5. Discussion

In this work, the problem of stability and control of step-down DC-DC converters have been addressed. The main contribution of this paper was to investigate the criteria that make the converter work properly. The analysis in section 3 shows that the condition for converter stability is that the supply voltage E must always be greater than the output voltage $v_o$ which is demonstrated by Eq. (16).

Simulation results show that the converter works properly and it's able to give the desired voltage despite variations in supply voltage and load value as long as the condition of Eq. (16) is satisfied.

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