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Theory of Mitigate Temperature Effect on the Equilibrium Point in Vertical Cavity Surface Emitting Lasers

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Abstract

This paper presents a way to mitigate the influence of temperature effects on the equilibrium point (Q-point) of vertical cavity surface emitting lasers (VCSELs) by investigating the effect of laser injection current (I_{inj}) and dc-bias level (I_{bias}) numerically using MATHCAD software. Results show that, by changing temperature 50 ^{o}C (i.e. from 10 to 60) with $I_{inj} = 3I_{th}$ and $I_{bias} = 0$, the photons density ($P(t)$) has decreased from 1.636 × 10^{16} cm^{-3} to 0.733 × 10^{16} cm^{-3}, the carrier density ($N(t)$) has increased from 2.367 × 10^{18} cm^{-3} to 2.669 × 10^{18} cm^{-3} and the laser output power (P_{out}) has decreased from the 2.366 mW to the 1.025 mW. In contrast, by increasing the I_{inj} from $3I_{th}$ to $5I_{th}$ and the I_{bias} from 0 to $1.5I_{th}$, the rate of the decreasing in the $P(t)$ and in the P_{out} have reduced more than 25%.

Keywords: Equilibrium point, semiconductor lasers, vertical cavity surface emitting lasers, temperature effect.

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نظرية التخفيف من تأثير درجة الحرارة على نقطة التوازن في ليزرات ألانبعاث السطحي ذات التجويف العمودي

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الملخص

تقدم هذه الورقة طريقة للتخفيف من تأثير تأثيرات درجة الحرارة على نقطة التوازن (Q-point) لليزر الانبعاث السطحي ذات التجويف العمودي (VCSEL) من خلال دراسة تأثير تيار الحقن (I_{inj}) ومستوى التحيز المستمر (I_{bias}) عدديًا باستخدام برنامج ال MATHCAD. تظهر النتائج أنه بتغيير درجة الحرارة 50 درجة مئوية (أي من 10 إلى 60) مع $I_{inj} = 3I_{th}$ و $I_{bias} = 0$ ، فإن كثافة الفوتونات $P(t)$ قد انخفضت من ($1.636\text{cm}^{-3} \times 10^{16}$) إلى ($0.733\text{cm}^{-3} \times 10^{16}$) و قد زادت الكثافة الحاملة $N(t)$ من ($2.367\ \text{cm}^{-3} \times 10^{18}$) إلى ($2.669\text{cm}^{-3} \times 10^{18}$) وانخفضت قدرة الليزر ($P_{out}$) من 2.366 ملي واط إلى 1.025 mW. في المقابل، عند زيادة I_{inj} من $3I_{th}$ إلى $5I_{th}$ ، و I_{bias} من 0 إلى I_{th} فإن معدل التناقص في $P(t)$ وفي P_{out} قد أنخفض ألى أكثر من 25%.

الكلمات الدالة: نقطة التوازن، ليزرات أشباه الموصلات، ليزر الانبعاث السطحي ذات الفجوة العمودية، تأثير درجة الحرارة.

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1. Introduction:

The huge increase in the amount of the data transferred enhances the use of wavelength division multiplexing (WDM) and dense WDM (DWDM) systems [1-3]. This requires laser with a single-longitudinal-mode (SLM) operation, fast response, low chirp and high wavelength stability [4]. However, the change in temperature leads to fluctuate in the lasing frequency, which makes it unsuitable for the WDM and DWDM requirements [5]. This instability is one of the most important limitations that affecting the performance of these systems [6].

Due to their superior features such as SLM operation, very low threshold current, direct laser-to-fiber connection and narrow circular output beam, VCSELs have attracted an exciting attention making them one of the most important and promising sources for high-speed fiber optic communication systems [7-16]. However, and despite all of their advantages, there are still number of unwanted features that make edge-emitting lasers more competitive. One of these, which can limit their characteristics, is the temperature behavior [6-8]. VCSEL is a highly stronger temperature-sensitive device [8,9]. Temperature changing directly affects lasing wavelength, laser threshold current, laser output, system efficiency, and operating lifetime [17-20].

In the last few decades, many theoretical and experiential studies on the characteristics of VCSELs have been reported [9-26]. However, to our best knowledge; there is no study has been reported on the Q-point characteristics. Equilibrium point plays an important role in determining the VCSEL,s performance. Where, any unexpected movement may lead to an increase in the threshold criteria and sometimes may push the laser to work in undesirable areas, which leading it to failure in operation [27-29]. Therefore, mitigating the thermal effect on the Q-point characteristics is an important and indispensable.

2. VCSEL Model Development:

Fig. 1 shows the model of VCSEL under the assumption of a uniform gain structure [8, 9, 27]. Due to the multi-visual feedbacks, the amplifications inside the active region of the laser will generate by providing top and bottom mirrors with Rt and Rb reflectivity, respectively. Then the output light will emit from the layers vertically [8, 9, 27].

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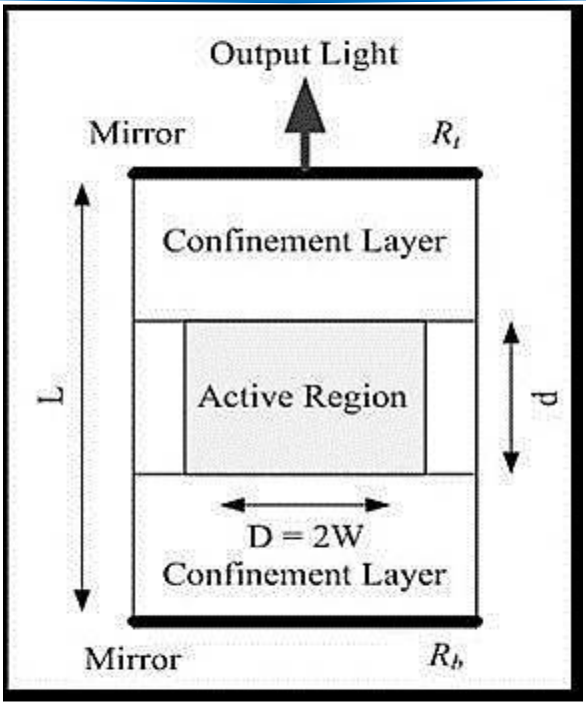

Fig. 1: VCSEL model [9].

