Stability Conditions for Flow Rate of Liquid Between Two Vessels

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ABSTRACT

In this paper we obtain the conditions under which the trivial solution is stable for the following perturbed differential system :

$$\varepsilon\frac{dx_1}{dt}=\left(e^{\alpha_1 t}-\sin\alpha_2 t\right)x_1+\sin\alpha_2 t\ x_2,$$

$$\varepsilon\frac{dx_2}{dt}=\sin\alpha_2 t\ x_1+\left(\alpha_3 t-\sin\alpha_2 t\right)x_2, \qquad 0<\varepsilon<1$$

where $\alpha_1,\alpha_2,\alpha_3$ _ constants.

which described the flow rate of liquid or gases between two vessels, we transform this differential system to auxiliary system which gives the stability conditions of solution by Perron principle, these conditions represent stability conditions for required differential system.

Keywords: stability, perron principle.

شروط الاستقرار لمعدل تدفق السائل بين وعائيين

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الملخص

في هذا البحث تم الحصول على الشروط التي تجعل الحل الصفري مستقرا لمنظومة معادلات تفاضلية مشوشة لها الشكل:

$$\varepsilon\frac{dx_1}{dt}=\left(e^{\alpha_1 t}-\sin\alpha_2 t\right)x_1+\sin\alpha_2 t\ x_2,$$

$$\varepsilon\frac{dx_2}{dt}=\sin\alpha_2 t\ x_1+\left(\alpha_3 t-\sin\alpha_2 t\right)x_2, \qquad 0<\varepsilon<1$$

حيث $\alpha_1,\alpha_2,\alpha_3$ _ ثوابت.

والتي تصف معدل الجريان للسوائل أو الغازات بين وعائيين وذلك بتحويلها إلى النظام المساعد والذي منه نجد شروط استقرارية الحل باستخدام مبدأ الاستقرارية لبيرون وهذه الشروط تمثل شروط استقرار الحل للمنظومة التفاضلية المطلوبة.

الكلمات المفتاحية: الاستقرارية, مبدأ بيرون.

1. Introduction:

The common problem in differential equations is to determine the behavior of the solutions near a given constant solution. Physically, a constant solution represents an equilibrium, and many problems of science and technology can be reduced to the question of the stability or instability of an equilibrium**[3]**. In the context of such problems, only those solutions are of interest whose initial values lie near the equilibrium, and these equation arises whether these solutions remain near the equilibrium, and possibly even tend toward it in the course of time. The common problems in applied mathematics and physics study the flow rate of liquid or gases between two or more vessels when the coefficients of mathematical model which is represented by differential system are constant or variables **[1,2,6]**.And many of these problems discussed the conditions under which these systems are stable or unstable**[4,8]**. In our problem we discuss these conditions when the differential system has variable coefficients w.r.t. t (coefficients depending on the time), and finding the effect of parameter ε which represents the allowance in perturbation on conditions of the stability by using Perron principle of stability **[7]**

2. Definitions:

Definition 1 [3]:

Given the autonomous system

$$\left.\begin{aligned} \frac{dx}{dt} &= P(x,y), \\ \frac{dy}{dt} &= Q(x,y) \end{aligned}\right\} \qquad \ldots(2.1)$$

a point (x_0,y_0) at which both $P(x_0,y_0)=0$ and $Q(x_0,y_0)=0$ is called equilibrium point (the terms critical point and singular point are also used) of (2.1).

Definition 2 [3]:

Let (x_0,y_0) be an equilibrium point of system (2.1), we say that (x_0,y_0) is stable if for every $\varepsilon>0$ there is $\delta>0$ such that every solution u,v which satisfies, for some t_0,

$$[u(t_0)-x]^2+[v(t_0)-y_0]^2<\delta^2 , \qquad \ldots(2.2)$$

also satisfies, for all $t>t_0$, and

$$[u(t)-x]^2+[v(t)-y_0]^2<\varepsilon^2 .$$

Definition 3 [3]:

If (x_0,y_0) is a stable point**,** and if δ can be chosen so that (2.1) implies that $(u(t),v(t))\to(x_0,y_0)$ as $t\to\infty$, we say that (x_0,y_0) is asymptotically stable.

