Geometric Analysis of Images for 3D Models Reconstruction

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ABSTRACT

Determining three-dimensional (x, y, z) to a point exists in the form of three-dimensional images of two-dimensional method is complex and requires high precision to derive the equations that were special to represent the three-dimensional geometric figure who is in fact in this research.

To derive the equations for this purpose rely on certain constants in order to properly process the account. These constants is the status of the camera, the reference point in the picture and the distance between the image the camera moves to another.

Applying the equations are extracting values triple with finding the least error rate, which represents the accuracy of work and comparing the result with the real values of reality.

Keywords: Geometric Analysis, Images for 3D Models Reconstruction.

التحليل الهندسي للصور ثلاثية الأبعاد

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الملخص

تحديد الإبعاد الثلاثية (x,y,z) لنقطة موجودة في شكل ثلاثي الإبعاد من صور ثنائية الأبعاد هي طريقة معقدة وتحتاج إلى دقة عالية لذلك تم اشتقاق معادلات خاصة لتمثيل الشكل الهندسي الثلاثي الإبعاد الموجود في الواقع في هذا البحث.

إن اشتقاق المعادلات الخاصة لهذا الغرض تعتمد على ثوابت معينة لكي تتم عملية الحساب. هذه الثوابت تتمثل بمركز الكاميرا , النقطة المرجعية في الصورة و المسافة التي تتحركها الكاميرا بين صورة وأخرى.

بتطبيق المعادلات يتم استخراج القيم الثلاثية مع إيجاد اقل نسبة خطأ والتي تمثل دقة العمل ومقارنة النتيجة مع القيم الحقيقة من الواقع.

الكلمات المفتاحية: التحليل الهندسي، صور ثلاثية الأبعاد.

1. Introduction:

Modeling of 3Dimension (3D) objects from image sequences is a challenging problem and has been a research topic for many years. Hartley and Faugeras in 1992 concluded the problem of camera calibration and 3D reconstruction can be approached in different ways. When both intrinsic and extrinsic parameters of a vision system are known, the 3D reconstruction can thus be simply realized by a traditional triangulation method.

But when the parameters of the vision system are totally uncalibrated, the 3D structure can be reconstructed up to a projective transformation from two uncalibrated images [1, 2]. Marc Pollefeys et al 2000 discussed an automatic 3D scene modeling technique that is capable of building models from uncalibrated image sequences. The technique was able to extract metric 3D models without any prior knowledge about the scene or the camera. The calibration is obtained by assuming a rigid scene and some constraints on the intrinsic camera parameter (e.g. square pixels). The goal of this work is achieved by combining state-of-the–art algorithms for uncalibrated projective reconstruction, self –calibration and dense correspondence matching [3]. Jin 2002 suggested a method of computing a shape from two images without using a calibration target. First, the camera system is constrained such that intrinsic and extrinsic parameters are limited to a minimum number. The parameter values of the restricted system are used as initial values to a more general configuration. This method is used when restricting the motion of two cameras [4]. Tang and Ng in 2006 proposed a 3D reconstruction line segment by minimizing the sum of orthogonal distances of the reprojected end points of the 3D segment from the measured 2Dimension (2D) line in all the images .The problem of line reconstruction is thus formulated purely on the basis of line correspondences in multiple views [5].

2. 2D to 3D coordinates transformation:

Camera calibration and 3D reconstruction have been studied for many years, but it is still an active research to pick in robot vision. This problem is related to structure of motion, pose determination, etc. This results in its application including object modeling, mobile robot navigation, localization and environments building [6]. The points in image represented in 2D coordinate (u,v) can be transformed to the 3D coordinate (x,y,z) in scene space by depending on the following aspects:

Given a point (u_0, v_0) in 2D an image that corresponds to point (x, y, z) in 3D real coordinates. The relationships between the 2D and 3D points are shown in the following equations:

$$u_{0(2D)} = L\left[\frac{X_{(3D)}}{L+Z_{(3D)}}\right] \qquad \ldots(1)$$

$$v_{0(2D)} = L\left[\frac{Y_{(3D)}}{L+Z_{(3D)}}\right] \qquad \ldots(2)$$

Where L is focal length of image plane.

