# Comparison of Finite Difference Solution Methods for Reaction Diffusion System in Two Dimensions

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## ABSTRACT

In this paper, we study three types of finite difference methods, to find the numerical solution of reaction difference systems of PDEs in two dimensions. These methods are ADE, ADI and Hopscotch, where Gray-Scott model in two dimensions has been considered. Our numerical results show that the ADI method produces more accurate and stable solution than ADE method and Hopscotch method is the best because does not involve any tridiagonal matrix. Also we studied the consistency, stability and convergence of the above methods.
**Keywords**: Gray-Scott model in two dimensions, finite difference methods, Alternating direction explicit method, Alternating direction implicit method, Hopscotch method.

**مقارنة طرائق حلول الفروقات المنتهية لنظام الانتشار التفاعلي في بعدين**

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## الملخص

في هذا البحث ندرس ثلاث أنواع من طرق الفروقات المنتهية، وبالتفصيل، لإيجاد الحل العددي لنظام من المعادلات التفاضلية الجزئية غير الخطية من نوع القطع المكافئ في بعدين، هذه الطرق هي طريقة المتجهات المتعاقبة الصريحة (ADE) وطريقة المتجهات المتعاقبة الضمنية (ADI) وطريقة (Hopscotch) لنموذج Gray-Scott في فضاء ذات بعدين. واستنتجنا من النتائج العددية أن طريقة المتجهات المتعاقبة الضمنية (ADI) هي أفضل من طريقة المتجهات المتعاقبة الصريحة (ADE) ولكن بالنتيجة إن طريقة Hopscotch هي أفضل من طريقة ADI لأنها لا تحتاج إلى خزن كبير بالإضافة إلى أنها لا تحتوي صيغة المصفوفات الثلاثية الأقطار Tridiagonal. كما درسنا ثبوتية واستقرارية وتقارب لكل من الطرق الثلاثة.

**الكلمات المفتاحية:** أنموذج گراي-سكوت, طرائق الفروقات المنتهية, طريقة المتجهات المتعاقبة الصريحة , طريقة المتجهات المتعاقبة الضمنية , طريقة هبسكوج.

## 1. Introduction

Many Mathematical models, such as partial differential equations, can be used to describe some chemical, physical, biological, fluid flow, electricity systems, etc [6].
Zegeling [16] discussed a Gray-Scott model in two-dimensions for adaptive grid method that is based an a tensor-product approach. Adaptive grids are a commonly used tool for increasing the accuracy and reducing costs when solving both PDEs and ODEs.

A traditional and widely used form adaptively is the concept of equidistribution. Which is well-defined and well-understood in one space dimension? The extension of the equidistribution principle to two or three dimensions, however, is far from trivial and has been the subject of investigation of many researches during the last decade. Besides the non-singularity of the transformation that defines the non-uniform adaptive grid, the smoothness of the grid (or transformation) plays an important role as well. The analysis of these properties and illustrate their importance with numerical experiments for a set of time-dependent PDE models with steep moving pulses, fronts, and boundary layers.

In this paper we study and apply the finite difference methods to approximate the solution and study the consistency, stability and convergence of the numerical solution of a model of nonlinear parabolic partial differential systems which is two dimensional Gray-Scott models [15]. These methods are combinations of finite difference method with

- Alternating direction explicit method (ADE)
- Alternating direction implicit method (ADI)
- Hopscotch method.

First we derive the finite differential form of ADE, ADI and Hopscotch methods for the given model and then present an algorithm for each method. Also we compare between them.

The consistency, stability and convergence for the above methods have been examined.

## 2. The Gray-Scott model in two Dimensions [3]

Reaction-diffusion models of chemical species can produce a variety of patterns, reminiscent of those often seen in nature. The Gray-Scott equations model can be consider as reaction. Numerical simulations of this model were performed in an attempt to find stationary lamellar patterns like those observed in earlier laboratory experiments on ferrocyanideiodate-sulphite reactions [13]. The chemical reactions for this situation are described by

$$U+2V \rightarrow 3V,$$

$$V \rightarrow P,$$

where U, V and P are chemical species. The system of reaction-diffusion equations for this situation is given by

$$u_t = \alpha_1\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)-uv^2+f\left(1-u\right),$$

$$v_t = \alpha_2\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)+uv^2-\left(f+g\right)v.$$

where u and v are concentrations of two reactions, $\alpha_1$ and $\alpha_2$ are the diffusion rates in the process, g represents the rate of conversion of V to P. And f is the rate of the process that feeds U and drains U, V and P ([15], [11]).

Then we choose the model parameters as $\alpha_1 = 8\times10^{-5}$, $\alpha_2 = 4\times10^{-5}$, f=0.02 and g=0.066 to get the model showed in equations (1) and (2)

From pattern formation the following reaction diffusion system [3] exhibits complicated solution behavior:

$$\frac{\partial u}{\partial t} = \alpha_1\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) - uv^2 + 0.02(1-u)\,, \tag{1}$$

$$\frac{\partial v}{\partial t} = \alpha_2\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) + uv^2 - 0.086\, v. \tag{2}$$

where $\alpha_1=8\times10^{-5}$ and $\alpha_2=4\times10^{-5}$
The initial conditions are

$u(x,y,0)=1 \qquad ,0 \le x ,y \le 1,$

$v(x,y,0)=1 \qquad ,0 \le x ,y \le 1,$

on the spatial domain $[0,1]\times[0,1]$.

In this model self-replicating spots have been observed. These are regions in which the (chemical) concentrations of some of the species exhibit large amplitude perturbations from a surrounding homogenous state.

## 3. Numerical Methods

We solve the mathematical model in (1) and (2) with the combination of the finite difference methods with ADE method, ADI method and Hopscotch method.

### 3.1 ADE Method [14]

This method is referred to as alternating direction since a single cycle of computation requires the solution of two different finite difference approximations written in different physical directions. The end result of the two cycles is taken as the answers on the (n+1) plane [7].