The rate equations, Eqs. (1) - (3) are the basic of the temperature VCSEL model analysis presented in this paper. These equations are simple, flexible, and are applicable for both direct and continuous modulated laser operations [27, 30].

$$\frac{dP(t)}{dt} = \Gamma g \frac{N(t)-N_o}{1+\varepsilon P(t)} - \frac{P(t)}{\tau_p} + \frac{\Gamma \beta N(t)}{\tau_n} \quad (1)$$

$$\frac{dN(t)}{dt} = \frac{I(t)}{qV} - g \frac{N(t)-N_o}{1+\varepsilon P(t)} P(t) - \frac{N(t)}{\tau_n} \quad (2)$$

$$P_{out}(t) = \frac{V\eta hv}{2\Gamma\tau_p} P(t) \quad (3)$$

Where, P(t) is the photon density, N(t) is the carrier density, No is the transparency carrier density, I(t) is the laser drive current, q is the electron charge, V = (πdW2) is the active region volume, τ_n is the electron lifetime, g is the gain slop constant, ε is the nonlinear gain coefficient, $\Gamma = \frac{d}{L}$ is the optical confinement factor, τ_p is the photon lifetime, η is the laser quantum efficiency, v is the laser wavelength, and β is the spontaneous emission factor. By considering

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the effect of the temperature (T) variation, the VCSEL threshold current Ith,VCSEL can be rewritten as [9, 27-29].

$$I_{th,VCSEL}(T) = qVN_{t,VCSEL}\Psi(T, N_{t,VCSEL}) \quad (4)$$

Where Ψ(T, Nt,VCSEL) is the carriers recombination rate [9, 28, 29], given as

$$\Psi(T, N_{t,VCSEL}) = A + BN_{t,VCSEL}(T) + CN^2_{t,VCSEL}(T) \quad (5)$$

where A, B , C(T) and $N_{t,VCSEL}$ are the non-radiative recombination, the radiaitive recombination coefficient, the temperature dependence (TD) Auger process and the threshold carrier density, respectively. The TD of the VCSEL model is assumed vary according to [1-6, 9]. The TD photon lifetime τp(T) can be calculated by [1-6, 9]:

$$\tau_p(T) = \frac{1}{v_g(T)\alpha_{T,VCSEL}(T)} \quad (6)$$

Where αT,VCSEL(T) is the TD total cavity loss that is defined as [9]

$$\alpha_{T,VCSEL}(T) = \alpha_{int}(T) + \frac{1}{L}\ln\left(\frac{1}{R}\right) + \Gamma\alpha_d \quad (7)$$

Where αint(T) is the TD internal cavity loss, $((1/L)\ln(1/R))$ is the mirror loss, R =(Rt.Rb)1/2 and α_d is the diffraction loss defined by [9]

$$\alpha_d = -\frac{1}{d}\ln\left(\frac{2}{\left(2+3\left(\frac{2(L-d)}{kW^2}\right)^2\right)+\left(\frac{2(L-d)}{kW^2}\right)^4}\right) \quad (8)$$

Where k (=2πn/λ) is the propagation constant, n is the effective refractive index and λ is the lasing wavelength.

3. Results and Discussion:

Table 1 shows the typical values of the VCSEL parameters that are used in analysis.

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Table 1: Parameters of VCSEL model [8, 9, 28, 29].

Parameters	Description
$N_o = 1\times10^{24}$ m^{-3}	Transparency carrier density
$A = 1\times10^{8}$ sec^{-1}	Non-radiative coefficient
$B = 1\times10^{-16}$ m^3/sec	Radiative coefficient
$C = 3\times10^{-41}$ m^6/sec	Auger coefficient
$\lambda=0.87\ \mu m$	Lasing wavelength
$d = 3\ \mu m$	Active region length
$D = 4\ \mu m$	Active region diameter
$\alpha_{int} = 18.571$ cm^{-1}	Internal cavity loss
$R = 0.99$	Mirror reflectivity
$I_{inj} = 3\ I_{th}$	Injection current

Fig. 2 and 3 show the effect of temperature variation on the VCSEL phase plane (dP/dN) and output power (P_{out}) response characteristics at zero bias (i.e. $I_{bias} = 0$) and $I_{inj} = 3I_{th}$, respectively. Results show a significant effect of temperature on the laser output performance. Where, by varying temperature 50 ^{o}C (i.e. from 10 to 60 ^{o}C), photons density $P(t)$ has decreased from 1.636×10^{16} cm^{-3} to 0.733×10^{16} cm^{-3}. In contrast, carrier density $N(t)$ has increased from 2.367×10^{18} cm^{-3} to 2.669×10^{18} cm^{-3}. This behavior has forced the equilibrium point (Q-point) to move away from its position. This change in the laser performance due to this nonlinear behavior has pushed the P_{out} to decrease from the 2.26 mW to the 1.16 mW as shown in Fig 4. The obtained results can be explained more precisely as: due to the strongly temperature dependence of the laser performance [28, 29], thus with temperature changing, the internal fluctuations will increase gradually which leads to increase the total cavity loss [1-6, 28, 29]. Due to the short laser cavity, the increment in the total loss results in reducing in the photon lifetime [8, 9, 27], and in turn this leads for increasing in the threshold carrier density (N_{th}) [8, 9, 27-29]. And as its known, any increase in the N_{th} (i.e. laser threshold current (I_{th})) at constant I_{inj}; meaning that the laser needs to more time to operate or may be fail to work if I_{th} increases with temperature to a value above from the I_{inj} [9, 28, 29]. The Q-point movement is very important; it may push the laser to operating in improper region leads to increase the total loss [28, 29] or make it in the off mode [9]. It can be seen that the Q- point takes an approximately a spiral path before reaching its steady state. This path represents the behavior of the laser during the transient

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period [28, 29]. This period is very important for laser performance especially for the fast response applications [1-6, 8, 9, 28, 29].