$u_{0(2D)}$,$v_{0(2D)}$: is coordinate in first image plane

$X_{(3D)}$,$y_{(3D)}$, $Z_{(3D)}$ is coordinate point in 3D space.

Equations (1) and (2) are used for deriving the all cases of camera motion. The first camera capturing point (x,y,z) is known as space scene. The following cases which were taken by camera for calculating the 3D point in scene from 2D coordinate space are shown below:

2.1 Translation camera as Horizontal Straight Line only:

In this case is supposed that after capturing the first image, the camera is translated to another location horizontally to capture the second image. The point that is needed to reconstruct 3D must be appeared in the two scenes. So the equation (3) is used in this case by transferring camera with (D) distance space that is measured by centimeter (cm). Figure (1) illustrates this case.

$$u_{1(2D)} = L\left[\frac{X_{(3D)}-D}{L+Z_{(3D)}}\right] \qquad \ldots(3)$$

$$v_{1(2D)} = L\left[\frac{Y_{(3D)}}{L+Z_{(3D)}}\right] \qquad \ldots(4)$$

Where:

L : represent focal length .

$u_{1(2D)}, v_{1(2D)}$: represent the width and height coordinate from second image.
$X_{(3D)}, Y_{(3D)}, Z_{(3D)}$: represent the width, height and depth coordinate from real world.

By making a beneficial use of equations (1 - 4), the result gives the following equations.

$$X_{3D} = \left[\frac{D\, u_{0(2D)}}{u_{0(2D)} - u_{1(2D)}}\right] \qquad \ldots(5)$$

$$Y_{3D} = \left[\frac{D\, v_{0(2D)}}{u_{0(2D)} - u_{1(2D)}}\right] \qquad \ldots(6)$$

$$Z_{3D} = \left[\frac{DL}{u_{0(2D)} - u_{1(2D)}} - L\right] \qquad \ldots(7)$$

2.2 Translation camera as Horizontal Straight Line with Rotation:

In this case it is supposed that after capturing the first image, the camera is translated to another location as horizontally and with rotation by any angle (θ) to capture the second image. Also the point that is needed to reconstruct 3D must be appeared in the two scenes. So the equations (8, 9) are used in this case and illustrated in Figure (2).

$$u_{1(2D)} = \frac{X_{1(3D)}}{Z_{1(3D)} + \frac{L}{\cos\theta}}\; L \qquad \ldots(8)$$

$$v_{1(2D)} = \frac{Y_{(3D)}}{Z_{1(3D)} + \frac{L}{\cos\theta}}\; L \qquad \ldots(9)$$

Where: $X_{1(3D)} = X_{(3D)}\cos\theta + Z_{(3D)}\sin\theta$

$Z_{1(3D)} = Z_{(3D)}\cos\theta - X_{(3D)}\sin\theta$

$\tan\theta = \frac{D}{L}$

$u_{1(2D)}, v_{1(2D)}$: are 2D coordinate represents width and height from second image
$X_{1(3D)}, Z_{1(3D)}$: are 3D coordinate represents width and depth after camera rotation and translation in scene

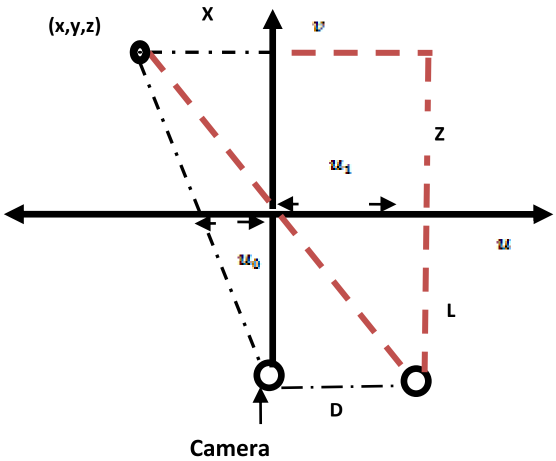

Figure (1) Translation camera as horizontal line

By applying the equations (1), (2) to (3), (4). The equations of 3D coordinate in scene are illustrated in the following:

$$X_{3D} = u_{0(2D)} + \frac{u_{0(2D)} L v_{1(2D)} - u_{0(2D)}^2 \sin\theta\cos\theta\, v_{1(2D)} - v_{0(2D)}\cos\theta\, L u_{0(2D)}}{\cos\theta\,(v_{0(2D)} L - v_{1(2D)}\cos\theta\, L + u_{0(2D)}\sin\theta\, v_{1(2D)})} \qquad \ldots(10)$$

$$Z_{3D} = \frac{L^2 v_{1(2D)} - u_{0(2D)} L\sin\theta\cos\theta\, v_{1(2D)} - v_{0(2D)}\cos\theta\, L^2}{\cos\theta\,(v_{0(2D)} L - v_{1(2D)}\cos\theta\, L + u_{0(2D)}\sin\theta\, v_{1(2D)})} \qquad \ldots(11)$$

$$y_{3D} = v_{0(2D)} + \frac{v_{0(2D)} L v_{1(2D)} - u_{0(2D)} v_{0(2D)} \sin\theta \cos\theta v_{1(2D)} - v_{0(2D)}^{2} \cos\theta L}{\cos\theta \left(v_{0(2D)} L - v_{1(2D)} \cos\theta L + u_{0(2D)} \sin\theta v_{1(2D)}\right)} \quad \ldots(12)$$

3. Interpolation point from two images:

In ideal nondiscrete images, if a point in first image is(x_{11}, y_{11}) and the same point appears in a horizontally transmit image as(x_{21}, y_{21}). The lines between camera locations and the point in corresponding image must intersect at specific point (x_1, y_1, z_1) which is the real coordinates of the point.In the following demonstrated case the two images are discrete so the two lines from camera location and the point in the corresponding image will not intersect. In state of calculating the point of intersection the nearest two points of the two lines are to be evaluated.

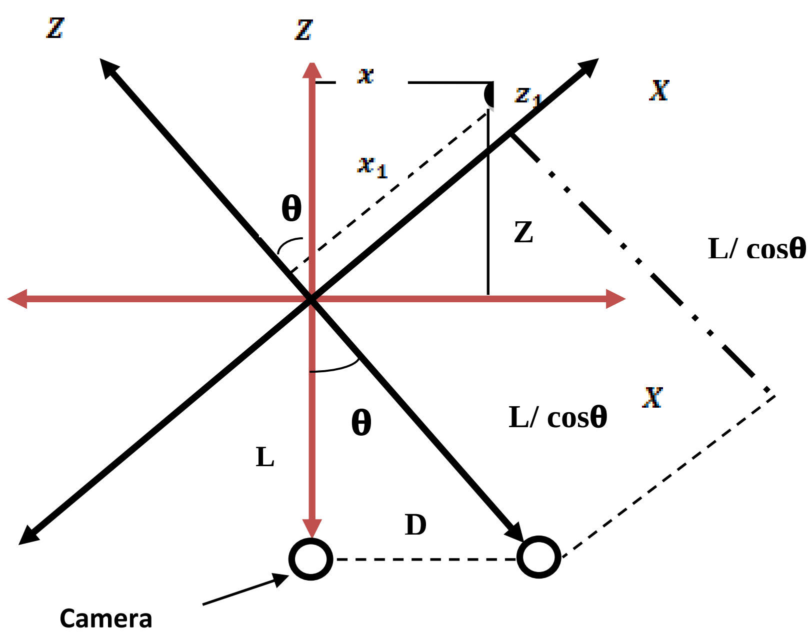

Figure (2) Translation camera horizontally with rotation

The first 3D coordinate is (x_f, y_f, z_f) from the line that passes the first camera location and the point (x_{11}, y_{11}), and the second 3D coordinate (x_g, y_g, z_g) is from the line that passes the second camera location and the point (x_{21}, y_{21}). The relation between these points as the following equations:

$$\frac{x_f}{y_f} = \frac{x_{11}}{y_{11}} \quad \ldots(13)$$

$$\frac{x_{11}}{L} = \frac{x_f}{z_f + L} \quad \ldots(14)$$

$$\frac{x_{21}}{y_{21}} = \frac{x_g - D}{y_g} \quad \ldots(15)$$

$$\frac{x_{21}}{L} = \frac{(x_g - D)}{z_g + L} \quad \ldots(16)$$

Where:

x_f, y_f, z_f: are represented the points width, height and depth from first camera .