When we consider a square region $0 \le x \le 1$, $0 \le y \le 1$ and u, v are known at all points within and on the boundary of the square region. We draw lines parallel to x, y, t– axis as

$x = ih$ , $i = 0, 1, 2, ...$

$y = jk$ , $j = 0, 1, 2, \ldots$

$t = n\delta t$ , $n = 0, 1, 2, \ldots$

Then the explicit finite difference approximation to Gary-Scott model in two-dimensions are given by

$$\frac{u_{i,j,n+1}-u_{i,j,n}}{\delta t} = \alpha_1\left(\frac{u_{i-1,j,n}-2u_{i,j,n}+u_{i+1,j,n}}{h^2}\right) + \alpha_1\left(\frac{u_{i,j-1,n}-2u_{i,j,n}+u_{i,j+1,n}}{k^2}\right) - u_{i,j,n}v_{i,j,n}^2 + 0.02(1-u_{i,j,n}) \tag{3}$$

$$\frac{v_{i,j,n+1}-v_{i,j,n}}{\delta t} = \alpha_2\left(\frac{v_{i-1,j,n}-2v_{i,j,n}+v_{i+1,j,n}}{h^2}\right) + \alpha_2\left(\frac{v_{i,j-1,n}-2v_{i,j,n}+v_{i,j+1,n}}{k^2}\right) + u_{i,j,n}v_{i,j,n}^2 - 0.086\, v_{i,j,n} \tag{4}$$

Multiply eq.(3) and eq.(4) by $\delta t$ and set h=k, $\frac{\alpha_1\delta t}{h^2} = r_1$ and $\frac{\alpha_2\delta t}{h^2} = r_2$ then we get

$$u_{i,j,n+1} = (1-4r_1-0.02\,\delta t)u_{i,j,n} + r_1(u_{i-1,j,n}+u_{i+1,j,n}+u_{i,j-1,n}+u_{i,j+1,n}) - u_{i,j,n}\,v_{i,j,n}^2\,\delta t + 0.02\delta t \tag{5}$$

and

$$v_{i,j,n+1} = (1 - 4r_2 - 0.086\,\delta t)v_{i,j,n} + r_2(v_{i-1,j,n} + v_{i+1,j,n} + v_{i,j-1,n} + v_{i,j+1,n}) + u_{i,j,n}\, v_{i,j,n}^2\, \delta t \quad (6)$$

### 3.2 ADI Method [8]

The ADI method is developed by peaceman and Rachford in 1955 [12] and is called Alternating Direction Implicit (ADI) Method. This two-step approach requires minimal computer storage and is quite accurate. Further, ADI method is unconditionally stable. The method involves the alternate of two different finite difference approximations to the two-dimensional space [1].

In the ADI approach, the finite difference equations are written in terms of quantities at two x levels. However, two different finite difference approximations are used alternately, one to advance the calculations from the plan n to a plane (n+1) and the second to advance the calculations from (n+1) plane to the (n+2) plane [7].

With this method, each of the two steps involves diffusion in both the x and y - directions. In the first step the diffusion in x is modeled implicitly while diffusion in y modeled explicitly with the roles reversed in the second step [9].

Then we advance the solution of the Gary-Scott model, from nth plane to (n+1)th plane by replacing $\frac{\partial^2 u}{\partial x^2}$ and $\frac{\partial^2 v}{\partial x^2}$ by implicit finite difference approximation at the (n+1)th plane. Similarly $\frac{\partial^2 u}{\partial y^2}$ and $\frac{\partial^2 v}{\partial y^2}$ are replaced by an explicit finite difference approximation at the nth plane. With these approximations eq.(1) and eq.(2) in Gary-Scott model can be written as.

$$\frac{u_{i,j,n+1} - u_{i,j,n}}{\delta t} = \alpha_1\left[\frac{u_{i-1,j,n+1} - 2u_{i,j,n+1} + u_{i+1,j,n+1}}{h^2}\right] + \alpha_1\left[\frac{u_{i,j-1,n} - 2u_{i,j,n} + u_{i,j+1,n}}{k^2}\right] - u_{i,j,n}v_{i,j,n}^2 + 0.02(1 - u_{i,j,n}) \quad (7)$$

and

$$\frac{v_{i,j,n+1} - v_{i,j,n}}{\delta t} = \alpha_2\left[\frac{v_{i+1,j,n+1} - 2v_{i,j,n+1} + v_{i+1,j,n+1}}{h^2}\right] + \alpha_2\left[\frac{v_{i,j-1,n} - 2v_{i,j,n} + v_{i,j+1,n}}{k^2}\right] + u_{i,j,n}v_{i,j,n}^2 - 0.086\, v_{i,j,n} \quad (8)$$

We set h=k, than we have a square region and multiply eq.(7) and eq.(8) by δt and let $r_1 = \frac{\alpha_1 \delta t}{h^2}$ and $r_2 = \frac{\alpha_2 \delta t}{h^2}$ then we get

$$-r_1 u_{i-1,j,n+1} + (1+2r_1)u_{i,j,n+1} - r_1 u_{i+1,j,n+1} = r_1 u_{i,j-1,n} + (1-2r_1)u_{i,j,n} + r_1 u_{i,j+1,n} - \delta t\, u_{i,j,n}v_{i,j,n}^2 + 0.02\,\delta t(1 - u_{i,j,n}) \quad (9)$$

and

$$-r_2 v_{i-1,j,n+1} + (1+2r_2)v_{i,j,n+1} - r_2 v_{i+1,j,n+1} = r_2 v_{i,j-1,n} + (1-2r_2)v_{i,j,n} + r_2 v_{i,j+1,n} + \delta t\, u_{i,j,n}\, v_{i,j,n}^2 - 0.086\,\delta t\, v_{i,j,n} \quad (10)$$

Now we advance the solution from the (n+1)th plane to (n+2)th plane by replacing $\frac{\partial^2 u}{\partial x^2}$ and $\frac{\partial^2 v}{\partial x^2}$ with explicit finite difference approximation at (n+1)th plane then $\frac{\partial^2 u}{\partial y^2}$ and $\frac{\partial^2 v}{\partial y^2}$ by an implicit finite approximation at the (n+2)th plane. Then eq.(1) and eq.(2) in Gary-Scott model becomes

$$\frac{u_{i,j,n+2}-u_{i,j,n+1}}{\delta t}=\alpha_1\left[\frac{u_{i-1,j,n+1}-2u_{i,j,n+1}+u_{i+1,j,n+1}}{h^2}\right]+\alpha_1\left[\frac{u_{i,j-1,n+2}-2u_{i,j,n+2}+u_{i,j+1,n+2}}{k^2}\right]$$
$$-u_{i,j,n}v_{i,j,n}^2+0.02\left(1-u_{i,j,n}\right) \tag{11}$$

and

$$\frac{v_{i,j,n+2}-v_{i,j,n+1}}{\delta t}=\alpha_2\left[\frac{v_{i-1,j,n+1}-2v_{i,j,n+1}+v_{i+1,j,n+1}}{h^2}\right]+\alpha_2\left[\frac{v_{i,j-1,n+2}-2v_{i,j,n+2}+v_{i,j+1,n+2}}{k^2}\right]$$
$$+u_{i,j,n}v_{i,j,n}^2-0.086\,v_{i,j,n} \tag{12}$$

