Solving Multi-Objective Complementary Programming Problem (MOCPP) by using Optimal Average

Najmaddin A. Sulaiman
College of Education
University of Salahaddin

Abdul-Qader O. Hamadamin
College of Sciences
University of Koya

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ABSTRACT

In this paper, we suggested an approach to solve Multi-Objective Complementary Programming Problem (MOCPP) by using optimal average (O_{AV}). The computer application of algorithm also has been demonstrated by flow-chart and solving a numerical examples by using MATHLABR2006a, and shown results in tables.

Key words: *Solving (MOCPP) by using Optimal Average* (O_{AV}).

حل مسألة البرمجة التكميلية متعددة الأهداف (MOCPP) باستخدام المعدل الامثل

نجم الدين سليمان
كلية التربية، جامعة صلاح الدين

عبد القادر حمد امين
كلية العلوم، جامعة كويا

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الملخص

في هذا البحث اقتراحنا تقنية لحل مسائل البرمجة التكميلية لمتعددة الأهداف باستخدام المعدل الامثل (O_{AV}). مع تطبيق بعض الأمثلة العددية لهذه الخوارزمية المسندة والمدعومة ب(فلو- كارت) وعلى الحاسوب (MATLAB R2006a).

الكلمات المفتاحية: البرمجة التكميلية متعددة الأهداف، المعدل الأمثل.

1. Introduction:

In (1971), Ibaraki, T. defined a new type of optimization, known as complementary problem with the addition (u.v=0) as follows [4], [7]:-

Maximize.Z=d.x+e.u+f.v

Subject to:

$A.x+B.u+C.v \le g$

u.v=0

$x, u, v \ge 0$

Where x, u.v are n, m and m dimensional vectors of variables respectively; d, e, f are n, m and m dimensional vectors of constants respectively; g is p-dimensional vectors of constants; A, B and C are $p\times n$, $p\times m$ and $p\times m$ matrices of constants, without the complementary condition (u.v=0) the above problem is an ordinary linear programming problem.

In (1984), Gary, K.C. & K.Swarup defined complementary programming with extreme point optimization [3].

In (1997), Sulaimam, N.A. searched and defined Upper-Bound cut for Extreme point Multi-Objective Complementary Problem [7].
In order to extend this work, we have defined a Multi-Objective Complementary Programming Problem (MOCPP), and investigated the algorithm to solve it, by using a new technique with optimal average (O_{AV}).
The computer application of our algorithm also has been discussed by solving numerical examples. Also, we have been shown results in tables.

2. Mathematical Form of the Multi-Objective Complementary Programming Problem (MOCPP):

A Multi-Objective Linear Programming Problem (MOLPP) is solved by Chandra Sen in (1983) [5]; Sulaiman& Othman (2007) [8] and suggested an approach to construct the multi-objective function.
As pointed in section (1), Ibaraki [4], defined a new type of optimization problem, known as complementary problem with the addition complementary condition (u.v=0); in addition to this we have Sulaimam, N.A.'s work[7].

By depending and searching of those scientists, experts researches the mathematical form of this type of problem (MOPP) is given as follows:

$$\left.\begin{array}{l} Max.Z1 = d1.x + e1.u + f1.v \\ Max.Z2 = d2.x + e2.u + f2.v \\ . \\ . \\ . \\ Max.Zr = dr.x + er.u + fr.v \\ Max.Z_{r+1} = d_{r+1}.x + er+1.u + fr+1.v \\ . \\ . \\ . \\ Max.Zs = ds.x + es.u + fs.v \end{array}\right\} \quad \dots (1.1)$$

Subject to:

$$A.x+B.u+C.v=b \quad \dots (1.2)$$

$$u.v=0 \quad \dots (1.3)$$

$$x, u, v \geq 0 \quad \dots (1.4)$$

Where; r is the number of objective functions to be maximized; s is the number of objective functions to be maximized and minimized, s-r is the number of objective functions to be minimized, x, u, v are n, m, and m dimensional vectors of variables respectively; d_i,c_i and f_i are vectors of constants; $\forall$ i= 1, 2,…, r, r+1,…, s ; b is p- dimensional vector of constants; A, B and C are $p\times n$, $p\times m$ and $p\times m$ matrices of constants respectively.

