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Parameters estimation of homogeneous gamma process via intelligence techniques

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Abstract

Recently, the Gamma process has been increasing used to model stochastic deterioration for optimizing maintenance because are well suited for modeling the temporal variability of deterioration. In this paper, we discussed two algorithms of the intelligent technique algorithms with moment method for estimating the parameters of the homogeneous gamma process. The application results demonstrate that the intelligent techniques estimation methods are considerably consistent in estimation compared to the moment method, using mean absolute error (MAE).

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1. Introduction

The study of stochastic processes is an important issue in real application, as it is the dynamic part of probability theory. The gamma process is one of the most important types of stochastic processes because of its importance in various fields. Abdel-Hameed (1975) is the first from introduce the gamma process as appropriate model of randomly occurring degradation in time. It is suitable to model gradual damage monotonically accumulating over time in a sequence of small increments such as crack growth, degrading health index, erosion, wear, corrosion, swell and creep (Van Noortwijk 2009). The Dufresne (1991) introduced the gamma process into actuarial literature and defined as a limit of compound Poisson processes and considered it as a model for aggregate claims process (Dickson and Waters 1993). It is has been found useful for analyzing degraduation data (Lawless and Crowder 2004, Wang 2009). In recent years, the nature inspired meta-heuristic algorithms has become widespread because they have the ability to deal with many of practical optimization problems both continuous and discrete in a reasonably time. The meta-heuristic algorithms use search strategies and concepts inspired from nature to explore several regions of the search space more effectively and focusing on some likely regions of the search space. Every meta-heuristic algorithm consists of a set of initial population or initial solutions, the sequence of solutions is then examined step by step based on randomization and some specified rules to reach the optimal solution (Osman 1996, Yang 2015, Faris, Aljarah et al. 2017, Haddad 2018). In this paper, two algorithms are proposed in addition to the moment method for estimating the parameters of the homogeneous gamma process and compare them through mean absolute error. The rest of the paper, is organized as follows. In section 2, we introduce properties the homogeneous gamma process. In section 3, we discuss estimation methods parameters of the homogeneous gamma process. In section 4, we implement the estimation methods used in this paper on real data. In section 5, we show application results.

2. Gamma Process

The gamma process is one of the stochastic processes with independent non-negative increments and having a gamma distribution with an conformable scale parameter (Van Noortwijk 2009, Zhou, Pan et al. 2010). It can be divided into two main types that are homogeneous gamma process and non-homogeneous gamma process, each process has its own properties and parameters. The difference between the two processes depends on the shape parameter. In this study, we will present the homogeneous gamma process.

The homogeneous gamma process is one of the stochastic processes which is the random variables that stationary independent increments and non-negative. Let $X(t)$ the homogeneous gamma process which is has the following properties (Roussignol 2009, Wang 2009):

1. $X(0) = 0$ with probability one.
2. $X(t)$ has independent increments and non-negative.
3. The probability density function (p.d.f) is

$$f_{X(t)}(x) = Ga\langle x \,|\alpha t, \beta\rangle = \frac{\beta^{\alpha t}}{\Gamma(\alpha t)} x^{\alpha t - 1} e^{-\beta x} \tag{1}$$

where α is the shape parameter and β is the scale parameter.

4. The mean and the variance for all $t \geq 0$ is

$$E(X(t)) = \frac{\alpha}{\beta} t, \qquad Var(X(t)) = \frac{\alpha}{\beta^2} t \tag{2}$$

3. Parameters Estimation Methods

3.1 Moment Method

This method is one of the oldest historical estimation methods that introduced by the world statistical Carl Pearson in 1894. It is equal to the moment of the society (about the origin) and the moment of its corresponding sample. The basis of this idea depends on the composition of the equations of random variable moments, thus obtaining several equations equal to the number of parameters estimated and their solution for the parameters of society are estimated for these parameters. The steps for estimating parameters of homogeneous gamma process in this method are as follows (Roussignol 2009).

$$\frac{\hat{\alpha}}{\hat{\beta}} = \frac{\sum_{i=1}^{n} \theta_i}{\sum_{i=1}^{n} \tau_i} = \frac{x_n}{t_n} = \bar{\theta} \tag{3}$$

where $\theta_i = x_i - x_{i-1}$ and $\tau_i = t_i - t_{i-1}$

$$\frac{\hat{\alpha}}{\hat{\beta}^2} = \frac{\sum_{i=1}^{n} (\theta_i - \bar{\theta}\tau_i)^2}{\sum_{i=1}^{n} \tau_i - \frac{\sum_{i=1}^{n} \tau_i^2}{\sum_{i=1}^{n} \tau_i}} \tag{4}$$

$$E\left(\frac{\hat{\alpha}}{\hat{\beta}}\right) = \frac{\alpha}{\beta} \qquad \textit{and} \qquad E\left(\frac{\hat{\alpha}}{\hat{\beta}^2}\right) = \frac{\alpha}{\beta^2}$$