Multiply eq.(11) and eq.(12) by δt and let $r_1=\frac{\alpha_1\delta t}{h^2}$ and $r_2=\frac{\alpha_2\delta t}{h^2}$ when h=k then we have for a square region,

$$-r_1u_{i,j-1,n+2}+(1+2r_1)u_{i,j,n+2}-r_1u_{i,j+1,n+2}=r_1u_{i-1,j,n+1}+(1-2r_1)u_{i,j,n+1}+r_1u_{i+1,j,n+1}$$
$$-\delta t\,u_{i,j,n}\,v_{i,j,n}^2+0.02\,\delta t\left(1-u_{i,j,n}\right) \tag{13}$$

and

$$-r_2v_{i,j-1,n+2}+(1+2r_2)v_{i,j,n+2}-r_2v_{i,j+1,n+2}=r_2v_{i-1,j,n+1}+(1-2r_2)v_{i,j,n+1}+r_2v_{i+1,j,n+1}$$
$$+\delta t\,u_{i,j,n}v_{i,j,n}^2-0.086\,\delta t\,v_{i,j,n} \tag{14}$$

These systems are a tridiagonal linear system of equations and can be solved by the LU algorithm.

### 3.3 Hopscotch method

This technique, which lies somewhere between explicit ADE and implicit ADI, was suggested by Gordeon [4] and marketed by Gourlay [5] under the name of Hopscotch [10].

The Hopscotch method is an interesting combination of the forward-time centered-space (FTCS) method and the backward-time centered-space (BTCS) method. The basic idea of the Hopscotch method is to make two sweeps through the solution domain at each time step. On the first sweep, the explicit FTCS method is applied at every other grid point. On the second sweep, the implicit BTCS method is applied at the remaining points. That is the explicit equation is first used for those points with (i+j) is even and then implicit equation for those points with (i+j) is odd [8].

Now we transform the Gary-Scott model in two dimensions to Hopscotch finite difference formula. Thus the explicit formula has the form:

$$\frac{u_{i,j,n+1}-u_{i,j,n}}{\delta t}=\alpha_1\left[\frac{u_{i-1,j,n}-2u_{i,j,n}+u_{i+1,j,n}}{h^2}\right]+\alpha_1\left[\frac{u_{i,j-1,n}-2u_{i,j,n}+u_{i,j+1,n}}{k^2}\right]$$
$$-u_{i,j,n}v_{i,j,n}^2+0.02\left(1-u_{i,j,n}\right) \tag{15}$$

and

$$\frac{v_{i,j,n+1}-v_{i,j,n}}{\delta t}=\alpha_2\left[\frac{v_{i-1,j,n}-2v_{i,j,n}+v_{i+1,j,n}}{h^2}\right]+\alpha_2\left[\frac{v_{i,j-1,n}-2v_{i,j,n}+v_{i,j+1,n}}{k^2}\right] \tag{16}$$
$$+u_{i,j,n}v_{i,j,n}^2-0.086\,v_{i,j,n}$$

multiply eq.(15) and eq.(16) by δt and when h=k, we assume that $\frac{\alpha_1\delta t}{h^2}=r_1$ and $\frac{\alpha_2\delta t}{h^2}=r_2$ then we get

$$u_{i,j,n+1}=(1-4r_1-0.02\,\delta t)u_{i,j,n}+r_1(u_{i-1,j,n}+u_{i+1,j,n}+u_{i,j-1,n}+u_{i,j+1,n}) \tag{17}$$
$$-u_{i,j,n}\,v_{i,j,n}^2\,\delta t+0.02\delta t$$

and

$$v_{i,j,n+1}=(1-4r_2-0.086\,\delta t)v_{i,j,n}+r_2(v_{i-1,j,n}+v_{i+1,j,n}+v_{i,j-1,n}+v_{i,j+1,n}) \tag{18}$$
$$+u_{i,j,n}\,v_{i,j,n}^2\,\delta t$$

and the implicit formula are

$$\frac{u_{i,j,n+1}-u_{i,j,n}}{\delta t}=\alpha_1\left[\frac{u_{i+1,j,n+1}-2u_{i,j,n+1}+u_{i-1,j,n+1}}{h^2}\right]+\alpha_1\left[\frac{u_{i,j-1,n+1}-2u_{i,j,n+1}+u_{i,j+1,n+}}{k^2}\right] \tag{19}$$
$$-u_{i,j,n}v_{i,j,n}^2+0.02(1-u_{i,j,n})$$

and

$$\frac{v_{i,j,n+1}-v_{i,j,n}}{\delta t}=\alpha_2\left[\frac{v_{i-1,j,n+1}-2v_{i,j,n+1}+v_{i+1,j,n+1}}{h^2}\right]+\alpha_2\left[\frac{v_{i,j-1,n+1}-2v_{i,j,n+1}+v_{i,j+1,n+1}}{k^2}\right]$$
$$+u_{i,j,n}v_{i,j,n}^2-0.086\,v_{i,j,n} \tag{20}$$

We multiply eq.(19) and eq.(20) by δt and also when h=k and assume $r_1=\frac{\alpha_1\delta t}{h^2}$ and $r_2=\frac{\alpha_2\delta t}{h^2}$ then we have

$$(1+4r_1)u_{i,j,n+1}=(1-0.02\delta t)u_{i,j,n}+r_1(u_{i-1,j,n+1}+u_{i+1,j,n+1}+u_{i,j-1,n+1}+u_{i,j+1,n+1}) \tag{21}$$
$$-\delta t\,u_{i,j,n}v_{i,j,n}^2+0.02\delta t$$

and

$$(1+4r_2)v_{i,j,n+1}=(1-0.086\delta t)v_{i,j,n}+r_2(v_{i-1,j,n+1}+v_{i+1,j,n+1}+v_{i,j-1,n+1}+v_{i,j+1,n+1}) \tag{22}$$
$$+\delta t\,u_{i,j,n}v_{i,j,n}$$

## 4. Numerical Consistency

A scheme is said to be consistent if the local truncation error tends to zero as h tends to zero [8]. In the context of the diffusion equation, the numerical scheme must be consistent with the original partial differential equation.

A numerical algorithm is said to be stable if the round off error does small errors in stay bounded during the numerical process.

A scheme converges if it is both consistent and stable as temporal and spatial discretisation is reduced [2].