3. Formulation of Multi-Objective Complementary Programming Problem Functions:

The formulation of multi-objective complementary programming problem functions given in the form: (1.1), subject to: (1.2), (1.3) & (1.4) is obtaining as; suppose we obtained a single value corresponding to each of objective functions of it being optimized individually as follows:-

$$\left.\begin{array}{l} Max.Zi = \phi i; \forall i = 1,2,\ldots,r \\ Min.Zi = \phi i; \forall i = r+1, r+2,\ldots,s \end{array}\right\} \quad \ldots(1.5)$$

Where; $\phi i = 1,2,\ldots,r, r+1, r+2,\ldots,s.$ the decision variables may not necessarily be common to all optimal solutions in the presence of conflicts among objectives [6]. But the common set of decision variables between objective functions is necessary in order to select the best compromise solution [2].

We can determine the common set of decision variable from the following combined objective function[5],[6],[1] which formulate the MOCPP given in (1.1) as:-

$$\text{Max.Z}= \sum_{k=1}^{r} Zk / |\phi k| - \sum_{k=r+1}^{s} Zk / |\phi k| \ , \ \forall \phi k \neq 0. \qquad \ldots (1.6)$$

Subject to the same constraints (1.2), (1.3) & (1.4), and the optimum value of $\phi k \in R$ - {0}, where R is the set of real numbers.

The equation (1.6) can be solving by Chandra Sen (C_A), [5], [1]; Sulaiman& Othman (2007) [8] Approach the solution of MOLPP (MOCPP without u.v=0… (1.3)) by using optimal average (O_{AV}) is better than the solution by C_A; so we solve (1.6) subject to the same constraints ;(1.2), (1.3) & (1.4), by using O_{AV}.

4. Solving the MOCPP by using Optimal Average (O_{AV}):

Optimal Average (O_{AV}), defined in [8] as:

$O_{AV}= (m_1+m_2)/2$; where:

m_1= min of the absolute value of the maximum value of Z_i, for all i= 1, 2… r.

m_2= min of the absolute value of the minimum value of Z_i, for all i= r+1, r+2… s.

The (MOPP)' s solution by using O_{AV}, is obtained by replacing O_{AV} in state of $|\phi k|$, in Eq. (1.6), subject to the same constraints ;(1.2), (1.3) & (1.4). Thus the formulation become as follows:-

$$\text{Max.Z}= \left(\sum_{i=1}^{r} Max.Zi - \sum_{i=r+1}^{s} Min.Zi\right)/O_{AV} \quad \Rightarrow$$

Max.Z=d.x+e.u+f.v … (1.7)

Where; x, u, v are n, m and m dimensional vectors of variables respectively; d, e, f are n, m and m dimensional vectors of constants respectively.

As, defined and known [4], [7] this type of optimization problem separable, that means without the complementary condition (u.v=0… (1.3)); the problem (1.7), subject to; ;(1.2), (1.3) & (1.4) is an ordinary LPP which can be solving by simplex method; after that verify or to checkout of the complementary condition (u.v=0… (1.3)) may be searching for.

5. Program Notations:

In this work we used the same notations in section (5.3) in [8], with external the complementary notations as:

Max.Z= d_1, with 0.v=0.and Max.Z= d_2, with u.0=0.

6. Algorithm:

The following algorithm is to obtain the optimal solution for MOCLPP defined previous can be summarized as follows:-

STEP1: Find the value of each of individual objective functions which is to be maximized or minimized.

STEP2: Solve the first objective problem by simplex method.

STEP3: Check the feasibility of the solution in step2. If it is feasible then go to step 4, otherwise, use dual simplex methods to remove infeasibility.

STEP4: assign a name to the optimum value of the first objective function Z_1 say ϕA_1

STEP5: Repeat the step2; i= 1,2,3,4 for the kth objective problem, $\forall$ k=2,3,…, r, r+1,…s.

STEP6: Select m_1= min $\{\phi A_i\}$, $\forall$ i= 1, 2… r.

m_2= min $\{\phi A_i\}$, $\forall$ i=r+1, r+2… s

Calculate $O_{AV} = \frac{1}{2}(m_1+m_2)$.