Remark[5]:

Investigating a solution of system (2.1)near the equilibrium point for stability can be reduced to investigating for stability zero (trivial) solution x=0 ,y=0 of (2.1).

3. Formulation of the Mathematical Model:

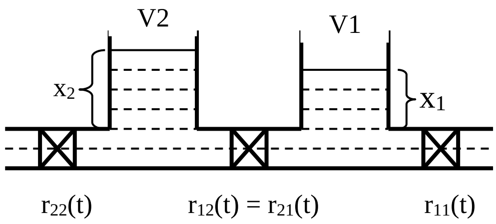
Assume that we have two vessels V1 and V2, and the heights of the liquid in V1 and V2 are x1 and x2, respectively **.** Each vessel is supplied with outflow valve r_{11}(t) , r_{22}(t) which are function of time (t) , and the two vessels are connected to a pipe that has a valve r_{12} (t) =r_{21} (t) which is also a function of time (t)**.**
So, we can choose an arbitrary function for each of r_{11}(t) , r_{12}(t) ,r_{21}(t), r_{22}(t) as in the following**:**

$r_{11}(t)=e^{\alpha_1 t}$

$r_{12}(t)=r_{21}(t)=\sin\alpha_2 t$

$r_{22}(t)=\alpha_3 t$

where $\alpha_1, \alpha_2, \alpha_3$ _ constants



The change of the heights of the liquid in the vessels, x_1,x_2 are proportional with the pressure in the outflow valve as follows:

$$\left(\frac{dx_1}{dt}\right)_1 \ \alpha \ x_1,$$

$$\left(\frac{dx_2}{dt}\right)_1 \ \alpha \ x_2.$$

The proportional constant in these cases depends on the outflow valve which are r_{11}(t), r_{22}(t), i.e.

$$\left.\begin{aligned}\left(\frac{dx_1}{dt}\right)_1 &= r_{11}(t) \quad x_1,\\ \left(\frac{dx_2}{dt}\right)_1 &= r_{22}(t) \quad x_2.\end{aligned}\right\} \qquad \dots(3.1)$$

In addition to this, the change in height of liquid in each vessel is proportional with flow rate between vessels through pipe and valve, i.e. the change is proportional with the difference of pressure between them that is

$$\left(\frac{dx_1}{dt}\right)_2 \ \alpha \ (x_2 - x_1),$$

$$\left(\frac{dx_2}{dt}\right)_2 \ \alpha \ (x_1 - x_2).$$

Since $r_{12}(t)=r_{21}(t)$, then

$$\left.\begin{aligned}\left(\frac{dx_1}{dt}\right)_2 &= r_{12}(t)\ (x_2-x_1),\\ \left(\frac{dx_2}{dt}\right)_2 &= r_{21}(t)\ (x_1-x_2).\end{aligned}\right\} \qquad \ldots(3.2)$$

Therefore

$$\left.\begin{aligned}\frac{dx_1}{dt} &= \left(\frac{dx_1}{dt}\right)_1 + \left(\frac{dx_1}{dt}\right)_2,\\ \frac{dx_2}{dt} &= \left(\frac{dx_2}{dt}\right)_1 + \left(\frac{dx_2}{dt}\right)_2.\end{aligned}\right\} \qquad \ldots(3.3)$$

Substituting (3.1) and (3.2) in (3.3),we get

$$\left.\begin{aligned}\frac{dx_1}{dt} &= r_{11}(t)\,x_1 + r_{12}(t)(x_2-x_1),\\ \frac{dx_2}{dt} &= r_{22}(t)\,x_2 + r_{21}(t)(x_1-x_2).\end{aligned}\right\} \qquad \ldots(3.4)$$

