Study of Dynamical Properties and Effective of a State *u* for Hyperchaotic Pan Systems

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ABSTRACT

In this paper, we propose a new four-dimensional continuous autonomous hyperchaotic system which is built by adding a nonlinear controller to a three-dimensional chaotic Pan system. The new system is analyzed both theoretically and numerically by studying dynamical behaviors for this system, including equilibrium points, Lyapunov exponents, stability and bifurcation, also we study the stability, bifurcation and symmetry for another four dimensional system which is generated by Pan in (2011) from the original system, we compared between two systems and found the difference between them. Finally, we show the effective of state u on the two systems.

Keywords: Hyperchaos, Lyapunov exponent, Pan system , Stability, Bifurcation.

دراسة الخصائص الديناميكية وتأثير المتغير u لنظامي Pan كثيري الاضطراب

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الملخص

في هذا البحث اقترحنا نظام جديد رباعي الإبعاد مستمر مستقل ذاتيا كثير الاضطراب تم بناءه بإضافة السيطرة الغير خطية على نظام Pan المضطرب ثلاثي الإبعاد. تم تحليل النظام الجديد نظرياً وعدديا بدراسة السلوك الديناميكي له متضمنا نقاط الاتزان، آسية ليبانوف، الاستقرارية والتشعب, كما تم دراسة الاستقرارية والتشعب والتناظر لنظام رباعي أخر مولد من قبل العالم Pan في (2011) من النظام الأصلي وقارنا بين النظامين ووجدنا الاختلاف بينهما, وأخيرا بينا تأثير المتغير u على النظامين.

الكلمات المفتاحية: شديد الاضطراب، اسية لاينوف، نظام Pan، الاستقرارية، التشعب.

1- Introduction:

In the recent years, hyperchaos generation and control have been extensively studied due to its theoretical and practical applications in the fields of communications, laser, neural work, nonlinear circuit, mathematics, and so on [6,7,10] historically, hyperchaos was firstly reported by *Rössler* . That is, the noted four-dimensional(4D) hyperchaoticsystem[7,10]. *Rössler*

Generally, a hyperchaotic system is classified as a chaotic system with more than one positive Lyapunov exponents[2,3 ,6,7,9], and has more complex dynamical behaviors than chaotic system[3,9].Very recently, hyperchaos was found numerically and experimentally by adding a simple state feedback controller[6,7,10].

[2] generated 4D hyperchaotic *Lü* system in 2006,and in 2009 generated another 4D hyperchaotic *Lü* system [3], while [6] generated 4D hyperchaotic Lorenz system in 2006,and in 2008 generated another 4D hyperchaotic Lorenz system [8],later more and more hyperchaotic systems are generated such as 4D hyperchaotic Liu system(2008)

[10], 4D four-wing hyperchaotic system(2010) [9]. Finally, 4D hyperchaotic Pan system(2011) [5].

In [5] Pan and etc propose four dimensional hyperchaotic pan system(2011) via state feedback control then transformation to fractional-order hyperchaotic system (FOHS) and studied chaos synchronization for this system.

In this paper, we generated a new modified hyperchaotic Pan system based on a three-dimensional Pan system by introducing a nonlinear state feedback controller, and we study some basic properties and behaviors for this system, also you study the dynamical behaviors of hyperchaotic Pan system(2011) and compare between two systems. Finally, we show the effect of state u on two systems.

2- Hyperchaotic Pan System

The Pan system or L$\ddot{u}$-like system(2010)[4,5] is described by

$$\left.\begin{array}{l} \dot{x} = a(y - x) \\ \dot{y} = cx - xz \\ \dot{z} = xy - bz \end{array}\right\} \qquad \ldots(1)$$

where c and a, b are real constants. When $a = 10, b = 2, c = 16$, Pan system has a chaotic attractor with Lyapunov exponents $LE_1 = 0.8311$, $LE_2 = 0.0000$, $LE_3 = -12.5113$ by Wolf Algorithm and Lyapunov dimension $D_{LE} = 2.0664$ [5]. The Lyapunov exponents spectrum and attractor of the system (1) is shown in Fig.1[5] ,and Fig.2 [5].
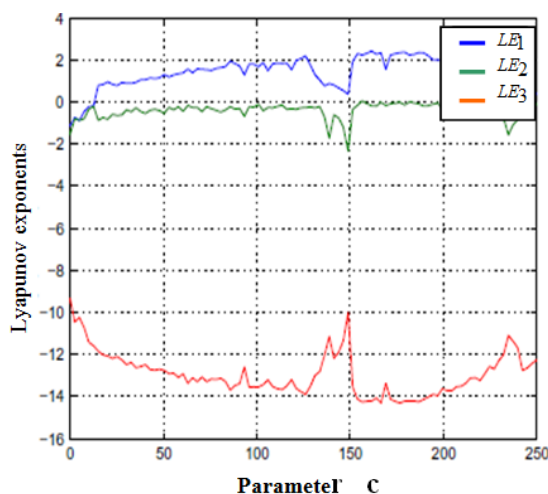

