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Design of an Optimal Backstepping Controller for Nonlinear System under Disturbance

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K E Y W O R D S

Backstepping Controller, Cuckoo Optimization Algorithm (COA), Nonlinear Pendulum System.

A B S T R A C T

The aim of the work for this paper is the design of an optimal backstepping controller for a nonlinear pendulum system to stabilize the position of pendulum's ball suspended in the desired position. The Cuckoo optimization algorithm (COA) has been utilized to get and tune the gain variables of the proposed backstepping controller in order to find the best torque action for the system. The numerical simulation results using (MATLAB package) show the robustness and the effectiveness of the proposed backstepping based COA controller in terms of obtaining the best torque control action without a saturation state that will stabilize the pendulum system performance. The simulation results show also that the proposed control system when compared with the other controller results has the capability of minimizing the pendulum's ball position tracking error to the zero value at the steady state response and speeding up the system response. Moreover, the fitness evaluation value is reduced.

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1. Introduction

In various nonlinear control system issues, the backstepping control method is recently popular since it gives a nonlinear robust controller with good performance. Since 1990, Peter V. Kokotovic and others have developed the backstepping strategy for designing stabilizing controls for a particular type of block strict-feedback nonlinear systems. The Backstepping approach is a recursive procedure using a systematic design approach and Lyapunov function for particular forms of the nonlinear dynamical systems. This methodology guarantees both global and local asymptotic convergence for regulation and tracking properties [1, 2].

Consequently, various types of backstepping control algorithms have been used for controlling various nonlinear systems such as adaptive backstepping controller [3], the integral backstepping controller [4], the optimal backstepping controller [5], the fuzzy backstepping controller [6], backstepping sliding mode controller [7], backstepping based PID controller [8], backstepping /nonlinear H∞ controller [9], and adaptive type-2 fuzzy backstepping controller [10].

Simple pendulums have been rigorously studied. Their nonlinear nature has turned them into an extensively used testbed for linear as well as nonlinear control methods. Next to serving as a control-theoretic testbed, simple pendulums have been successfully utilized to approximate complex mechanisms such as robotic brachiating or the manipulation of pendulum-like objects, robotic walking. Therefore, a robust nonlinear controller is needed for reaching stable limit cycles when the ground, changes, or manipulated objects are unknown [11]. This motivates the design of a robust nonlinear controller for controlling a simple pendulum.

The contribution of this paper is described as follows:

- Utilizing the COA which has the ability of fast off-line searching in global regions to obtain and tune the best gain variables for the backstepping controller. These gain variables are responsible for generating the best torque control action. Thus, the output position of the pendulum will quickly reach the desired output position in the transient response.
- Investigating the robustness performance of the backstepping based COA controller by adding an external torque as a disturbance effect to the pendulum system.

The residual of this work is established as follows: Section two contains the mathematical model of the simple pendulum system. Section three demonstrates the design structure of the backstepping controller. In section four, the Cuckoo optimization algorithm is explained. Section five presents the simulation results of the proposed backstepping-based COA controller in the absence and presence of the disturbance. Finally, the conclusions are explained in section six.

2. MODELING OF THE SIMPLE PENDULUM SYSTEM

The simple pendulum system is described as a suspended weight from a hinge (o) that enables it to freely left and right swing. If a simple pendulum is pushed sideways from its equilibrium resting position, it will be exposed to a restoring force because of the gravity (g) that accelerates it back toward the position of equilibrium. When the pendulum is released, the pendulum's ball mass (m) combined with the restoring force allows it to oscillate about the position of equilibrium, swinging forth and back. Consider the length of the rod (L) is massless [12]. The simple pendulum system is shown in Figure 1 [13].

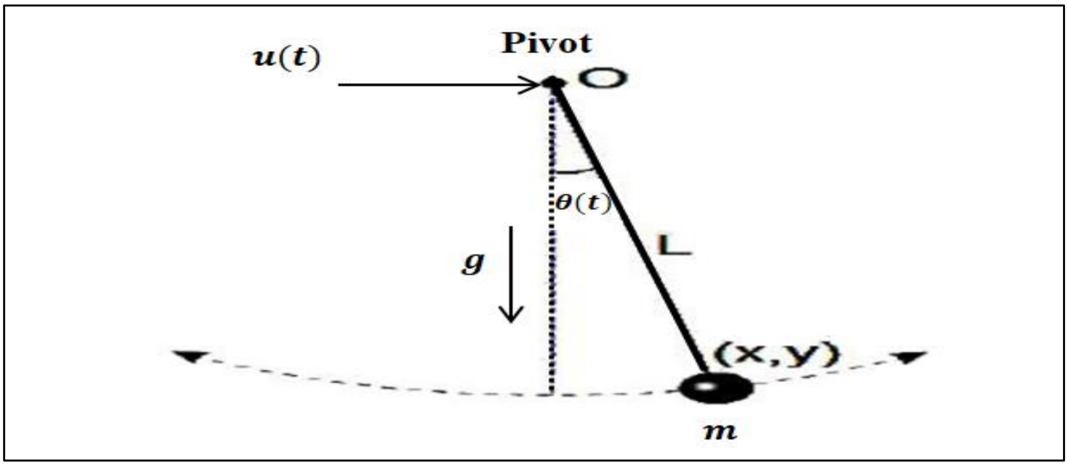

Figure 1: The simple pendulum system [13].