Then,

$$\left.\begin{aligned}\frac{dx_1}{dt} &= (r_{11}(t)-r_{12}(t))x_1 + r_{12}(t)\,x_2,\\ \frac{dx_2}{dt} &= r_{21}(t)\,x_1 + (r_{22}(t)\,x_1 - r_{21}(t))x_2.\end{aligned}\right\} \qquad \ldots(3.5)$$

Let ε be the parameter appearing in the equations governing the problem, here ε represents (allowance in perturbation).Then we get the following differential system:

$$\left.\begin{aligned}\varepsilon\,\frac{dx_1}{dt} &= p_{11}(t)\,x_1 + p_{12}(t)\,x_2,\\ \varepsilon\,\frac{dx_2}{dt} &= p_{21}(t)\,x_1 + p_{22}(t)\,x_2.\end{aligned}\right\} \qquad \ldots(3.6)$$

where

$$p_{11}(t) = r_{11}(t) - r_{12}(t) = e^{\alpha 1t} - \sin\alpha_2 t,$$

$$p_{12}(t) = r_{12}(t) = \sin\alpha_2 t,$$

$$p_{21}(t) = r_{21}(t) = \sin\alpha_2 t,$$

$$p_{22}(t) = r_{22}(t) - r_{21}(t) = \alpha_3 t - \sin\alpha_2 t.$$

and $\alpha_1, \alpha_2, \alpha_3$ _ constants.

Thus, the differential system in (3.6) becomes

$$\left.\begin{aligned} \varepsilon \frac{dx_1}{dt} &= \left(e^{\alpha_1 t} - \sin\alpha_2 t\right) x_1 + \sin\alpha_2 t \; x_2, \\ \varepsilon \frac{dx_2}{dt} &= \sin\alpha_2 t \; x_1 + \left(\alpha_3 t - \sin\alpha_2 t\right) x_2 . \end{aligned}\right\} \qquad ...(3.7)$$

4. Stability Conditions

In order to find stability conditions of (3.7), we rewrite (3.7) in vector form:

$$x^{\bullet} = p_1(t)\, x \;, \qquad \ldots(4.1)$$

where

$$x^{\bullet} = \begin{bmatrix} x_1^{\bullet} \\ x_2^{\bullet} \end{bmatrix}, \; p_1(t) = \frac{1}{\varepsilon}\begin{bmatrix} e^{\alpha_1 t} - \sin\alpha_2 t & \sin\alpha_2 t \\ \sin\alpha_2 t & \alpha_3 t - \sin\alpha_2 t \end{bmatrix}.$$

Applying the transformation

$$x = \frac{1}{\varepsilon} A(t)\, y,$$

$$\Rightarrow y = \varepsilon\, A^{-1}(t)\, x$$

to the differential system (4.1), we get the following auxiliary system:

$$y^{\bullet} = \left[D(t) - A^{-1}(t)\, A'(t)\right] y \,, \qquad \ldots(4.2)$$

where

$$D(t) = A^{-1}(t)\, p_1(t) A'(t) = \begin{bmatrix} \lambda_1(t) & 0 \\ 0 & \lambda_2\ (t) \end{bmatrix} \;, \; \lambda_1(t) \neq \lambda_2(t). \qquad \ldots(4.3)$$

The system (4.1) has roots of the form

$$\lambda_1(t) = \frac{\alpha_3 t - 2\sin\alpha_2 t + e^{\alpha_1 t}}{2\varepsilon} + \frac{\sqrt{\left(\alpha_3 t - e^{\alpha_1 t}\right)^2 + 4\sin^2\alpha_2 t}}{2\varepsilon},$$

$$\lambda_2(t) = \frac{\alpha_3 t - 2\sin\alpha_2 t + e^{\alpha_1 t}}{2\varepsilon} - \frac{\sqrt{\left(\alpha_3 t - e^{\alpha_1 t}\right)^2 + 4\sin^2\alpha_2 t}}{2\varepsilon}.$$