### 4.1 Consistency of ADE Algorithm

The ADE algorithm for equation (5) where $r_1=\frac{\alpha_1\delta t}{h^2}$

$$u_{i,j,n+1} = u_{i,j,n} + r_1\left(u_{i-1,j,n} - 2u_{i,j,n} + u_{i+1,j,n}\right) + r_1\left(u_{i,j-1,n} - 2u_{i,j,n} + u_{i,j+1,n}\right) - u_{i,j,n}\, v_{i,j,n}^2\, \delta t + 0.02\delta t\left(1 - u_{i,j,n}\right) \quad (23)$$

When we examine the consistency we eliminate all terms that not effect on the consistency, non linear terms and constant terms.
Thus the eq.(23) becomes

$$u_{i,j,n+1} - u_{i,j,n} = r_1\left(u_{i-1,j,n} - 2u_{i,j,n} + u_{i+1,j,n}\right) + r_1\left(u_{i,j-1,n} - 2u_{i,j,n} + u_{i,j+1,n}\right) - 0.02\delta t\, u_{i,j,n} \quad (24)$$

Now each term in eq.(24) is replaced by its Taylor series expansion taken about $(x_i, y_j)$ at time $t_n$.

$$u_{i,j,n+1} - u_{i,j,n} = r_1\delta_x^2\, u_{i,j,n} + r_1\delta_y^2\, u_{i,j,n} - 0.02\delta t\, u_{i,j,n} \quad (25)$$

divided eq.(25) by $\delta t$ and replaced $r_1$ by $\frac{\alpha_1 \delta t}{h^2}$ then

$$\frac{u_{i,j,n+1} - u_{i,j,n}}{\delta t} = \frac{\alpha_1}{h^2} u_{xx}^n + O\left(h^2\right) + \frac{\alpha_1}{h^2} u_{yy}^n + O\left(h^2\right) - 0.02u_{i,j,n}$$

i.e.

$u_t - \frac{\alpha_1}{h^2} u_{xx}^n - \frac{\alpha_1}{h^2} u_{yy}^n = -0.02u_{i,j,n}^n + O\left(h^2\right) + O\left(\delta t\right)$, when h=k any implementation of ADE algorithm in which h individual tend to zero with $\frac{\alpha_1}{h^2} = c$, c is a constant, thus the ADE algorithm is consistent.

In the manner we can show that ADE algorithm for the equation (4) that is bellow.

$$v_{i,j,n+1} = v_{i,j,n} + r_2\left(v_{i-1,j,n} - 2v_{i,j,n} + v_{i+1,j,n}\right) + r_2\left(v_{i,j-1,n} - 2v_{i,j,n} + v_{i,j+1,n}\right) + \delta t\, u_{i,j,n}\, v_{i,j,n}^2 - 0.086\,\delta t\, v_{i,j,n}$$

Is consistent.

Thus the ADE algorithm for eqs.(1) and (2) is consistent and the local truncation error is of $O\left(h^2 + \left(\delta t\right)\right)$.

### 4.2 Consistency of ADI

the ADI finite difference form of Gary-Scott model in two-dimensions are

$$-r_1 u_{i-1,j,n+1} + \left(1 + 2r_1\right)u_{i,j,n+1} - r_1 u_{i+1,j,n+1} = r_1 u_{i,j-1,n} + \left(1 - 2r_1\right)u_{i,j,n} + r_1 u_{i,j+1,n} - \delta t\, u_{i,j,n} v_{i,j,n}^2 + 0.02\delta t\left(1 - u_{i,j,n}\right) \quad (26)$$

and

$$-r_1 u_{i-1,j,n+2} + \left(1 + 2r_1\right)u_{i,j,n+2} - r_1 u_{i+1,j,n+2} = r_1 u_{i,j-1,n+1} + \left(1 - 2r_1\right)u_{i,j,n+1} + r_1 u_{i,j+1,n+1} - \delta t\, u_{i,j,n} v_{i,j,n}^2 + 0.02\delta t\left(1 - u_{i,j,n}\right) \quad (27)$$

Now we write the eqs.(26) and (27) in the forms in which we dropped the subscripts i and j from u and write the time at in the super subscripts, and let $r_1 = \frac{\alpha_1 \delta t}{h^2}$ then we have the following form

$$\left(1 - r_1\delta_x^2\right)u^{n+1} = \left(1 + r_1\delta_y^2\right)u^n - 0.02\delta t\, u^n \quad (28)$$

$$\left(1-r_1\delta_y^2\right)u^{n+2}=\left(1+r_1\delta_x^2\right)u^{n+1}-0.02\delta t\,u^n \tag{29}$$

The quantity $u^{n+1}$ is first eliminated by multiplying equation (29) by the operator $\left(1-r_1\delta_x^2\right)$ and thus (29) becomes

$$\left(1-r_1\delta_x^2\right)\left(1-r_1\delta_y^2\right)u^{n+2}=\left(1+r_1\delta_x^2\right)\left(1-r_1\delta_x^2\right)u^{n+1}-0.02\delta t\left(1-r_1\delta_x^2\right)u^n$$

$$=\left(1+r_1\delta_x^2\right)\left(1+r_1\delta_y^2\right)u^n-0.02\delta t\left(1-r_1\delta_x^2\right)u^n$$

thus we can verity that

$$\left(1+r_1^2\delta_x^2\delta_y^2\right)\left(u^{n+2}-u^n\right)=r_1\left(\delta_x^2+\delta_y^2\right)\left(u^{n+2}-u^n\right)-0.02\delta t\,u^n\left(1-r_1^2\delta_x^2\right), \tag{30}$$

if we divide eq.(30) by $2\delta t$ and replace $r_1$ by $\frac{\alpha_1\delta t}{h^2}$ to obtain

$$\left(1+r_1^2\delta_x^2\delta_y^2\right)\frac{u^{n+2}-u^n}{2\delta t}=\alpha_1\left(\delta_x^2+\delta_y^2\right)\frac{u^{n+2}-u^n}{2h^2}-0.01\left(1-r_1^2\delta_x^2\right)u^n \tag{31}$$

each term appearing in eq.(31) is replaced by its Taylor series expansion taken about $(x_i,y_j)$ at time $t_{n+1}$ . It is easy to see that $(u^{n+2}-u^n)/2\delta t$ is the central difference formula for $u_t$ at time $t_{n+1}$ and therefore

$$\frac{u^{n+2}-u^n}{2\delta t}=u_t^{n+1}+O((\delta t)^2),$$

$$u^{n+2}-u^n=2\delta t u_t^{n+1}+O((\delta t)^2),$$

Consequently

$$\alpha_1\left(\delta_x^2+\delta_y^2\right)\frac{u^{n+2}-u^n}{2h^2}=\frac{\alpha_1\left(\delta_x^2+\delta_y^2\right)2\delta t u_t^{n+1}}{2h^2}+O\left((\delta t)^2\right)$$