STEP7: Optimize the combined objective function order the same constraints; (1.2), (1.3) and (1.4) as:

Max.Z= $(\sum_{i=1}^{r} Max.Zi \, . - \sum_{i=r+1}^{s} Min.Zi)/O_{AV}$, say:

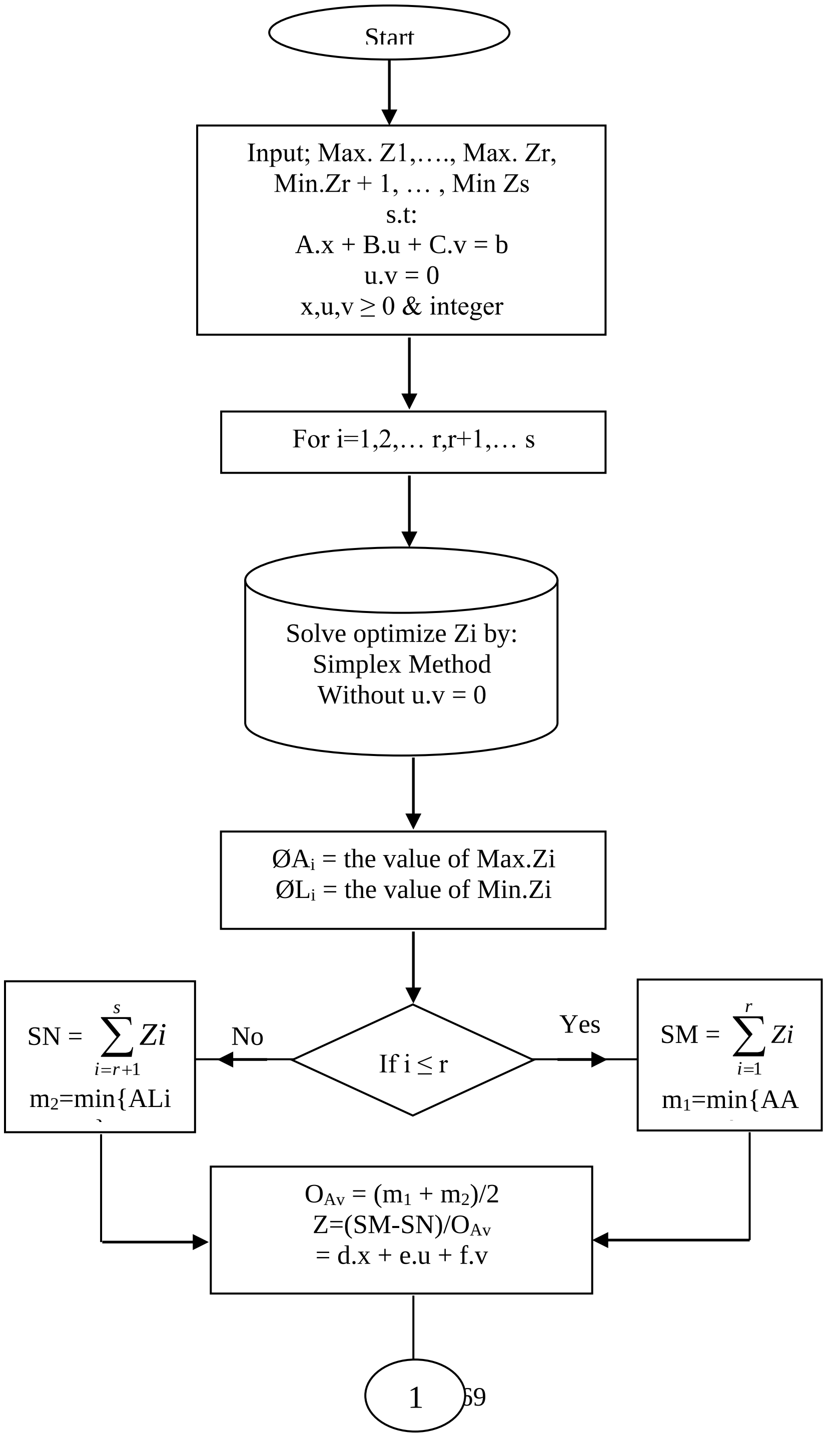
Max.Z=d.x+e.u+f.v … (1.7).