The equation of motion for the simple pendulum model is given by Eq. (1) as follows [13]:

$$\ddot{\theta}(t) = -a\sin\theta\,(t) - b\dot{\theta}(t) + c(u(t) + T_d(t) \quad (1)$$

Where, $\theta(t)$ represents the angle caused by the rod with the vertical axis, $u(t)$ represents the torque control action, $T_d(t)$ represents an external torque disturbance, $a = 10$, $b = 1$, and $c = 10$.
The $x_1(t)$ state and its time derivative $x_2(t)$ are defined as follows:

$$x_1(t) = \theta(t) \quad (2)$$

$$x_2(t) = \dot{x_1}(t) = \dot{\theta}(t) \quad (3)$$

According to the Eqs. (2) and (3), the Eqs. (1) and (3) are written as follows:

$$\dot{x_1}(t) = x_2(t) \quad (4)$$

$$\dot{x_2}(t) = -a\sin x_1(t) - bx_2(t) + c(u(t) + T_d(t)) \quad (5)$$

3. BACKSTEPPING CONTROLLER DESIGN

In this paper, the control goal is to determine the control law $u(t)$ that makes the angular position $x_1(t) = \theta(t)$ of the pendulum's ball tracks the desired angular position $x_d(t) = \theta_d(t)$ [3, 14]. The output position error ($Z_1(t)$) and its time derivative ($\dot{Z}_1(t)$) are described as in Eqs. (6) and (7).

$$Z_1(t) = x_1(t) - x_d(t) \quad (6)$$

$$\dot{Z}_1(t) = \dot{x}_1(t) - \dot{x}_d(t) \quad (7)$$

Where, $x_d(t)$ represents the desired output position.
The stability function is defined according to the virtual controller ($\lambda(t)$) as in Eq. (8).

$$\lambda(t) = -a_1Z_1(t) + \dot{x}_d(t) \quad (8)$$

Where, a_1 represents a constant positive value.
The velocity of the pendulum ball is defined as in Eq. (9).

$$Z_2(t) = \dot{x}_1(t) - \lambda(t) \quad (9)$$

The acceleration of the pendulum ball is defined as Eq. (10) by taking the first derivative of Eq. (9).

$$\dot{Z}_2(t) = \ddot{x_1}(t) - \dot{\lambda}(t) \quad (10)$$

Therefore, Eq. (10) can be rewritten according to time derivative of Eq. (8) as in Eq. (12).

$$\dot{\lambda}(t) = -a_1\dot{Z}_1(t) + \ddot{x_d}(t) \quad (11)$$

$$\dot{Z}_2(t) = -a\sin x_1(t) - bx_2(t) + c(u(t) + T_d) + a_1\dot{Z}_1(t) - \ddot{x_d}(t) \quad (12)$$

To confirm the closed-loop control law for the nonlinear pendulum system is asymptotically stable, the Lyapunov criterion is used as follows:

$$V(t) = 0.5Z_1^2(t) + 0.5Z_2^2(t) \quad (13)$$

The time derivative of Eq. (13) is taken to give Eq. (14).

$$\dot{V}(t) = Z_1(t)\dot{Z}_1(t) + Z_2(t)\dot{Z}_2(t) \quad (14)$$

Then substituting Eqs. (7), (9), and (8) in Eq. (14) and with some rearrangement gives Eq. (15).

$$\dot{V}(t) = -a_1{Z_1}^2(t) + Z_1(t)Z_2(t) + Z_2(t)\dot{Z}_2(t) \quad (15)$$

Put equation (15) is asymptotically stable form as Eq. (16).

$$\dot{V}(t) = -a_1{Z_1}^2(t) - a_2{Z_2}^2(t) \leq 0 \quad (16)$$

Where: a_2 represents a constant positive value.
Substituting Eq. (12) into Eq. (15) leads to Eq. (17).

$$\dot{V}(t) = -a_1{Z_1}^2(t) + Z_1(t)Z_2(t) + Z_2(t)(-a\sin x_1(t) - bx_2(t) + c(u(t) + T_d(t)) + a_1\dot{Z}_1(t) - \ddot{x_d}(t)) \quad (17)$$

Finally, the control law became as Eq. (18) to make the operation of the system asymptotically stable as follows:

$$u(t) = \frac{1}{c}((-a_1a_2 - 1)x_1(t) - (a_2 - b + a_1)x_2(t) + (a_1a_2 + 1)x_d(t) + a\sin x_1(t) - cT_d(t) + (a_1+a_2)\dot{x}_d(t) + \ddot{x}_d(t) \quad (18)$$

It is clear, when $Z_1(t)$ and $Z_2(t)$ are equal to zero $V(t)$ and $\dot{V}(t)$ are equal to zero too and when $Z_1(t)$ and $Z_2(t)$ is not equal to zero $V(t)$ is a positive (not equal to zero) value and $\dot{V}(t)$ is a negative (not equal to zero) value and this means the closed-loop feedback control of the pendulum system is asymptotically stable. Finally the control gain parameters a_1 and a_2 are tuned and obtained by using COA.