$$=\alpha_1\delta t\left(u_{txx}^{n+1}+u_{tyy}^{n+1}\right)+O((\delta t)^2,h^2)$$

and

$$\frac{r_1^2\delta_x^2\delta_y^2\left(u^{n+2}-u^n\right)}{2\delta t}=\frac{\alpha_1^2(\delta t)^2}{h^4}\delta_x^2\delta_y^2u_t^{n+1}+O\left((\delta t)^2\right)$$

$$=\alpha_1^2(\delta t)^2u_{txxyy}^{n+1}+O\left((\delta t)^2,h^2\right)$$

the component are now assembled to give

$$-0.01u^n+0.01\,r_1\delta_x^2u^n=-0.01u^n+0.01\frac{\alpha_1\delta t}{h^2}\left(u_{i-1,j,n}-2u_{i,j,n}+u_{i+1,j,n}\right)$$

$$=-0.01u^n+0.01\alpha_1(\delta t)u_{xx}^n+O(h^2)$$

thus eq.(31) becomes

$$u_t^{n+1}+O\left((\delta t)^2\right)+\alpha_1^2(\delta t)^2u_{txxyy}^{n+1}+O\left(h^2,(\delta t)^2\right)=\alpha_1\delta t u_{xx}^{n+1}+\alpha_1\delta t u_{yy}^{n+1}$$

$$-0.01\delta t\,u^n+0.01(\delta t)\alpha_1\,u_{xx}^n+O\left((\delta t)^2,h^2\right) \tag{32}$$

then eq.(32) may be rearranged to give

$$u_t^{n+1}-\alpha_1\delta t\left(u_{xx}^{n+1}+u_{yy}^{n+1}\right)=-0.01\delta t\,u^n+0.01(\delta t)\alpha_1\,u_{xx}^n-\alpha_1^2(\delta t)^2u_{txxyy}^{n+1}$$

$$+O\left((\delta t)^2,h^2\right)$$

Thus the ADI algorithm for Gray-Scott model, in two-dimensions, is consistent. Also we show that the concentration v for eq. (8) and (12) is also consistent. Consequently the ADI method of Gary-Scott model, in two –dimension, is unconditionally consistent since h tend to zero. Thus the local truncation error is $O((\delta t)^2 + h^2)$.

### 4.3 Consistent of Hopscotch Method

The Hopscotch finite difference method for Gary-Scott model in two-dimensions has the following explicit form

$$u_{i,j,n+1} = u_{i,j,n} + r_1\left(u_{i-1,j,n} - 2u_{i,j,n} + u_{i+1,j,n}\right) + r_1\left(u_{i,j-1,n} - 2u_{i,j,n} + u_{i,j+1,n}\right) - \delta t u_{i,j,n}\, v_{i,j,n}^2 + 0.02\delta t - 0.02\delta t u_{i,j,n} \tag{33}$$

and the implicit form is

$$u_{i,j,n+1} = u_{i,j,n} + r_1\left(u_{i-1,j,n+1} - 2u_{i,j,n+1} + u_{i+1,j,n+1}\right) + r_1\left(u_{i,j-1,n+1} - 2u_{i,j,n+1} + u_{i+1,j,n+1}\right) - \delta t u_{i,j,n}\, v_{i,j,n}^2 + 0.02\delta t - 0.02\delta t u_{i,j,n} \tag{34}$$

the linear terms of eq.(33) and (34) without constant are

$$u_{i,j,n+1} - u_{i,j,n} = r_1\delta_x^2 u_{i,j,n} + r_1\delta_y^2 u_{i,j,n} - 0.02\delta t u_{i,j,n} \tag{35}$$

$$u_{i,j,n+1} - u_{i,j,n} = r_1\delta_x^2 u_{i,j,n+1} + r_1\delta_y^2 u_{i,j,n+1} - 0.02\delta t u_{i,j,n} \tag{36}$$

If we divide (35) and (36) by $\delta t$ and replace $r_1$ by $r_1 = \frac{\alpha_1 \delta t}{h^2}$ then we have

$$\frac{u_{i,j,n+1} - u_{i,j,n}}{\delta t} = \frac{\alpha_1}{h^2}\delta_x^2 u_{i,j,n} + \frac{\alpha_1}{h^2}\delta_y^2 u_{i,j,n} - 0.02\, u_{i,j,n} \tag{37}$$

$$\frac{u_{i,j,n+1} - u_{i,j,n}}{\delta t} = \frac{\alpha_1}{h^2}\delta_x^2 u_{i,j,n+1} + \frac{\alpha_1}{h^2}\delta_y^2 u_{i,j,n+1} - 0.02\, u_{i,j,n} \tag{38}$$

each term in (37) and (38) is now replaced by its Taylor expansion to obtain from (37)

$$\frac{\partial u_{i,j,n}}{\partial t} + \frac{\delta t}{2}\frac{\partial^2 u_{i,j,n}}{\partial t^2} + O\left((\delta t)^3\right) = \alpha_1 u_{xx}^n + O\left(h^2\right) + \alpha_1 u_{yy}^n + O\left(h^2\right) - 0.02u_{i,j,n} \tag{39}$$

and from (38)

$$\frac{\partial u_{i,j,n}}{\partial t} + \frac{\delta t}{2}\frac{\partial^2 u_{i,j,n}}{\partial t^2} + O\left((\delta t)^3\right) = \alpha_1 u_{xx}^{n+1} + O\left(h^2\right) + \alpha_1 u_{yy}^{n+1} + O\left(h^2\right) - 0.02u_{i,j,n} \tag{40}$$

from (39) and (40) we have

$$u_t - \alpha_1\left(u_{xx} + u_{yy}\right) = O\left(h^2, (\delta t)^3\right) - 0.02u_{i,j,n} \tag{41}$$

thus the Hopscotch algorithm is unconditionally consistent since h and $\delta t$ tend to zero and the local truncation error is $O\left(h^2 + (\delta t)^3\right)$.

In the same manner we can show that the Hopscotch algorithm of eq. (18) and (20) is also unconditionally consistent in the end the Hopscotch algorithm of Gray-Scott model in two dimension is unconditionally consistent.

### 5. Numerical Stability

There two methods, we used here one including the effect of boundary conditions and the other excluding the effect of boundary conditions are used to investigate stability.

Both methods are attributed to John von Neumann. These approaches are Fourier method and matrix method.

Fourier method, the primary observation in the Fourier method is that the numerical scheme is linear and therefore it will have solution in the form $u(x,t)=\lambda^t e^{i\alpha x}$. Thus numerical scheme is stable provided $|\lambda|<1$ and unstable whenever $|\lambda|>1$ [14].