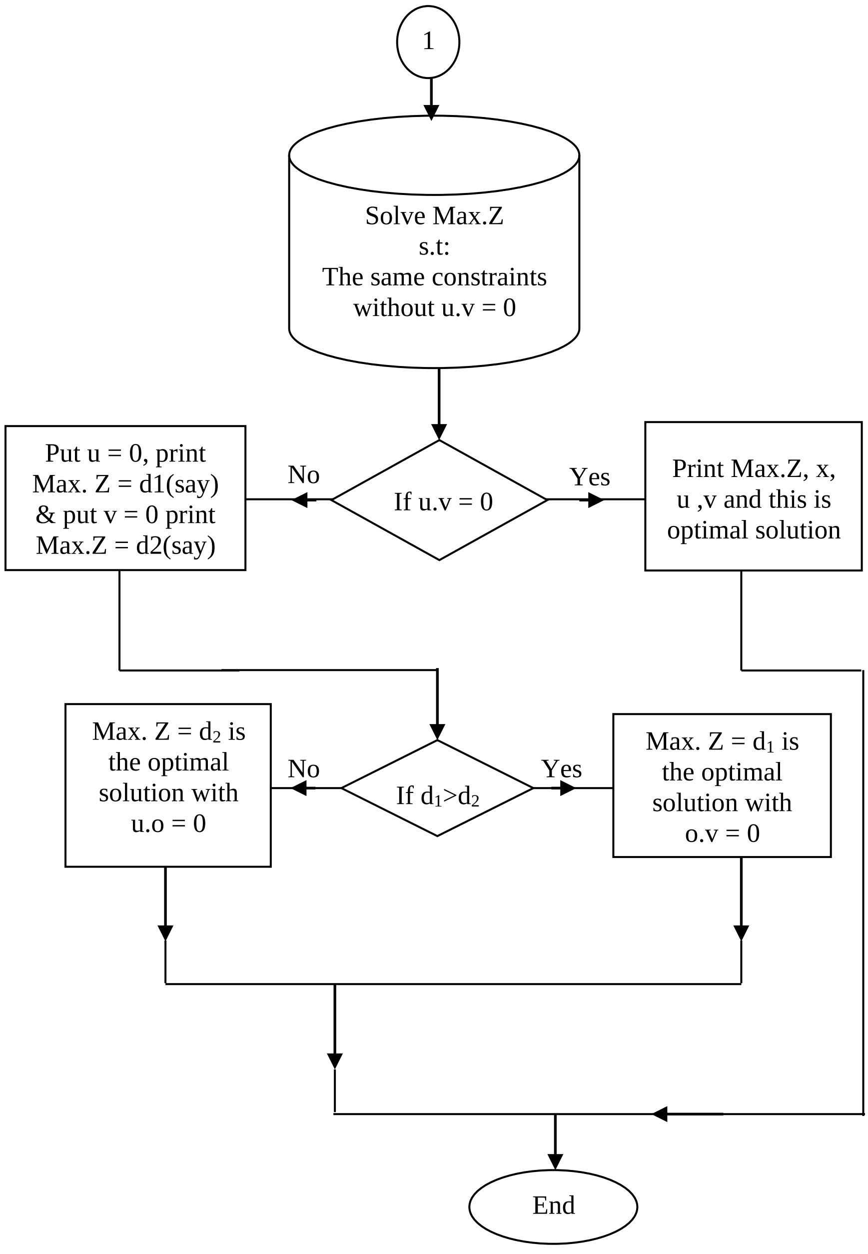
STEP8: Solve (1.7) subject to: (1.2), and (1.4), by simplex method.

STEP9: If u.v=0, the optimal solution obtained and print Max.Z, x, u, and v; otherwise go to STEP10.

STEP10: Satisfy u.v=0, once by putting u=0 and print Max.Z=d_1 (say); others by putting v=0 and print Max.Z=d_2 (say); if $d_1 > d_2$ that means Max.Z, with the complementary condition u=0and v has its values is the optimal solution and vice versa.

7. Flow-Chart:



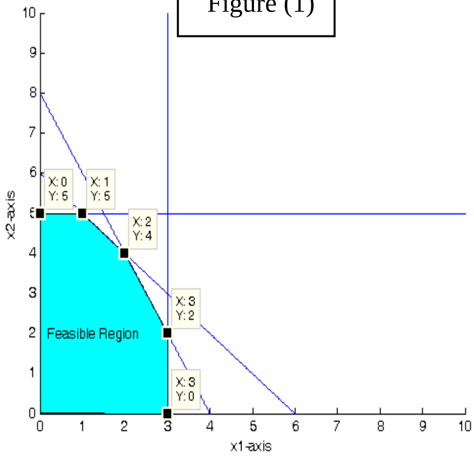

1
Solve Max.Z
s.t:
The same constraints
without u.v = 0
If u.v = 0
No
Yes
Put u = 0, print
Max. Z = d1(say)
& put v = 0 print
Max.Z = d2(say)
Print Max.Z, x,
u ,v and this is
optimal solution
If d1>d2
No
Yes
Max. Z = d2 is
the optimal
solution with
u.o = 0
Max. Z = d1 is
the optimal
solution with
o.v = 0
End