The proposed block diagram for controlling the angular position of the simple pendulum model is depicted in Figure 2 and is consists of the backstepping controller, COA, and simple pendulum system.

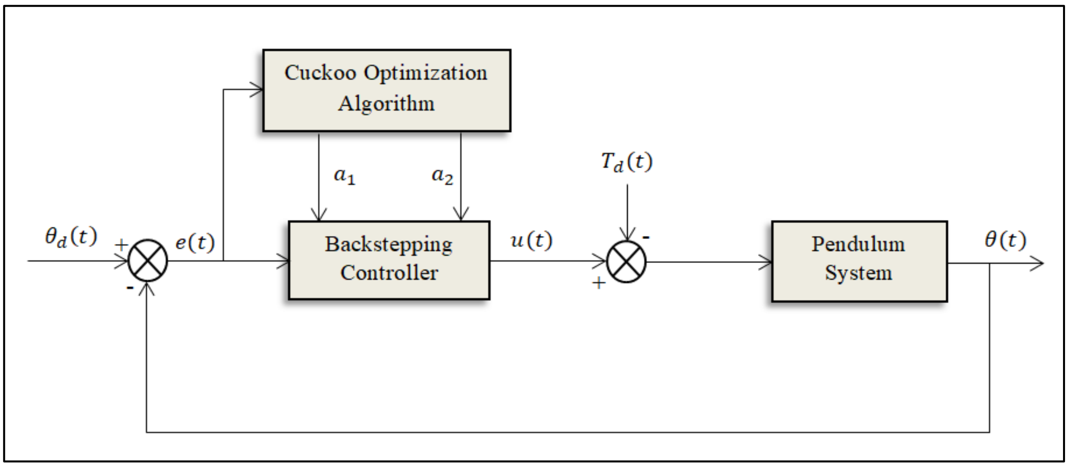

Figure 2: The block diagram of the optimal backstepping controller for the pendulum system.

4. CUCKOO OPTIMIZATION ALGORITHM

The cuckoo algorithm is an optimization algorithm inspired by the cuckoo bird's special lifestyle; there is no bird of the cuckoo that gives birth to live young. The mature cuckoos must locate a nest of the other birds where they can lay their eggs safely and hatch in the nests of the host birds. Afterward, the host bird feed will be responsible for feeding. Some of the chicks have come out or eggs are deposited in a bad nest will be killed or destroyed. Only some of the cuckoo's eggs have the opportunity for growing up and become mature birds. All of these mature cuckoos will be heading toward the best Habitat. After some iteration, the cuckoo's population will converge with the best profit values in a habitat [15].

Similar to the different types of evolutionary algorithms, the cuckoo algorithm starts with an initial cuckoos' population. These initial cuckoos have some eggs to lay in the nests of some host birds. Some of these eggs that are more analogous to the host birds' eggs have the chance to grow and become mature birds. While the other eggs will be detected by the host birds and are killed. The grown eggs reveal the appropriateness of nests in that region. As a result, the greater the eggs survive in a region, the greater the profit is acquired in that region. Thus, the concept that the COA is going to optimize is defined by the habitat in which greater eggs will survive [16, 17].

The steps of COA are summarized as follows:

- **Step 1:** Initialize the cuckoo algorithm parameters such as the dimension of the problem (d), the population of host nests (N), maximum number of iterations ($Iter$), the probability of alien eggs discover (p_a), and initial cost function value for each nest ($costn$).
- **Step 2:** In the cuckoo algorithm, each nest represents a feasible solution and the initial population for each nest can be generated randomly as in Eq. (19).

$$nest_{ij} = Lb_j + rand()(Ub_j - Lb_j) \quad (19)$$

Where, $i = 1, \dots, N$, $j = 1, \dots, d$, and Lb_j and Ub_j are the lower and upper boundaries of dimension j respectively.

- **<u>Step 3:</u>** Calculate the cost function ($ITAE$) as in equation (20) for each nest as in Eq. (20).

$$cost_i = \int_0^T t|e(t)|\, dt \quad (20)$$

Where, $t = 1, \dots, T$, and T represents the maximum simulation time.