### 5.1 Stability Analysis of ADE Method

The von-Neumann method has been used to study the stability analysis of Gary-Scott model in two dimensions.

We can apply this method by substituting the solution in finite difference method at the time t by $\psi(t)e^{m\beta x}\ e^{m\gamma y}$, when $\beta,\gamma>0$ and m= $\sqrt{-1}$ [14].

To apply von-Neumann on eq.(1) we have to linearlize the problem and thus we get after we eliminate non-linear term the following:

$$\frac{\partial u}{\partial t}=\alpha_1\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)+0.02(1-u) \tag{42}$$

we have the following finite difference scheme for eq.(42)

$$\begin{aligned} u_{i,j,n+1} &= u_{i,j,n}+r_1[u_{i-1,j,n}+u_{i+1,j,n}+u_{i,j+1,n}+u_{i,j-1,n}-4u_{i,j,n}] \\ &\quad +0.02\,\delta t-0.02\delta t\,u_{i,j,n} \end{aligned} \tag{43}$$

where $r_1=\frac{\alpha_1\delta t}{h^2}$, and (h=k). We assume $u_{i,j,n}=\psi(t)e^{m\beta x}e^{m\gamma y}$, substituting in eq.(43) then we have

$$\psi(t+\Delta t)e^{m\beta x}e^{m\gamma y}=\psi(t)e^{m\beta x}e^{m\gamma y}+r_1\Big[\psi(t)e^{m\beta(x-\Delta x)}e^{m\gamma y}+\psi(t)e^{m\beta(x+\Delta x)}e^{m\gamma y}+$$

$$\psi(t)e^{m\beta x}e^{m\gamma(y+\Delta y)}+\psi(t)e^{m\beta x}e^{m\gamma(y-\Delta y)}-4\psi(t)e^{m\beta x}e^{m\gamma y}\Big]+0.02\Delta t-0.02\Delta t\psi(t)e^{m\beta x}e^{m\gamma y}$$

or $$\frac{\psi(t+\Delta t)}{\psi(t)}=1+r_1\left[e^{-m\beta\Delta x}+e^{m\beta\Delta x}+e^{-m\gamma\Delta y}+e^{m\gamma\Delta y}-4\right]+0.02\Delta t$$

$$\frac{\psi(t+\Delta t)}{\psi(t)}=\xi=1-4r_1\left[\sin^2\left(\frac{\beta\Delta x}{2}\right)+\sin^2\left(\frac{\gamma\Delta y}{2}\right)\right]+0.02\Delta t$$

For stable situation, we need $|\xi|\le 1$, and hence we have

$$-1\le 1-4r_1\left[\sin^2\left(\frac{\beta\Delta x}{2}\right)+\sin^2\left(\frac{\gamma\Delta y}{2}\right)\right]-0.02\Delta t\le 1$$

Considering the left-side inequality (as the right-side inequality is always true), we have

$$-1\le 1-0.02\Delta t-4r_1\left[\sin^2\left(\frac{\beta\Delta x}{2}\right)+\sin^2\left(\frac{\gamma\Delta y}{2}\right)\right]$$

for some $\beta$ and $\gamma$, $\sin^2\left(\frac{\beta\Delta x}{2}\right)$ and $\sin^2\left(\frac{\gamma\Delta y}{2}\right)$ is unity. Hence, we have

$$-1\le 1-0.02\Delta t-4r_1(2)$$

$$r_1\le 0.25-0.0025\Delta t$$

This is the condition for stability, in a square region h=k, when we use ADE method for eq.(1). Thus the ADE method for eq.(1) is conditionally stable.

Now, Also the stability condition of eq.(2) can be found, for h=k, which is $r_2 \le \frac{1}{4} - \frac{0.086}{8}\Delta t$. Thus the ADE method is conditionally stable.

## 5.2 Stability of ADI Method for Gray-Scott in Two Dimensions

The ADI finite difference form for (9) is

$$\begin{aligned} -r_1 u_{i-1,j,n+1} + (1+2r_1)u_{i,j,n+1} - r_1 u_{i+1,j,n+1} = r_1 u_{i,j-1,n} + (1-2r_1)u_{i,j,n} + r_1 u_{i,j+1,n} \\ - \Delta t\, u_{i,j,n}\, v_{i,j,n}^2 + 0.02\,\Delta t(1-u_{i,j,n}) \end{aligned} \tag{44}$$

Assuming

$$u_{i,j,n} = \psi(t) e^{m\beta x} e^{m\gamma y} \tag{45}$$

And substitute (45) in (44) we have

$$\begin{aligned} &-r_1\psi(t+\Delta t)e^{m\beta(x-\Delta x)}e^{m\gamma y} + (1+2r_1)\psi(t+\Delta t)e^{m\beta x}e^{m\gamma y} - r_1\psi(t+\Delta t)e^{m\beta(x+\Delta x)}e^{m\gamma y} \\ &= r_1\psi(t)e^{m\beta}e^{m\gamma(y-\Delta y)} + (1-2r_1)\psi(t)e^{m\beta x}e^{m\gamma y} + r_1\psi(t)e^{m\beta x}e^{m\gamma(y+\Delta y)} - 0.02\Delta t\psi(t)e^{m\beta x}e^{m\gamma y} \end{aligned}$$

i.e. $\left[-r_1 e^{-m\beta\Delta x} + (1+2r_1) - r_1 e^{m\beta\Delta x}\right]\psi(t+\Delta t) = \left[r_1 e^{-m\gamma\Delta y} + (1-2r_1) + r_1 e^{m\gamma\Delta y}\right]\psi(t) - 0.02\Delta t\psi(t)$

i.e. $\left[-r_1(\cos(\beta\Delta x) - m\sin(\beta\Delta x)) + 1 + 2r_1 - r_1(\cos(\beta\Delta x) + m\sin(\beta\Delta x))\right]\psi(t+\Delta t) =$
$\left[r_1\cos(\gamma\Delta y) - m\sin(\gamma\Delta y) + 1 - 2r_1 - 0.02\Delta t + r_1(\cos(\gamma\Delta y) - m\sin(\gamma\Delta y))\right]$

i.e. $\left[-2r_1\cos(\beta\Delta x) + 1 + 2r_1\right]\psi(t+\Delta t) = \left[2r_1\cos(\gamma\Delta y) + 1 - 2r_1 - 0.02\Delta t\right]\psi(t)$

$$\frac{\psi(t+\Delta t)}{\psi(t)} = \frac{2r_1\cos(\gamma\Delta y) + 1 - 2r_1 - 0.02\Delta t}{-2r_1\cos(\beta\Delta x) + 1 + 2r_1}$$

$$= \frac{2r_1\left(1 - 2\sin^2\left(\frac{\gamma\Delta y}{2}\right)\right) + 1 - 2r_1 - 0.02\Delta t}{-2r_1\left(1 - 2\sin^2\left(\frac{\beta\Delta x}{2}\right)\right) + 1 + 2r_1}$$

$$= \frac{2r_1 - 4r_1\sin^2\left(\frac{\gamma\Delta y}{2}\right) + 1 - 2r_1 - 0.02\Delta t}{-2r_1 + 4r_1\sin^2\left(\frac{\beta\Delta x}{2}\right) + 1 + 2r_1}$$

$$\frac{\psi(t+\Delta t)}{\psi(t)} = \frac{1 - 4r_1\sin^2\left(\frac{\gamma\Delta y}{2}\right) - 0.02\Delta t}{1 + 4r_1\sin^2\left(\frac{\beta\Delta x}{2}\right)} = \xi_I$$

