DOI: https://doi.org/10.33103/uot.ijccce.18.2.4

Design of L_1 Adaptive Controller for Position Control of Permanent Magnet Linear Synchronous Motor (PMLSM)

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Abstract— *In the present work, the design of an L_1 adaptive controller for position control of a linear servo motor for X-Y table application has been developed. The AC Permanent Magnet Linear Synchronous Servo Motor (PMLSM) is considered. A comparative study between L_1 adaptive control and Model Reference Adaptive Control (MRAC) has been made. The effectiveness of the L_1 adaptive controller against uncertain parameters is analyzed based on simulated results. Robustness characteristics of both L_1 adaptive controller and model reference adaptive controller to different input reference signals and different structures of uncertainty have been evaluated. The L_1-adaptive controller could ensure uniformly bounded transient and asymptotic tracking for input and output signals. Simulations based on MATLAB of an x-y table based on PMLSM with time-varying friction and disturbance are presented to verify the theoretical findings. The simulation results within the environment of MATLAB/SIMULINK showed that L_1-adaptive controller could give better tracking performance, dynamic and steady-state characteristics, than that obtained from MRAC for considered types of input and for various structures of uncertainties.*

Index Terms— *L_1-Adaptive Control, MRAC, X-Y table, PMLSM, Position control*

I. INTRODUCTION

Slow adaptation and lack of robustness in airplane control strategy were the main cause of the famous accident happened in 1989. This incidence has motivated many researchers to develop adaptive controllers with fast adaptation, strong robustness and high performance in transient characteristics [1]. The l_1-adaptive controller is one of the modern and efficient adaptive controllers that has been successfully tested on NASA's AirSTAR test vehicle. On June 2010, a test flight of the AirSTAR was performed with an all-adaptive flight control system. The L_1-adaptive controller guaranteed safe operation of the vehicle during the flight, and the pilot satisfactorily flew the specified tasks. In recent years, the L_1-adaptive controller has gained a high interest in many applications such as acrobat, airplanes, robotics and biomedical control due to its characteristic features listed below [1-5]:

1. It guarantees robustness separated from other adaptive failures.
2. It can strongly enable the system response to track the desired trajectory with zero steady state error.
3. It can cope with time-varying uncertainties.
4. It can establish a compromise between robustness and tracking performance.
5. It permits decoupling of robustness and adaptation.

Received 26 Feb 2018; Accepted 9 May 2018
 ISSN (Print) 1811-9212 *ISSN (Online) 2617-3352*

6. It is capable of handling nonlinear constrained systems with fast adaptation.
7. Its structure does not need any persistent excitation, high-gain feedback, or gain scheduling.

The main differences in structure between L_1 adaptive control strategy and model reference adaptive control (MRAC) are the introduction of state predictor instead of reference model and the inclusion of low pass filter in the feedback loop, which is required to attenuate undesirable frequencies and chattering at the control input channel resulting from high rate learning [1, 3, 4].

Implementing L_1-adaptive controller requires three main laws; namely state prediction law, control law, and adaptive law. The main task of state predictor is to estimate the desired behavior of the system, while the adaptation law works to match the actual states with estimated states. On the other hand, the control law tries to eliminate the wanted frequencies or chattering at the control channel using a low pass linear filter [1, 2].

Permanent magnet linear motors (PMLSMs) are characterized by high thrust density, low losses, and small electrical time constant (rapid response). They become the main part in many automation factories, which requires linear actuating processes [6]. Position control of the PMLSM gained a wide space of interest in recent literature.

Optimal control theory was presented by (Cheema and et.al 2016) [7, 8]; adaptive backstepping were introduced by (Ting and et.al 2014) [9]; combined sliding mode observer assessed by (Cheema and et.al 2014)[10]; lumped disturbance compeonsation was presented by (Kim and et.al 2016)[11]; robust control based H_∞ was introduced by (Zhang and et.al 2011)[12]; Robustness improvement of predictive current control with integrating adaptive internal model was originated by (Yang, et al. 2017)[13]; Adaptive Sliding Mode Control was assessed by (Yahiaoui, et al. 2017)[14]; Dynamic surface backstepping sliding mode position control was presented by (Xiaoying, et al. 2017)[15]; Disturbance rejection using direct thrust control was introduced by (Su, et al. 2016)[16]; Passivity-based control under EL equation was rolled out by (Chen, et al. 2016)[17]; periodic adaptive disturbance observer was presented by (Cho, et al. 2015)[18]; adaptive variable speed back-stepping sliding mode controller was applicated by (Chen and Lu 2014)[19]

This paper is highly motivated by the recent studies in a precise position and speed control of electrical machines and aircraft rolling which represent recent contributions of this paper author in the field of Robust adaptive control theory presented by (Humaidi and Hameed and et.al 2016-2017) [20-27]

The main objective of this paper can be summarized by:

- To design of an L1-adaptive controller for position control of PMLSM.
- To make a comparative study between the performances of position-controlled-systems based on L1-adaptive controller and that which is based on the classical model reference adaptive controller. The performance of each controller is evaluated in terms of robustness (against parameter variation) and disturbance rejection capability.

II. SIMPLIFIED DYNAMIC MODEL OF PMLSM

In this section, the dynamic model of PMLSM is developed so that it will be processed by the L_1-adaptive controller. To deal with close-to-real PMLSM, the following assumptions are considered [28];

- Eddy current losses and magnetic hysteresis are not considered effective.
- No squirrel cage or short circuit ring in the mover.
- Induced EMF is sinusoidal.
- Permanent magnet saturation is ignored and it can be considered as a parameter variation.
- No current excitation dynamics.

The mathematical model of the motor in the three-phase reference frame is difficult and complicated, which is not easy for control design and simulation. The three-phase reference frame passes through

Received 26 Feb 2018; Accepted 9 May 2018
 ISSN (Print) 1811-9212 ISSN (Online) 2617-3352

Park's transformation so that it is transformed into a two-axis reference frame. Taking the benefits of motor unification theory, the dynamical characteristics of a PMLSM is described by synchronous Park's equations in the d-q coordinate system [28, 29],

$$\begin{aligned} v_q &= R\, i_q + p\, \lambda_q + \omega_s \lambda_d \\ v_d &= R\, i_d + p\, \lambda_d - \omega_s \lambda_d \\ \lambda_q &= L_q\, i_q \\ \lambda_d &= L_d\, i_q + \lambda_{af} \end{aligned} \tag{1}$$

Where v_q, v_d represent the stator direct and quadrature axises voltages respectively, i_d,i_q are stator direct and quadrature axises currents respectively, R represents stator resistance, λ_q, λ_d represent stator direct and quadrature axises flux linkages, respectively, ω_s represent the synchronous rotary angular equivalent velocity, L_d, L_qrepresent stator direct and quadrature axises inductances respectively and λ_{af}assigned to permanent magnet flux linkage.