- **<u>Step 4:</u>** For the i^{th} nest, if its new cost function value ($cost_i$) is smaller than its previous cost function value ($costn_i$) then:

$$costn_i = cost_i \quad (21)$$

$$nest_{ij} = nest_{ij} \quad (22)$$

- **<u>Step 5:</u>** Calculate the minimum cost function value (f_{min}) and the best nest ($bestn_{ij}$) as the nest with the minimum cost function value as follows:

$$[f_{min}, index] = min(costn) \quad (23)$$

$$bestn_{ij} = nest_{ij}(index, :) \quad (24)$$

- **<u>Step 6:</u>** A Lévy distribution is generated using Mantegna's algorithm as follows:

$$nest_{ij}{}^{k+1} = nest_{ij}{}^{k} + \alpha \oplus Le'vy(\lambda) \quad (25)$$

$$Le'vy(\gamma) = 1^{-\gamma} \quad (26)$$

Where, $\mathrm{k} = 1, 2, \dots, \mathrm{Iter}$, $\alpha = 1$, and $1 < \gamma \leq 3$.

- **<u>Step 7:</u>** Repeat steps 3, 4, and then calculate the new minimum cost function value (f_{new}) and the best nest ($bestn_{ij}$) as the nest with the minimum cost function value as follows:

$$f_{new} = min(costn) \quad (27)$$

$$bestn_{ij} = nest_{ij} \quad (28)$$

- **<u>Step 8:</u>** For each nest choose a random value and if its greater than the probability of alien eggs discover (p_a) then:

$$nest_{ij} = nest_{ij} + step\ size.rand() \quad (29)$$

Where, $step\ size = 0.1$.

- **<u>Step 9:</u>** Repeat steps 7.
- **<u>Step 10:</u>** If the new minimum cost function value (f_{new}) is smaller than the minimum cost function value (f_{min}) then:

$$f_{min} = f_{new} \quad (30)$$

$$bestn = bestn_{ij} \quad (31)$$

- **<u>Step 11:</u>** Stop if the maximum number of iterations ($Iter$) is reached. Otherwise, Step 6 to Step 11 is repeated.

5. SIMULATION RESULTS

In this section, an optimal backstepping controller for controlling the output position of the pendulum ball is simulated using MATLAB package. The simulation results for the optimal backstepping controller with the initial conditions as ($x_1 = 0.85\,rad$) and ($x_2 = 0\,rad/sec$) with the desired output position as ($\pi/4\,rad$) and the maximum simulation time (T) equals 10 sec are described as follows:

To investigate the optimal backstepping controller as shown in Figure 2 for controlling the output position of the pendulum ball, cuckoo tuning control methodology has been used to find and tune the optimal parameters of the backstepping controller (a_1) and (a_2). These parameters lead to finding the optimal control action and minimizing the position tracking error with the minimum number of fitness evaluation, the parameters of the control methodology based on COA algorithm is defined in Table I.

TABLE I: The parameters of the Cuckoo optimization algorithm.

Description and Symbol	Value
$Population\ of\ host\ nests\ (N)$	20
$Maximum\ number\ of\ iteration\ (Iter)$	100
$Dimension\ of\ the\ problem\ (d)$	2
$Probability\ of\ alien\ eggs\ discover\ (p_a)$	0.25
Lower boundary of the dimension j	0
Upper boundary of the dimension j	12

The error ($Z_1(t)$) and the derivative of error ($x_2(t)$) signals are equal to zero value at 1.4 sec as depicted in Figures (3) and (4) respectively. This means that the controller can make the system asymptotically stable.

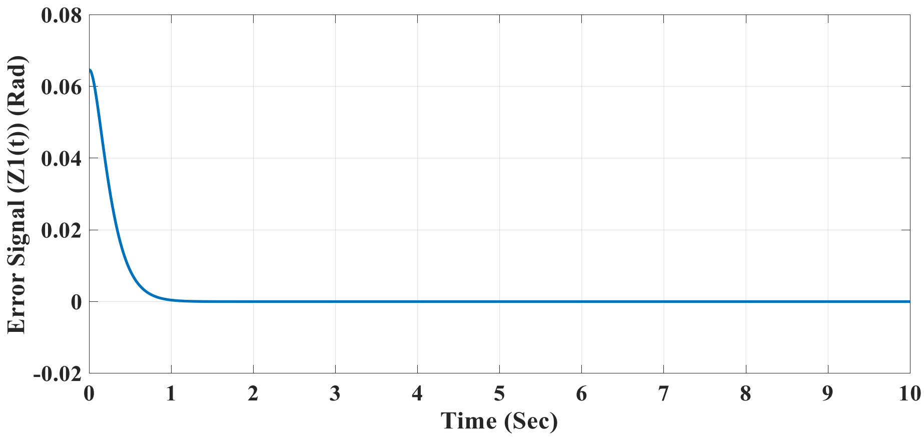

Figure 3: The error signal of the optimal backstepping controller for the pendulum system.