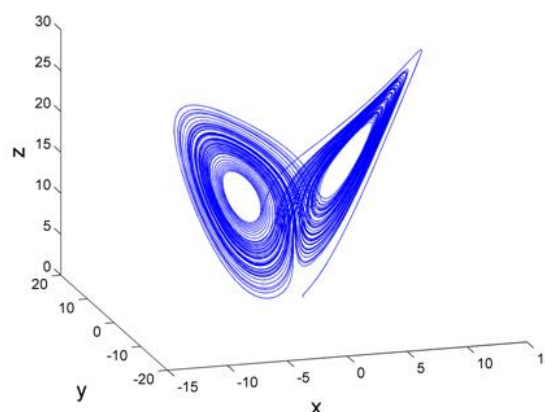| Fig. 1. Lyapunov exponents spectrum of Pan chaotic system versus parameter c	Fig. 2. the attractor of Pan system in the x-y-z space;

In order to obtain hyperchaotic , the important requirements as follows [2,5,7,8, 9,10 ,11] :

  (1) The minimal dimension of the phase space of an autonomous system is at least four.
  (2) The number of terms in the coupled equations giving rise to instability should be, at least, two, of which, at least, one should have a nonlinear function.

Based on pan system and above two basic requirements, one can construct fourth-order hyperchaot system ,by introducing a state feedback controller, as follows:

$$\left.\begin{array}{l} \dot{x} = a(y-x)+u \\ \dot{y} = cx - xz \\ \dot{z} = xy - bz \\ \dot{u} = -xz + du \end{array}\right\} \qquad \ldots(2)$$

where $(x, y, z, u) \in R^4$, and $a, b, c, d \in R$ are constant parameters. this system is called hyperchaotic Pan system, When $a = 10, b = 8/3, c = 28, d = 1.3$ [5], the Lyapunov exponents $LE_1 = 0.7340$, $LE_2 = 0.2492$, $LE_3 = 0.0000, LE_4 = -11.3437$ by Wolf Algorithm[5]. The Lyapunov exponents spectrum and attractor of the system (2) are shown in Fig.3[5] , and Fig.4 [5].
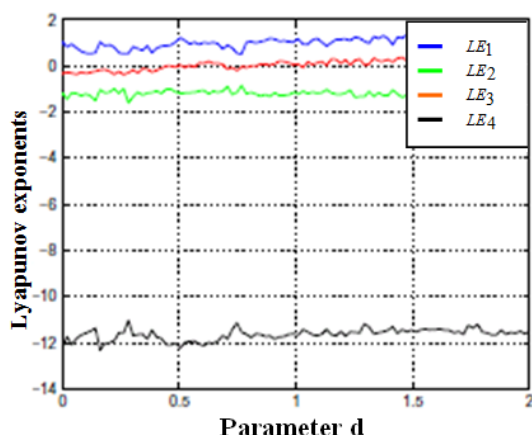
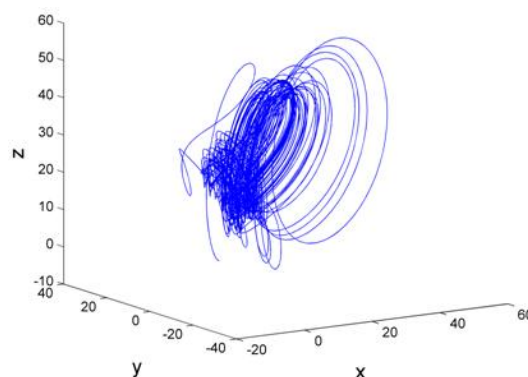Fig. 3. Lyapunov exponents spectrum of hyperchaotic Pan system versus parameter d

Fig. 4. the attractor of hyperchaotic Pan system in the x –y-z space;

3- Helping Results:

Remark 1 (Routh– Hurwitz Test) [1]:

All the roots of the indicated polynomial have negative real parts precisely when the given conditions are met.