Now, to find the elements of the matrix A(t), we have from (4.3)

$$p_1(t)\, A(t) = A(t)\, D(t).$$

Then

$$\left(\frac{e^{\alpha_1 t} - \alpha_3 t}{2\varepsilon} - \frac{\sqrt{\left(\alpha_3 t - e^{\alpha_1 t}\right)^2 + 4\sin^2\alpha_2 t}}{2\varepsilon}\right) a_{11}(t) + \frac{1}{\varepsilon}\sin\alpha_2 t\; a_{21}(t) = 0, \qquad \ldots(4.4)$$

$$\left(\frac{e^{\alpha_1 t}-\alpha_3 t}{2\varepsilon}+\frac{\sqrt{\left(\alpha_3 t-e^{\alpha_1 t}\right)^2+4\sin^2\alpha_2 t}}{2\varepsilon}\right)a_{12}(t)+\frac{1}{\varepsilon}\sin\alpha_2 t\ a_{22}(t)=0, \qquad \ldots(4.5)$$

$$\left(\frac{\alpha_3 t-e^{\alpha_1 t}}{2\varepsilon}-\frac{\sqrt{\left(\alpha_3 t-e^{\alpha_1 t}\right)^2+4\sin^2\alpha_2 t}}{2\varepsilon}\right)a_{21}(t)+\frac{1}{\varepsilon}\sin\alpha_2 t\ a_{11}(t)=0\,, \qquad \ldots(4.6)$$

$$\left(\frac{\alpha_3 t-e^{\alpha_1 t}}{2\varepsilon}+\frac{\sqrt{\left(\alpha_3 t-e^{\alpha_1 t}\right)^2+4\sin^2\alpha_2 t}}{2\varepsilon}\right)a_{22}(t)+\frac{1}{\varepsilon}\sin\alpha_2 t\ a_{12}(t)=0. \qquad \ldots(4.7)$$

If we take the minors for the coefficients of equations (4.4) and (4.6), we get:

$$a_{11}(t)=\frac{\alpha_3 t-e^{\alpha_1 t}}{2\varepsilon}-\frac{\sqrt{\left(\alpha_3 t-e^{\alpha_1 t}\right)^2+4\sin^2\alpha_2 t}}{2\varepsilon},$$

$$a_{21}(t)=-\frac{1}{\varepsilon}\sin\alpha_2 t.$$

Also, if we take the minors for the coefficients of equations (4.5) and (4.7), we get:

$$\Rightarrow a_{12}(t)=\frac{\alpha_3 t-e^{\alpha_1 t}}{2\varepsilon}+\frac{\sqrt{\left(\alpha_3 t-e^{\alpha_1 t}\right)^2+4\sin^2\alpha_2 t}}{2\varepsilon}\,,$$

$$a_{22}(t)=-\frac{1}{\varepsilon}\sin\alpha_2 t.$$

therefore,

$$A(t)=\begin{bmatrix}\dfrac{\alpha_3 t-e^{\alpha_1 t}}{2\varepsilon}-\dfrac{\sqrt{\left(\alpha_3 t-e^{\alpha_1 t}\right)^2+4\sin^2\alpha_2 t}}{2\varepsilon} & \dfrac{\alpha_3 t-e^{\alpha_1 t}}{2\varepsilon}+\dfrac{\sqrt{\left(\alpha_3 t-e^{\alpha_1 t}\right)^2+4\sin^2\alpha_2 t}}{2\varepsilon}\\ -\dfrac{1}{\varepsilon}\sin\alpha_2 t & -\dfrac{1}{\varepsilon}\sin\alpha_2 t\end{bmatrix}.$$