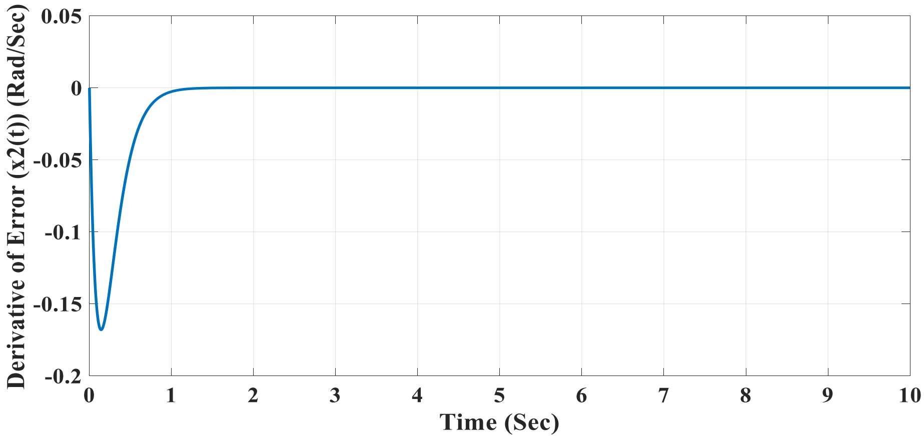

Figure 4: The derivative of the error signal of the optimal backstepping controller for the pendulum system.

The response of the pendulum output position ($\theta(t)$) reached the steady-state value at 1.4 sec and this means that the settling time equals 1.4 sec as shown in Figure 5.

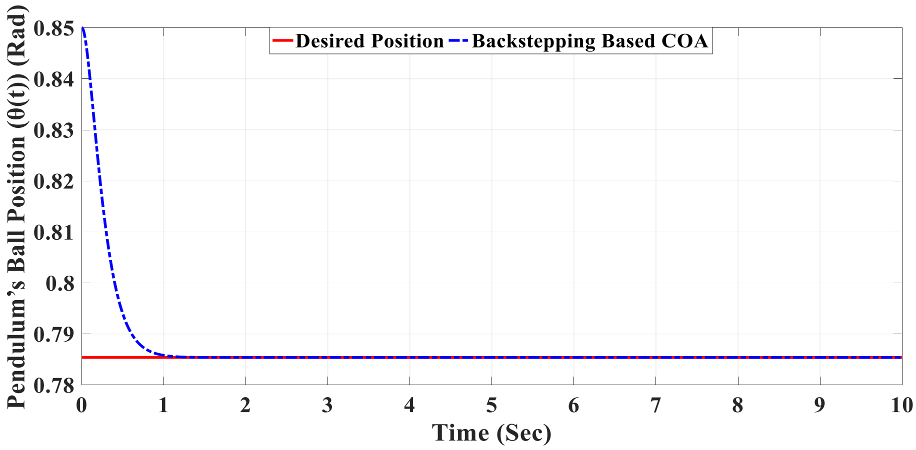

Figure 5: The output response of the optimal backstepping controller for the pendulum system.

The torque control action was smooth without oscillation response, no spikes behavior and the action response did not exceed (0.757 $N.m$) as depicted in Figure 6.

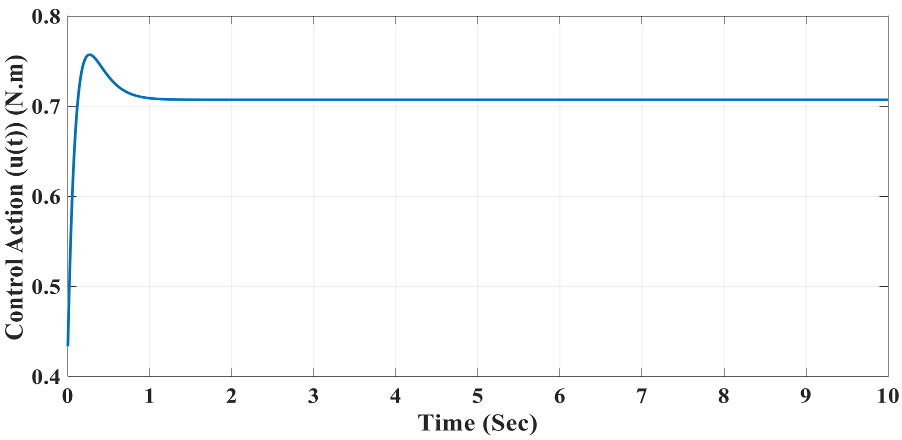

Figure 6: The torque control action of the optimal backstepping controller for the pendulum system.

The control action is directed to force the derivative of the error ($x_2(t)$) signal equals to the virtual controller ($\lambda(t)$) signal. As a result, they have coincided and reached the zero value at 1.4 sec even if they are started from different values as depicted in Figure 7.