Eq. (4) can be rewritten as

$$\frac{\sum_{i=1}^{n} \theta_i}{\hat{\beta}} \left(1 - \frac{\sum_{i=1}^{n} (\tau_i)^2}{\left(\sum_{i=1}^{n} \tau_i\right)^2} \right) = \sum_{i=1}^{n} \left(\theta_i - \bar{\theta}(\tau_i)\right)^2 \tag{5}$$

$$\hat{\beta}=\frac{\sum_{i=1}^{n}\theta_i\left(1-\frac{\sum_{i=1}^{n}(\tau_i)^2}{\left(\sum_{i=1}^{n}\tau_i\right)^2}\right)}{\sum_{i=1}^{n}\left(\theta_i-\bar{\theta}(\tau_i)\right)^2} \quad (6)$$

We can obtain $\hat{\alpha}$ from Eq. (3) as

$$\hat{\alpha}=\hat{\beta}\times\frac{\sum_{i=1}^{n}\theta_i}{\sum_{i=1}^{n}\tau_i} \quad (7)$$

Compensate the estimated value $\hat{\beta}$ in Eq. (7), we get the estimated value $\hat{\alpha}$ shown in Eq. (8)

$$\hat{\alpha}=\frac{\sum_{i=1}^{n}\theta_i\left(1-\frac{\sum_{i=1}^{n}(\tau_i)^2}{\left(\sum_{i=1}^{n}\tau_i\right)^2}\right)}{\sum_{i=1}^{n}\left(\theta_i-\bar{\theta}(\tau_i)\right)^2}\times\frac{\sum_{i=1}^{n}\theta_i}{\sum_{i=1}^{n}\tau_i} \quad (8)$$

3.2 The Proposed Methods

In this paper, we proposed two methods: particle swarm optimization and firefly algorithm to estimate the parameters of the homogeneous gamma process. Further, we used the mean absolute error to the purpose of comparison between the proposed methods and the moment method.

3.2.1 Particle Swarm Algorithm

Particle swarm optimization is a stochastic optimization algorithm proposed originally for solving continuous optimization problem by Kennedy and Eberhart (1995). The PSO algorithm simulates the social behavior of birds and fish in the way of searching food. PSO algorithm based on population this population is known as swarm, each individuals in the swarm are called the particle N which fly in D-dimensional search space. Each particle has a position $y_i=(y_{i1},y_{i2},.......,y_{iD})$ and velocity $v_i=(v_{i1},v_{i2},.......,v_{iD})$ to movement within search space where $i=1,2,.....,N$. Each particle is updated through each iteration based on two values: the first value is the best solution $(pbest)$, which has been obtained by the particle, and the second value is the current best value $(Gbest)$, which has been obtained in the swarm. The new velocity and position vectors in each iteration are updated according to the following equations (Rao 2009, Parsopoulos K and Vrahatis M 2010, Wang 2018).

$$v_i^{t+1}=W\,v_i^t+\varepsilon_1.R_1.\left(Pbest_i^t-y_i^t\right)+\varepsilon_2.R_2.\left(Gbest^t-y_i^t\right) \quad (9)$$

$$y_i^{t+1}=x_i^t+y_i^{t+1} \quad (10)$$

where t is the current iteration, W refers to the inertia weight. ε_1 , ε_2 are the acceleration coefficients and R_1 , R_2 are the random numbers $\in[0,1]$.

In this paper, a PSO algorithm is proposed to estimate the parameters of the homogeneous gamma process. The parameters are estimated using the suggested method as follows:

1- The number of particles, N is set to 50 and the maximum number of iterations is $t_{max}=200$. The acceleration coefficients ε_1 and ε_2 are set to 2.05. Further, the minimum and the maximum values for the inertial weight are: $w_{\min} = 0.4$ and $w_{\max} = 0.9$. The inertial weight is updated according to the following equation:

$$W = w_{\max} - \frac{(w_{\max} - w_{\min})}{t_{\max}} t \qquad (11)$$

2- The initial positions of the particles are randomly generated from a uniform distribution in range [0,100] where the position of a particle represents the parameters of the homogeneous gamma process.
3- The initial velocities of each particle are generated from a uniform distribution within the range [0, 4].
4- The fitness function represents the mean absolute error, which can be defined as

$$MAE = \frac{\sum_{i=1}^{n} \left| y_i - y_i \right|}{n} \qquad (12)$$

The fitness function are calculated for all particles, the $Pbest$ and $Gbest$ vectors are determined.