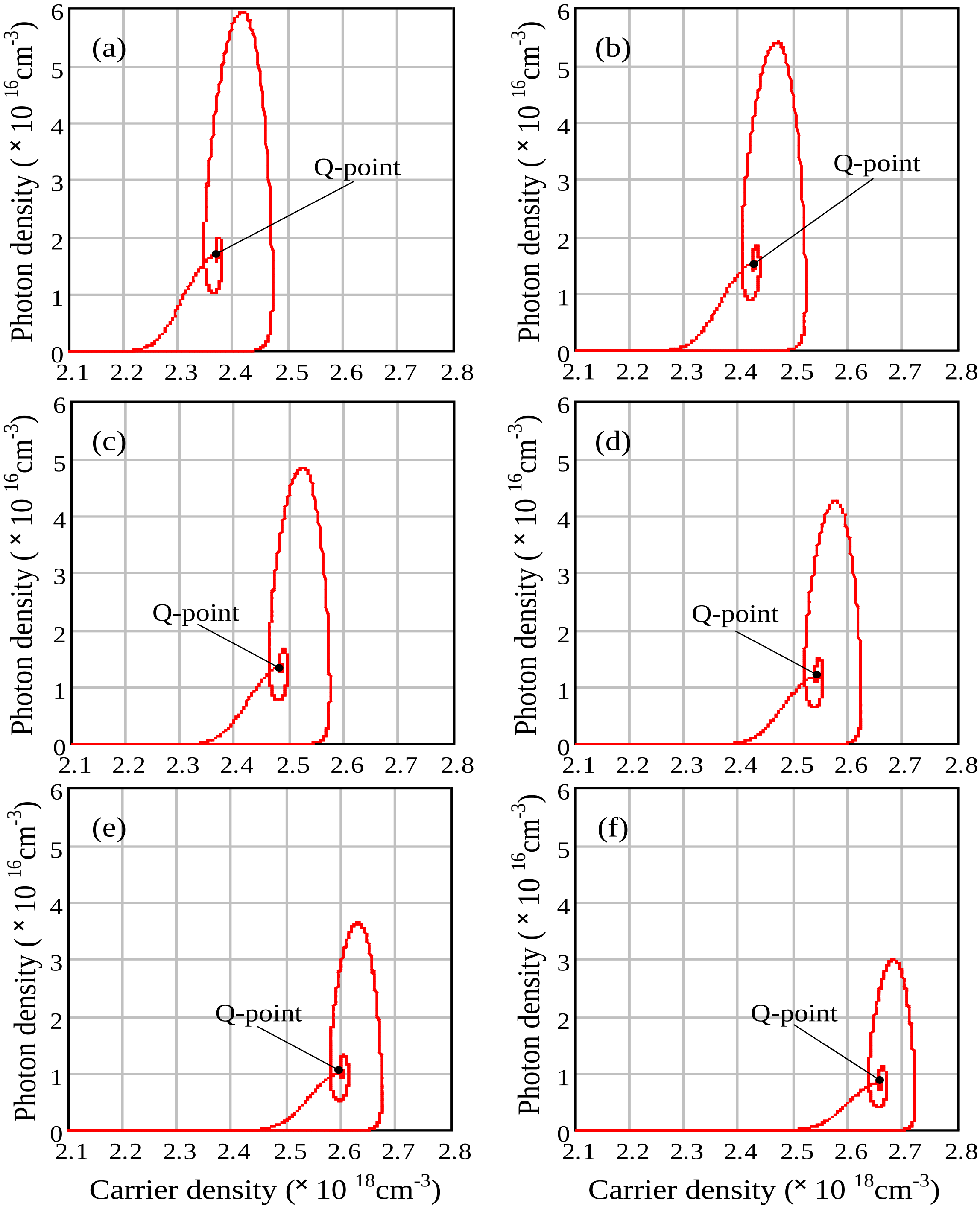

Fig. 2: VCSEL phase plane at I_{bias} = 0 and $I_{inj} = 3I_{th}$: (a) T = 10 °C, (b) T = 20 °C, (c) T = 30 °C, (d) T = 40 °C, (e) T= 50 °C, (f) T = 60 °C.