Similarity for eq.(10) we obtain

$$\xi_{II} = \frac{1 - 4r_1\sin^2\left(\frac{\gamma\Delta y}{2}\right) - 0.086\Delta t}{1 + 4r_1\sin^2\left(\frac{\beta\Delta x}{2}\right)}$$

where $\xi_I$ and $\xi_{II}$ stands for the I plane and II plane. Each of the above terms $\xi_I$ and $\xi_{II}$ are conditionally stable. However, the combined two-level has the form:

$$\xi_{ADI} = \xi_I \cdot \xi_{II} = \left[\frac{1-4r_1\sin^2\left(\frac{\gamma\Delta y}{2}\right)-0.02\Delta t}{1+4r_1\sin^2\left(\frac{\beta\Delta x}{2}\right)}\right]\cdot\left[\frac{1-4r_1\sin^2\left(\frac{\gamma\Delta y}{2}\right)-0.086\Delta t}{1+4r_1\sin^2\left(\frac{\beta\Delta x}{2}\right)}\right]$$

Thus the above scheme is unconditionally stable, each individual equation is conditionally stable by itself, and the combined two-level is completely stable.

### 5.3 Stability of Hopscotch Method

The stability of explicit form of Hopscotch is the same as the stability of ADE method i.e. is conditionally stable.
Now we study the stability of implicit form of Hopscotch formula

$$(1+4r_1)u_{i,j,n+1} = (1-0.02\Delta t)u_{i,j,n} + r_1\left(u_{i-1,j,n+1} + u_{i+1,j,n+1} + u_{i,j-1,n+1} + u_{i,j+1,n+1}\right) \tag{46}$$

we assume that:

$$u_{i,j,n} = \psi(t)e^{m\beta x}e^{m\gamma y} \tag{47}$$

Substitute (47) in (46) we have

$$(1+4r_1)\psi(t+\Delta t)e^{m\beta x}e^{m\gamma y} = (1-0.02\Delta t)\psi(t)e^{m\beta x}e^{mxy} + r_1\left[\psi(t+\Delta t)e^{m\beta(x-\Delta x)}e^{m\gamma y}\right.$$
$$\left.+\psi(t+\Delta t)e^{m\beta(x+\Delta x)}e^{m\gamma y} + \psi(t+\Delta t)e^{m\beta x}e^{m\gamma(y-\Delta y)} + \psi(t+\Delta t)e^{m\beta x}e^{m\gamma(y+\Delta y)}\right]$$

or

$$(1+4r_1)\psi(t+\Delta t) = (1-0.02\Delta t)\psi(t) + r_1\psi(t+\Delta t)\left[e^{-m\beta\Delta x} + e^{m\beta\Delta x} + e^{-m\gamma\Delta y} + e^{m\gamma\Delta y}\right]$$
$$= (1-0.02\Delta t)\psi(t) + 2r_1\psi(t+\Delta t)\left[\cos(\beta\Delta x)+\cos(\gamma\Delta y)\right]$$

i.e.

$$(1+4r_1)\psi(t+\Delta t) = (1-0.02\Delta t)\psi(t) + 2r_1\psi(t+\Delta t)\left[\cos(\beta\Delta x)+\cos(\gamma\Delta y)\right]$$
$$= (1-0.02\Delta t)\psi(t) + 2r_1\psi(t+\Delta t)\left[1-2\sin^2\left(\frac{\beta\Delta x}{2}\right)+1-2\sin^2\left(\frac{\gamma\Delta y}{2}\right)\right]$$

or $$(1+4r_1)\psi(t+\Delta t) = (1-0.02\Delta t)\psi(t) + 4r_1\psi(t+\Delta t)\left[1-\sin^2\left(\frac{\beta\Delta x}{2}\right)-\sin^2\left(\frac{\gamma\Delta y}{2}\right)\right]$$

or $$\left[(1+4r_1)-4r_1\left(1-\sin^2\left(\frac{\beta\Delta x}{2}\right)-\sin^2\left(\frac{\gamma\Delta y}{2}\right)\right)\right]\psi(t+\Delta t) = (1-0.02\Delta t)\psi(t)$$

$$\xi_{Hop.I} = \frac{\psi(t+\Delta t)}{\psi(t)} = \frac{1\text{-}0.02\Delta t}{1+4r_1\text{-}4r_1(-1)} = \frac{1\text{-}0.02\Delta t}{1+8r_1},\quad \left|\xi_{Hop.I}\right| \le 1$$

thus $$\left|\frac{1\text{-}0.02\Delta t}{1+8r_1}\right| \le 1$$

Thus the Hopscotch implicit form is conditionally stable. Thus the combination of the two forms is conditionally stable

$$\xi_{\text{Hop.}}=\xi_{\text{Hop.E}}.\xi_{\text{Hop.I}}=\frac{r_1}{(0.25\text{-}0.02\Delta t)}\frac{1\text{-}0.02\Delta t}{1+8r_1}$$

then we conclude that the Hopscotch method is unconditionally stable.

