Digital Image Watermarking using Singular Value Decomposition

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ABSTRACT

This paper presents a digital image watermarking that applied theory of linear algebra called "Singular Value Decomposition (SVD)" to digital image watermarking .SVD watermarking scheme, which successfully embeds watermarks into images imperceptible way .SVD method can transform matrix A into product *USV* .

Watermarking, is the process of embedding data into a multimedia cover , and can be used primarily for copyright protection and other purposes. Schemes that have recently been proposed modify the pixel values or Transform domain coefficients. The Singular Value Decomposition (SVD) is a practical numerical tool with applications in a number of signal processing fields including image compression. In an SVD-based watermarking scheme, the singular values of the cover image are modified to embed the watermark data. This method has been proposed an optimal SVD-based watermarking scheme that embeds the watermark in two steps. In the first step, the cover image is divided into smaller blocks and a piece of the watermark is embedded in each block. In the second step extracting the watermark from the watermarked image.

All tests and experiments are carried out using MATLAB as computing environment and programming language.

Keywords: Watermarking, Singular Value Decomposition, Digital Watermarking, Image Processing.

العلامة المائية الصورية الرقمية باستخدام تحليل القيمة المفردة

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الملخص

يقدم هذا البحث تضمين العلامة المائية الصورية الرقمية باستخدام نظرية الجبر الخطي المسماة تحليل القيمة المفردة SVD. أن نموذج تضمين العلامة المائية SVD يضمن بنجاح وبشكل غير مرئي تضمين العلامة المائية في الصور. أن طريقة SVD بإمكانها تحويل المصفوفة الى ثلاثة مصفوفات هي USV. أن تضمين العلامة المائية هو عملية دمج البيانات في عنصر من عناصر الوسائط المتعددة والتي تستخدم بشكل اساسي لحماية حقوق النشر والتأليف واغراض أخرى. أن النماذج المقترحة حاليا" تحوير قيم البيكسل أو معاملات التحويل . تحليل القيمة المفردة SVD هي عبارة عن التحليل العددي المستخدم في العديد من مجالات معالجة الاشارة بضمنها كبس الصور . في نموذج العلامة المائية المبني على اساس استخدام تحليل القيمة المفردة يتم تحوير القيم المفردة للصورة المغطاة لدمج بيانات العلامة المائية .هذه الطريقة تقترح نموذج أمثل من تضمين العلامة المائية المبني على اساس SVD والذي يقوم بدمج العلامة المائية في خطوتين في الخطوة الاولى يتم تقسيم الصورة المغطاة الى أجزاء صغيرة وتضمين بيانات العلامة المائية في كل جزء من هذه الصورة اما في الخطوة الثانية فيتم أستخلاص بيانات العلامة المائية من الصورة الرقمية المتضمنة للعلامة المائية. تم تنفيذ جميع الاختبارات باستخدام لغة MATALB .

الكلمات المفتاحية: العلامات المائية ، تحليل القيمة المفردة ، علامات مائية رقمية ، معالجة الصور.

1. Introduction

In recent days, usage of computer networks for communication and for information sharing leads to increase in size of Internet. As the size of the Internet grows, the volume of multimedia data (images, text, video / audio) floating around also increases day by day. As many advanced tools are readily available to duplicate and modify those data in the Internet easily, Security is the major concern, which requires some mechanisms to protect digital multimedia data .Thus watermarking is a technique which supports with feasible solution.

Image processing is computer imaging where the application involves a human being in the visual loop .in other words, the image are to be examined and acted upon ,An image can be defined as a two dimension function f (x, y) (2D image), where x and y are spatial coordinates, and the amplitude off at any pair of (x, y) is the gray level of the image at that point. For example, a grey level image can be represented as:

$$f_{ij} \equiv f(x_i, y_j)$$

Where x, y and the amplitude value of *f* are finite, discrete quantities, the image is called “a digital image”.[1][2]

Watermarking (data hiding) is the process of embedding data into a multimedia element such as image, audio or video. This embedded data can later be extracted from, or detected in, the multimedia for several purposes including copyright protection, access control and broadcast monitoring.

