Techniques of Finding Lower Bounds in Multi Objective Functions

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الملخص

تناولنا في هذا البحث جدولة n من النتاجات على ماكنة واحدة مع دالة الهدف المركبة .مسالتان تمت دراستهما وهما مجموع الإتمام الكلي مضافا أليها اكبر تأخير($\sum_{i \in N} c_i + T_{max}$)ومجموع الإتمام الكلي مع اكبر تأخير($\sum_{i \in N} c_i \;\; and \; T_{max}$).

المسالة الأولى لها حل امثل عن طريق تقنية التفرع والتقيد أما المسالة الثانية فلها حلول كفوءة وجدت بطريقة خوارزمية فان وازنهوف .

قدمت نظرية تبين العلاقة بين الحلول الكفوءة , القيد الأدنى والحل الأمثل . هذه النظرية تحدد مدى القيد الأدنى الذي يعتبر العامل الرئيسي لإيجاد الحل الأمثل , كذلك تقدم مفاهيم و عمليات جبرية لإيجاد قيود دنيا جديدة.

ABSTRACT

In this paper, the problm of sequencing n jobs on one machine is considered with a multi objective function.Two problems have been studied, sum of completion times added with the maximum tardiness ($\sum_{i \in N} c_i + T_{max}$) and sum of completion times with the maximum tardiness ($\sum_{i \in N} c_i \;\; and T_{max}$), the first one has optimal solution solved by Branch and bound technique, the second has efficient solutions founded by Van Wassenhove algorithm.A theorem is presented to show a relation between the number of efficient solutions, lower bound (LB) and optimal solution.This theorem restricts the range of the lower bound, which is the main factor to find the optimal solution.Also the theorem opens algebraic operations and concepts to find new lower bounds.

Keywords: Lower Bound,Multi Objective,Efficient Solution function , Optimal Value.

1.Introduction :

Although there are a lot of published results on single machine problems with tardiness (T_i), there are only some papers dealing with multi objective function[Lauff and Werner , 2004]. The problem class considered is as follows :

n jobs 1,2,3,…,n have to be processed on a single machine (m=1) and become available at time zero , require a positive processing time P_i [Potts,1991].For each job i ,a processing time P_i ,a due date d_i , are specified .Given a schedule,we can compute for each job i the completion time c_i , the tardiness T_i = max{ $c_i - d_i$, 0 } and

$T_{\max}$ =max { T_i }.Many sequencing problems have a combinatorial nature and they are very difficult to solve to optimality within acceptable computation time. We consider a multi objective function which is the sum of completion time($\sum_{i \in N} C_i$) and the maximum

tardiness ($T_{\max}$) [Abdul-Razaq,2001].

2.Notations and Definitions :

N=the set {1,2,3,…,n}.

P_i =processing time for job i .

d_i =Due date for job i.

c_i =Completion time for job i.

L_i =Lateness of job i.

T_i =Tardiness of job i.

EDD- rule: (Early due date) meaning the jobs are sequenced in non-decreasing order

of d_i

SPT-rule: (Short processing time) meaning the jobs are sequenced in non-decreasing order of p_i .

LB: (Lower bound) is a value of objective function, which is less than or equal to optimal value.

UB: (Upper bound) is a value of objective function, which is greater than or equal to optimal value.

Example:

i	1	2	3
P_i	3	5	4
d_i	9	8	2

For this schedule (1,2,3) we find c_i and T_{max} as follows :
$c_1 = p_1$, $c_2 = c_1 + p_2$, $c_3 = c_2 + p_3$ and $T_i = \max\{c_i - d_i , 0\}$.

i	1	2	3
P_i	3	5	4
d_i	9	8	2
c_i	3	8	12
T_i	0	0	10

Therefore $\sum_{i=1}^{3} c_i = 23$ and $T_{\max} = 10$.

3.Van Wassenhove Algorithm

In 1978 Van Wassenhove and Gelders[Van Wassenhove and Gelders,1980] present an algorithm to find all efficient solutions for the problem

$$\sum_{i\in N} c_i \ \ and\, T_{\max} \quad \text{……….} \ (1)$$

The Algorithm :

Step(0) : Put $\Delta = \sum_{i\in N} p_i$

Step(1) : Let $D_i = d_i + \Delta$ for all i.

Step(2) : Solve using modified smith rule , if a solution exists then it is efficient.Else,go to step (4).