The total input power can be calculated by summing the individual input powers of all phases, i.e, [28]

$$P_{in} = v_a i_a + v_b i_b + v_c i_c = \frac{3}{2}\left(v_d i_d + v_q i_q\right) \tag{2}$$

From Eq. (1), the total power can be extended and the term belongs to converted mechanical power can be deduced. This part of power is directly related to synchronous speed ω_s and can be described by

$$P_m = (3/2)\, \omega_s\, [\lambda_{af}\ i_q + (L_d - L_q)\, i_d i_q] \tag{3}$$

Since the synchronous and mechanical speeds are related by$\omega_s = P\, \omega_r$, then the expression of mechanical power is written as;

$$P_m = P\, (3/2)\, [\lambda_{af}\ i_q + (L_d - L_q)\, i_d\, i_q]\ v \tag{4}$$

Where, P is the number of pole pairs. Transforming from the rotary rotation into linear is performed via the relation
$\omega_r = v\pi/\tau_p$such that the mechanical power becomes [28, 30],

$$P_m = (3/2)\, (\pi P/\tau_p)\ \ [\lambda_{af}\ i_q + (L_d - L_q)\, i_d\, i_q]\ v \tag{5}$$

where τ_p represents the pole pitch and v is the mover linear velocity. It is well-known that the mechanical power equation in linear motion is defined by $P_m = v\, F_e$, where$F_e$is the developed electromagnetic thrust force which will be described by;

$$F_e = (3/2)\, (\pi P/\tau_p)\ \ [\lambda_{af}\ i_q + (L_d - L_q)\, i_d\, i_q] \tag{6}$$

This developed thrust force has to overcome the load motion and friction, i.e;

$$F_e = f_{uc} + B_v\, v + M\, \dot{v} \tag{7}$$

where B_v stands for the velocity damping coefficient, $M$is the mass of moving part. The load force and friction force are lumped into$f_{uc} = f(v) + F_L$, which accounts for friction and load uncertainties. The friction uncertainty part $f(v)$, which stands for uncertainty due to Coulomb friction, viscous friction, and Stribeck effect, can be written as:

$$f(\,v) = k_v\, v + (F_s - F_c)\ sgn\,(v)\, e^{-(v/v_s)^2} + F_c\, sgn\ (v)$$

Using Eq.(1), Eq.(6), and Eq.(7), the dynamic model is reformulated in terms of motor variables i_d, $i_q$and $v$as follows [29-31],

$$d\, i_d/dt = -(R/L_d)\ i_d + (P\, \pi\, L_q/\tau_p\, L_d)\ v\, i_q + (1/L_d)\ v_d \tag{8}$$

Received 26 Feb 2018; Accepted 9 May 2018
 ISSN (Print) 1811-9212 ISSN (Online) 2617-3352

$$d\, i_q/dt = -(R/L_q)\ i_q - (P\ \pi\, \lambda_{af}\, L_d/\tau_p\, L_q)\ v\, i_d - (P\ \pi\, \lambda_{af}\, L_d/\tau_p\, L_q)\ v + (1/L_d)\ v_q \quad (9)$$

$$d\, v/dt = (3\, \lambda_{af}\, P\, \pi/2\, \tau_p\, M)\, i_q + (3\, \lambda_{af}\, P\, \pi/2\, \tau_p\, M)(L_d - L_q)\, i_d i_q - (B_v/M)\ v\ -\ f_{uc}/M \quad (10)$$

It is clear that the above system of equations is nonlinear model due to the presence of product items of velocity v and currents i_d, i_q

To maximize the thrust current in the quadrature axis, the concept of field-oriented control is included by setting the current of the direct axis to zero $i_d^* = 0$. Accordingly, the control signal is completely governed by quadrature current i_q^* such that the electromagnetic force of Eq.(6) can be simplified to the following

$$F_e = K_f\, i_q^* \quad (11)$$

$$d\, v/dt = (K_f/M)\, i_q^* - (B_v/M)\ v\ -\ F_{uc}/M \quad (12)$$

where $K_f = 1.5\, P\, \pi\, \lambda_{af}/\tau_p$. If the state variables x_1 and x_2 are assigned to position$x$and velocity$v$, respectively, Eq.(12) can be arranged in the form;

$$\dot{x}_2 = -(\tfrac{B_v}{M})\, x_2 + (\tfrac{K_f}{M}) i_q^* - (1/M)\, f_{uc} \quad (13)$$

In matrix form, the state space representation of the simplified model is given by;

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ 0 & -\frac{B_v}{M}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ 1\end{bmatrix}(\tfrac{K_f}{M} u - \tfrac{1}{M} f_{uc}(t)),\ y = [1 \quad 0]\begin{bmatrix}x_1\\ x_2\end{bmatrix} \quad (14)$$

The above equation has to be standardized with the class of equation in the analysis of L1-adaptive control,

$$\dot{x}(t) = A_m\, x(t) + b(\omega\ u(t) + \theta^{\mathrm{T}}(t)\, x(t) + \sigma(t)) \quad (15)$$

where ω, $\sigma(t)$, θ^{T} and b can easily be found to be

$\omega = K_f/M,\ \sigma(t) = -\frac{1}{M}(f(v) + F_{uc}),\ \theta^{\mathrm{T}} = [0 \quad 0],\ b = [0 \quad 1]^T$

The numerical values of parameters for PMLSM are listed in Table (1).

TABLE (1) SYSTEM MODEL PARAMETERS FOR PMLSM [30]

Parameter	Parameter definition	value
K_f	Thrust coefficient	20 (N/Amp)
M	The total mass of the mover	1.97 (kg)
B_v	Viscous friction and iron loss coefficient	83.2245 (kg/s)

III. L1-ADAPTIVE CONTROL

In the sense of the L1-adaptive control design, two different adaptive control structure will be discussed: direct MRAC and state predictor-based direct MRAC. The latter architecture can be modified to synthesize the structure of L1-adaptive control. In what follows, direct MRAC is first analyzed and then the main constituent elements of the L1–adaptive control will be described.