Digital Watermarking is defined as the process of hiding a piece of digital data in the cover data which is to be protected and extracted later for ownership verification .[3][4].A digital watermarking can be visible or invisible .A visible watermark typically consists of a conspicuously visible message or a company logo indicating the ownership of the image .On the other hand a invisible watermarked image appears very similar to the original .The existence of an invisible watermark can only be determined using an appropriate watermark extraction or detection algorithm.[5].A watermarking algorithm consists of the watermark structure, an embedding algorithm and an extraction, or a detection algorithm. Watermarks can be embedded in the pixel domain in the transform domain such as the DCT or wavelet.[6] In most multimedia applications, three desired attributes for a watermarking scheme are invisibility, robustness and high capacity. Invisibility refers to the degree of distortion introduced by the watermark and its effect on the viewers or listeners. Robustness is the resistance of an embedded watermark against intentional attacks, and normal audio/visual processes such as noise, filtering, and resembling, scaling, rotation, cropping and lossycompression.The watermark can be easily erased by lossy image compression. [8][9]

Singular value decomposition is a numerical technique used to diagonalize matrices in numerical analysis. It is an algorithm developed for a variety of applications. Any matrix ‘M’ is decomposed into three sub matrices [u, s, v]such that: $M=u \times s \times v^T$ Where ‘u’ and ‘v’ are the orthogonal matrices $u \times u^T = I$ and $v \times v^T = I$ where ‘I’ is the Identity matrix and ‘s’ is the diagonal matrix$(s_1,s_2, s_3 \ldots s_N)$ where $s_1>=s_2>=s_3...s_{(N-1)}>=s_N$. [10]

These values are known as singular values, and matrices u and v are known as corresponding singular vectors [11]. The above decomposition is termed as Singular Value Decomposition.

A SVD, applied to the image matrix, provides singular values (diagonal matrix's') that represent the luminance or color intensity of the image while the matrices 'u' and 'v' represents the geometry of the image. It has been scientifically proved that slight variation in the singular values doesn't change the visual perception of the image. [10]

There are many researches about watermarking, Like in 2003 a research for an optimal watermarking scheme are built based on singular value decomposition [4]. In 2006, A nonnegative matrix factorization scheme are used for digital image watermarking [7]. In 2009, digital image watermarking built using complex wavelet transform [14]. In this paper he implements the SVD . so that the hidden digital image watermarks can be removed and high image quality can be provided in restored images.

Matlab is used as a platform of programming and experiments in this project, since MATLAB is a high-performance in integrating computation, visualization and programming.

The rest of this paper is organized as follows. In Section 2, we describe theory of singular value decomposition SVD. Several experimental results are illustrated and discussed in Section 3. Finally, conclusion and future work are stated in Section 5.

2. Theory of Singular Value Decomposition

2.1 Singular Value Decomposition

Singular value decomposition (SVD) is a linear algebra technique that decomposes a given matrix into three component matrices [15]: (1) the left singular vectors; (2) a set of singular values; and (3) and right singular vectors. The two matrices that are made up of singular vectors provide information about the structure of the original matrix. The singular Values describe the strength of the given components of the original matrix. The SVD theorem [13] states that given a matrix M, then there exists a decomposition of M such that $A=USV$ see the figure 1 .

The SVD of a matrix can also be described geometrically. The SVD shows that the values of any matrix A can be reconstructed by a rotation (*U*), followed by increasing the matrix values (*S*), followed by another rotation (*V*) [16]

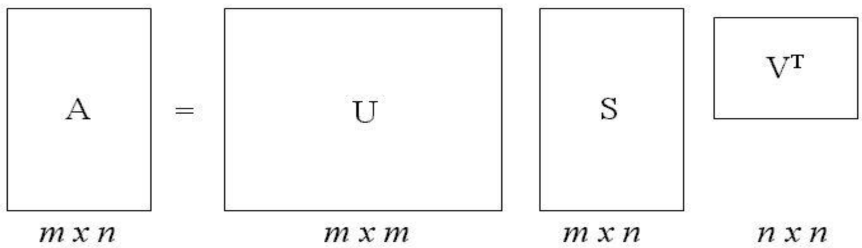


Figure1: llustration of Factoring *A* to *USV*.[19]