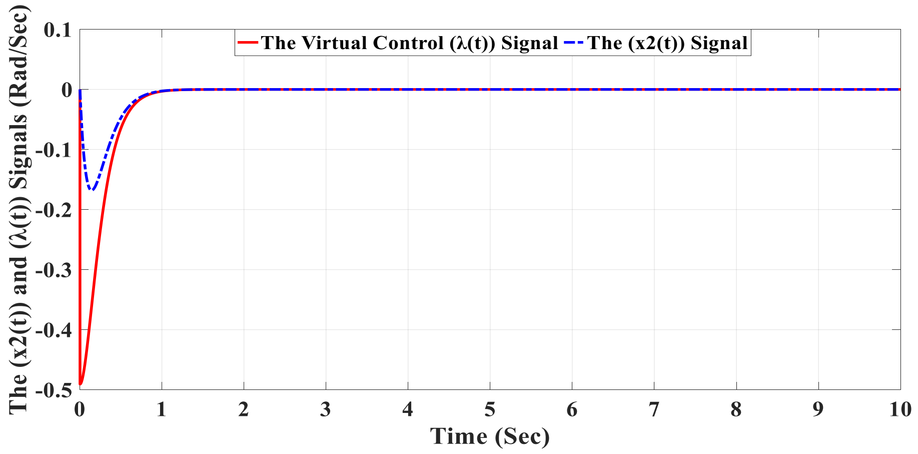

Figure 7: The virtual controller and the derivative of error signals of the optimal backstepping controller for the pendulum system.

The error ($Z_1(t)$) and the derivative of error ($x_2(t)$) go to the origin in the final trajectory of the phase plane. This means that the system is asymptotically stable as shown in Figure 8.

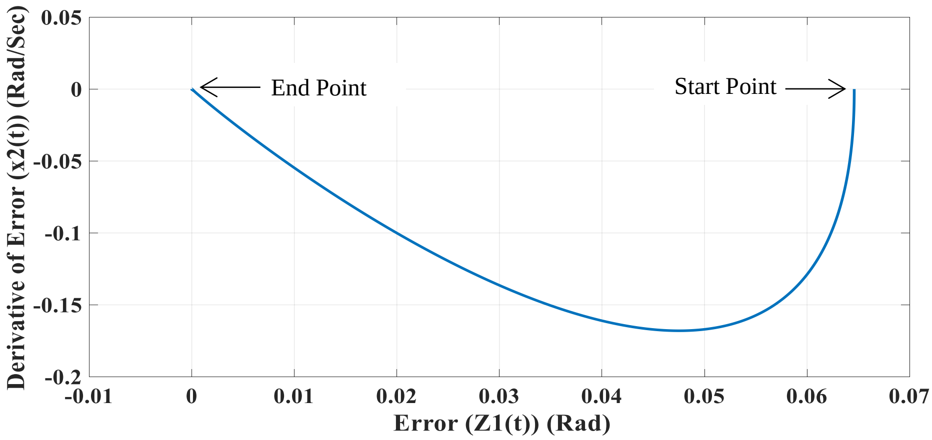

Figure 8: The plot of the phase plane of the error against the derivative of the error.

Figure 9 clearly shows the improved performance indices of the optimal backstepping controller based on the Integral Time Absolute Error ($ITAE$).

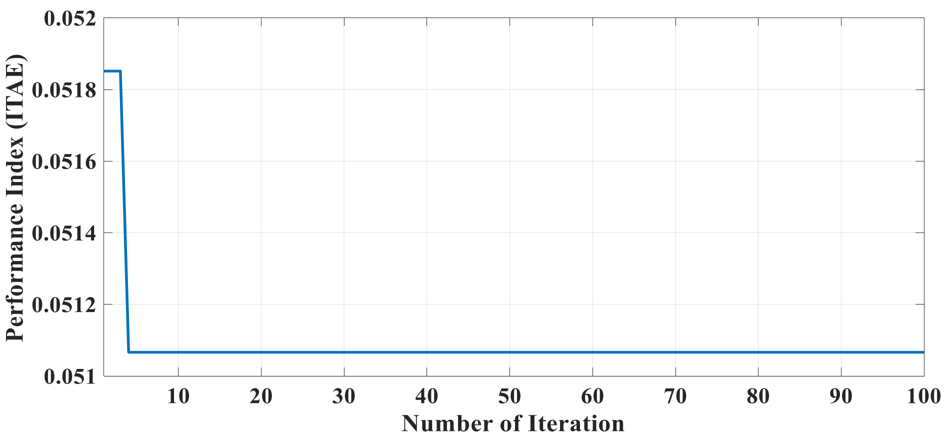

Figure 9: The performance index ($ITAE$) of the optimal backstepping controller for the pendulum system.

The optimal parameters values (a_1) and (a_2) and the dynamic behavior of the backstepping based COA controller such as the time it takes for the error to reach the zero value (T_e), the time it takes for the derivative of the error to reach the zero value ($T_{\dot{e}}$), the settling time (T_s), and the steady-state error ($E_{s.s}$) in comparison with the other controller result are summarized in Table II.