A. Direct Model-Reference Adaptive Control (MRAC)

Consider the following system dynamics described by the general structure [4]:

$$\dot{x} = A_m\, x + \ b\, (u + k_x^{\mathrm{T}}\, x)\ \ , y = \ c^{\mathrm{T}}\, x \quad (16)$$

Received 26 Feb 2018; Accepted 9 May 2018

 ISSN (Print) 1811-9212 ISSN (Online) 2617-3352

where $x \in \mathbb{R}^n$ is measured the state of the system, $A_m \in \mathbb{R}^{n\times n}$represent a known Hurwitz matrix of the desired dynamics for the closed-loop system with negative real eigenvalues, $b, c \in \mathbb{R}^n$ denotes a known constant vectors, $u \in \mathbb{R}$ the control input, $y \in \mathbb{R}$ referred to the regulated output, and $K_x \in \mathbb{R}^n$ is the vector of unknown constant parameters. The development of the MRAC algorithm can be started by suggesting a nominal controller of the form;

$$u_{nom} = -k_x^{\mathrm{T}} x + k_g r \tag{17}$$

where $r \in \mathbb{R}$ is bounded reference input trajectory and k_g is given by

$$k_g = \frac{1}{c^T A_m^{-1} b} \tag{18}$$

The structure of control law is given by,

$$u(t) = -\hat{k}_x^{\mathrm{T}}(t)\, x(t) + k_g\, r(t) \tag{19}$$

where $\hat{k}_x \in \mathbb{R}^n$ is a continuous estimation of k_x (dynamic feedback). Substituting Eq.(19) into Eq.(16) yields the closed loop system dynamics [4]:

$$\dot{x} = (A_m - b\,\tilde{k}_x^{\mathrm{T}})\, x + b\, k_g\, r \tag{20}$$

where $\dot{e} = A_m\, e + b\,\tilde{k}_x^{\mathrm{T}}\, x$represent the state error dynamics and the parametric estimation error is denoted by$\tilde{k}_x = \hat{k}_x - k_x$. The tracking error is defined by:

$$e(t) \triangleq x_m(t) - x(t) \tag{21}$$

The adaptive law of the parametric estimate is given by [5]:

$$\dot{\hat{k}}_x = -\Gamma\, x\, e^{\mathrm{T}}\, P\, b, \tag{22}$$

where $\Gamma \in \mathbb{R}^+$ is the learning rate. The matrix $P = P^{\mathrm{T}} > 0$ is the solution of the algebraic Lyapunov equation [2, 32]:

$$A_m^{\mathrm{T}}\, P + P\, A_m = -\,Q \tag{23}$$

where$Q = Q^T > 0$. The lyapunov candidate is chosen as:

$$V(e, \tilde{k}_x) = e^{\mathrm{T}}\; P\; e + \Gamma^{-1}\, \tilde{k}_x^T\, \tilde{k}_x \tag{24}$$

The time derivative of the Lyapunov candidate is given by

$$\dot{V}(t) = -e^T\, Q\; e \;\leq 0 \tag{25}$$

The asymptotic convergence of error to zero requires the second derivative of candidate function; i.e,

$$\ddot{V}(t) = -2\, e^{\mathrm{T}}\, Q\; \dot{e} \tag{26}$$

This indicates that $\dot{e}$ is uniformly bounded, so $\ddot{V}$ is bounded, which in turn result that $\dot{V}$ is uniformly continuous. Barbalat's lemma shows [4]:

$$\lim_{t\to\infty} \dot{V}(t) = 0 \tag{27}$$

which lead to the fact $e \to 0$ as $t \to \infty$. Thus, x asymptotically tracks x_m.

B. Direct MRAC with State Predictor

One can re-parameterize the above argument by introducing a state predictor given by [2]

$$\dot{\hat{x}} = A_m\hat{x} + b\,(\,u + \hat{k}_x^{\mathrm{T}}\, x) \quad , \hat{y} = c^{\mathrm{T}}\hat{x} \tag{28}$$

Received 26 Feb 2018; Accepted 9 May 2018
 ISSN (Print) 1811-9212 ISSN (Online) 2617-3352

so $\hat{x} \in \mathbb{R}^n$ is the state vector of the predictor. From Eq.(16) and Eq.(28), the prediction error dynamics can be obtained,

$$\dot{\tilde{x}} = A_m \tilde{x} + b\, \tilde{k}_x^{\mathrm{T}}\, x \tag{29}$$

where $\tilde{x} = \hat{x} - x$ and $\tilde{k}_x = \hat{k}_x - k_x$. The adaptive law for $\hat{k}_x$ is given as

$$\dot{\hat{k}}_x = -\Gamma\, x\, \tilde{x}^{\mathrm{T}} P\ b \tag{30}$$

This adaptive law is identical to Eq. (22) except that e is replaced by $\tilde{x}$. The Lyapunov candidate is selected as

$$V(\tilde{x}, \tilde{k}_x) = \tilde{x}^{\mathrm{T}}\, P\, \tilde{x} + \Gamma^{-1} \tilde{k}_x^{\mathrm{T}} \tilde{k}_x \tag{31}$$

This will yield

$$\dot{V} = -\tilde{x}^{\mathrm{T}} Q\, \tilde{x}\ \le 0 \tag{32}$$

Thereby, the uniform boundedness of $\tilde{x}$ and $\tilde{k}_x$can be guaranteed. However, the uniform boundedness of $\tilde{x}$ does not mean the asymptotic stability of $\hat{x}$and x; as they may diverge at the same rate keeping $\tilde{x}$uniform bounded. Therefore, the asymptotic convergence of $\tilde{x}$ to zero has to be proven using Barbalat's lemma by inclusion of the adaptive feedback control action u. Since the state predictor mimics the model reference of the system:

$$\dot{\hat{x}} = A_m \hat{x} + b\, k\, r \quad , \ \hat{y} = c^{\mathrm{T}} \hat{x} \tag{33}$$

then, Barbalat's lemma can prove that $\tilde{x} \rightarrow\ 0$ as $t \rightarrow \infty$.