For example, if A represented coordinates that generated a three-dimensional shape, then that shape could be constructed from the rotational information in *U* and *V*, along with stretching the shape out to its proper size with the information in *S* [16]. This type of decomposition can be important and useful in that the rotational matrices isolate the key components of the original matrix, finding relationships between the various data points, while the strength matrix indicates which of the key components illuminated in the rotational matrices are the most important [15, 16,17]. In this research, the core idea of isolating key components of the original matrix is the basis for using the SVD. When the matrix is comprised of change records, fault information, or some other data from the development process, these key components highlight underlying structures in the code base. The SVD has other uses in computing. For instance, this technique can be used in Image and signal compression. A gray scale image could be represented as a two dimensional matrix made up of intensity values, indicating the darkness of a particular pixel. In this instance, we could treat the image matrix itself as the original *M* matrix and perform a SVD on it. Once the decomposition is completed, the resulting matrices, *USV*, can also be represented as the sum of component matrices, as shown in Equation

$$M = U_1 S_{1.1} V_1 + U_2 S_{2.2} V_2 + \ldots\ldots + U_K S_{K.K} V_K \qquad \ldots(1)$$

Where *k* is less than or equal to the rank of matrix *M*. This provides a rank-*k* approximation of the matrix. The first component of this factorization indicates the component that has the largest singular value and contributes the most variability to the overall matrix. As subsequent components are added together, the matrix, and thus the image itself, begins to re-take its original form. [15][17].

2.2 Singular Value Decomposition watermarking

SVD watermarking is designed to work on binary .for an image of $N \times N$ pixels and a binary watermark of $p \times p$ pixels , divided the image into $(N/4) \times (N/4)$ non overlapping blocks whose size is 4×4 pixels [20].Which is based to decide the positions of embedded blocks for each watermark bit.The watermarking embedding procedure show in this figure [19].

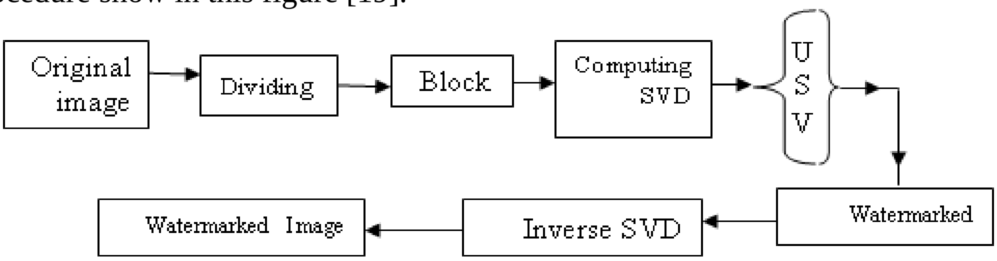


Figure 2: Watermark embedding procedure.

Using SVD on different blocks of the image .for each generated Sj matrix of each block Bj , where $1 \leq j \leq \left(\frac{N}{4}\right) \times \left(\frac{N}{4}\right)$,as below , they let s3 be equal to s2 and set s2 be equal to $s2 + \alpha \times wi$ for embedding the binary value of the watermark, where α is a constant and wi is the watermark bit .Each Sj matrix of a block Bj contains value of the watermark .After embedding watermark into sj matrix of block Bj,the embedded block $B'j$ is obtained by inversing its corresponding U,V.

The proposed watermarking can be broken into two procedures: embedding, and extracting and restoring. [20].