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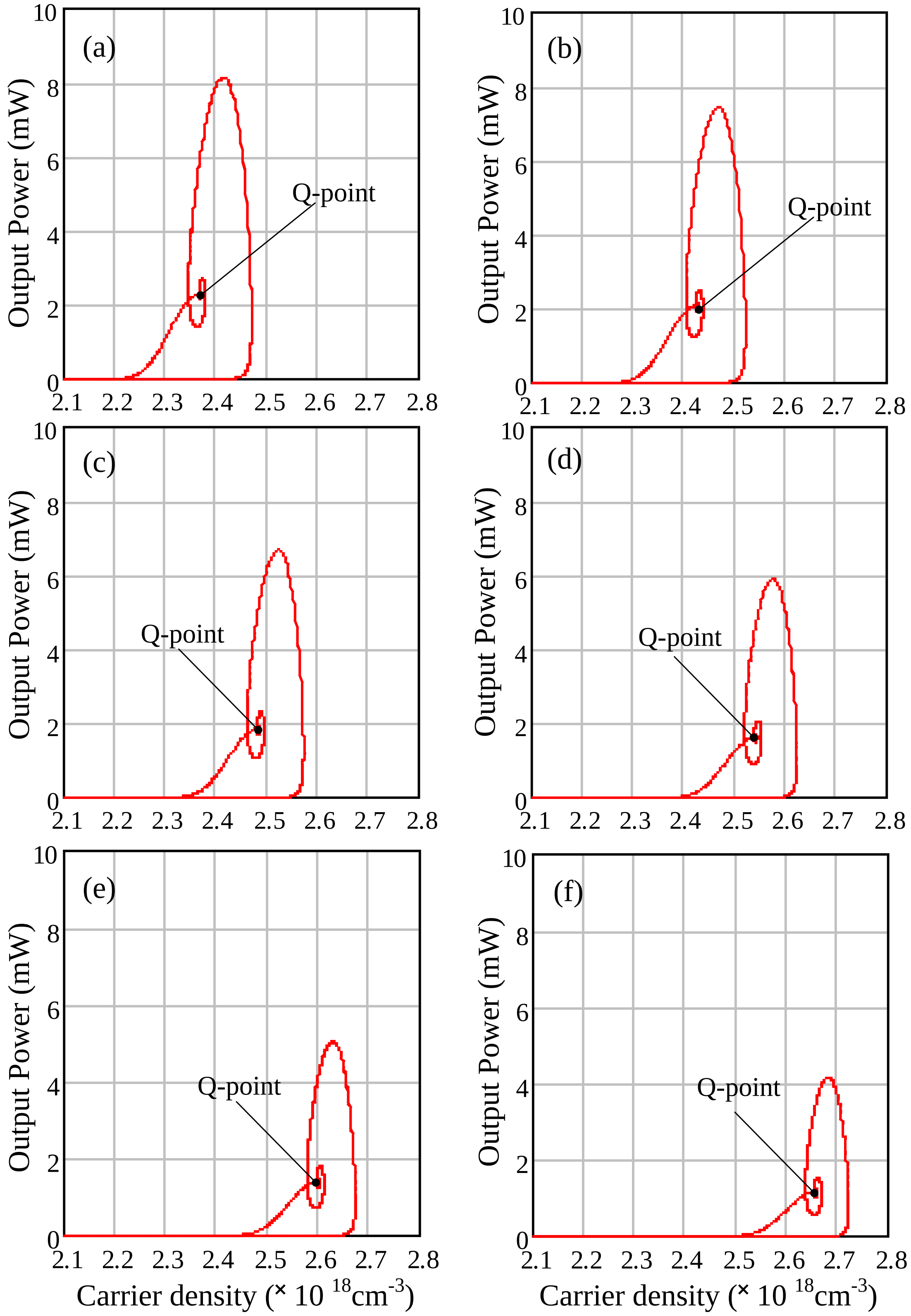

Fig. 3: VCSEL output power vs carrier density at I_{bias} = 0 and $I_{inj} = 3I_{th}$: (a) T = 10 °C, (b) T = 20 °C, (c) T = 30 °C, (d) T = 40 °C, (e) T= 50 °C, (f) T = 60 °C.

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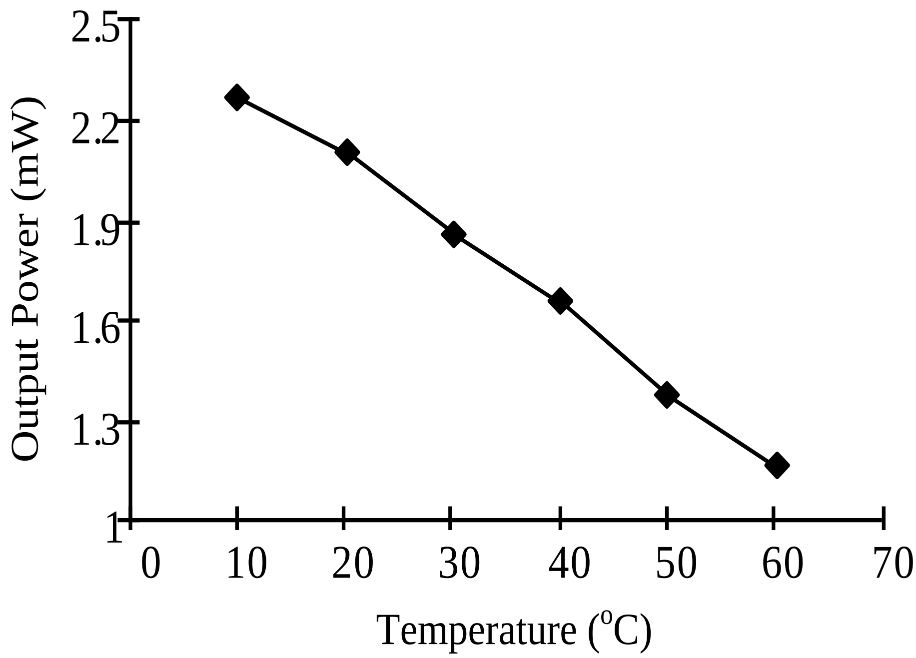

Fig. 4: Effect of temperature variation on VCSEL output power at I_{bias} = 0 and I_{inj} = $3I_{th}$:

Fig. 5 , 6 show the effect of temperature variation on the VCSEL phase plane (dP/dN) and output power (P_{out}) response characteristics at $I_{bias} = 1.5I_{th}$ and $I_{inj} = 3I_{th}$, respectively. We can see that, the operating with input biasing level has affected positively on the laser performance. By comparing the results have obtained in Fig. 5 (a) with that have given in Fig. 2 (a) it can be seen that $P(t)$ has increased more than the doubled at the same temperature operation and by changing temperature 50 ^{o}C, the rate of the decreasing in the $P(t)$ and in the P_{out} (as shown in Fig. 7) has reduced approximately more than by 25% compared to what is given in Fig. 4. In contrast, the rate of the increase in the $N(t)$ is not the most influential. This behavior can be explained as: increasing the basing level leads to increase the photons number inside the active region [28, 29], which leading to a reduction in the laser threshold level and thus hastens from the lasing emitting, thus increases P_{out} [28, 29].

