Comparison between the Heun's and Haar Wavelet Methods for solution Differential-Algebraic Equations (DAEs)

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ABSTRACT

In this paper, We solved the system of differential-algebraic equation (DAEs) of index one numerically with Heun's method and operational matrices of Haar wavelet method, When we compared the results of the two methods with the exact solution, show that the operational matrices of Haar wavelet method is more efficiency and it's numerical results near to the exact solution more than the Heun's method, and the solution accuracy of this method is increasing and the error decreases when the number of mesh points and size of matrices increase.

Keywords: Differential-Algebraic Equation, Heun's method**,** Haar wavelet method**.**

مقارنة بين طريقة هيون وطريقة موجة هار لحل المعادلات التفاضلية الجبرية

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الملخص

في هذا البحث تم حل نظام من المعادلات التفاضلية الجبرية ذات الدليل الواحد عدديا باستخدام طريقة Heun وكذلك طريقة مصفوفات العوامل لموجات Haar القصيرة وبعد مقارنة نتائج الطريقتين مع الحل المضبوط تبين أن طريقة مصفوفات العوامل لمويجة Haar ذات كفاءة عالية ونتائجها العددية اقرب إلى الحل المضبوط من طريقة Heun , وان دقة الحل لهذه الطريقة تزداد والخطأ يتناقص كلما ازدادت عدد نقاط الشبكة أو سعة المصفوفة (m).

الكلمات المفتاحية: المعادلات التفاضلية الجبرية, طريقة هيون, طريقة موجة هار.

1. Introduction:

In this paper we consider implicit differential equations

$$f\left(y'(t), y(t), t\right) = 0 \qquad \ldots(1)$$

on an interval $I \subset R$. If $\frac{\partial f}{\partial y'}$ is non singular, then it possible to formally solve (1) for y' in order to obtain an ordinary differential equation. However, if $\frac{\partial f}{\partial y'}$ is singular, this no longer possible and the solution y has satisfy certain algebraic constraints. Thus equations (1) where $\frac{\partial f}{\partial y'}$ is singular are referred to as differential algebraic equations (DAEs)[6].

In this paper we take the special case of eq.(1) which is a semi-explicit DAE

$$y_1' = f(t, y_1, y_2) \qquad \ldots(2a)$$

$$0 = g(t, y_1, y_2) \qquad \ldots(2b)$$

With conditions:

$$y_1(t_0) = A_1 \text{ and } y_2(t_0) = A_2$$

Where A_1 and A_2 are constants.

The index is one if $\frac{\partial g}{\partial y_2}$ is non singular, because then on differentiation of (2b) yields y_2' in principle. [7].

1.1 Definition: Index of the DAE is the number of differentiations needed for transformation the algebraic equation to differential equation. for example, let $q(t)$ be a given, smooth function, then the following problems for $y(t)$.

The scalar equation $y = q(t)$ is trivial index-1 DAE, because it takes one differentiation to obtain an ODE for y.

for the system

$$y_1 = q(t)$$

$$y_2 = y_1'$$

we differentia the first equation to get

$$y_2 = y_1' = q'(t)$$

and $\quad y_2' = y_1'' = q''(t)$

The index is 2 because to differentiation of $q(t)$ where needed [1].

Let equation (1) is DAEs, the index along a solution y(t) is the minimum number of differentiations of the system which required to solve for y' uniquely in terms of y and t (i.e. to define an ODE for y). Thus, the index is defined in terms of the over determined system

$$\begin{aligned} &F(t, y, y') = 0 \\ &\frac{\partial F}{\partial t}(t, y, y', y'') = 0 \\ &\vdots \\ &\frac{\partial^p F}{\partial t^p}(t, y, y', \cdots, y^{(p+1)}) = 0 \end{aligned} \qquad \ldots(3)$$

to be the smallest integer p so that y' in (3) can be solved for in terms of y and t.

Haar wavelets have become an increasingly popular tool in the computational sciences. They have had numerous applications in a wide rang of areas such as signal analysis, data compression and many others[8].

Wu and Chen (2003) [8] studied the numerical solution for partial differential equations of first order via operational matrices , they used the Haar wavelets in the solution with constant initial and boundary conditions.