2.3 Properties of the SVD

There are many properties and attributes for SVD, here some of these properties are presented

a. The singular value $\propto_1, \propto_2, \propto_n$ are unique, however, the matrices U and V are not unique.

b. Since $A^T A = VS^T SV^T$, so V diagonalizes $A^T A$, it follows that the V_j s are the eigenvector of $A^T A$.

c. Since $AA^T = USS^TU^T$, so it follows that U diagonalizes AA^T and that the U_i's are the eigenvectors of AA^T .

d. If A has rank of r then $v_j, v_j, \ldots V_r$ form an orthonormal basis for range space of A^T , R(A^T),and $u_j, u_j \ldots u_r$ form an orthonormal basis for .range space A, R(A).

e. The rank of matrix A is equal to the number of its nonzero singular values.[19]

2.4 The proposed Algorithm

The watermark is the signal which is actually added to the cover image A(*k* x *k). A* cover image *A* is divided into *nxn* blocks. For each *nxn* block, λ max denotes the largest SVD of the cover image. where *r*, the rank of *W*, is less than or equal to *l*. The order of the SVs of W is randomized, and each λ *wi* is embedded into one *nxn* block of the cover image.

2.4.1 Steps of the Embedding Process

The block diagram for embedding watermark using SVD technique is shown in fig. 4 and fig.5.The host image is divided into smaller blocks then SVD of blocks, variable scaling factor and distributed over the image blocks .let (A) the cover image is the size of *N*x*N* and (*W*)is the watermark.

a. Input a cover image of size *N* × *N* and watermark image .

b. Convert the RGB components of color image to YCbCr . luminance (Y) and chrominance (Cb and Cr) color values as columns, rgbmap is returned as an M- by-3 matrix that contains the Red, Green, and Blue values equivalent to those colors.

c. The SVD technique is applied on each block of cover data image as well as on watermark using SVD .

d. Combine the watermark with the *SVD* of the selected block using appropriate scaling factor; α , as following Equation

$$D = S + \alpha \times W \qquad \ldots(2)$$

e. Inverse SVD transform technique is applied to get the watermarked image [14].As shown this in fig5 illustration steps of the Embedding Process.