C. L$_1$-ADAPTIVE CONTROL

The following structure of system dynamics will be considered throughout the analysis of L$_1$-adaptive controller [4]:

$$\dot{x} = A_m\, x + b\, (\, \omega\ u + \theta^{\mathrm{T}}\, x + \sigma\,)\ \ , y = c^{\mathrm{T}} x \tag{31}$$

where $\theta \in \mathbb{R}^n$ represents time-varying unknown parameters vector, $\omega \in \mathbb{R}$ represents the unknown constant with a known sign and $\sigma\,(t) \in \mathbb{R}$ is the input disturbances. The control objective is to ensure the output y tracks the reference r by utilizing full state feedback adaptive control. The L$_1$-adaptive controller includes three main parts; state predictor, adaptation law and control law. In what follows, a mathematical description of each part will be briefly explained.

1. State Predictor

The state predictor can be described by the following model structure[1-5],

$$\dot{\hat{x}} = A_m\, \hat{x} + b\, (\, \hat{\omega}\, u + \hat{\theta}^{\mathrm{T}} x + \hat{\sigma}), \quad \hat{y}(t) = c^{\mathrm{T}}\, \hat{x}(t) \tag{32}$$

which is the same as Eq.(20) with exception of replacement of ω, θ, and σ by their adaptive estimates $\hat{\omega}$, $\hat{\theta}$, and $\hat{\sigma}$, respectively.

2. Adaptation Laws

Let us consider a set with convexity and compactness properties with boundary given by [4, 23];

$$\Omega_c = \{\theta \in \Re^n | f(\theta) \le c\} \qquad\qquad 0 \le c \le 1, \tag{33}$$

where $f : \mathbb{R}^n \rightarrow \mathbb{R}$ is a convex function of the form,

$$f(\theta) = \frac{\|\theta\|_2^2 - \theta_{max}^2}{\varepsilon_\theta\, \theta_{max}^2}\ \ ,\ \ 0 < \varepsilon_\theta\ \le 1 \tag{34}$$

Received 26 Feb 2018; Accepted 9 May 2018
 ISSN (Print) 1811-9212 ISSN (Online) 2617-3352

where $\theta_{max}$is the maximum allowable value for the root of the squared sum of vector θ. ε_θ denotes the tolerance of the adaptive parameter to exceed its maximum value. If the function$f(\theta) \leq 1$ is defined as the boundaries of the outer set, then one can get that $\theta^T\theta \leq (1+\varepsilon_\theta)\ \theta_{max}^2$. The projection operator can be defined as [4]:

$$Proj(\theta, y) = \begin{cases} y & if \quad f(\theta) < 0 \\ y & if \quad f(\theta) \geq 0 \wedge \nabla f^T y \leq 0 \\ y - \frac{\nabla f}{\|\nabla f\|} < \frac{\nabla f^T}{\|\nabla f\|}, y > & if \quad f(\theta) \geq 0 \wedge \nabla f^T y > 0 \end{cases} \tag{35}$$

Generally, the projection operator algorithm has employed to get bounded adaptive gains from the adaptive law. For the present work, the projection operator is used to confining the bound of $\hat{\theta}$, $\hat{\sigma}$ and$\hat{\omega}$ as follows;

$$\begin{aligned} \dot{\hat{\theta}} &= \Gamma \text{ Proj } (\hat{\theta}, -\tilde{x}^{\mathrm{T}} P\, b) \\ \dot{\hat{\sigma}} &= \Gamma \text{ Proj } (\hat{\sigma}, -\tilde{x}^{\mathrm{T}} P\, b) \\ \dot{\hat{\omega}} &= \Gamma \text{ Proj } (\hat{\omega}, -\tilde{x}^{\mathrm{T}} P\, b) \end{aligned} \tag{36}$$

where $\tilde{x} = \hat{x} - x$ $\Gamma \in \mathbb{R}^+$ represents the learning rate and $P = P^{\mathrm{T}} > 0$ is the Lyapunov equation solution given by Eq. (23).

3. Control Law

The control signal resulting from system feedback is given by [4, 33]:

$$u(s) = -k\ D(s)\ \ (\hat{\eta}(s) - k_r r(s)) \tag{37}$$

where $r\ (s)$ and $\hat{\eta}\ (s)$ represents the Laplace transforms of r and $\hat{\eta}$, respectively. The expressions for $\hat{\eta}$ and k are given by

$$\hat{\eta} = \hat{\omega}\ \ u + \hat{\theta}^{\mathrm{T}}\, x + \hat{\sigma}, \tag{38}$$

$$k \ = -1/c^{\mathrm{T}} A_m^{-1}\ b \tag{39}$$

where $k > 0$and $D(s)$ are feedback gains, where$D(s)$ is a strictly proper transfer function designed to give a strictly proper stable filter:

$$C(s) = \frac{\omega\, k\, D\,(s)}{1+\omega\, k\, D\,(s)} \tag{40}$$

The DC gain can be obtained by setting $C(0) = 1$. Choosing $D(s) = 1/s$ will produce a simple strictly proper first order filter$C(s)$of the form

$$C(s) = \frac{\omega\ k}{s+\omega\ k} \tag{41}$$

The L_1-adaptive controller is conditioned by the following L_1-norm inequality [2, 4]:

$$G(s) \quad L \leq 1 \tag{42}$$

where $L = \max_{\theta\in\Theta} \theta_1$ and $G(s) = H(s)\ \ (1 - C(s))$, where $H(s) = (s\mathbb{I} - A_m)^{-1} b$ and $\theta \in \Theta$. For the special selection of $D(s) = 1/s$, the closed-loop system matrix Acan be considered as;

$$A = \begin{bmatrix} A_m + b\,\theta^{\mathrm{T}} & b\,\omega \\ -k\,\theta^{\mathrm{T}} & -k\omega \end{bmatrix}, \tag{43}$$

where$A$ must be Hurwitz with negative real eigenvalues for all $\theta \in \Theta$and $\omega \in \Omega_0$. The matrix $A_m$is given, $A_m = A - bK_m$ where $K_m = [k_1 \quad k_2]$ is the state feedback gain matrix, whose elements are required to make the state matrix A a Hurwitz; that is, all real parts of all its eigenvalues have real

Received 26 Feb 2018; Accepted 9 May 2018
 ISSN (Print) 1811-9212 ISSN (Online) 2617-3352

values. Completely state controllable of the systems is a pre-requisite for applying pole placement. The state and input matrix of PMLSM are given, respectively,

$A = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{B_v}{M} \end{bmatrix}$ and $b = [0 \quad 1]^T$.The controllability matrix is given by $P_c = [b \quad Ab] = \begin{bmatrix} 0 & 1 \\ 1 & -\frac{B_v}{M} \end{bmatrix}$.