Wu and Chen (2004) [9] studied the numerical solution for fractional calculus and the fractional differential equation by using the operational matrices of orthogonal functions. The fractional derivatives of the four typical functions and two classical fractional differential equations solved by the new method and they are compared the results with the exact solutions, they are found the solutions by this method is simple and computer oriented.

Lepik and Tamme (2007) [3] derived the solution of nonlinear Fredholm integral equations via the Haar wavelet method, they are find that the main benefits of the Haar wavelet method are sparse representation, fast transformation, and possibility of implementation of fast algorithms especially if matrix representation is used.

Lepik Uio (2007) [4] studied the application of the Haar wavelet transform to solve integral and differential equations, he demonstrated that the Haar wavelet method is a powerful tool for solving different types of integral equations and partial differential equations. The method with far less degrees of freedom and with smaller CPU time provides better solutions then classical ones.

Numerical approaches for the solution of DAEs can be divided roughly into two classes: direct discretizations of the given system and methods which involve a reformulation (i.e. index reduction), combined with a discretization [1].

In this paper, we will study the numerical solution for Differential-Algebraic equations (DAEs) by using Heun's method and Haar wavelets method and we will compare the results of these methods with the exact solution.

2. Heun’s Method:

We will use the Heun’s Method to solve Eq.(2a) and (2b) [5]
Then the general steps of Heun’s Method to eq.(2a) and (2b) are

$$y' = f(t_k, y_{1,k}, y_{2,k})$$

$$p_{k+1} = y_{1,k} + hf(t_k, y_{1,k}, y_{2,k})$$

and

$$q_{k+1} = y_{2,k} \text{ since } g(t_k, y_{1,k}, y_{2,k}) = 0$$

then

$$y_{1,k+1} = y_{1,k} + \frac{h}{2}\left[f(t_k, y_{1,k}, y_{2,k}) + f(t_{k+1}, p_{k+1}, q_{k+1})\right] \quad \ldots(4)$$

$$y_{2,k+1} = y_{1,k+1} + t_{k+1} \quad \ldots(5)$$

where h is step size and $t_{k+1} = t_k + h$. then we illustrate this method in an example in numerical results

3. Review of the operational matrices and Haar wavelets:

The main characteristic of the operational method is to convert a differential equation into an algebraic one, and the core is the operational matrix for integration. The integral property of the basic orthonormal matrix, $\phi(t)$. we write the following approximation:

$$\int_0^t\int_0^t\int_0^t \ldots\ldots \int_0^t \phi(t)(dt)^k \cong Q_\phi^k \phi(t) \quad \ldots(6)$$

where $\phi(t) = [\vec{\varphi}_0(t) \ \ \vec{\varphi}_1(t) \ \ \ldots \ \ \vec{\varphi}_{m-1}(t)]^T$ in which the elements $\vec{\varphi}_0(t), \vec{\varphi}_1(t), \ldots, \vec{\varphi}_{m-1}(t)$ are the discrete representation of the basis functions which are orthogonal on the interval [0,1) and Q_ϕ is the operational matrix for integration of $\phi(t)$ [8,9].

The operational matrix of an orthogonal matrix $\phi(t)$, Q_ϕ can be expressed by:

$$[Q_\phi] = [\phi] \cdot [Q_B] \cdot [\phi]^{-1} \quad \ldots(7)$$

where $[Q_B]$ is the operational matrix of the block pulse function:

$$Q_{B_m} = \frac{1}{m}\begin{bmatrix} 1/2 & 1 & \ldots & \ldots & 1 \\ 0 & 1/2 & 1 & \ldots & 1 \\ 0 & \ldots & 1/2 & \ldots & 1 \\ 0 & \ldots & 0 & 1/2 & 1 \\ 0 & \ldots & \ldots & 0 & 1/2 \end{bmatrix} \quad \ldots(8)$$