8: Numerical Examples:

<u>Example (1):</u> solve the following MOCPP:

$$\left.\begin{aligned}
&Max.Z1 = 2x1 + 1x2\\
&Max.Z2 = 4x1 - 1x2\\
&Max.Z3 = 1x1 + 2x2\\
&Max.Z4 = 3x1 + 1x2\\
&Max.Z5 = 3x1 + 2x2\\
&Max.Z6 = 2x1 - 3x2\\
&Min.Z7 = -x1 + 1x2\\
&Min.Z8 = 1x1 - 1x2\\
&Min.Z9 = -2x1 - 1x2\\
&Min.Z10 = 1x1 - 3x2\\
&s.t:\\
&1x1 + 1x2 \le 6\\
&8x1 + 4x2 \le 32\\
&1x1 + 0x2 \le 3\\
&0x1 + x2 \le 5\\
&x1.x2 = 0\\
&x1, x2 \ge 0 \;\&\; \text{int}.
\end{aligned}\right\} \quad \ldots(1.8).$$

Figure (1)



First we solve each objective function in (1.8), subject to the given constraints without the complementary condition $x_1.x_2=0$ individually the results as in the table (1) below:

i	Z_i	X_i	ϕi	AA_i	AL_i	m_1	m_2	O_{AV}
1	8	(3,2)	8	8		6	3	4.5
2	12	(3,0)	12	12				
3	11	(1,5)	11	11				
4	11	(3,2)	11	11				
5	14	(2,4)	14	14				
6	6	(3,0)	6	6				
7	-3	(3,0)	-3		3			
8	-5	(0,5)	-5		5			
9	-8	(3,2)	-8		8			
10	-15	(0,5)	-15		15			

Table:(1);Results of the values of the objective functions, and the value of O_{AV}.

Now; by using (1.7) subject to the given constraints we get:

$$\left.\begin{array}{l} Max.Z = 3.5556x1 + 1.3333x2 \\ s.t. \\ 1x1 + 1x2 \le 6 \\ 8x1 + 4x2 \le 32 \\ 1x1 + 0x2 \le 3 \\ 0x1 + 1x2 \le 5 \\ x1.x2 = 0 \\ x1, x2 \ge 0, \& \text{ int}. \end{array}\right\} \quad \ldots(1.9)$$

Let's, solve (1.9), without the complementary condition ($x_1.x_2=0$), by simplex method we get that:

```
>> simplex('max',[3.5556 1.3333],[1 1;8 4;1 0;0 1],[6;32;3;5],'y')
```

Tableaux of the Simplex Algorithm

Initial tableau

A =

1.0000	1.0000	1.0000	0	0	0	6.0000
8.0000	4.0000	0	1.0000	0	0	32.0000
1.0000	0	0	0	1.0000	0	3.0000
0	1.0000	0	0	0	1.0000	5.0000
-3.5556	-1.3333	0	0	0	0	0

Press any key to continue...

x =

3 2

Final tableau

A =

0	0	1.0000	-0.2500	1.0000	0	1.0000
0	1.0000	0	0.2500	-2.0000	0	2.0000
1.0000	0	0	0	1.0000	0	3.0000
0	0	0	-0.2500	2.0000	1.0000	3.0000
0	0	0	0.3333	0.8890	0	13.3334

Press any key to continue...

Max.Z= 13.3334 at the extreme point (3, 2). Since $x_1.x_2 \neq 0$, hence this result isn't optimal solution to the MOCPP.

If $x_1=0$, $x_2=2 \Rightarrow$ Max.Z=2.6666.

If $x_2=0$, $x_1=3 \Rightarrow$ Max.Z=10.6668.

Since 10.6668>2.6666, hence Max.Z=10.6668 at the extreme point (3, 0) is optimal solution to the MOCPP.