**Lax Equivalence Theorem**

Given a properly posed initial value problem,

$Lu = f(x, t, u, u_x)$ in $D + \partial D$

$u(x,0)= \psi(x)$, $t=0$

A finite-difference approximation that satisfies the consistency condition, stability is the necessary and sufficient condition for convergence.
Proof : [2]

**6. Numerical results**

The exact solution of the Gray-Scott model problem in two dimensions cannot be found by using analytic method, thus we need numerical methods such as FDADE method, FDADI method and FD Hopscotch method. We take the parameter $\alpha_1 = 8\times10^{-5}$ and $\alpha_2 = 4\times10^{-5}$. Also we compute the stability of each above methods and we conclude that the FDADE method is conditionally stable where $r_1 \le 0.25-\frac{0.02\Delta t}{8}$ and $r_2 \le 0.25-\frac{0.086\Delta t}{8}$ compared with FDADI and FD Hopscotch method that are unconditionally stable. Also we prove that FDADE method is conditionally consistent but FDADI method and Hopscotch methods are unconditionally consistent. Using Lax Equivalence Theorem then FDADE method, FDADI method and Hopscotch method are convergence. However, our numerical results show that the region of convergence of the FDADI and FD Hopscotch methods are bigger than FDADE method. Thus we can use FDADI and FD Hopscotch methods with any values of the space step size h and k and time step size δt but in FDADE method must the condition $r_1 \le 0.25-\frac{0.02\Delta t}{8}$ and $r_2 \le 0.25-\frac{0.086}{8}\Delta t$, Table (1) contains the numerical solution of the Gray-Scott model in two dimensions by using the above three methods with space step size h=k=0.1 and time step size $\Delta t = 0.1$. Also we present comparison figures for values of concentrations u and v by the methods.

**Table (1).** comparison between the methods FDADE, FDADI and FD Hopscotch for the values of concentrations u and v that computed at time step size δt=0.1 and space step size h=k=0.1.

| Point (i,j,n) | ADE method | ADI method | Hopscotch method |
|---|---|---|---|
| | Concentration u | | |
| (6,8,1) | 1 | 1 | 1 |
| (9,4,2) | 0.9 | 0.89999942583805 | 0.89856459330144 |

| | | | |
|---|---|---|---|
| (5,3,3) | 0.7929955036 | 0.79299550359847 | 0.79299097460478 |
| (4,9,4) | 0.68125995063869 | 0.68125994821486 | 0.67857021869709 |
| (10,7,5) | 0.56678772627119 | 0.56709825767783 | 0.56587046171125 |
| (2,5,6) | 0.45758846348854 | 0.45794347250011 | 0.45619039859748 |
| (3,2,7) | 0.35702127347594 | 0.35702298348826 | 0.35663165633968 |
| (10,5,8) | 0.26938672979787 | 0.26974325366916 | 0.26988367809689 |
| (7,10,9) | 0.19728167818297 | 0.19728267819338 | 0.19816561442277 |
| (8,6,10) | 0.14054354179992 | 0.14054355158273 | 0.14055604868282 |
| (10,10,11) | 0.10022376177821 | 0.08561961389106 | 0.010023046907569 |
| | Concentration v | | |
| (6,8,1) | 1 | 1 | 1 |
| (9,4,2) | 1.0914 | 1.09139982565501 | 1.09052827476038 |
| (5,3,3) | 1.1892176564 | 1.18921765639976 | 1.18921624710288 |
| (4,9,4) | 1.29113846978462 | 1.29113846841503 | 1.28723963348004 |
| (10,7,5) | 1.39066262285382 | 1.39054460703718 | 1.38749874608211 |
| (2,5,6) | 1.48776132420422 | 1.48764753175995 | 1.47899897905992 |
| (3,2,7) | 1.57565081627314 | 1.57564735519975 | 1.56418588597058 |
| (10,5,8) | 1.65011689132299 | 1.65008668455253 | 1.64306636189790 |
| (7,10,9) | 1.70861958154437 | 1.70861958150998 | 1.70044504584434 |
| (8,6,10) | 1.76025514009007 | 1.76025512184087 | 1.76011226769665 |
| (10,10,11) | 1.7680883916045 | 1.79210211432531 | 1.76804705226718 |

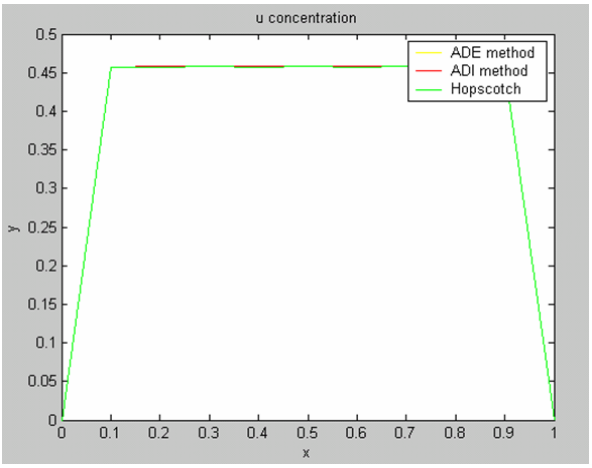


**Figure (1)**. show that the comparison between ADE, ADI and Hopscotch methods for finding the concentration values u(6,:,6) at level n=6, row i=6 and for all columns j When $\alpha_1 = 8\times10^{-5}$, $\alpha_2 = 4\times10^{-5}$, and h=k=δt=0.1.

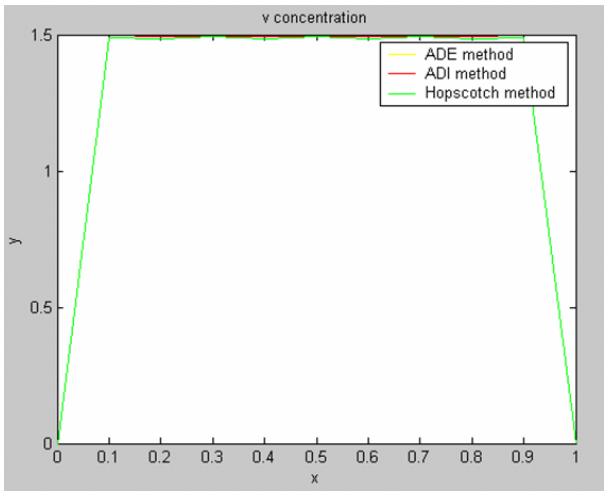


**Figure (2).** shows that the comparison between ADE, ADI and Hopscotch methods for finding the concentration values v(6,:,6) at level n=6, row i=6 and for all columns j. When $\alpha_1 = 8\times10^{-5}$, $\alpha_2 = 4\times10^{-5}$, and h=k=δt=0.1.

## 7. Conclusion

When we compared the results of ADE algorithm with an ADI algorithm on a number of complex PDE, our results observes, that ADI yielded significantly greater accuracy ( the local truncation error of ADI is smaller than ADE) but required more computing time. on the other hand the Hopscotch method has minimal storage requirements compare with ADI method and Hopscotch method also does not involve any tridiagonal matrix solver, but ADI algorithm for the Gray-Scott model in two dimensions have four tridiagonal matrix form. Both methods are unconditionally stable, consistent and convergence compared with ADE method.

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