If the transform matrix $[\phi]$ is unitary ,that is $[\phi]^{-1} = [\phi]^T$, then the equation (7) can be rewritten as [8,9]:

$$[Q_\phi] = [\phi] \cdot [Q_B] \cdot [\phi]^T \quad \ldots(9)$$

The Haar functions are an orthogonal family of switched rectangular waveforms where amplitudes can differ from one function to another. They are defined in the interval [0,1] by [8,9]:

$$h_0(t) = \frac{1}{\sqrt{m}}$$

$$h_i(t) = \frac{1}{\sqrt{m}} \begin{cases} 2^{\frac{J}{2}} & \frac{k-1}{2^J} \le t < \frac{k-1/2}{2^J} \\ -2^{\frac{J}{2}} & \frac{k-1/2}{2^J} \le t < \frac{k}{2^J} \\ 0 & \text{otherwise in } [0,1) \end{cases} \qquad \ldots(10)$$

where i=0,1,2,…..,m-1, $m = 2^{\alpha}$ and α is a positive integer. J and k represent the integer decomposition of the index i , i.e. $i = 2^J + k - 1$.

Any function $y(t)$ which is square integrable in the interval $0 \le t < 1$ can be expanded into Haar series by:

$$y(t) = \sum_{i=0}^{\infty} c_i \, h_i(t) \qquad \ldots(11)$$

where $c_i = \int_0^1 y(t)\, h_i(t)$ [8].

Usually the series expansion of equation (11) contains infinite terms for a general smooth function y(t). However, if y(t) is approximated as piecewise constant during each subinterval, equation (11) will be terminated at finite terms, i. e. :

$$y(t) = \sum_{i=0}^{m-1} c_i \, h_i(t)$$

The equation (11) can be written into the discrete form by:

$$[\vec{Y}]^T = [\vec{C}]^T \cdot [H(t)] \qquad \ldots(12)$$

where $[\vec{Y}]^T = [y_0 \quad y_1 \quad \ldots \quad y_{m-1}]$ is the discrete form of the continuous function y(t), and m is the dimension and usually $m = 2^{\alpha}$ α is a positive integer.

$[\vec{C}]^T = [c_0 \quad c_1 \quad \ldots \quad c_{m-1}]$ is called coefficient vector of $y(t)$ calculated by:

$$\left[\vec{C}\right]^T = [\vec{Y}]^T [H]^{-1} \qquad \ldots(13)$$

Since the Haar wavelet matrix H is unitary, $[H]^{-1} = [H]^T$, Thus:

$$\left[\vec{C}\right]^T = [\vec{Y}]^T [H]^T$$

For deriving the operational matrix of Haar wavelets, we let $[\phi] = [H]$ in the equation (9), and obtain:

$$[Q_H] = [H] \cdot [Q_B] \cdot [H]^T \qquad \ldots(14)$$

where $[Q_H]$ is the operational matrix for integration of $[H]$.

For example, the operational matrix of the Haar wavelet in the case of m=4 is given by:

$$[Q_H]=[H]_{4*4}\cdot[Q_B]\cdot[H]^T_{4*4}$$

$$=\begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}\cdot\frac{1}{4}\begin{bmatrix} \frac{1}{2} & 1 & 1 & 1 \\ 0 & \frac{1}{2} & 1 & 1 \\ 0 & 0 & \frac{1}{2} & 1 \\ 0 & 0 & 0 & \frac{1}{2} \end{bmatrix}\cdot\begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}^T$$

$$=\begin{bmatrix} 0.5 & -0.25 & -0.0884 & -0.0884 \\ 0.25 & 0 & -0.0884 & 0.0884 \\ 0.0884 & 0.0884 & 0 & 0 \\ 0.0884 & -0.0884 & 0 & 0 \end{bmatrix}$$

4. Haar wavelet method:

We will use the operational matrices of the Haar wavelets to solve the differential-algebraic equations (1) numerically.