Then

$$A^{-1}(t)\,A'(t)=\frac{\varepsilon}{\sqrt{\left(\alpha_3 t-e^{\alpha_1 t}\right)^2+4\sin^2\alpha_2 t}}\begin{bmatrix}z_{11} & z_{12}\\ z_{21} & z_{22}\end{bmatrix}, \qquad \ldots(4.8)$$

where

$$z_{11}=\frac{-\alpha_3+\alpha_1 e^{\alpha_1 t}}{2\varepsilon}+\frac{1}{2\varepsilon}\left(\frac{\alpha_3^2 t-\alpha_1\alpha_3\, t\, e^{\alpha_1 t}-\alpha_3 e^{\alpha_1 t}+\alpha_1 e^{2\alpha_1 t}+4\sin\alpha_2 t\cos\alpha_2 t}{\sqrt{\left(\alpha_3 t-e^{\alpha_1 t}\right)^2+4\sin^2\alpha_2 t}}\right)$$
$$+\frac{\alpha_2}{\varepsilon}\cos\alpha_2 t\left(\frac{\alpha_3 t-e^{\alpha_1 t}+\sqrt{\left(\alpha_3 t-e^{\alpha_1 t}\right)^2+4\sin^2\alpha_2 t}}{2\sin\alpha_2 t}\right),$$

$$z_{12}=\frac{-\alpha_3+\alpha_1 e^{\alpha_1 t}}{2\varepsilon}-\frac{1}{2\varepsilon}\left(\frac{\alpha_3^2 t-\alpha_1\alpha_3\, t\, e^{\alpha_1 t}-\alpha_3 e^{\alpha_1 t}+\alpha_1 e^{2\alpha_1 t}+4\sin\alpha_2 t\cos\alpha_2 t}{\sqrt{\left(\alpha_3 t-e^{\alpha_1 t}\right)^2+4\sin^2\alpha_2 t}}\right)$$
$$+\frac{\alpha_2}{\varepsilon}\cos\alpha_2 t\left(\frac{\alpha_3 t-e^{\alpha_1 t}+\sqrt{\left(\alpha_3 t-e^{\alpha_1 t}\right)^2+4\sin^2\alpha_2 t}}{2\sin\alpha_2 t}\right),$$

$$z_{21}=\frac{\alpha_3-\alpha_1 e^{\alpha_1 t}}{2\varepsilon}-\frac{1}{2\varepsilon}\left(\frac{\alpha_3^2 t-\alpha_1\alpha_3\, t\, e^{\alpha_1 t}-\alpha_3 e^{\alpha_1 t}+\alpha_1 e^{2\alpha_1 t}+4\sin\alpha_2 t\cos\alpha_2 t}{\sqrt{\left(\alpha_3 t-e^{\alpha_1 t}\right)^2+4\sin^2\alpha_2 t}}\right)-$$
$$-\frac{\alpha_2}{\varepsilon}\cos\alpha_2 t\left(\frac{\alpha_3 t-e^{\alpha_1 t}-\sqrt{\left(\alpha_3 t-e^{\alpha_1 t}\right)^2+4\sin^2\alpha_2 t}}{2\sin\alpha_2 t}\right),$$

$$z_{22}=\frac{\alpha_3-\alpha_1 e^{\alpha_1 t}}{2\varepsilon}+\frac{1}{2\varepsilon}\left(\frac{\alpha_3^2 t-\alpha_1\alpha_3\, t\, e^{\alpha_1 t}-\alpha_3 e^{\alpha_1 t}+\alpha_1 e^{2\alpha_1 t}+4\sin\alpha_2 t\cos\alpha_2 t}{\sqrt{\left(\alpha_3 t-e^{\alpha_1 t}\right)^2+4\sin^2\alpha_2 t}}\right)-$$
$$-\frac{\alpha_2}{\varepsilon}\cos\alpha_2 t\left(\frac{\alpha_3 t-e^{\alpha_1 t}-\sqrt{\left(\alpha_3 t-e^{\alpha_1 t}\right)^2+4\sin^2\alpha_2 t}}{2\sin\alpha_2 t}\right),$$

5. The Main Results:

We can not find stability conditions directly of (3.7), therefore, we first find stability conditions of auxiliary system (4.2), these conditions represent the stability conditions of the differential system (3.7). For this purpose we formulate the following theorem.