It is evident that the rank of the controllability matrix is equal to 2, which is equal to the system order. Therefore, the system is completely controllable and the pole placement could be applied. Numerically, if the system has the following values,

M=1.97 and B_v=83.2245

The eigenvalues for this system is $s_1 = 0$and $s_2 = -42.246$. Since the system completely states controllable, one can arbitrarily select the desired poles to be $s_{1,2} = -16 \mp 10.677\, i$. The elements of state feedback gain K_m which performs pole placement requirements are given by

$K_m = [k_1 \quad k_2] = [370 \quad -10.2459]$

This transformation is achieved by $A_m = A - b\, K_m = \begin{bmatrix} 0 & 1 \\ -370 & -32 \end{bmatrix}$.

IV. SIMULATED RESULTS

The uncertainty $\sigma(t)$ considered in this system has the following form, $\sigma(t) = -(1/M)(f(v) + F_L)$. Also, $\theta^T = [0 \ \ 0]$, $\omega = K_f/M$. Substituting the values of friction model parameters, the uncertainty bound and the value of the parameter ω can be given by $\sigma(t) \in \Delta = [-1.0769, 0.6091]$, $\omega = 10.1523(N/Amp.Kg)$. It is worthy to mention here that the voltage/position scale used in the simulated results has the value *1V=63.662m*.

For simulation purposes, two different architectures of adaptive control were taken: L_1-adaptive control and MRAC. Three types of inputs: ramp, sinusoidal, and step inputs were used to compare the two architectures. In the design of the L_1-adaptive controller, the filter of the controller is selected as $D(s) = K/s$ and the parameter of gain K and adaptation gain Γ has been set to $K = 100$ and $\Gamma = 10^4$ using the trial-and-error procedure.

To show the robustness of the L_1 adaptive control, four cases of different values of uncertainties and disturbances were listed in Table (2),

TABLE (2) CASES FOR DISTURBANCE AMPLITUDE AND FREQUENCY [34]

Parameter	Case 1	Case 2	Case 3	Case 4
$\sigma(t)$	$0.002\sin(t) + f(v)$	$\sin(t) + f(v)$	$0.002\sin(10t) + f(v)$	$\sin(10t) + f(v)$

where $f(v)$ is the friction force given by,

$$f(v) = k_v v(t) + (F_s - F_c)sgn(v(t))e^{-(v(t)/v_s)^2} + F_c sgn(v(t)).$$

The following friction parameters have been considered for simulation [29, 30];

$$k_v = 0.8\ N.s/mm\ ,\ F_c = 0.08\ \text{N},\ \ F_s = 1.2\ \text{N},\ v_s = 0.08\ mm/s$$

A. Results based on Ramp input

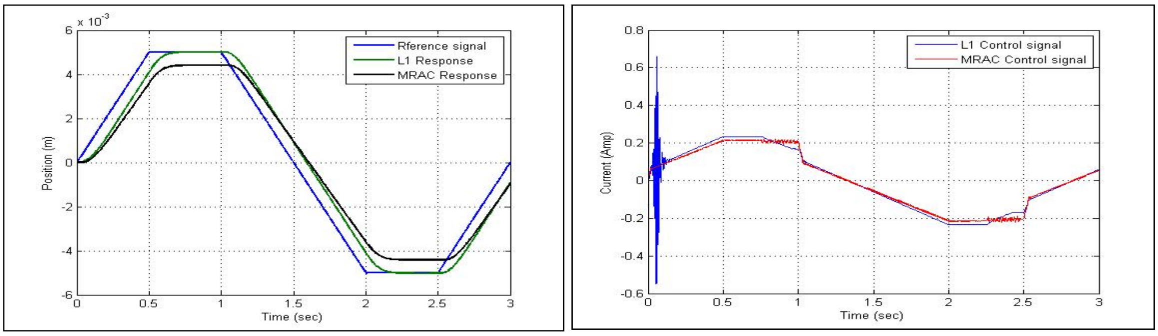
For case (1), the position behaviors and the control signals are illustrated in Fig. (1). The figure shows that L_1-adaptive control has better tracking performance for the ramp input as compared to that of MRAC. The steady-state error for L_1 adaptive control response is 0.017 *mm* while for the MRAC is 0.5832 *mm*.

For case (2), the responses of positions and control signals are depicted in Fig. (2). In this case, the disturbance amplitude has been changed, while its frequency was fixed at the value of the previous case. From the figure, it is evident that L_1-adaptive controller could also give better tracking performance

Received 26 Feb 2018; Accepted 9 May 2018
 ISSN (Print) 1811-9212 ISSN (Online) 2617-3352

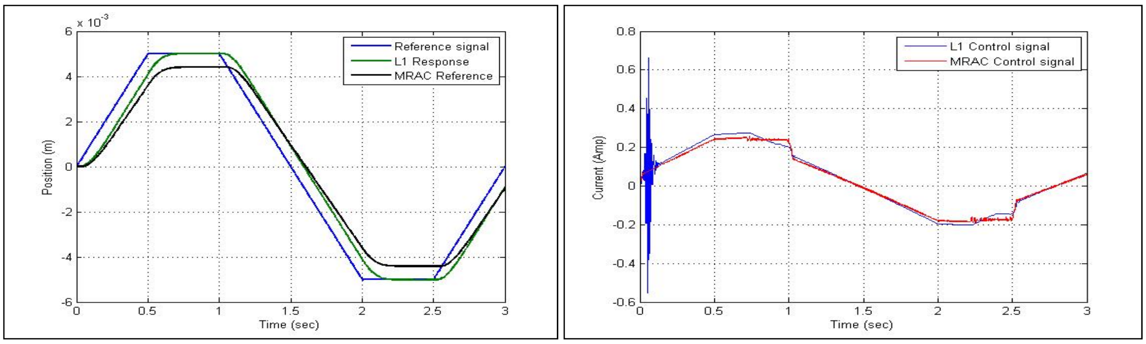
with smaller time delay than MRAC. Moreover, the response based on MRAC could not overlap with ramp input, on the contrary of the response resulting from the L_1-adaptive controller, where there is a complete coincidence between the response and the prescribed input. The steady-state errors between the input and the responses based on for L_1-adaptive controller and MRAC are 0.0173 mm and 0.584mm, respectively.