By using the equation (6), the integration of equation (12) with respect to variable t yields [8]:

$$\int_0^t [\vec{y}(t)]^T\,dt = \int_0^t [\vec{C}]^T\cdot H(t)\,dt = [\vec{C}]^T\cdot\int_0^t H(t)\,dt$$
$$= [\vec{C}]^T\cdot[Q_H]\cdot[H] \qquad \ldots(15)$$

Further the double integration of y(t) with respect to variable (t) and by using equation (6), we get:

$$\int_0^t\int_0^t [\vec{y}(t)]^T\,dt\,dt = \int_0^t\int_0^t [\vec{C}]^T\cdot H(t)\,dt\,dt$$
$$= [\vec{C}]^T\cdot\int_0^t\int_0^t H(t)\,dt\,dt \qquad \ldots(16)$$
$$= [\vec{C}]^T\cdot[Q_H]^2\cdot[H]$$

Now, we consider the differential-algebraic equations (DAEs) (2a) and (2b) of the form [2]:

$$y'(t) = f(t, y_1(t), y_2(t))$$
$$0 = g(t, y_1(t), y_2(t))$$

with

$$y_1(t_0) = A_1$$
$$y_2(t_0) = A_2 \qquad \ldots(17)$$

where A_1 and A_2 are constants.
by integrating equation (2a) with respect to (t), we get:

$$\int_0^t y_1'(t)dt = \int_0^t f\left(t, y_1(t), y_2(t)\right)dt$$

$$0 = g\left(t, y_1(t), y_2(t)\right)$$

$$\Rightarrow \left[y_1(t) - y_1(0)\right] = \int_0^t f\left(t, y_1(t), y_2(t)\right)dt \qquad \ldots(18a)$$

$$0 = g\left(t, y_1(t), y_2(t)\right) \qquad \ldots(18b)$$

we transform the equations (18a) and (18b) into the matrices forms by using equation (12), we get:

$$\left[\overrightarrow{Y_1}\right]^T = \left[\overrightarrow{C_1}\right]^T \cdot [H]$$

$$\left[\overrightarrow{Y_2}\right]^T = \left[\overrightarrow{C_2}\right]^T \cdot [H]$$

then

$$\left[\overrightarrow{C_1}\right]^T \cdot [H] - \left[\overrightarrow{Y_1(t_0)}\right]^T = \int_0^t f\left(t_i, \left[\overrightarrow{C_1}\right]^T \cdot [H], \left[\overrightarrow{C_2}\right]^T \cdot [H]\right)dt \qquad \ldots(19a)$$

$$0 = g\left(t_i, \left[\overrightarrow{C_1}\right]^T \cdot [H], \left[\overrightarrow{C_2}\right]^T \cdot [H]\right) \qquad \ldots(19b)$$

such that by using the initial condition (17), we get:

$$\left[\overrightarrow{y_1(t_0)}\right]^T = \left[\overrightarrow{A_1}\right]^T \qquad \ldots(20)$$

where

$$t_i = \frac{1}{2m} + \frac{i}{m} \qquad i = 0,1,2,\cdots$$

m is the dimension of the matrix.
Now, by using the integration (15), the equations (19a) and (19b) becomes:

$$\left[\overrightarrow{C_1}\right]^T \cdot [H] - \left[\overrightarrow{A_1}\right]^T = f\left(t_i, \left[\overrightarrow{C_1}\right]^T \cdot [Q_H] \cdot [H], \left[\overrightarrow{C_2}\right]^T \cdot [Q_H] \cdot [H]\right) \qquad \ldots(21a)$$

$$0 = g\left(t_i, \left[\overrightarrow{C_1}\right]^T \cdot [H], \left[\overrightarrow{C_2}\right]^T \cdot [H]\right) \qquad \ldots(21b)$$

such that the dimension for all matrices are $m \times m \cdot [H]$ is Haar wavelets matrix, $[Q_H]$ is the operational matrix of the Haar wavelets. $\left[\overrightarrow{C_1}\right]^T$ and $\left[\overrightarrow{C_2}\right]^T$ are the coefficient vectors of $y_1(t)$ and $y_1(t)$ respectively:

$$\left[\overrightarrow{C_1}\right]^T = \left[C_{10} \quad C_{11} \quad C_{12} \quad \cdots \quad C_{1(m-1)}\right]$$

$$\left[\overrightarrow{C_2}\right]^T = \left[C_{20} \quad C_{21} \quad C_{22} \quad \cdots \quad C_{2(m-1)}\right]$$