<u>Example (2):</u> solve the following MOCPP:-

$$\left.\begin{aligned}
&Max.Z1 = 12x1 + 20x2 + 18x3\\
&Max.Z2 = 20x1 + 12x2 + 18x3\\
&Max.z3 = 12x1 + 18x2 + 20x3\\
&Min.Z4 = 18x1 - 20x2 - 12x3\\
&Min.Z5 = -18x1 + 20x2 - 12x3\\
&s.t:\\
&1x1 + 3x2 + 2x3 \le 9\\
&3x1 + 2x2 + 1x3 \le 8\\
&2x1 + 1x2 + 3x3 \le 7\\
&x2.x3 = 0\\
&x\,i \ge 0\ \&\ \text{int}.;\forall\, i = 1,2,3.
\end{aligned}\right\}. \qquad \ldots(1.10)$$

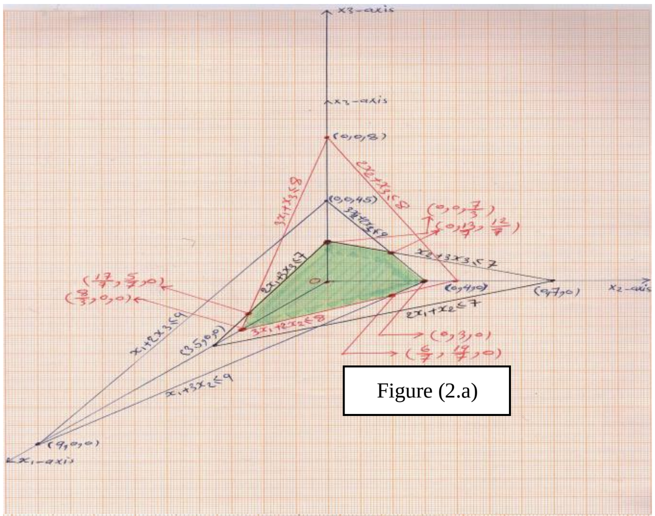


Figure (2.a)

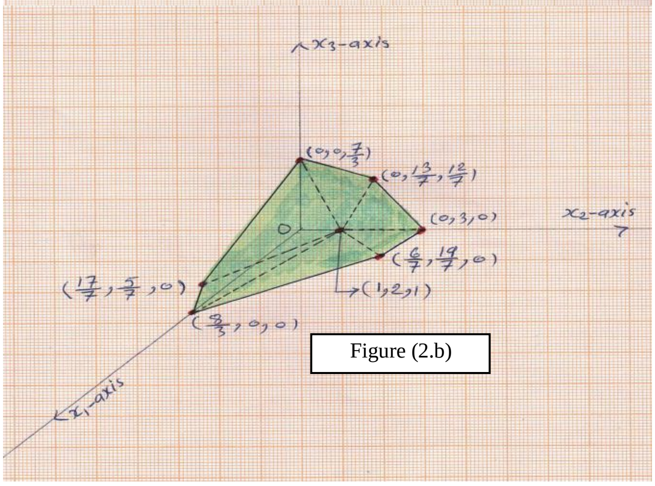


Figure (2.b)

Solution: First we solve each objective function in (1.10), subject to the given constraints without the complementary condition $x_2.x_3=0$ individually the results as in the table (2) below:

i	Z_i	$X_i(x_1,x_2,x_3)$	ϕi	AA_i	AL_i	m_1	m_2	O_{AV}
1	70	(1,2,1)	70	70		62	52. 2857	57. 1428
2	62	(1,2,1)	62	62				
3	68	(1,2,1)	68	68				
4	-60	(0,3,0)	-60		60			
5	-52. 2857	(2.4286, 0,0.7143)	-52. 2857		52. 2857			

Table: (2); Results of the values of the objective functions, and the value of $O_{AV.}$

Now; by using (1.7) subject to the given constraints we get:

$$\left.\begin{aligned}&Max.z = 0.77x1 + 0.875x2 + 1.4x3\\&s.t:\\&1x1 + 3x2 + 2x3 \le 9\\&3x1 + 2x2 + 1x3 \le 8\\&2x1 + 1x2 + 3x3 \le 7\\&s.t: i \ge 0, \&\text{int}.; \forall i = 1,2,3.\end{aligned}\right\}. \qquad \ldots(1.11)$$

Let's, solve (1.11), without the complementary condition ($x_2.x_3=0$), by simplex method we get that:

```
>> Simplex ('max',[0.77 0.875 1.4],[1 3 2;3 2 1;2 1 3],[9;8;7],'y')
________________________________________________________
Tableaux of the Simplex Algorithm
________________________________________________________
Initial tableau
A =
   1.0000   3.0000   2.0000   1.0000   0        0        9.0000
   3.0000   2.0000   1.0000   0        1.0000   0        8.0000
   2.0000   1.0000   3.0000   0        0        1.0000   7.0000
  -0.7700  -0.8750  -1.4000   0        0        0        0
Press any key to continue...
x =
       0    1.8571    1.7143
```

Final tableau

A =

-0.1429	1.0000	0	0.4286	0	-0.2857	1.8571
2.5714	0	0	-0.7143	1.0000	0.1429	2.5714
0.7143	0	1.0000	-0.1429	0	0.4286	1.7143
0.1050	0	0	0.1750	0	0.3500	4.0250

Press any key to continue...