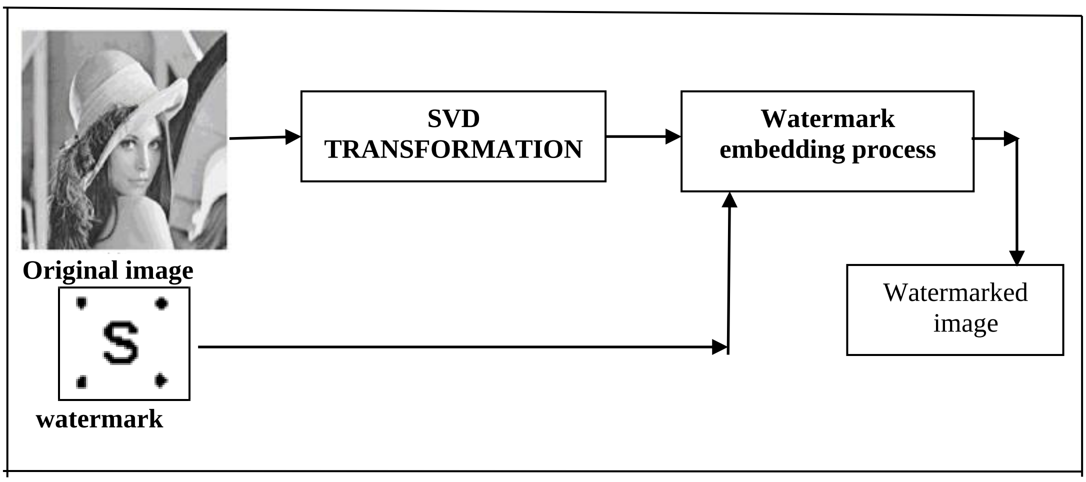


Figure3: Process of Embedding Watermark Image

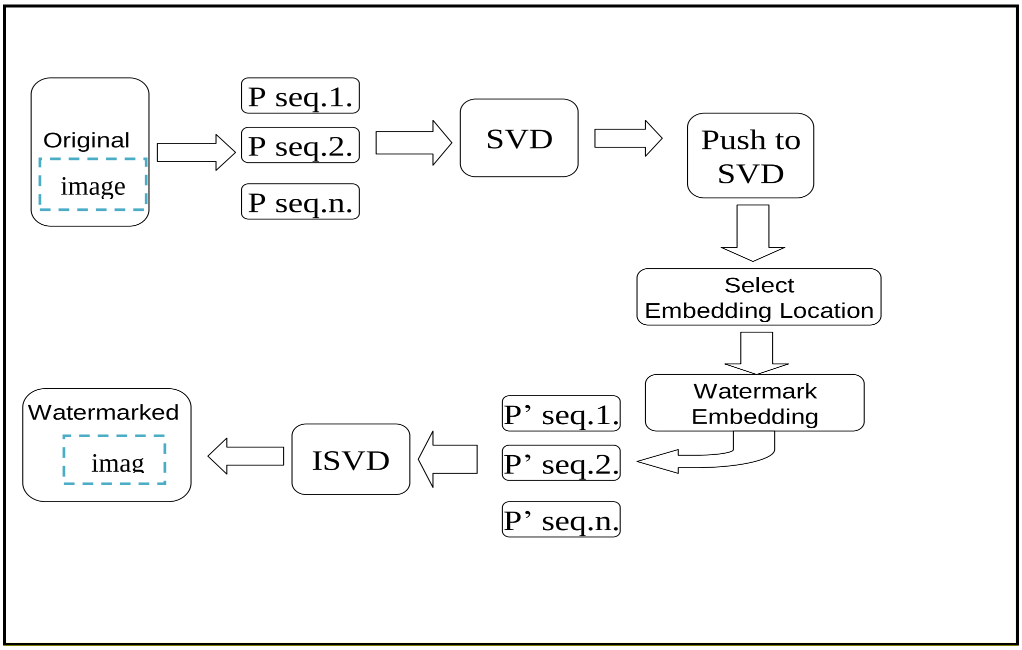


Figure 4: Diagram of Watermark Embedding

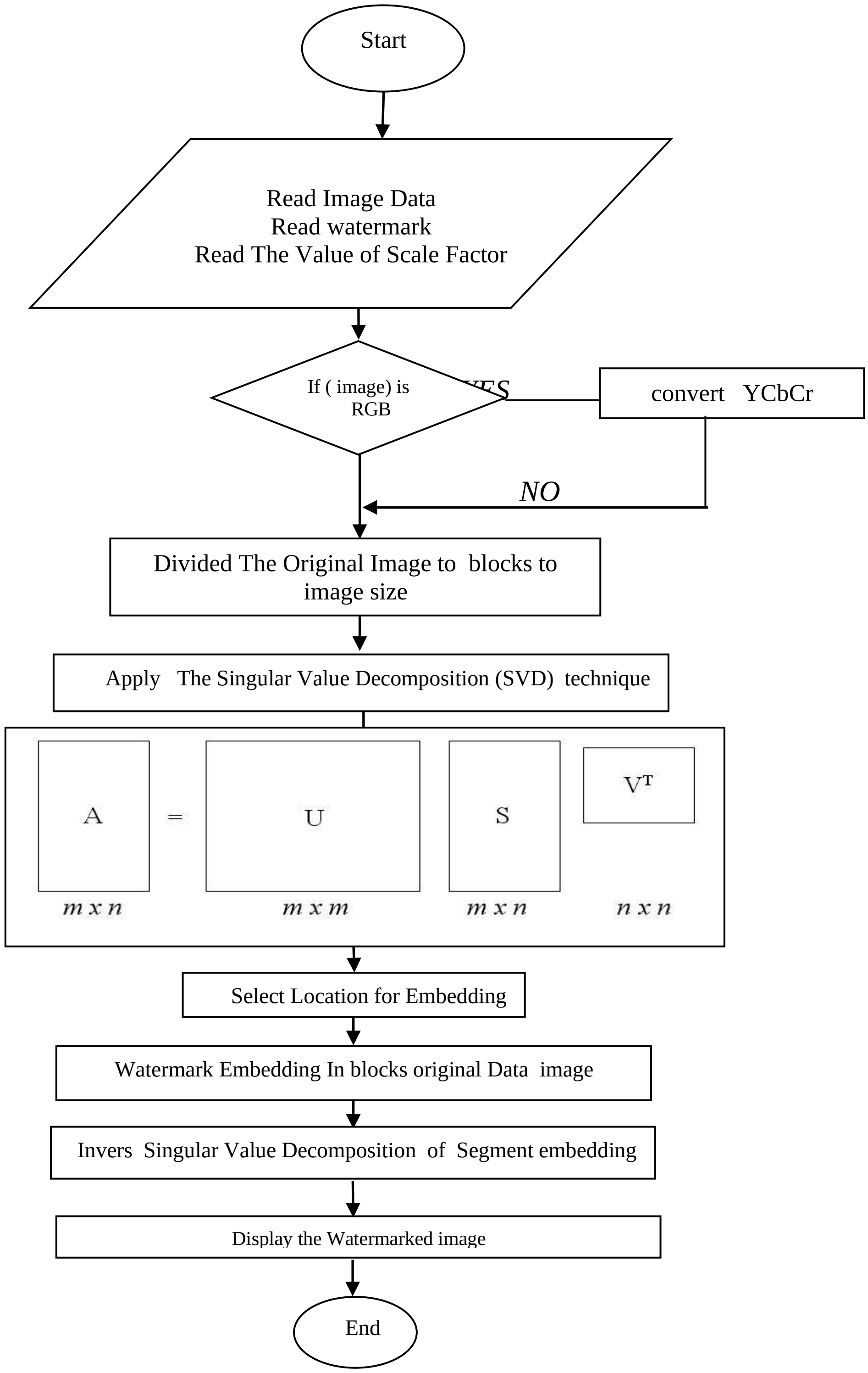


Figure 5: Flow Chart of the Embedding Process

2.4.2 Steps of the extraction process

In general, the extraction process is the inverse of the embedding procedure. In watermark extraction, a possibly distorted watermark *(W)* is extracted from the possibly distorted watermarked image by essentially reversing the above watermark embedding steps.

Let (D) the watermark image is the size of N x N and (W) is the watermark. The watermark extraction can be described as follows:

a. Let j=1:N , α is the value of scale factor.

b. Compute the SVD of the corresponding block, obtain three matrices which are Uj , Sj , and Vj . Assume that

$$Sn= \begin{bmatrix} S\alpha 1 & 0 & \cdots & 0 \\ \vdots & & \ddots & \vdots \\ 0 & & \cdots & S\alpha n \end{bmatrix} \quad \ldots(4)$$

c. Compute the diagonal (S).

d. Extract the watermark from watermark image by using Equation

$$W= (D-S)/\alpha \quad \ldots(5)$$

Where W:Watermark

D : watermarked image

S: Diagonal matrix of original image.

α :is a scale factor. See fig. (6) and fig. (7).