To find the coefficient matrix $\left[\overrightarrow{C_1}\right]^T$ and $\left[\overrightarrow{C_2}\right]^T$ which have m of the elements respectively, we solve the system (21a) and (21b) which given linear system of the equations such that the variables number are $2*m$ and we will can be solved this linear system by Gauss-Jordan method, after this we find the vectors solution $\left[\overrightarrow{Y_1}\right]^T$ and $\left[\overrightarrow{Y_2}\right]^T$ by using the equation (12) that is:

$$\left[\overrightarrow{Y_1}\right]^T = \left[\overrightarrow{C_1}\right]^T \cdot [H]$$
$$\left[\overrightarrow{Y_2}\right]^T = \left[\overrightarrow{C_2}\right]^T \cdot [H]$$

5. Example :

we take the system of DAEs bellow :

$$y_1'(t) = -y_1(t) + y_2(t) \qquad \ldots(22a)$$
$$0 = -\frac{1}{2} y_1(t) + y_2(t) - 2 \qquad \ldots(22b)$$

with

$$y_1(0) = \frac{1}{2}$$
$$y_2(0) = \frac{9}{4}$$

which has the analytic solutions [2]:

$$y_1(t) = 4 - (4 - y_1(0))e^{-t/2} \qquad \ldots(23a)$$
$$y_2(t) = 4 - (4 - y_2(0))e^{-t/2} \qquad \ldots(23b)$$

by using the equation (21a) and (21b), the system (22a) and (22b) becomes:

$$\left[\overrightarrow{C_1}\right]^T \cdot [H] - \left[\overrightarrow{y_1(0)}\right]^T = -\left[\overrightarrow{C_1}\right]^T \cdot [Q_H] \cdot [H] + \left[\overrightarrow{C_2}\right]^T \cdot [Q_H] \cdot [H] \qquad \ldots(24a)$$
$$0 = -\frac{1}{2}\left[\overrightarrow{C_1}\right]^T \cdot [H] + \left[\overrightarrow{C_2}\right]^T \cdot [H] - \left[\vec{2}\right]^T \qquad \ldots(24b)$$

where

$$\left[\overrightarrow{y_1(0)}\right]^T = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \cdots & \frac{1}{2} \end{bmatrix}_{m-1}$$

and

$$\left[\vec{2}\right]^T = \begin{bmatrix} 2 & 2 & \cdots & 2 \end{bmatrix}_{m-1}$$

when $m = 4$ then:

$$\left[\overrightarrow{C_1}\right]^T = \begin{bmatrix} C_{10} & C_{11} & C_{12} & C_{13} \end{bmatrix}$$
$$\left[\overrightarrow{C_2}\right]^T = \begin{bmatrix} C_{20} & C_{21} & C_{22} & C_{23} \end{bmatrix}$$
$$\left[\overrightarrow{y_1(0)}\right]^T = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

$$\left[\vec{2}\right]^{T} = \begin{bmatrix} 2 & 2 & 2 & 2 \end{bmatrix}$$

from the equation (10), we get:

$$[H] = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$$

from the equation (14), we get:

$$[Q_H] = \begin{bmatrix} 0.5 & -0.25 & -0.0884 & -0.0884 \\ 0.25 & 0 & -0.0884 & 0.0884 \\ 0.0884 & 0.0884 & 0 & 0 \\ 0.0884 & -0.0884 & 0 & 0 \end{bmatrix}$$

Now, by substitute the matrices $[Q_H]$ and $[H]$ and the vectors $\left[\overrightarrow{C_1}\right]^T$, $\left[\overrightarrow{C_2}\right]^T$, $\left[\overrightarrow{y_1(0)}\right]^T$ and $\left[\vec{2}\right]^T$ in the system (24a) and (24b) we get:

$$\begin{bmatrix} C_{10} & C_{11} & C_{12} & C_{13} \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix} - \begin{bmatrix} 1/2 & 1/2 & 1/2 & 1/2 \end{bmatrix}$$