TABLE II: The optimal parameters of the controller and the dynamic behavior of the pendulum system output position with different controllers.

Parameter	Backstepping Based COA Controller	SLM Controller using GA [13]
a_1	7.594	-
a_2	6.348	-
T_e	1.4 Sec	22 Sec
$T_{\dot{e}}$	1.4 Sec	22 Sec
T_s	1.4 Sec	22Sec
$E_{s.s}$	0	0

The robustness performance of the proposed backstepping based COA controller is investigated by adding an external torque $T_d(t)$ equals ($0.1\ N.m$) as a step disturbance at the moment ($4 - 5\ sec$). The error ($Z_1(t)$) and the derivative of error ($x_2(t)$) signals have a very small overshoot at the moment ($4 - 5\ sec$) during adding disturbance and they have a zero value at the steady-state response as depicted in Figures (10) and (11), respectively. This means that the controller has also the ability to make the system asymptotically stable in the presence of the disturbance.

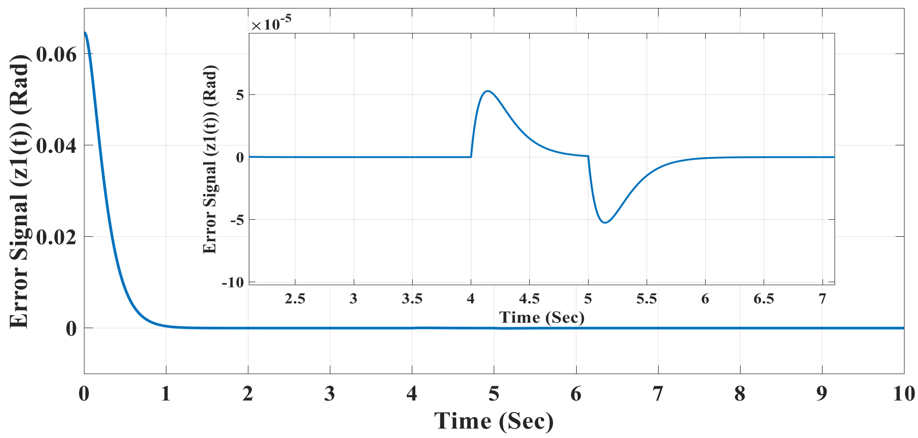

Figure 10: The error signal of the optimal backstepping controller for the pendulum system under disturbance.

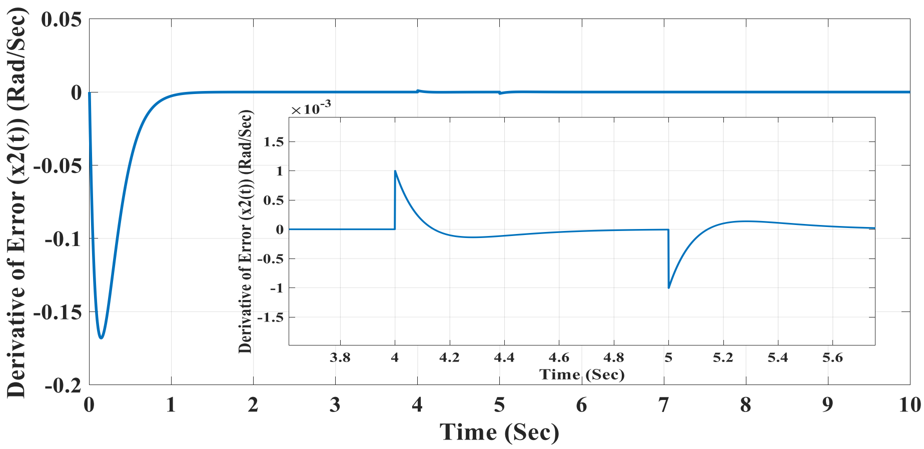

Figure 11: The derivative of the error signal of the optimal backstepping controller for the pendulum system under disturbance.

The output position response of the pendulum system ($\theta(t)$) has a very small overshoot at the moment ($4-5\ sec$) during adding disturbance and the error equals zero value at the steady-state response. Besides, the settling time also equals 1.4 sec as shown in Figure 12.

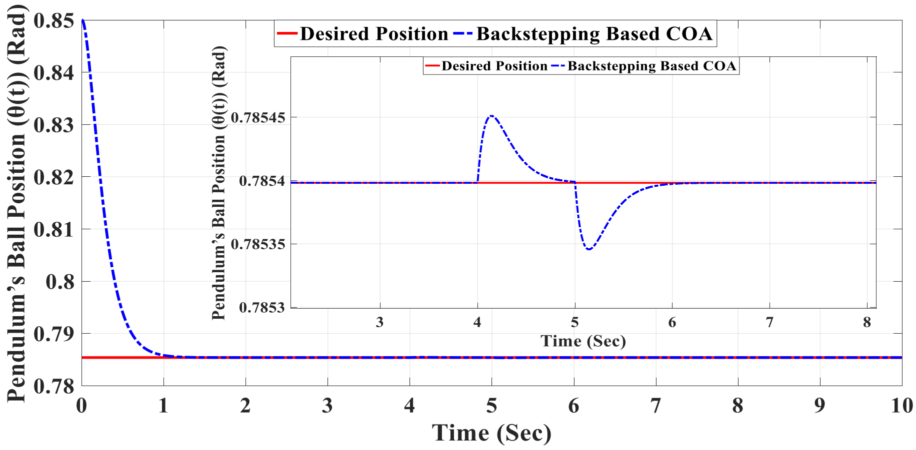

Figure 12: The output response of the optimal backstepping controller for the pendulum system under disturbance.