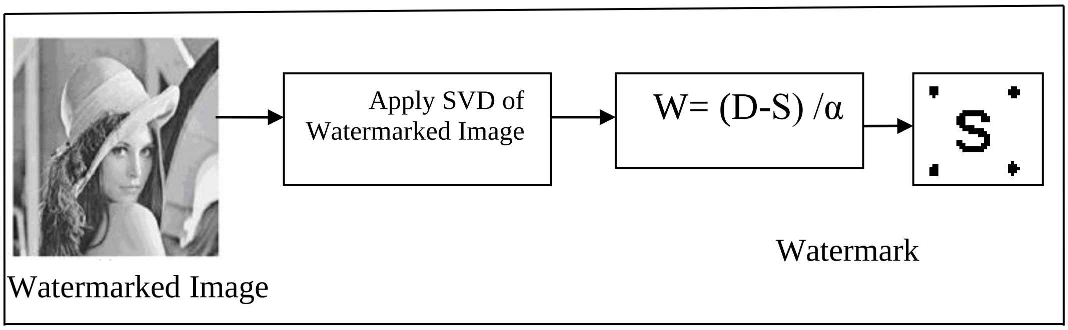


Figure 6: Extraction Watermark Process

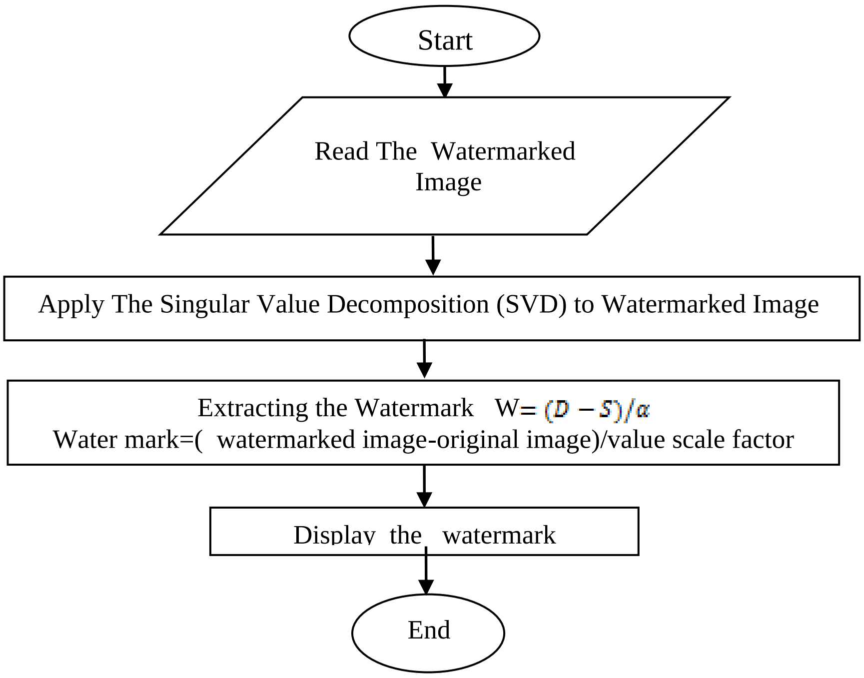
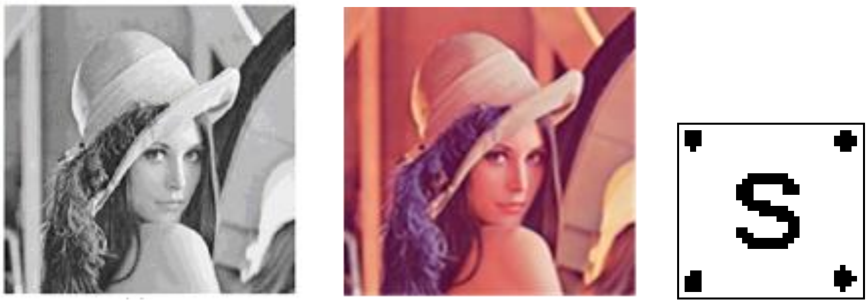


Figure 7: Flow chart of the extraction process

3. RESULTS

In this section, we have presented the experimental results we have implemented in MATLAB. Gray scale Lena image with size 256 × 256 pixels and RGB Lena image with size 512×512 pixels which were used as the cover images, and the watermark image with size 32 × 32 pixels are used. See fig. (8).



(a) **(b)** **(c)**

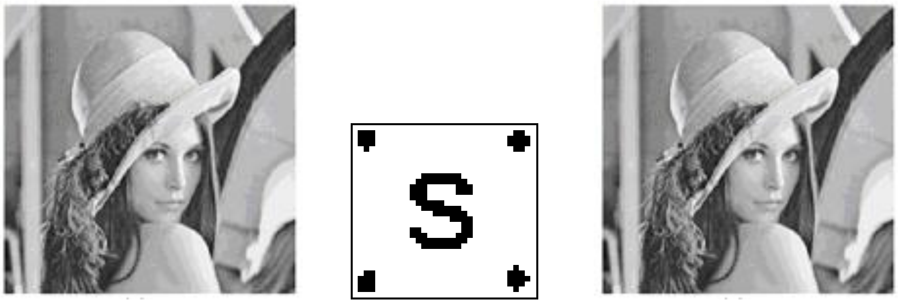
Figure 8: Different size of images:

(a) Original Image

(b) Original Image

(c)Watermark Image

Fig. 9 shows the program results which displayed the original image and the watermark image, then obtained the watermarked image. This figure shows the extraction watermark images where get successful execution.