$$= -\begin{bmatrix} C_{10} & C_{11} & C_{12} & C_{13} \end{bmatrix} \cdot \begin{bmatrix} 0.5 & -0.25 & -0.0884 & -0.0884 \\ 0.25 & 0 & -0.0884 & 0.0884 \\ 0.0884 & 0.0884 & 0 & 0 \\ 0.0884 & -0.0884 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$$

$$+ \begin{bmatrix} C_{20} & C_{21} & C_{22} & C_{23} \end{bmatrix} \cdot \begin{bmatrix} 0.5 & -0.25 & -0.0884 & -0.0884 \\ 0.25 & 0 & -0.0884 & 0.0884 \\ 0.0884 & 0.0884 & 0 & 0 \\ 0.0884 & -0.0884 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix} \quad ..(25a)$$

$$0=-0.5\begin{bmatrix}C_{10} & C_{11} & C_{12} & C_{13}\end{bmatrix}.\begin{bmatrix}\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2}\\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2}\\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0\\ 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}}\end{bmatrix} \quad \ldots(25b)$$

$$+\begin{bmatrix}C_{20} & C_{21} & C_{22} & C_{23}\end{bmatrix}.\begin{bmatrix}\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2}\\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2}\\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0\\ 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}}\end{bmatrix}-\begin{bmatrix}2 & 2 & 2 & 2\end{bmatrix}$$

By solving the system (25a) and (25b) we get a linear system consist of 8 equations and 8 variables which are represents the vectors $\left[\overrightarrow{C_1}\right]^T$ and $\left[\overrightarrow{C_2}\right]^T$ respectively and by solving this system by Gauss-Jordan method, we obtain:

$$\left[\overrightarrow{C_1}\right]^T=\begin{bmatrix}2.48588978 & -0.68658182 & -0.27403446 & -0.21334863\end{bmatrix}$$

$$\left[\overrightarrow{C_2}\right]^T=\begin{bmatrix}5.24294489 & -0.34329091 & -0.13701723 & -0.10667432\end{bmatrix}$$

Now, by using the equation (12), we get:

$$\left[\overrightarrow{Y_1}\right]^T=\left[\overrightarrow{C_1}\right]^T\cdot[H]$$
$$=\begin{bmatrix}0.70588235 & 1.09342561 & 1.43537553 & 1.73709606\end{bmatrix}$$

$$\left[\overrightarrow{Y_2}\right]^T=\left[\overrightarrow{C_2}\right]^T\cdot[H]$$
$$=\begin{bmatrix}2.35294118 & 2.54671280 & 2.71768777 & 2.86854803\end{bmatrix}$$

when $m=8$ and $m=16$ the results are illustration in the tables (1) and (2).

Table (1). A comparison between the operational matrix of the Haar wavelets method and Heun's method with exact solution for DAEs (22a) and (22b) with: m=8 , $y_1(0)=\frac{1}{2}$.

The value of (t)	The numerical solution of Haar wavelets y_1	The numerical solution of Heun's method y_1	The exact solution for y_1
0.0625	0.6060606060606	0.605957031250000	0.60768367933280
0.1875	0.811753902662993	0.808345150668174	0.81321373516988
0.3125	1.004980938865236	0.998664786788180	1.00629135442402
0.4375	1.186497245600676	1.177635588023851	1.18767099208829
0.5625	1.357012564049120	1.345934289870578	1.35806139303847
0.6875	1.517193620773416	1.504197273819535	1.51812836146911
0.8125	1.667666734665936	1.653022973682860	1.66849736253779
0.9375	1.809020265898303	1.792974138428698	1.80975596638393

Table (2). A comparison between the operational matrix of the Haar wavelets method and Heun's method with exact solution for DAEs (22a) and (22b) with: m=8 , $y_2(0)=\frac{9}{4}$.