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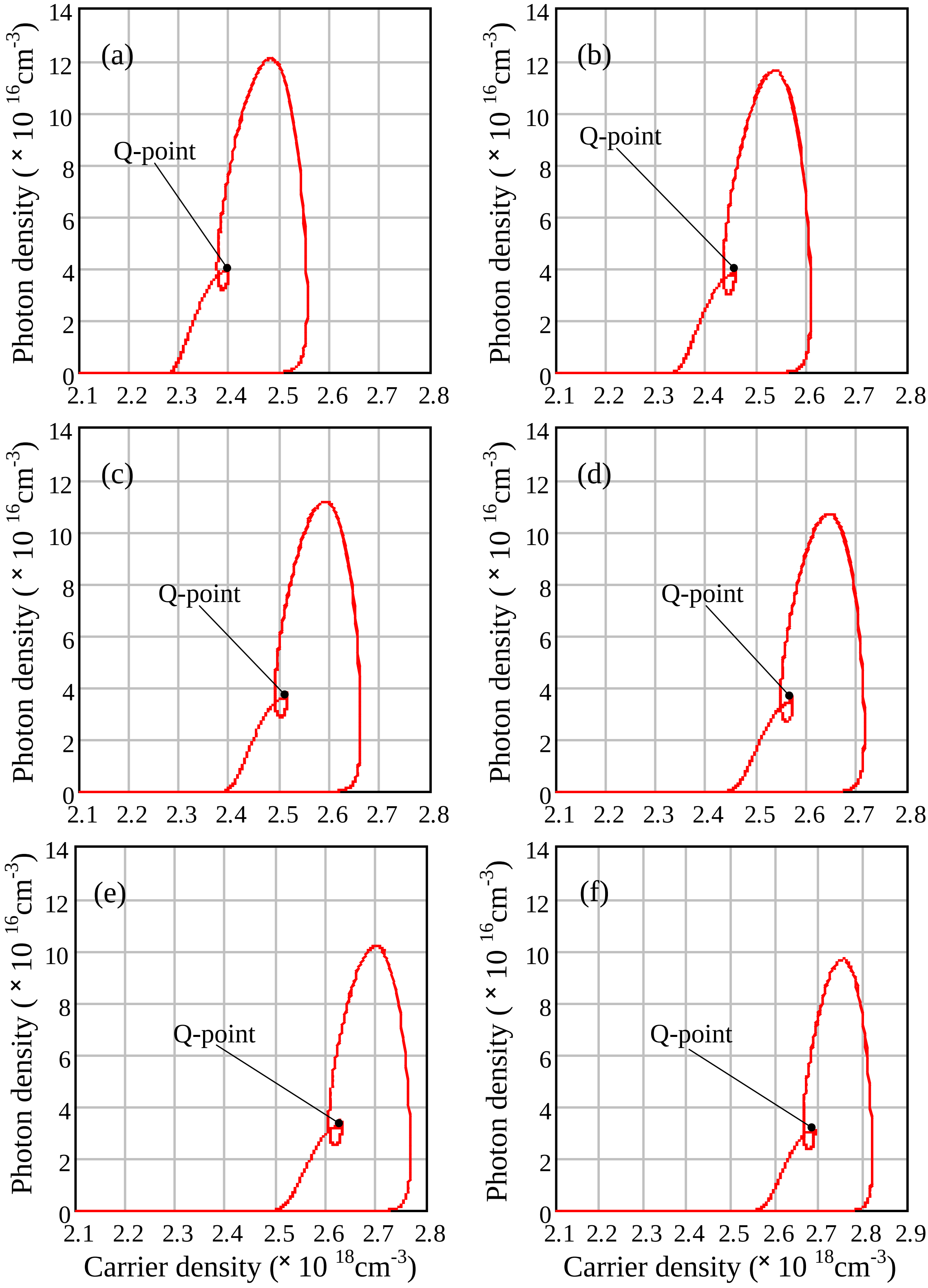

Fig. 5: VCSEL phase plane at $I_{bias} = 1.5I_{th}$ and $I_{inj} = 3I_{th}$: (a) $T = 10$ °C, (b) $T = 20$ °C, (c) $T = 30$ °C, (d) $T = 40$ °C, (e) $T= 50$ °C, (f) $T = 60$ °C.