The torque control action response in Figure 13 of the Backstepping-based COA controller is smooth without oscillation response, no spikes behavior, and has a small change in its value at the steady-state response at the moment ($4-5\ sec$) during adding disturbance.

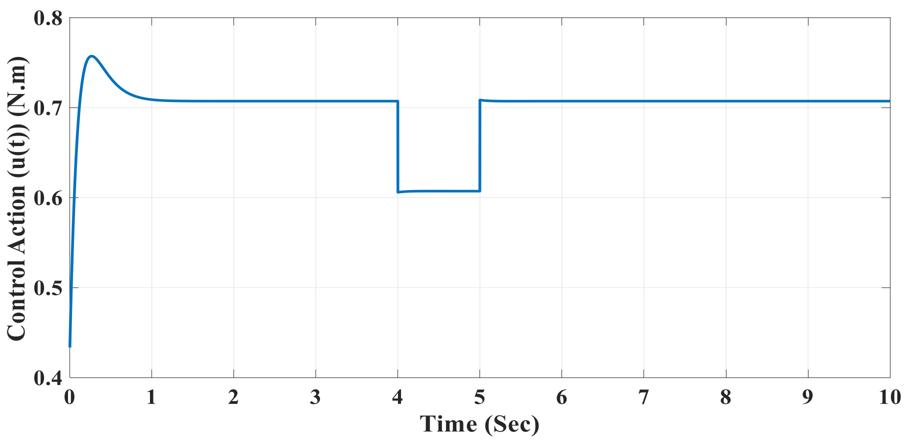

Figure 13: The torque control action of the optimal backstepping controller for the pendulum system under disturbance.

The derivative of the error ($x_2(t)$) and the virtual controller ($\lambda(t)$) signals have a very small overshoot at the moment ($4-5\ sec$) during adding disturbance and they have a zero value at the steady-state response as depicted in Figure 14.

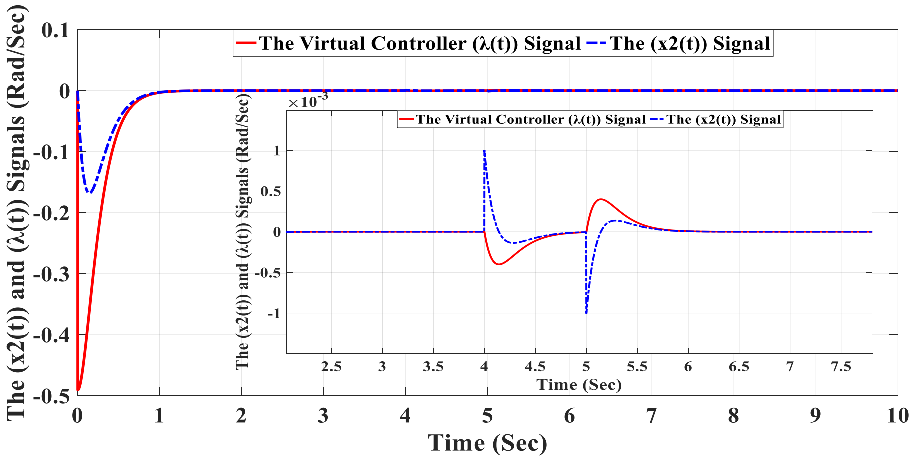

Figure 14: The virtual controller and the derivative of error signals of the optimal backstepping controller for the pendulum system under disturbance.

6. CONCLUSIONS

The numerical Matlab simulation results based on the off-line tuning Cuckoo optimization algorithm of the backstepping controller have been demonstrated in this work for the nonlinear pendulum system. The proposed control system has the following capabilities:

- The off-line Cuckoo control algorithm can fast finding and tuning the optimal gains of the controller with the minimum fitness evaluation number.
- A proper control action was obtained as a smooth without oscillation response and no spikes behavior occurs.
- The numerical simulation results for the backstepping controller based on the Cuckoo tuning control algorithm show that the controller can give excellent performance in terms of speeding up the system response and reducing the settling time in comparison with the other controller result. Moreover, the fitness evaluation number is reduced.
- High robustness performance was obtained by adding an external torque as a disturbance effect to the pendulum system using a backstepping controller based on the Cuckoo optimization algorithm.

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