Max.Z= 4.0250 at the point (0, 1.8571, 1.7143), which is fractional in x_2 and x_3.

So, the best integral solution which we can obtain by using the Branch-and-Bound Procedure (Method) is:

>> Max.Z=0.77*1+0.875*2+1.4*1

Max =

Z: 3.9200, at the extreme point (1, 2, 1).

Now, we are going to satisfy the complementary condition ($x_2.x_3=0$), because $x_2.x_3 \neq 0$.

If $x_2=0$, then:

>> Max.Z=0.77*1+0.875*0+1.4*1

Max =

Z: 2.1700, at the point (1, 0, 1).

If $x_3=0$, then:

>> Max.Z=0.77*1+0.875*2+1.4*0

Max =

Z: 2.5200, at the point (1, 2, 0).

Since, 2.52>2.17, hence the optimal solution is:

Z: 2.5200, at the point (1, 2, 0).

Example (3): solve the following MOCPP:-

$$\left.\begin{array}{l} Let: x=(x1), u=(x2), v=(x3,x4) \\ Max.Z1 = 2x1+1x2+4x3+1x4 \\ Max.Z2 = -5x1+8x2-3x3+2x4 \\ Max.Z3 = 1x1+10x2+7x3-3x4 \\ Max.Z4 = 4x1+1x2+3x3+4x4 \\ Max.Z5 = 6x1+3x2+1x3+2x4 \\ Min.Z6 = 4x1-7x2+1x3-1x4 \\ Min.Z7 = 1x1+4x2-2x3-3x4 \\ s.t: \\ 2x1+3x2-1x3+1x4 \le 18 \\ -3x1+1x2+2x3+4x4 \le 12 \\ 1x1+1x2+1x3+5x4 \le 22 \\ 2x1+3x2+4x3-2x4 \le 14 \\ u.v=0 \\ x,u,v \ge 0 \;\&\; \text{int}. \end{array}\right\}. \qquad \ldots(1.12)$$

Solution: After solving each objective function in (1.12), without the complementary condition (u.v=0) individually, subject to the given constraints by simplex method, results obtained as following in table (3) below:

i	Z_i	$X_i(x_1,x_2,x_3,x_4)$	ϕi	AA_i	AL_i	m_1	m_2	O_{AV}
1	23.3673	(2.6531,0,3.7347,3.1224)	23.3673	23.3673		23.3673	14.1837	18.7755
2	47.1111	(0,5.5556,0,0,1.3333)	47.1111	47.1111				
3	51.5556	(0,5.5556,0,0,1.3333)	51.5556	51.5556				
4	45.1481	(8.0741,0,0.7778,2.6296)	45.1481	45.1481				
5	54.4815	(8.0741,0,0.7778,2.6296)	54.4815	54.4815				
6	-40.2222	(0,5.5556,0,0,1.3333)	-40.2222		40.2222			
7	-14.1837	(2.6531,0,3.7347,3.1224)	-14.1837		14.1837			

Table: (3); Results of the values of the objective functions, and the value of O_{AV}.

Now; by using (1.7) subject to the given constraints we get:

$$\left.\begin{aligned}
&Let: x=(x1), u=(x2), v=(x3, x4)\\
&Max.Z=0.1598x1+1.3848x2+0.6924x3+0.5326x4\\
&s.t:\\
&2x1+3x2-1x3+1x4\le 18\\
&-3x1+1x2+2x3+4x4\le 12\\
&1x1+1x2+1x3+5x4\le 22\\
&2x1+3x2+4x3-2x4\le 14\\
&u.v=0\\
&x,u,v\ge 0\,\&\,\text{int}.
\end{aligned}\right\}. \qquad \ldots(1.13)$$

Let's, solve (1.13), without the complementary condition (u.v=0), by simplex method we get that:

>> simplex('max',[0.1598 1.3848 0.6924 0.5326],[2 3 -1 1;-3 1 2 4;1 1 1 5;2 3 4 -2],[18;12;22;14],'y')