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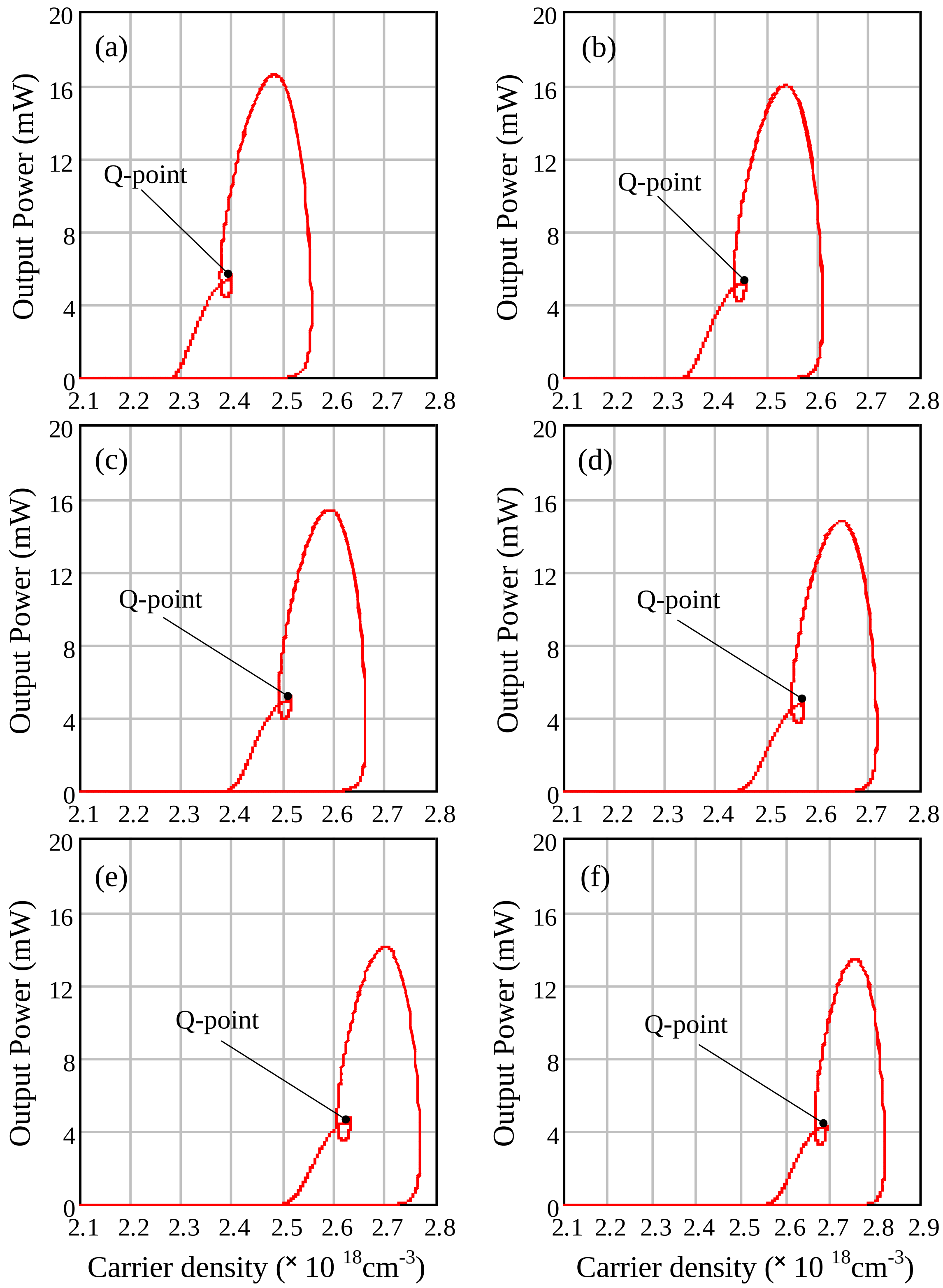

Fig. 6: VCSEL output power vs carrier density at $I_{bias} = 1.5I_{th}$ and $I_{inj} = 3I_{th}$: (a) T = 10 °C, (b) T = 20 °C, (c) T = 30 °C, (d) T = 40 °C, (e) T= 50 °C, (f) T = 60 °C.

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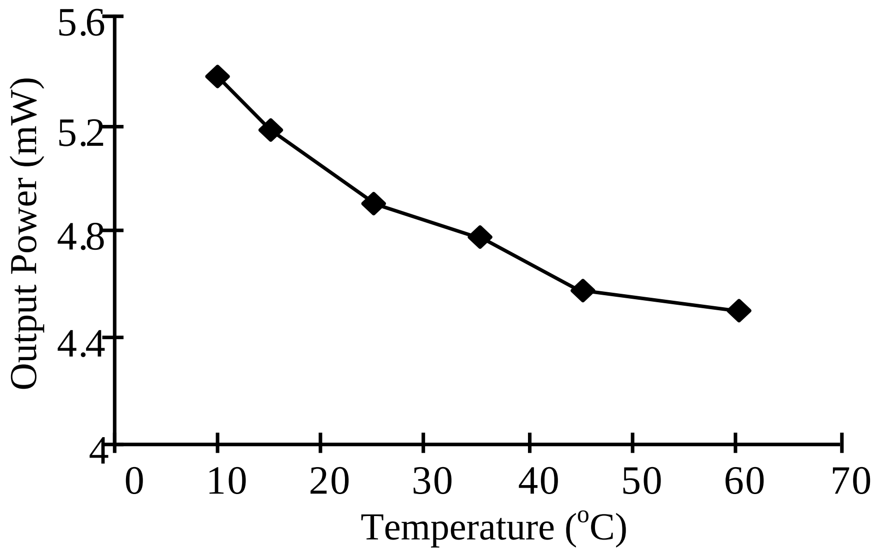

Fig. 7: Effect of temperature variation on VCSEL output power at $I_{bias} = 1.5I_{th}$ and $I_{inj} = 3I_{th}$

Fig. 8, 9 show the effect of temperature variation on the VCSEL phase plane (dP/dN) characteristics at $I_{bias} = 0$ and for $I_{inj} = 4I_{th}$ and $I_{inj} = 5I_{th}$ respectively. Results show, for the same range in temperature change, the moving in the Q-point is less than that given in the Fig. 2. More increasing in the I_{inj} value leads to an overall improvement in the behavior. Where, $P(t)$ has decreased from 2.738×10^{16} cm^{-1} to the 2.065×10^{16} cm^{-1}, this means that the decreasing range in the $P(t)$ has reduced more than 25% compare with what is in the Fig. 2, and this is reflected significantly on system performance as shown in Fig. 10. Where, the decreasing rate in the P_{out} has reduced from 1.51 mW to the 0.97 mW with increasing the I_{inj} value from $4I_{th}$ to $5I_{th}$.