Theorem: The differential system (3.7) is stable if

1. $\mu_1\int_T^t\frac{-\varepsilon z_{12}\,\mu_2}{\sqrt{\left(\alpha_3\tau-e^{\alpha_1\tau}\right)^2+4\sin^2\alpha_2\tau}}\,d\tau<1,$

2. $c_1\mu_1=c_1\,\mathrm{e}^{\int_T^t\left(\frac{\alpha_3\tau-2\sin\alpha_2\tau+e^{\alpha_1\tau}}{2\varepsilon}+\frac{\sqrt{\left(\alpha_3\tau-e^{\alpha_1\tau}\right)^2+4\sin^2\alpha_2\tau}}{2\varepsilon}-\frac{\varepsilon z_{11}}{\sqrt{\left(\alpha_3\tau-e^{\alpha_1\tau}\right)^2+4\sin^2\alpha_2\tau}}\right)d\tau}$

$=0\ (1)\ ,\ t\to\infty,$

3. $\mu_3 \int_T^t \frac{-\varepsilon z_{21}}{\sqrt{\left(\alpha_3 t - e^{\alpha_1 t}\right)^2 + 4\sin^2 \alpha_2 t}} \mu_4 \, dt < 1,$

4. $c_2 \mu_3 = c_2 \, e^{\int_T^t \left(\frac{\alpha_3 \tau - 2\sin\alpha_2 \tau + e^{\alpha_1 \tau}}{2\varepsilon} - \frac{\sqrt{\left(\alpha_3 \tau - e^{\alpha_1 \tau}\right)^2 + 4\sin^2 \alpha_2 \tau}}{2\varepsilon} - \frac{\varepsilon z_{22}}{\sqrt{\left(\alpha_3 \tau - e^{\alpha_1 \tau}\right)^2 + 4\sin^2 \alpha_2 \tau}} \right) d\tau}$

$= 0(1)$, $t \to \infty$,

where

$$\mu_2 = e^{-\int_T^\tau \left(\frac{\alpha_3 t - 2\sin\alpha_2 t + e^{\alpha_1 t}}{2\varepsilon} + \frac{\sqrt{\left(\alpha_3 t - e^{\alpha_1 t}\right)^2 + 4\sin^2 \alpha_2 t}}{2\varepsilon} - \frac{\varepsilon z_{11}}{\sqrt{\left(\alpha_3 t - e^{\alpha_1 t}\right)^2 + 4\sin^2 \alpha_2 t}} \right) d\tau},$$

and

$$\mu_4 = e^{-\int_T^\tau \left(\frac{\alpha_3 t - 2\sin\alpha_2 t + e^{\alpha_1 t}}{2\varepsilon} - \frac{\sqrt{\left(\alpha_3 t - e^{\alpha_1 t}\right)^2 + 4\sin^2 \alpha_2 t}}{2\varepsilon} - \frac{\varepsilon z_{22}}{\sqrt{\left(\alpha_3 t - e^{\alpha_1 t}\right)^2 + 4\sin^2 \alpha_2 t}} \right) dt}$$

proof:

In order to find stability conditions of (3.7),we substitute (4.3) and (4.9) in (4.2):