Step(3) : Compute $T_{\max}$ .Put $\Delta = T_{\max} - 1$, go to step(1).

Step(4) : Stop.

The algorithm finds only the efficient solutions for (1). After that several attempts were done to solve this problm [Ramadhan and Abdul-Razaq , 2001].In 1993 using branch and bound technique, the problm solved up to 30-jobs [Abdul-Razaq,1993].This technique used upper bound (UB) and lower bound (LB), where

UB= $\sum_{i\in N} c_i(SPT) + T_{\max}\ (SPT)$ and LB= $\sum_{i\in N} c_i(SPT)\ + T_{\max}\ (EDD)$.

4.Relation Between Optimal and Efficient Solutions :

We know that a lower bound is less than the optimal solution.The question is: " What is the difference between lower bound and the optimal solution ?"of course , this depends on the lower bound and the objective function , our objective function is ($\sum_{i \in N} c_i + T_{\max}$) and the lower bound is given as LB $= \sum_{i \in N} c_i (SPT) + T_{\max} (EDD)$. The relation between the optimal value , LB and efficient solutions is given in the following theorem .

Theorem :

There exists a non-negative integer M such that LB + M = 0ptimal value and
M ε [N_1-1,N_2+1] , where :
N_1=number of efficient solutions .
$N_2 = T_{\max}(SPT) - T_{\max}(EDD)$.

Proof :

Since $LB \leq$ optimal value , so there exists a non-negative integer M such that
LB + M = optimal value which proves the first part of the theorem.It remains to show that M ε [N_1-1,N_2+1] or to show $N_1 - 1 \leq M \leq N_2 + 1$.
Now LB + M = optimal value,thus M = optimal value – LB ≤ UB-LB $= \sum_{i \in N} c_i (SPT) + T_{\max}(SPT) - \sum_{i \in N} c_i (SPT) - T_{\max} (EDD) =$
$T_{\max}(SPT) - T_{\max}(EDD) = N_2 \leq N_2 + 1$.

Hence M ≤ N_2 + 1.We will prove N_1-1 ≤ M by induction on N_1.

If N_1 = 1,that is there is only one efficient solution which is SPT as well as EDD then
M=0ptimal value –LB = $\sum_{i \in N} c_i (opt.) + T_{\max}(opt.) - \sum_{i \in N} c_i (SPT) -$
$T_{\max}(EDD) = \sum_{i \in N} c_i (SPT) + T_{\max}(SPT) - \sum_{i \in N} c_i (SPT) - T_{\max}(EDD) = 0$.
Thus N_1-1 ≤ M ≤ N_2 + 1.

That is M ε [N_1-1,N_2+1], and so the theorem is true for N_1 = 1.

If N_1 = 2, i.e, the number of efficient solutions is two which are SPT and σ , say. N_1=2 implies that N_1-1=1, if SPT is optimal then
M = $\sum_{i \in N} c_i (opt.) + T_{\max}(opt.) - \sum_{i \in N} c_i (SPT) - T_{\max}(EDD)$
$= \sum_{i \in N} c_i (SPT) + T_{\max}(SPT) - \sum_{i \in N} c_i (SPT) - T_{\max}(EDD) \geq 1 = N_1 - 1$.

Hence N_1-1 $\leq M \leq N_2$+1.

And now if σ is optimal then

M = $\sum_{i\in N} c_i(\sigma) + T_{\max}(\sigma) - \sum_{i\in N} c_i(SPT) - T_{\max}(EDD)$ =

$\sum_{i\in N} c_i(\sigma) + \sum_{i\in N} c_i(SPT)$ $\geq$ 1=N_1-1,thus again N_1-1$\leq$ M$\leq$ N_2+1 , and so M ε [N_1-1 , N_2+1] and hence the theorem is true for N_1 = 2.

If N_1 = 3, i.e.,there are three efficient solutions SPT, σ and σ_1, say . N_1 = 3→N_1-1 =2, if SPT is optimal , then

M =$\sum_{i\in N} c_i(SPT)$ + $T_{\max}(SPT) - \sum_{i\in N} c_i(SPT) - T_{\max}(EDD)$ =

$T_{\max}(SPT) - T_{\max}(EDD) \geq 2$ = N_1-1.

Hence N_1-1 $\leq$ M $\leq$ N_2+1 or M ε [N_1-1,N_2+1].