For case (3), the position responses and the control signals are shown in Fig. (3). In the present case, the disturbance amplitude is also fixed at the value of the previous scenario and the frequency is allowed to be changed 10 times of the first case. It is evident from the figure that the responses based L_1-adaptive controller shows better tracking characteristics than MRAC. The steady-state error given by L_1-adaptive controller is equal to 0.0165 mm, while MRAC yields a steady-state error of value 0.6463 mm.



(A) STEP RESPONSES (B) CONTROL SIGNALS

FIGURE (1) TRANSIENT RESPONSES AND CONTROL SIGNALS BASED ON L1 ADAPTIVE CONTROLLER AND MRAC FOR RAMP INPUT (CASE 1)

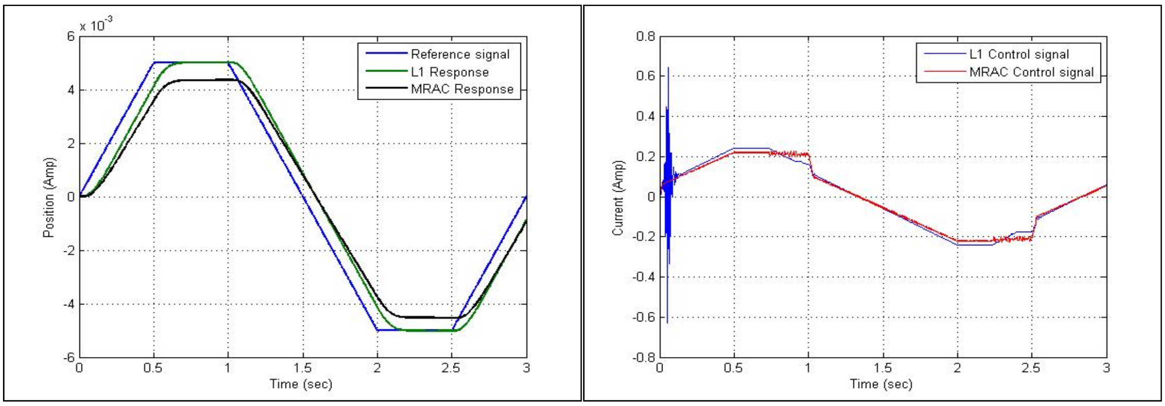


(A) STEP RESPONSES (B) CONTROL SIGNALS

FIGURE (2) TRANSIENT RESPONSES AND CONTROL SIGNALS BASED ON L1 ADAPTIVE CONTROLLER AND MRAC FOR RAMP INPUT (CASE 2)

For case (4), the responses and the control signals are shown in Fig. (4). In such case, both amplitude and frequency were changed. Again, the response based on the L_1-adaptive controller has better transient and tracking performance than that for MRAC. The steady-state errors based on L_1-adaptive controller and MRAC are 0.022 mm and 0.5478 mm, respectively. For comparison purposes, Table (3) shows the summary of all steady-state errors for all considered cases.

Received 26 Feb 2018; Accepted 9 May 2018
 ISSN (Print) 1811-9212 ISSN (Online) 2617-3352

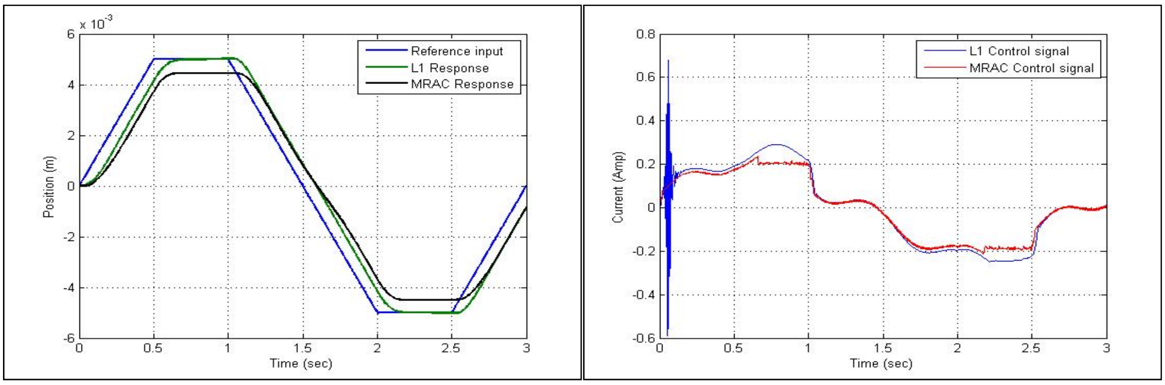


(A) STEP RESPONSES (B) CONTROL SIGNALS

FIGURE (3) TRANSIENT RESPONSES AND CONTROL SIGNALS BASED ON L1 ADAPTIVE CONTROLLER AND MRAC FOR RAMP INPUT (CASE 3)

TABLE (3) STEADY-STATE ERROR FOR DIFFERENT CASES

	Steady state error (mm)			
	Case 1	**Case 2**	**Case 3**	**Case 4**
L_1-controller	0.017	0.0171	0.0165	0.022
MRAC	0.5832	0.584	0.6463	0.5478



(A) STEP RESPONSES (B) CONTROL SIGNALS

FIGURE (4) TRANSIENT RESPONSES AND CONTROL SIGNALS BASED ON L1 ADAPTIVE CONTROLLER AND MRAC FOR RAMP INPUT (CASE 4)