$$\left.\begin{aligned} \frac{dy_1}{dt} &= \left(\frac{\alpha_3 t - 2\sin\alpha_2 t + e^{\alpha_1 t}}{2\varepsilon} + \frac{\sqrt{\left(\alpha_3 t - e^{\alpha_1 t}\right)^2 + 4\sin^2 \alpha_2 t}}{2\varepsilon} \right. \\ &\quad \left. - \frac{-\varepsilon z_{11}}{\sqrt{\left(\alpha_3 t - e^{\alpha_1 t}\right)^2 + 4\sin^2 \alpha_2 t}} \right) y_1 - \frac{\varepsilon z_{12} y_2}{\sqrt{\left(\alpha_3 t - e^{\alpha_1 t}\right)^2 + 4\sin^2 \alpha_2 t}}, \\ \frac{dy_2}{dt} &= \frac{-\varepsilon z_{21}}{\sqrt{\left(\alpha_3 t - e^{\alpha_1 t}\right)^2 + 4\sin^2 \alpha_2 t}} y_1 + \left(\frac{\alpha_3 t - 2\sin\alpha_2 t + e^{\alpha_1 t}}{2\varepsilon} \right. \\ &\quad \left. - \frac{\sqrt{\left(\alpha_3 t - e^{\alpha_1 t}\right)^2 + 4\sin^2 \alpha_2 t}}{2\varepsilon} - \frac{\varepsilon z_{22}}{\sqrt{\left(\alpha_3 t - e^{\alpha_1 t}\right)^2 + 4\sin^2 \alpha_2 t}} \right) y_2 . \end{aligned}\right\} \quad ...(5.1)$$

From Leibniz's formula [5],the first equation of (5.1) has a solution of the form:

$$y_1 = \mu_1 \int_T^t \frac{\varepsilon\, z_{12}}{\sqrt{\left(\alpha_3\tau - e^{\alpha_1\tau}\right)^2 + 4\sin^2\alpha_2\tau}}\ \zeta_2\, \mu_2\, d\tau + \mu_1 c_1,$$

where $c_1$is an arbitrary constant.
Now, by using the principle stability of Perron [7], we get

$$y_1 \le \mu_1 \int_T^t \frac{-\varepsilon\, z_{12}\, \varepsilon_0}{\sqrt{\left(\alpha_3\tau - e^{\alpha_1\tau}\right)^2 + 4\sin^2\alpha_2\tau}}\mu_2\, d\tau + \mu_1\, c_1 \le \varepsilon_0.$$

Hence the zero solution of first equation of (5.1) is stable if

1. $\mu_1 \int_T^t \frac{-\varepsilon\, z_{12}\, \mu_2}{\sqrt{\left(\alpha_3\tau - e^{\alpha_1\tau}\right)^2 + 4\sin^2\alpha_2\tau}} d\tau < 1,$

and

2. $c_1\mu_1 = c_1\, e^{\int_T^t \left(\frac{\alpha_3\tau - 2\sin\alpha_2\,\tau + e^{\alpha_1\tau}}{2\varepsilon} + \frac{\sqrt{\left(\alpha_3\tau - e^{\alpha_1\tau}\right)^2 + 4\sin^2\alpha_2\tau}}{2\varepsilon} - \frac{\varepsilon\, z_{11}}{\sqrt{\left(\alpha_3\tau - e^{\alpha_1\tau}\right)^2 + 4\sin^2\alpha_2\tau}} \right) d\tau}$
$= O(1)\ ,\ t \to \infty.$

Similarly, the zero solution of the second equation of (5.1) is stable if

3. $\mu_3 \int_T^t \frac{-\varepsilon\, z_{21}}{\sqrt{\left(\alpha_3 t - e^{\alpha_1 t}\right)^2 + 4\sin^2\alpha_2 t}} \mu_4\ dt < 1,$

and

4. $c_2\mu_3 = c_2\, e^{\int_T^t \left(\frac{\alpha_3\tau - 2\sin\alpha_2\,\tau + e^{\alpha_1\tau}}{2\varepsilon} - \frac{\sqrt{\left(\alpha_3\tau - e^{\alpha_1\tau}\right)^2 + 4\sin^2\alpha_2\tau}}{2\varepsilon} - \frac{\varepsilon\, z_{22}}{\sqrt{\left(\alpha_3\tau - e^{\alpha_1\tau}\right)^2 + 4\sin^2\alpha_2\tau}} \right) d\tau}$
$= 0\ (1)\ ,\ t \to \infty,$

where c_2 is an arbitrary constant.
Hence the zero solution of auxiliary system(4.2) is stable if it is satisfying the conditions 1,2,3,4 and these gives the stability conditions of the trivial solution of the differential system (3.7).

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