x_g, y_g, z_g: are represented the points width, height and depth from second camera .

x_{11}, y_{11} : are represented the coordinate from first image.

x_{21}, y_{21} : are represented the coordinate from second image.

The following equation is to find the distance between any a point (f) from first image and point (g) from second image:

$$Q = \left(\left(x_g - x_f\right)^2 + \left(y_g - y_f\right)^2 + \left(z_g - z_f\right)^2 \right) \quad \ldots(17)$$

From equations (13-16) the following equations results are:

From image 1 the following equations are obtained

$$y_f = x_f \frac{y_{11}}{x_{11}} \quad \ldots(18)$$

$$z_f = \frac{x_f L}{x_{11}} - L \quad \ldots(19)$$

From image 2 the following equations:

$$y_g = \left(x_g - D\right) \frac{y_{21}}{x_{21}} \quad \ldots(20)$$

$$z_g = \frac{\left(x_g - D\right) L}{x_{21}} - L \quad \ldots(21)$$

To minimize the distance $\frac{\partial Q}{\partial x + k}$ should be equal to zero for all x. The equations below are result:

$$\frac{\partial Q}{\partial x_f} = -2\left(x_g - x_f\right) + 2\left(y_g - y_f\right)\left(y'_g - y'_f\right) + \left(z_g - z_f\right)\left(z'_g - z'_f\right) \quad \ldots(22)$$

$$\frac{\partial Q}{\partial x_g} = 2\left(x_g - x_f\right) + 2\left(y_g - y_f\right)\left(y^{*}_g - y^{*}_f\right) + \left(z_g - z_f\right)\left(z^{*}_g - z^{*}_f\right) \quad \ldots(23)$$

In simplicity the equation (22) and (23) and equal to zero the result are:

$$-x_g \left(1 + \frac{y_{21} y_{11} + L^2}{x_{21} x_{11}}\right) + x_f \left(1 + \frac{y_{11} y_{11} + L^2}{x_{11} x_{11}}\right) + D \frac{y_{21} y_{11} + L^2}{x_{21} x_{11}} = 0 \quad \ldots(24)$$

$$x_g \left(1 + \frac{y_{21} y_{21} + L^2}{x_{21} x_{21}}\right) - x_f \left(1 + \frac{y_{11} y_{21} + L^2}{x_{11} x_{21}}\right) - D \frac{y_{21} y_{21} + L^2}{x_{21} x_{21}} = 0 \quad \ldots(25)$$

From equations (24) and (25) the value of x_f and x_g was:

$$x_f = \frac{(a-c) x_{11} D}{x_{21}(e+f)} \quad \ldots(26)$$

$$x_g = \frac{D\left(a_2 - b_2\right)}{c_2 - d_2} \quad \ldots(27)$$

Where:

$$a = (y_{11} y_{21} + L^2)(x_{11} x_{21} + y_{11} y_{21} + L^2)(x_{21}^2 + y_{21}^2 + L^2)$$

$$c = (y_{21}^2 + L^2)(x_{21} x_{11} + y_{21} y_{11} + L^2)^2$$

$$e = (x_{11} x_{21} + y_{21} y_{11} + L^2)^3$$

$$f = (x_{21}^2 + y_{21}^2 + L^2)(x_{11}^2 + y_{11}^2 + L^2)(x_{11} x_{21} + y_{11} y_{21} + L^2)$$

$$a_2 = y_{21} y_{11} x_{11} x_{21} + x_{11} x_{21} L^2 + (y_{21} y_{11} + L^2)^2$$