B. Results based on Step input

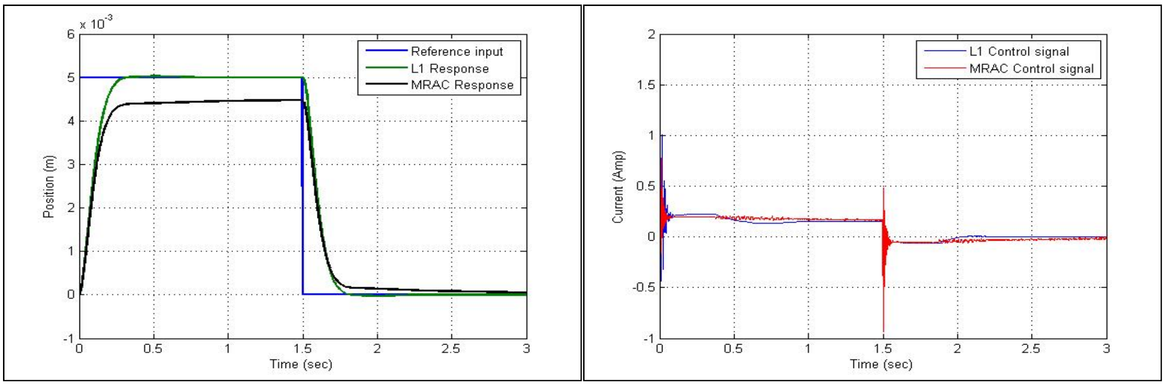
It is interesting to examine the effectiveness of both controllers as the system is subjected to a unit step. For the present scenarios, a step input of 5 mm height is fed to the system. This step input is inverted after 1.5 sec. such that a square wave input is repeated for every 3 sec. The performance of both controllers for the situations listed in Table (2) will be considered again here.

The position responses and the control signals based on Case (1) are shown in Fig. (5). One can easily see that the response based on the L_1-adaptive controller could give better performance in terms of transient characteristics than those based on MRAC. However, a small peak over-shoots ($Mp = 0.025$) has been seen in time response based on the L_1-adaptive controller.

In the next scenario, the structure of the uncertainty of case (2) is considered. Figure (6) shows the position and control signal behaviors based on both controllers. The control effort of L_1 adaptive controller shows a better tracking for the desired step input rather than MRAC. A zero steady-state error

Received 26 Feb 2018; Accepted 9 May 2018
 ISSN (Print) 1811-9212 ISSN (Online) 2617-3352

is found for the case of the L_1-adaptive controller, while it has a value of 0.596 mm for the MRAC. Figure (7) shows that the position response for MRAC is broken down at 2.92 sec. and the system will be blown up. This indicates that MRAC could not cope with this structure of uncertainty (case 2).



(A) STEP RESPONSES (B) CONTROL SIGNALS

FIGURE (5) TRANSIENT RESPONSES AND CONTROL SIGNALS BASED ON L1 ADAPTIVE CONTROLLER AND MRAC FOR STEP INPUT (CASE 1)

For case 3, the position responses and the control signals for both controllers are shown in Fig. (8). It is clear from the figure that the L_1 adaptive controller outperforms MRAC. The first controller could give zero mm steady-state error, while the latter one yields 0.52 mm value of steady-state error. However, a peak overshoot of value $M.P. = 0.0243$ has appeared at the response of the first controller.

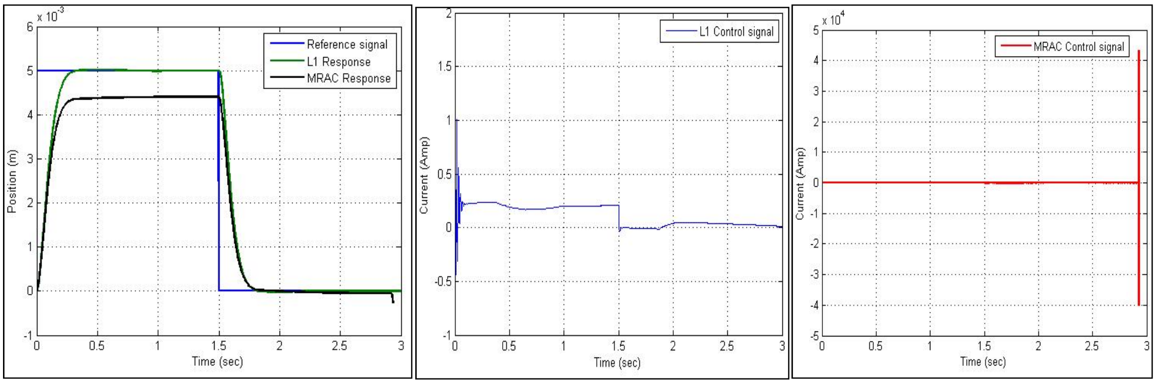
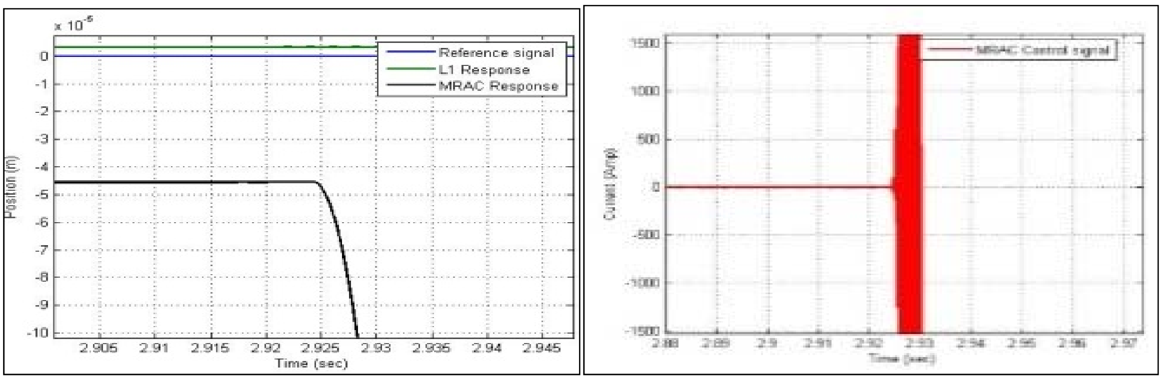


FIGURE (6) TRANSIENT RESPONSES AND CONTROL SIGNALS BASED ON L1 ADAPTIVE CONTROLLER AND MRAC FOR STEP INPUT (CASE 2)



(A) RESPONSE NEAR 2.92 SEC. (B) ZOOMED OF MRAC CONTROL SIGNAL

FIGURE (7) BREAKING DOWN OF TRANSIENT RESPONSE BASED ON MRAC AFTER 2.92 SEC.