If σ is optimal , then

M=$\sum_{i\in N} c_i(\sigma) + T_{\max}(\sigma) - \sum_{i\in N} c_i(SPT) - T_{\max}(EDD) = \sum_{i\in N} c_i(\sigma) -$
$\sum_{i\in N} c_i(SPT) + T_{\max}(\sigma) - T_{\max}(EDD) \geq$ 1+1 = 2 = N_1-1.

Hence N_1-1 $\leq M \leq N_2$+1 or M ε [N_1-1,N_2+1]. Finally if σ_1 is optimal, then

M = $\sum_{i\in N} c_i(\sigma_1) + T_{\max}(\sigma_1) - \sum_{i\in N} c_i(SPT) - T_{\max}(EDD)$ =

$\sum_{i\in N} c_i(\sigma_1) - \sum_{i\in N} c_i(SPT) \geq 2 = N_1 - 1$.Hence N_1-1$\leq M \leq N_2$+1 or M ε [N_1-1,N_2+1].Thus the theorem is true for N_1 = 3.

Suppose the theorem is true for N_1 = k , i.e., the theorem is true for the k efficient solutions SPT, σ , σ_1 , …, σ_{k-2} ,that is for these k efficient solutions N_1-1$\leq M \leq N_2$+1.

Let N_1 = k+1,that is , there is k+1 efficient solutions SPT , σ , σ_1,…, σ_{k-2}, σ_{k-1}, if any one of the first k efficient solutions SPT , σ , σ_1,…., σ_{k-2} , is optimal then since the theorem is true for N_1=k , we get N_1-1 $\leq$ M , and hence N_1-1$\leq M \leq N_2$+1 and if the last efficient solution σ_{k-1} is optimal , then

M =$\sum_{i\in N} c_i(\sigma_{k-1}) + T_{\max}(\sigma_{k-1}) - \sum_{i\in N} c_i(SPT) - T_{\max}(EDD)$=

$\sum_{i\in N} c_i(\sigma_{k-1}) - \sum_{i\in N} c_i(SPT) \geq$ k = k+1-1 = N_1-1 ,thus N_1-1 $\leq M \leq N_2$+1 or M ε [N_1-1,N_2+1].

Thus the theorem is true for N_1 = k+1 which completes the proof.

Example

i	1	2	3	4
P_i	2	4	3	1
d_i	1	2	4	6

Using Van Wassenhove algorithm for this example we find three efficient solutions,i.e., $N_1 = 3$. $T_{\max}(SPT) = 8$, $T_{\max}(EDD) = 5$, and then $N_2 = T_{\max}(SPT) - T_{\max}(EDD) = 3$.

Thus $[N_1-1,N_2+1] = [2,4]$. $\sum_{i=1}^{4} C_i(SPT) = 20$, LB = 20 +5 = 25, optimal value = 27. Therefore M = optimal value – LB = 27 – 25 = 2 , and clearly 2 ε [2,4].

5.Conclutions and Suggestions

At the end of this paper , we conclude that the lower bound of a problem is one of the important factors to understand the nature of objective function and the method which is used to solve the problem .Also the efficient solutions used to find optimal solution ,but in our objective function , the relation between them will lead to a new area of study , that is the difference between optimal value and lower bound with the help of efficient solutions . This study opens algebraic operations and concepts to solve any problem of this type.

Lastly , using the new lower bound of this objective function certainly leads to other results.

REFERENCES

[1] Abdul-Razaq, T.S. (2001) “Local search heuristic for single machine scheduling with batching to minimize the multiple objective function”,**J.of Al-Mustansiriya** Vol. 1.

[2] Abdul-Razaq, T. S. (1993) “Near optimal solution of scheduling problems using heuristic methods” , **J. of Basrah university** No.(437), pp.87-104.

[3] Lauff, V.and F. Werner, (2004) “Scheduling with common due date, earliness and tardiness penalties for multi-machine problems :a survey” , **Compu.Oper**.31,pp.317-345.

[4] Potts , C.N. (1991) “Scheduling two jobs classes on a single machine”, **Oper.Res**. 18, pp.411-415.

[5] Ramadhan, A.M. and T.S. Abdul-Razaq, (2001) “A new algorithm for optimality” , (KAJ)1A , No. 1 , 75-78.

[6] Van Wassenhove , L.N. and Gelders , L.F. (1980) “Solving a bicriterion-scheduling problem” , **European J. Oper.Res**. Vol.4 , pp.42-48.