$$b_2 = (y_{21}^2 + L^2) x_{11}^2 - (y_{11}^2 + L^2)(y_{21}^2 + L^2)$$

$$c_2 = (x_{21} x_{11} + y_{21} y_{11} + L^2)^2$$

$$d_2 = (x_{21}^2 + y_{21}^2 + L^2)(x_{11}^2 + y_{11}^2 + L^2)$$

Result value from equation (26) is applyed to the equations (18, 19) to find the values of y_f, z_f and from equation (27) is applyed to the equations (20, 21) to find the values of y_g, z_g . The values must be appeared in nearest together by:

$$y_f \cong y_g , \qquad z_g \cong z_f \qquad \text{and} \qquad x_g \cong x_f$$

4. Reconstruct the point from 2D to 3D coordinates:

In this stage the restructuring points (or locate the points) of the existing images and representation to reality. Will need two photos, at least to define this point. It required picture-taking format engineering from the fact by relying on installing the camera and identify a reference point to the picture taken with the camera.

Taking the second image of the same geometric figure with moving the camera certain distance measured by a centimeter. With attention to be the reference point for the second picture is the same in the first image.

For explaining this step, supposed the line endpoints which needs to be reconstructed from two images are illustrated in Table (1) and these lines are illustrated in Figures (3.a, 3.b). The camera center must be known, so that the centre values as illustrated in Table (1). The distance between two camera centers was 90cm (D). The focal length for these images was 390cm and the scale between the image and scene was (90cm per 323 pixels).

Table -1- endpoints of line from two images.

Images	Start point as(x,y)	End point as (x,y)	Center
First image	**451,102**	**563,316**	**640,467**
Second image	**172,112**	**428,324**	**613,478**

The calculation about this method was, firstly subtracting the endpoints from image

Images / Endpoints	first image	pixel	second image	Pixel
Start points	$x_{11} = x_1 - O_{x1}$	**-189**	$x_{21} = x_1{}' - O_{x2}$	**-441**
	$y_{11} = y_1 - O_{y1}$	**365**	$y_{21} = y_1{}' - O_{y2}$	**366**
End points	$x_{12} = x_2 - O_{x1}$	**-77**	$x_{22} = x_2{}' - O_{x2}$	**-185**
	$y_{12} = y_2 - O_{y1}$	**151**	$y_{22} = y_2{}' - O_{y2}$	**154**

centre as illustrated in table (2).

Table-2-different between pixel coordinate and center coordinate

a b

Figure (3) a. Line location in first image. b. Line location in Second image

To find the minimum value of error for reconstructing 3D points in real scene, the equations that are illustrated previously was applied.

Notice that these points' values are not accurate because the location of center camera is chosen with low accuracy. Table 4 shows the comparison of the resultant points with real point in scene.

Table-3- Error between two images.

Endpoints in first image	Point as x_f, y_f, z_f in cm	Point as x_g, y_g, z_g in cm
Start point	$x_f = -67.51955921$ $y_f = 130.394915$ $z_f = 110.0205603$	$x_g = -67.5188$ $y_g = 130.333$ $z_g = 109.9$
End point	$x_f = -66.28892819$ $y_f = 129.9951709$ $z_f = 814.9274576$	$x_g = -69.19334663$ $y_g = 132.5177271$ $z_g = 814.3807809$

Table -4- Comparison between resultant and real point's results

Points	Actual point from scene		Reconstruction point from two image	
	Start point	End point	Start point	End point
X(width)	**-72cm**	**-72cm**	**-67.519cm**	**-66.193346cm**
Y(height)	**130cm**	**130cm**	**130.333cm**	**129.9951709cm**
Z(depth)	**130cm**	**830cm**	**110.0205cm**	**814.927457cm**

5. Conclusion:

Equations for pretext the 3D point are derived by minimizing the distance between the lines connecting camera positions with the point in the corresponding image. This method needs to be accurate in choosing the points in the picture where there may be a percentage of error and thus may display incorrect values.

It was used to determine how the error rate found in selected points and take the smallest percentage points error and mind are the points that represent the correct values and the real three-dimensional

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