This effect can be summarized as: the change in temperature leads to an increase in total system losses [1-6, 9 28, 29] resulting in a reduction in the photon density value and thus reduction in P_{out} [28, 29]. In contrast, the increase in the I_{inj} value leads to an excessive increases in the photons concentration inside the active region and thus to the strengthening of constructive interactions between the carriers [28, 29], leading to a fold effect of the change in temperature. This effect may not be clearly observed from the first glance, but when looking at the results in Figs. 8 and 9 from (a) to (f) and comparing them with what is shown in Fig. 2, the effect is evident. These results are very important, and they can be considered as a guideline for this type of laser designers to avoid undesirable work areas.

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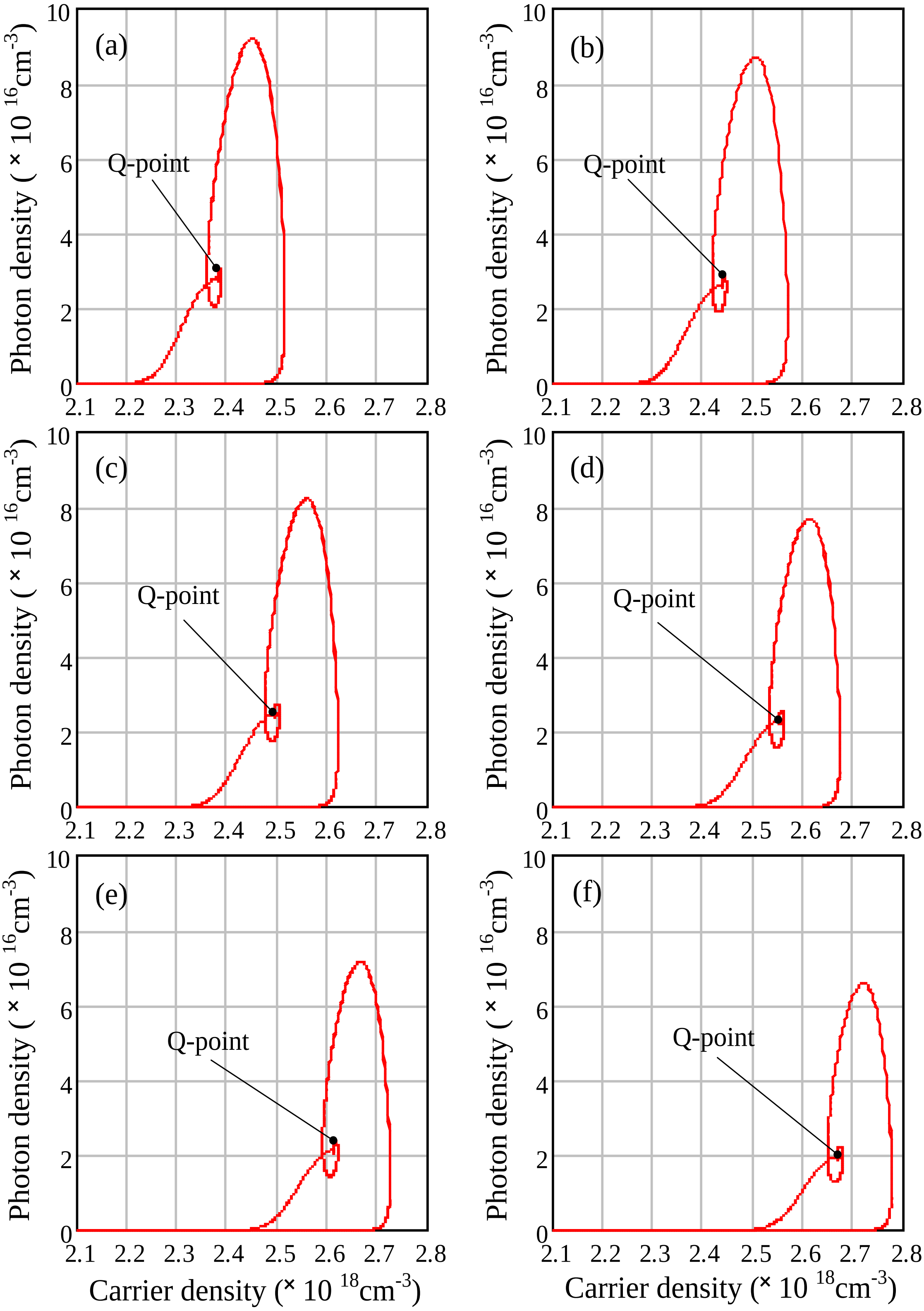

Fig. 8: VCSEL phase plane at I_{bias} = 0 and I_{inj} = 4I_{th}: (a) T = 10 °C, (b) T = 20 °C, (c) T = 30 °C, (d) T = 40 °C, (e) T= 50 °C, (f) T = 60 °C.