(a) **(b)** **(c)**

Figure 9: Gray scale Lena Image Embedded Process
(a) Original Image

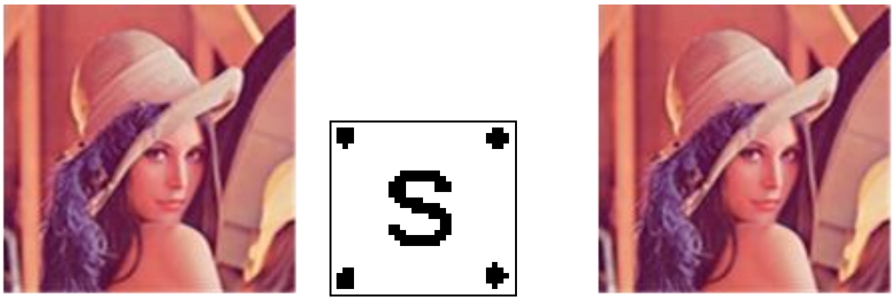
(b) Watermark

(c) Watermarked Image

(a) (b) (c)

Figure 10: Gray scale Lena Image Extraction Process
(a) Watermarked Image
(b) Original Image
(c) Watermark

Fig. 11 shows the program results which displayed the colored original image and the watermark image, then obtained the watermarked image. This figure shows the extraction watermark images where get successful execution.



(a) (b) (c)

Figure 11: Color Lena Image Embedded Process

(a) Original Image

(b)Watermark

(c)Watermark Image

The quality of the watermarked image is measured through PSNR('Peak Signal to Noise Ratio') and the MSE ('Mean Square Error') which is defined as below:

$$MSE=(1/NM)*\sum |k_{ij}-host_{ij} \quad \ldots (6)$$

In the previous equation, *M* and *N* are the number of rows and columns in the input images; respectively .Then the block computes the PSNR using the following equation

$$PSNR=10*\log(R^2/MSE) \quad \ldots(7)$$

Where R is the maximum fluctuation in the input image data type. For example, if the input image has a double-precision floating-point data type, then *R* is 1. If it has an 8-bit unsigned integer data type, *R* is 255,

The PSNR block computes the peak signal-to-noise ratio, in decibels, between two images. This ratio is often used as a quality measurement between the original and a compressed image.

The *Mean Square Error (MSE)* and the *Peak Signal to Noise Ratio (PSNR)* are the two error metrics used to compare image compression quality. The MSE represents the cumulative squared error between the compressed and the original image, whereas PSNR represents a measure of the peak error. The lower the value of MSE, the lower the error. shows that inserting the watermarked image and the calculated of PSNR value . Also the extracted watermark is exactly similar to original watermark image . The performance of the proposed technique is tested in Matlab software version 7.4

The PSNR for fig.9 was equal to 46.4357db, and the MSE was equal to 1.4774. The PSNR for fig. 11 was equal to 43.9351db, and the MSE was equal to 2.6277.

To improve the watermark technique strength different attachment processes can be used for this improvement. In this research an attachment for type rotate used to attach watermarked image, the results ensure the watermark program strength. As shown in fig. 12.

Figure 12: Gray scale Lena Image Attack

(a) Rotation Corrected

(b) Extract Attack

From the above figure, it is clear that the rotation attachment success in watermark extraction, which prove the power of suggested SVD watermarking technique.

4. Conclusion and Future Work

This work applied the technique of linear algebra “singular value decomposition (SVD)”to digital image watermarking processing. The proposed algorithm depends on embedding the watermark into SVD of original image after dividing it into blocks. According to our experimental results, our proposed scheme is a good embedding because the difference between the original and watermarked images is in vision. In addition, can extract the watermark correctly even though the watermarked image.
The experimental results show that the proposed algorithm found that SVD is effective method to execute the algorithm, each of them is carrying own data (information) to contribute to the system .SVD has the advantage of providing a good method of embedding watermark .

In the future work, the detection system will be extend to more transform domain watermarking approaches such as DWT-SVD and DCT -SVD. .

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