>>____

Tableaux of the Simplex Algorithm

____

Initial tableau

A =

2.0000	3.0000	-1.0000	1.0000	1.0000	0	0	0	18.0000
-3.0000	1.0000	2.0000	4.0000	0	1.0000	0	0	12.0000
1.0000	1.0000	1.0000	5.0000	0	0	1.0000	0	22.0000
2.0000	3.0000	4.0000	-2.0000	0	0	0	1.0000	14.0000
-0.1598	-1.3848	-0.6924	-0.5326	0	0	0	0	0

Press any key to continue...

x =

0 5.5263 0.1316 1.5526

Final tableau

A =

-0.7237	0	0	1.0000	0.0263	0.1974	0	-0.0921	1.5526
-0.4342	0	1.0000	0	-0.1842	0.1184	0	0.1447	0.1316
4.2895	0	0	0	-0.2105	-1.0789	1.0000	0.2368	8.5789
0.7632	1.0000	0	0	0.2632	-0.0263	0	0.0789	5.5263
0.2109	0	0	0	0.2509	0.1507	0	0.1605	8.5709

Press any key to continue...

Max.Z= 8.5709 at the point (0, 5.5263, 0.1316, 1.5526), which is fractional in x_2, x_3 and x_4.

So, the best integral solution which we can obtain by using the Branch-and-Bound Procedure (Method) is:

>> Max.Z=0.1598*2+1.3848*4+0.6924*1+0.5326*3

Max =

Z: 8.1490, at the extreme point (2, 4, 1, 3).

Now, we are going to satisfy the complementary condition (u.v=0), because u.v≠0.
If, u=(x_2) =0, then:
 >> Max.Z=0.1598*2+1.3848*0+0.6924*1+0.5326*3
Max =
 Z: 2.6098, at the (2, 0, 1, 3).
If, v=(x_3, x_4) =0, then:
>> Max.Z=0.1598*2+1.3848*4+0.6924*0+0.5326*0
Max =
 Z: 5.8588, at the (2, 4, 0, 0).
Since, 5.8588 >2.6098, hence the optimal solution is:
 Z: 5.8588, at (2, 4, 0, 0).

9: Comparing Results:

We compare the results which were obtained by solving the numerical examples as ordinary MOLPP, MOPP with integer variables and MOCPP as following in the table (4):

Example	MOLPP		MOLPP		MOCPP	
			with integer variables			
	Max.Z	X	Max.Z	X	Max.Z	X
(1)	13.3334	(3,2)	13.3334	(3,2)	10.6668	(3,0)
(2)	4.0250	(0,1.8571, 1.7143)	3.9200	(1,2,1)	2.5200	(1,2,0)
(3)	8.5709	(0,5.5263, 0.1316,1.5526)	8.1490	(2,4,1,3)	5.8588	(2,4,0,0)

Table (4) Comparing results between: MOLPP and MOLPP& MOCPP with integer variables.

REFERENCES

[1] Abdil-Kadir, M. S. & Sulaiman, N. A. (1993) "*An Approach for Multiobjective Fractional Programming*" Journal of College of Education, Uni. Salahaddin, Vol.3, No.1, pp (1-5).

[2] Cheney, W. & Kincaid, D. (2004) "*Numerical Mathematics and Computing*" Fifth Edition, Thomson.

[3] Gary, K. C. & Swarup, K. *(1983) "Complementary with Extreme Point Optimization*", Proceeding Operations Research Society of India.

[4] Ibaraki, T. (1971) "*Complementary Programming*" Operational Research, Vol.19, No.6, pp (1523-1529).

[5] Sen, Ch. (1983) "*A new approach objective planning*", The Indian Economic Journal Vol.30, no.4, pp (91-96).

[6] Sulaiman, N. A.& Sadiq, G. W.(2006) " *Solving the Multiobjective Programming Problem; Using Mean and Median Value*" Ref. J. of Com.& Math's, Vol.3, No.1.

[7] Sulaiman, N. A. (1997) "*Upper-Bound Cut for Extreme Point Multi-Objective Complementary Problem*", Zanco special Issue (1) proceeding of Third scientific.

[8] Sulaiman, N.A. & Othman, A.H. (2007)" Optimal Transformation Technique for Solve Multi-Objective Linear Programming Problems (MOLPP)", Journal of University of Kirkuk, Vol.2, No.2.