The value of (t)	The numerical solution of Haar wavelets y_2	The numerical solution of Heun's method y_2	The exact solution for y_2
0.0625	2.303030303030302	2.302978515625000	2.30384183966640
0.1875	2.405876951331496	2.404172575334087	2.40660686758494
0.3125	2.502490469432617	2.49933239394090	2.50314567721201
0.4375	2.593248622800338	2.588817794011926	2.59383549604414
0.5625	2.678506282024559	2.672967144935289	2.67903069651924
0.6875	2.758596810386707	2.752098636909768	2.75906418073455
0.8125	2.833833367332967	2.826511486841430	2.83424868126890
0.9375	2.904310132949151	2.896487069214349	2.90487798319197

Table (3). A comparison between the operational matrix of the Haar wavelets method and Heun's method with exact solution for DAEs (22a) and (22b) with: m=16 , $y_1(0)=\frac{1}{2}$.

The value of (t)	The numerical solution of Haar wavelets y_1	The numerical solution of Heun's method y_1	The exact solution for y_1
0.03125	0.553846153846153	0.553833007812500	0.554262470481071
0.09375	0.659881656804733	0.659027764988423	0.660276669107841
0.15625	0.762654528903049	0.761011436641680	0.763029153743283
0.21875	0.862265158782955	0.859882041632818	0.862620276776974
0.28125	0.958810846205018	0.955734606782168	0.959147303080049
0.34375	1.052385897091017	1.048661258202313	1.052704504998162
0.40625	1.143081715642063	1.138751309842622	1.143383254421811
0.46875	1.230986893622307	1.226091349330951	1.231272112023964
0.53125	1.316187296895467	1.310765321195007	1.316456913752130
0.59375	1.398766149298683	1.392854607543371	1.399020854659346
0.65625	1.478804113935647	1.472438106283725	1.479044570155961
0.71875	1.556379371968396	1.549592306953445	1.556606214761560
0.78125	1.631567698984753	1.624391364235466	1.631781538433949
0.84375	1.704442539015991	1.696907169230061	1.704643960549745
0.90625	1.775075076277038	1.767209418551028	1.775264641608818
0.96875	1.843534304699283	1.835365681312723	1.843712552732626

Table (4). A comparison between the operational matrix of the Haar wavelets method and Heun's method with exact solution for DAEs (22a) and (22b) with: m=16 , $y_2(0)=\frac{9}{4}$.

The value of (t)	The numerical solution of Haar wavelets y_2	The numerical solution of Heun's method y_2	The exact solution for y_2
0.03125	2.276923076923076	2.276916503906250	2.277131235240535
0.09375	2.329940828402366	2.329513882494211	2.330138334553920
0.15625	2.381327264451524	2.380505718320840	2.381514576871641
0.21875	2.431132579391477	2.429941020816409	2.431310138388487
0.28125	2.479405423102508	2.477867303391084	2.479573651540024
0.34375	2.526192948545508	2.524330629101156	2.526352252499081
0.40625	2.571540857821030	2.569375654921311	2.571691627210905
0.46875	2.615493446811153	2.613045674665476	2.615636056011982
0.53125	2.658093648447733	2.655382660597503	2.658228456876065
0.59375	2.699383074649341	2.696427303771686	2.699510427329673
0.65625	2.739402056967823	2.736219053141862	2.739522285077981
0.71875	2.778189685984197	2.774796153476722	2.778303107380780
0.78125	2.815783849492376	2.812195682117733	2.815890769216974
0.84375	2.852221269507995	2.848453584615030	2.852321980274872
0.90625	2.887537538138518	2.883604709275514	2.887632320804409
0.96875	2.921767152349641	2.917682840656362	2.921856276366313

Figure (1). A comparison between the operational matrix of the Haar wavelets method and Heun's method with exact solution for DAEs (22a) and (22b) with: m=8 , $y_1(0)=\frac{1}{2}$, $y_2(0)=\frac{9}{4}$.