- $\lambda^2 + A\lambda + B$: $A > 0$, $B > o$
- $\lambda^3 + A\lambda^2 + B\lambda + C$: $A > 0$, $C > 0$, $AB - C > 0$.
- $\lambda^4 + A\lambda^3 + B\lambda^2 + C\lambda + D$: $A > 0$, $AB - C > 0$, $(AB - C)C - A^2D > 0$, $D > 0$.

Remark 2 (Lyapunov Exponents) [2,7]:

The dynamical behaviors of this system can be classified as follows:

(1) For $LE_1 > LE_2 > 0, LE_3 = 0, LE_4 < 0$ and $LE_1 + LE_2 + LE_4 < 0$, the system is hyperchaos.

(2) For $LE_1 > 0, LE_2 = 0, LE_4 < LE_3 < 0$ and $LE_1 + LE_3 + LE_4 < 0$, the system is chaos.

(3) For $LE_1 = 0, LE_4 < LE_3 < LE_2 < 0$, the system is a periodic orbit.

(4) For $LE_4 < LE_3 < LE_2 < LE_1 < 0$, the system is a equilibrium point.

In the context of ordinary differential equations ODEs, the word "Bifurcation" has come to mean any marked change in the structure of the orbits of a system (usually nonlinear) as a parameter passes through a critical value[1].

Remark 3(Hopf Bifurcation) [9]:

Any system has a Hopf bifurcation if the following condition is satisfied:

    1- The Jacobian matrix has two purely imaginary roots and no other

roots with zero real parts.

2- $\dfrac{d}{d\mu}(\text{Re}(\lambda(\mu)))\big|_{\mu=\mu 0} \neq 0$

4- Main Results:

Based on Pan system and generating conditions, we can construct a new four dimension hyperchaotic system by introducing a state feedback controller, as follows:

Add controller u to the second equation of system (1), let $\dot{u} = -dy$, then we obtain a new hyperchaotic system

$$
\left.\begin{aligned}
\dot{x} &= a(y-x) \\
\dot{y} &= cx - xz + u \\
\dot{z} &= xy - bz \\
\dot{u} &= -dy
\end{aligned}\right\} \qquad \qquad …(3)
$$

where $(x, y, z, u) \in R^4$, and $a, b, c, d \in R$ are constant parameters. For simplification, system(3) is called a new modified hyperchaotic Pan system in this paper.

In the following we briefly describe some dynamical behaviors of the new hyperchaotic system (3).

4.1- Dissipative of System(3):

The divergence of the new four dimensional system (3) is

$$
div\vec{V} = \frac{\partial \dot{x}}{\partial x} + \frac{\partial \dot{y}}{\partial y} + \frac{\partial \dot{z}}{\partial z} + \frac{\partial \dot{u}}{\partial u} = -(a+b) \quad , \qquad …(4)
$$

Thus, when $a + b > 0$ the new system(3) is dissipative, consequently, all trajectories of the new system ultimately arrive at an attractor .

4.2- Equilibrium Point of System(3) :

The equilibrium of system (3) satisfies the following equations:

$$a(y-x) = 0, \qquad cx - xz + u = 0, \qquad xy - bz = 0, \qquad -dy = 0 \qquad …(5)$$

From the above equations, we obtain only equilibrium point $O(0,0,0,0)$

4.3- Lyapunov Exponents and Lyapunov Dimension :

We calculate the Lyapunov exponents for a new modified hyperchaotic system with the Wolf Algorithm by using MATLAB software, the numerical simulation is carried out with $a = 10$, $b = 8/3$, $c = 28$, $d = 10$ for the initial value (1 , 1 , 1 , 1) and the four Lyapunov exponents of a new modified hyperchaotic system (3) are $LE_1 = 0.38352$, $LE_2 = 0.12714$, $LE_3 = -0.00044514$, and $LE_4 = -13.1741$. The Lyapunov exponents spectrum and attractor of the system (3) are shown in Fig.5, and Fig.6 respectively.

So, we can obtain the Lyapunov dimension of the new modified hyperchaotic system(3), it is described as follows:

$$D_{LE} = j + \frac{1}{|LE_{j+1}|} \sum_{i=1}^{j} LE_i = 3 + \frac{LE_1 + LE_2 + LE_3}{|LE_4|} \qquad [5]$$