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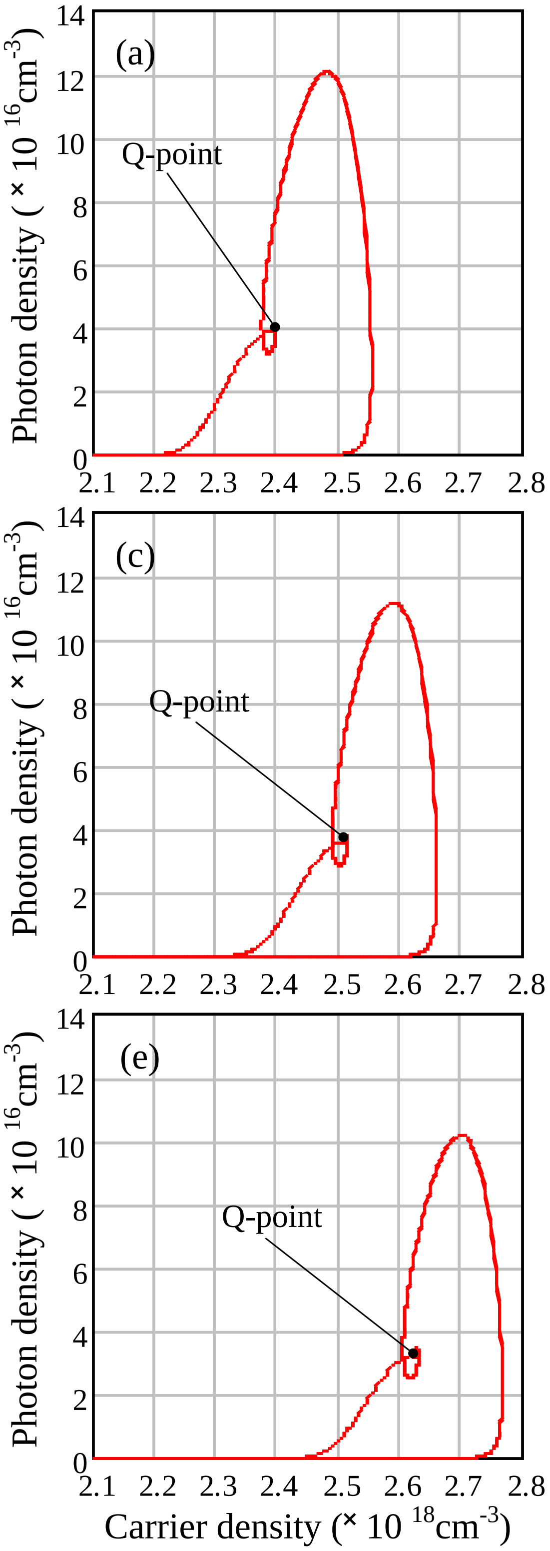

Fig. 9: VCSEL phase plane at $I_{bias} = 0$ and $I_{inj} = 5I_{th}$: (a) T = 10 °C, (b) T = 20 °C, (c) T = 30 °C, (d) T = 40 °C, (e) T= 50 °C, (f) T = 60 °C.

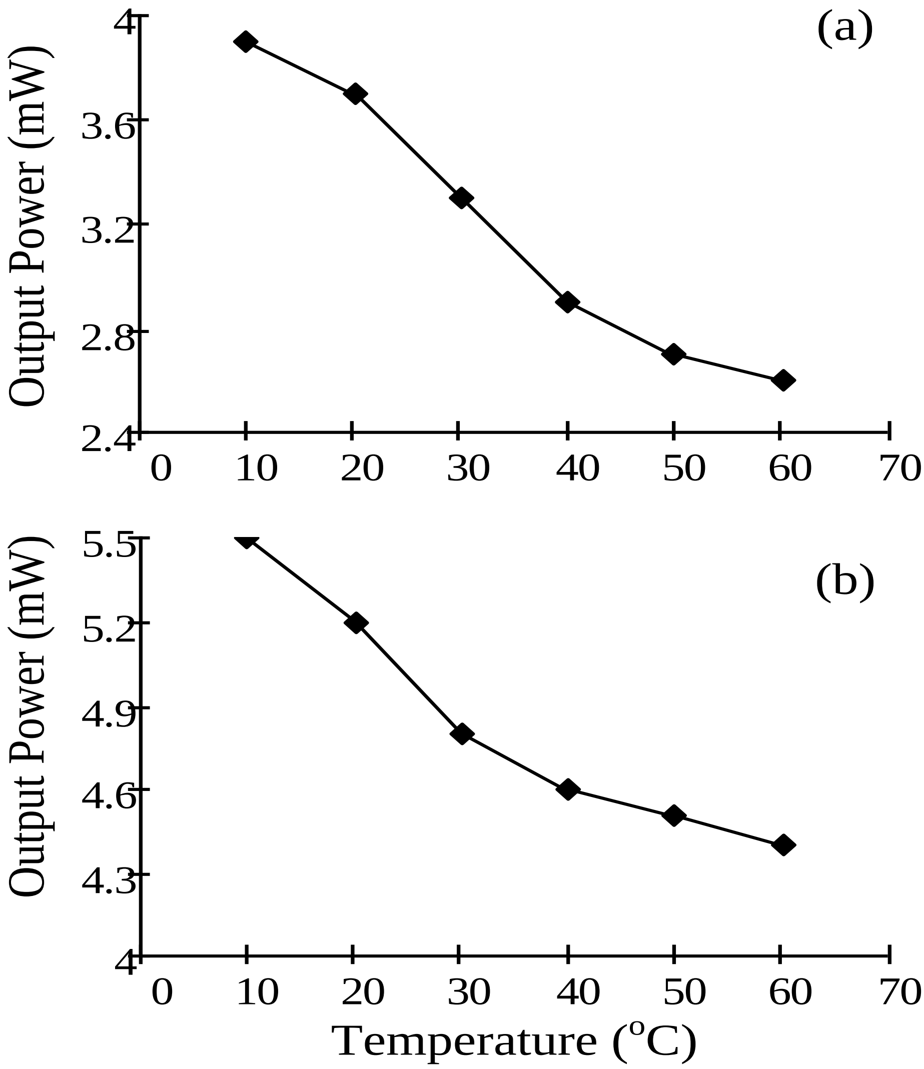

Fig. 10: Effect of temperature variation on VCSEL output power at I_{bias} = 0: (a) $I_{inj} = 4I_{th}$ and (b) $I_{inj} = 5I_{th}$

4. Conclusion:

A numerical analysis on mitigating the effect of temperature (T) variation on the Q-point characteristics of a VCSEL is conducted successfully by considering the effect of injection current (I_{inj}) and dc-bias level (I_{bias}). Results show that, Q-point response is affected by T significantly, where at high T variation; photon density and laser output power are reduced. However, the decreasing rate in the output power with T can be mitigating significantly by increasing the I_{inj} and/or the I_{bias}. The obtained results can be used as a guideline for designing and operating the VCSELs in high speed optical networks.

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