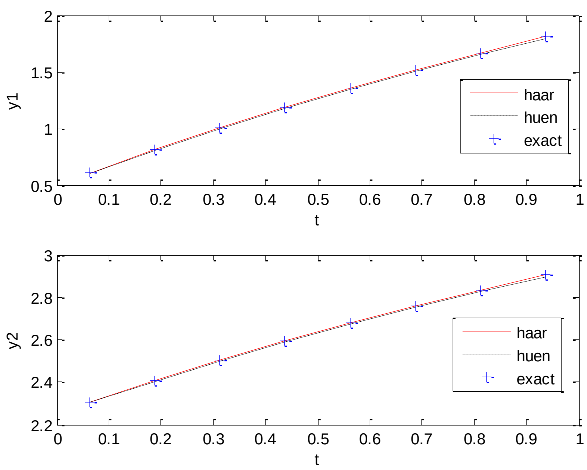
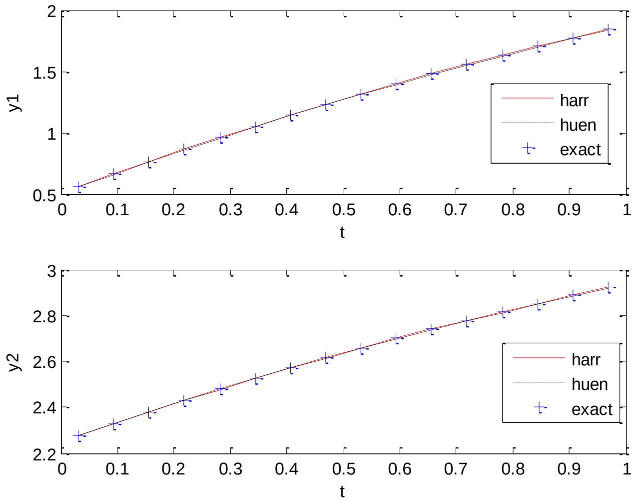


Figure (2). A comparison between the operational matrix of the Haar wavelets method and Heun's method with exact solution for DAEs (22a) and (22b) with: m=16 , $y_1(0)=\frac{1}{2}$, $y_2(0)=\frac{9}{4}$.



6. Conclusions:

The main goal of this paper was to demonstrate that the Haar wavelet method can be used to solve differential-algebraic equations (DAEs). The method is give results better then the classical (Heun's) method with small computation costs, As shown in table (1) and (2) and figure (1), when m=8, (m is size of matrices or mesh points).

When we increasing the values of (m) that obtained is more accuracy, i.e. when m=16, the results that obtained with Haar wavelet method it's show that in table (3) and (4) and figure (2) is more accurate and near to exact solution and the error is decrease as (m) is large.

The numerical solutions of these equations had been found using MATLAB which has the ability to approaches to the solution in high speed and accuracy and in less possible time.

REFERENCES

[1] Ascher, V.M. and Petzold, L.R., (1998), “Computer Methods for Ordinary Differential Equations and Differential-Algebraic Equations”, SIAM, Philadelphia, PA.

[2] B. Wu, and R. E. White, (2004), “ One implementation variant of the finite difference method for solving ODEs/DAEs ”, Computers and Chemical Engineering 28,pp. 303-309.

[3] Lepik, U. and Tamme, E., (2007), “Solution of nonlinear Fredholm integral equations via the Haar wavelet method”, Proc. Estonian Acad. Sci. Phys. Math., Vol. 56, No. 1, pp.17-27.

[4] Lepik, U., (2007), “Application of the Haar wavelet transform to solving integral and differential equations”, Proc. Estonian Acad. Sci. Phys. Math., Vol. 56, No. 1, pp.28-46.

[5] Mathews, J. H. and Fink. K. D., (2004), Numerical Methods using Matlab, prentice-Hall, Inc.

[6] Steffen Schulz, (2003), “Four Lectures on Differential-Algebraic Equations” , Report Humboldt University in Berlin.

[7] U. Ascher, (1997), “Stabilization of invariants of discredited differential system”, Numerical Algorithms, 14:1-24.

[8] Wu, J. L. and chen, C. H., (2003), "A novel numerical method for solving partial differential equations via operational matrices", proceeding of the 7th world multi conference on systemic, cybernetics and Informatics, Orlando, USA, Vol.5, pp.276-281.

[9] Wu, J. L. and chen, C. H., (2004), "A new operational approach for solving fractional calculus and fractional differential equations numerically", IEICE Trans. On fundamentals of Electronics, Communications and computer sciences, special Section on Discrete mathematics and It's Applications, Vol. E87-A, No. 5, pp.1077-1082.