For the last case, the position responses and the control signals are shown in Fig. (9). It has been mentioned earlier that both the amplitude and frequency of uncertainty structure were changed. One

Received 26 Feb 2018; Accepted 9 May 2018
 ISSN (Print) 1811-9212 ISSN (Online) 2617-3352

can easily see that L_1-adaptive controller could keep higher performance than its counterpart. In the situation, the peak overshoot of the response based on the L_1-adaptive controller has a little value of *M.P*.=0.0001. The steady-state error for L_1 adaptive controller-based response has the value 0.0166 mm, while for the other controller is equal to 0.451 mm.

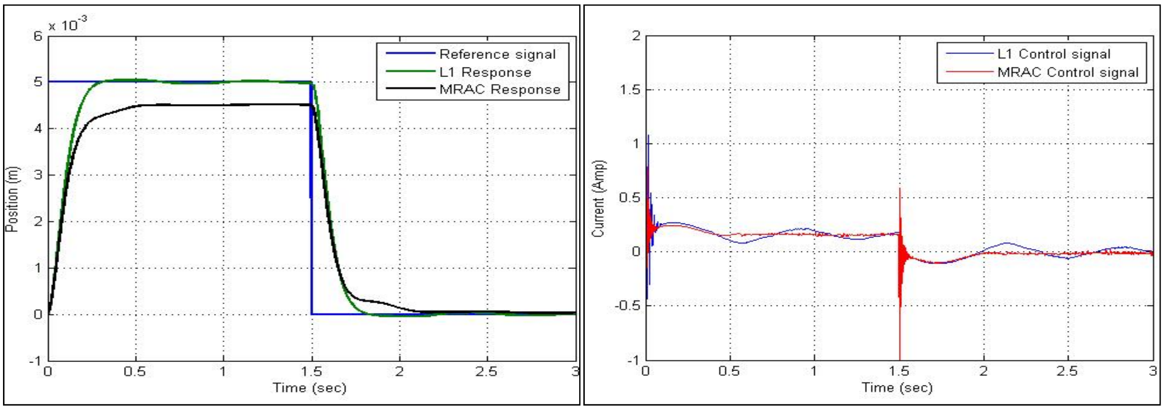


FIGURE (8) TRANSIENT RESPONSES AND CONTROL SIGNALS BASED ON L1 ADAPTIVE CONTROLLER AND MRAC FOR STEP INPUT (CASE 3)

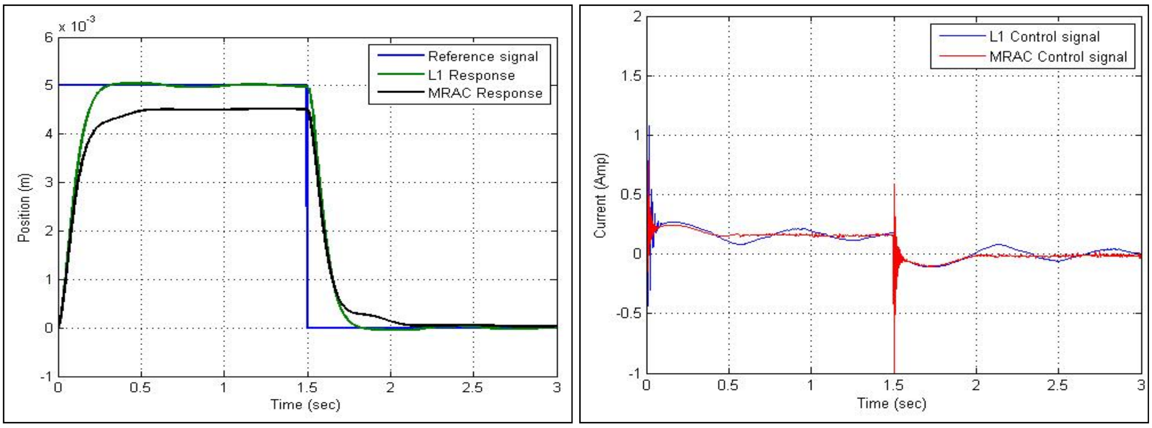


FIGURE (9) TRANSIENT RESPONSES AND CONTROL SIGNALS BASED ON L1 ADAPTIVE CONTROLLER AND MRAC FOR STEP INPUT (CASE 4)

Table (4) lists the summary of steady-state errors resulting from both controllers for all four cases. It can be concluded that steady-state error based on the L_1-adaptive controller for all considered cases has nearly zero value. On the other hand, MRAC gives considerably large steady-state error for all studied cases.

TABLE (4) STEADY-STATE ERROR FOR DIFFERENT CASES

	Steady state error (mm)			
	Case 1	**Case 2**	**Case 3**	**Case 4**
L_1-controller	0	0	0	0.0166
MRAC	0.52	0.596	0.52	0.451

Received 26 Feb 2018; Accepted 9 May 2018
 ISSN (Print) 1811-9212 ISSN (Online) 2617-3352

TABLE (5) SETTLING TIME FOR DIFFERENT CASES OF STEP INPUT SIGNAL

	Settling time (sec)			
	Case 1	**Case 2**	**Case 3**	**Case 4**
L_1-controller	0.8	0.75	0.83	0.65
MRAC	1.12	1.07	1.11	1.035

Table (5) reports the settling time due to the step input response of 5 mm for all cases. It is clear that L_1-adaptive controller could give lower values of settling time compared to those resulting from MRAC for all cases. This means that L_1-adaptive controller has a faster adaptation rate than MRAC.

V. CONCLUSION

In the present work, linear servo motor for x-y table application have been considered; which is Permanent Magnet Linear Synchronous Motor (PMLSM). Also, two adaptive controllers have been suggested for position controlling of the PMLSM under different structures of uncertainties and different types of inputs. Three types of inputs have been taken into account; ramp, step and sinusoidal input.

For the sake of clarity, the conclusions, based on the observations from simulated results can be highlighted as:

- For all types of inputs and for all structures of uncertainties, the L1-adaptive controller gives less steady state errors (nearly zero) than the MRAC, in case of step input,
- In case of step input and for all cases of uncertainties, the L1-adaptive controller gives faster transient responses than MRAC. This means that the adaptation rate of the L1-adaptive controller is faster than that of MRAC.

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Received 26 Feb 2018; Accepted 9 May 2018

 ISSN (Print) 1811-9212 *ISSN (Online) 2617-3352*