5- The particles In each iteration adjust the velocity according to Eq.(9) which is used to update the positions of particles according to Eq.(10).

6- Steps 4 and 5 are repeated until a $t_{\max}$ is reached.

3.2.2 Firefly Algorithm

Firefly algorithm is a meta-heuristic algorithm inspired by the flashing behavior of fireflies. It is proposed originally by (Yang 2008) for solving optimization problems with continuous variables. In firefly algorithm three main rules are used (Yang 2014, Tilahun and Nnotchouyge 2017):

1- All fireflies are unisex so that one firefly can be attracted to any other firefly.
2- The brightness of firefly is determined by the objective function.
3- The attractiveness of firefly is proportional to brightness. Thus, for any two flashing fireflies the less bright one moves towards brighter one, while it will move randomly when there is no brighter one than a specific firefly.

The brightness β of a firefly with the distance r can be defined as:

$$\beta = \beta_0 e^{-\gamma r^2} \qquad (13)$$

where r is the distance between two fireflies, γ is a light absorption coefficient and β_0 is the brightness when the distance is zero.
The distance between two fireflies i and j is computed using the Euclidean distance:

$$r_{ij} = \left\| y_i - y_j \right\| = \sqrt{\sum_{k=1}^{d} \left(y_{i,k} - y_{j,k} \right)^2} \qquad (14)$$

where $y_{i,k}$ is the k^{th} component of the i^{th} firefly. The movement of a firefly i attracted by another firefly j that is brighter is computed as:

$$y_i^{t+1} = y_i^t + \beta_0 e^{-\gamma r_{ij}^2} \left(y_j^t - y_i^t \right) + \alpha \left(rand - 0.5 \right) \qquad (15)$$

where α is a parameter which controls the step and rand is a random number $\in [0,1]$. In this paper, a firefly algorithm is proposed to estimate the parameters of the homogeneous gamma process. The parameters are estimated using the suggested method as follows:

1- The population size N is set to 50 and the maximum number of iterations is $t_{\max} = 200$. The firefly parameters was set as $\alpha = 0.1,\ \ \beta = 0.2$ and $\gamma = 0.9$.
2- The initial positions of the fireflies are randomly generated from a uniform distribution in range [0,100] where the position of a firefly represents the parameters of the homogeneous gamma process.

3- Evaluate the fitness function that is represents the mean absolute error, which can be defined as in Eq. (12).

4- The attractiveness β of a firefly is computed according to Eq. (13).

5- The movement of a firefly i attracted by another firefly j that is brighter according to Eq. (15).

6- Steps 3 and 5 are repeated until a $t_{\max}$ is reached.

4. Application Results

In this section, five datasets with different sample sizes are used. First to Four datasets , the number of operating (hours) between successive failures of air conditioning equipment in 4 Boeing 720 aircraft (Cox and Lewis 1966). And the Five dataset, the operating time for a filtration system for the General Company for Fertilizer industry in the Southern Region. By using the Kolmogorov goodness of fit test to test the appropriateness of datasets for gamma process according to the hypothesis as

H_0 : Data fit for the gamma process.

H_1 : Data not fit for the gamma process.

Table 1 represents the Kolmogorov test results. As shown, all the datasets used are fit for the gamma distribution under significant level 0.05. Moreover, all the datasets used are appropriateness for the gamma process.

Table 1: Test fit of the data for gamma distribution

Data	n	Kolmogorov	Critical value	P-value
Data 1	24	0.08888	0.29931	0.98255
Data 2	27	0.12604	0.25438	0.73788
Data 3	29	0.07855	0.24571	0.98771
Data 4	30	0.13432	0.2417	0.60398
Data 5	37	0.16039	0.21826	0.26728

The estimated parameters of the homogeneous gamma process using MO, PSO, and FA are shown in Table 2

Table 2: The estimated parameters of the homogeneous gamma process

Data	$\hat{\alpha}_{MO}$	$\hat{\beta}_{Mo}$	$\hat{\alpha}_{PSO}$	$\hat{\beta}_{PSO}$	$\hat{\alpha}_{FA}$	$\hat{\beta}_{FA}$
Data 1	1.0476	0.0163	2.0000	0.4205	3.0000	0.6998
Data 2	1.4536	0.0189	3.5000	3.4905	4.0000	3.9953
Data 3	1.3913	0.0167	1.5000	0.0449	2.5000	0.0851
Data 4	0.6874	0.0115	1.0000	0.1250	1.5000	0.1635
Data 5	4.9781	0.5923	5.7000	0.7175	6.2000	0.7757

Table 3: MAE values of the estimation methods

Data	MO	PSO	FA
Data 1	64.1172	64.1088	64.1061*
Data 2	76.8089	76.7878	76.7859*
Data 3	83.5113	83.5096	83.5086*
Data 4	59.5901	59.5812	59.5800*
Data 5	8.3316	8.3289	8.3275*

In Table 3 summarized the MAE values of the estimation methods, we can be seen that the FA and PSO yields the least MAE value from the Mo method, therefore the intelligent techniques are better than the moment method in estimating parameters this process for all the datasets.

5. Conclusions

By evaluating the results, we concluded that the all data are suitable for the homogeneous gamma process. The intelligent techniques algorithms are the best comparing with moment method and the FA is the best methods used in estimating parameters.

References

1. Abdel-Hameed, M. (1975). "A gamma wear process." IEEE Transactions on Reliability 24(2): 152–153.
2. Cox, D. R. and Lewis P.A.W (1966). "The Statistical Analysis of Series of Events." Mathuen, London.
3. Dickson, D. C. M. and H. R. Waters (1993). "Gamma Processes and Finite Time Survival Probabilities." Astin Bulleti 23(2).
4. Dufresne, F., Gerber, H.U. and Shiu, E.S.W (1991). "Risk theory and the gamma process." ASTIN Bulletin 22: 177-192.
5. Faris, H., I. , Aljarah and a. S. Mirjalili (2017). "Improved monarch butterfly optimization for unconstrained global search and neural network training." Applied Intelligence 48(2): 445-464.
6. Haddad, O. B. (2018). "Advanced Optimization by Nature-Inspired Algorithms." Springer Nature Singapore Pte Ltd.
7. Kennedy, J. and a. R. C. Eberhart (1995). "Particle swarm optimization." Proceedings of IEEE Conference on Neural Network 4: 1942–1948.
8. Lawless, J. and M. Crowder (2004). "Covariates and random effects in a gamma process model with application to degradation and failure." Lifetime Data Anal 10(3).
9. Osman, I. H., and J. P. Kelly (1996). "Meta-heusirtics: theory and applications." Kluwer Academic Publishers.
10. Parsopoulos K, E. and N. Vrahatis M (2010). "Particle Swarm Optimization and Intelligence Advances and Applications." United States by America, IGI Global.
11. Rao, S. S. (2009). "Engineering Optimization Theory and Practice." John Wiley and Sons, Inc. 4th ed.
12. Roussignol, M. (2009). "Gamma stochastic process and application to maintenance." University Paris-EST, Marne-la-vallee.
13. Tilahun, S. L. and J. M. T. Nnotchouyge (2017). "Firefly algorithm for discrete optimization problems: A survey." KSCE Journal of Civil Engineering 21(2): 535-545.
14. Van Noortwijk, J. M. (2009). "A survey of the application of gamma processes in maintenance." Reliability Engineering & System Safety 94(1): 2-21.
15. Wang, D., Tan D., and Liu, L. (2018). "Particle swarm optimization algorithm an overview." Springer-Verlag Berlin Heidelberg. Soft Comput 22: 387-408.
16. Wang, X. (2009). "Nonparametric estimation of the shape function in a Gamma process for degradation data." Canadian Journal of Statistics 37(1): 102–118.
17. Yang, X., S (2008). "Nature-Inspired Metaheuristic Algorithms." Luniver Press, UK.
18. Yang, X. S. (2014). "Cuckoo Search and Firefly Algorithm Theory and Applications." Springer International Publishing Switzerland.
19. Yang, X. S. (2015). "Recent Advances in Swarm Intelligence and Evolutionary Computation." Springer International Publishing Switzerland.
20. Zhou, J., Z. Pan, Member, IAENG and Q. Sun (2010). "Bivariate Degradation Modeling Based on Gamma Process." Proceedings of the World Congress on Engineering , London, U.K. 3.

تقدير المعلمات لعملية جاما المتجانسة عبر تقنيات الذكاء

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الخلاصة:

في الآونة الأخيرة ، تم استخدام عملية جاما بشكل متزايد لنمذجة التدهور العشوائي لتحسين الصيانة لأنها مناسبة تمامًا لنمذجة التباين الزمني للتدهور. ناقشنا في هذا البحث خوارزميات التقنية الذكية مع الطريقة اللحظية لتقدير معاملات غاما المتجانسة. تظهر نتائج التطبيق أن طرق تقدير التقنيات الذكية متسقة إلى حد كبير في التقدير مقارنة بالطريقة اللحظية ، باستخدام متوسط الخطأ المطلق.(MAE)

الكلمات المفتاحية: عملية جاما المتجانسة ، سرب الجسيمات ، اليراع ، تقدير العزم.