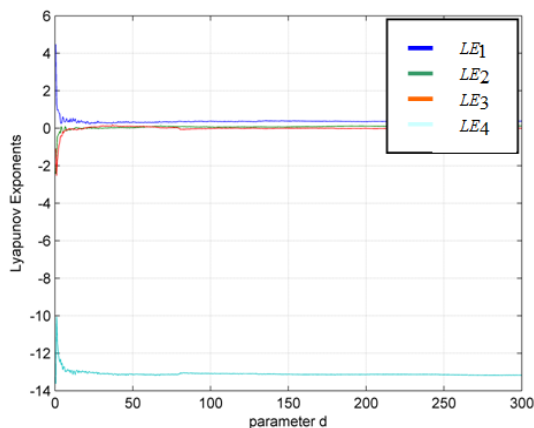$$= 3 + \frac{0.38352 + 0.12714 - 0.00044514,}{|-13.1741|} = 3.038728631.$$

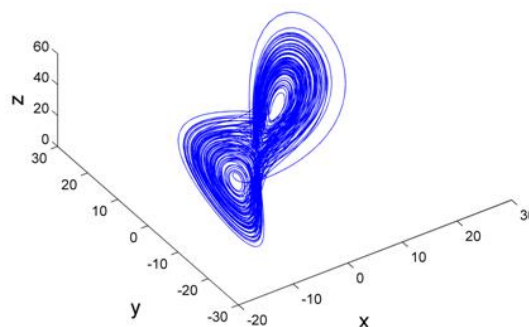**Fig. 5. Lyapunov exponents spectrum of modified hyperchaotic Pan system versus parameter d**

Fig. 6. the attractor of a new modified hyperchaotic Pan system in the x –y-z space;

Theorem1: The solution of system (3) when $a, b, d > 0$ has the following cases**:**

    1) Asymptotically stable   if $c \in (-\infty, 0)$ **,**

    2) Unstable   if $c \in (0, \infty)$ **,**

    3) critical case   if $c = 0$.

$$J = \begin{bmatrix} -a & a & 0 & 0 \\ c & 0 & 0 & 1 \\ 0 & 0 & -b & 0 \\ 0 & -d & 0 & 0 \end{bmatrix} \qquad \dots(6)$$

and its characteristic equation

$$f(\lambda) = \lambda^4 + (a+b)\lambda^3 + (ab + d - ac)\lambda^2 + (ad + bd - cab)\lambda + abd = 0 \qquad \dots(7)$$

Solving equation (7) gives $\lambda_1 = -b$, and the following equation:

$$f(\lambda) = \lambda^3 + a\lambda^2 + (d - ac)\lambda + ad = 0 \qquad \dots(8)$$

    By using Routh-Hurwitz method, the equation (8) has all roots with negative real parts if and only if $A > 0$, $C > 0$, $AB - C > 0$ where $A = a$, $B = d - ac$ and $C = ad$, so, it is clear that $A, C > 0$ we must show that $AB > C \Rightarrow a(d - ac) > ad \Rightarrow ac < 0$ since $a > 0$ hence $c < 0$ consequently, the solution of system (3) is Asymptotically stable if $c < 0$, unstable if $c > 0$ and critical case if $c = 0$.

Proposition 1: Equation (7) has purely imaginary roots if and only if $a, b, d > 0$, $c = 0 = c_0$.(c_0 critical value). In this case, the solutions of equation (7) are

$\lambda_1 = -b$ $\lambda_2 = -a$, $\lambda_{3,4} = \pm i\sqrt{d}$.

Proof: First get one root $\lambda_1 = -b$ from equation(7) then obtain cubic equation (equation 8) If $\lambda_{3,4} = \pm iw$ are the complex solutions and λ_2 the real solution of equation (7) then, from $\lambda_2 + \lambda_3 + \lambda_4 = -a \Rightarrow \lambda_2 = -a$. This easily leads to $a, b, d > 0$, $c = 0 = c_0$ and $\lambda_2 = -a$, $\lambda_{3,4} = \pm i\sqrt{d}$, $\lambda_1 = -b$.

In the following we will prove that the system (3) displays a Hopf bifurcation at the point $O(0,0,0,0)$. For $c = c_0 = 0$ the point $0(0,0,0,0)$ loses its stability.

Theorem 2: If $c = 0$, equation (7) has negative solutions $\lambda_1 = -b < 0$ $\lambda_2 = -a < 0$ together with a pair of purely imaginary roots $\lambda_{3,4} = \pm i\sqrt{d}$ such that $\text{Re}(\lambda'_c(c_0)) \neq 0$, therefore, the system (3) displays a Hopf bifurcation at the point $O(0,0,0,0)$.

Proof: If $c = 0$ the equation (8) is transformed into $(\lambda + a)(\lambda^2 + d) = 0$ with solutions $\lambda_2 = -a$, $\lambda_{3,4} = \pm i\sqrt{d}$

$$\lambda'_c = \frac{a\lambda}{3\lambda^2 + 2a\lambda + d - ca} \quad , \quad \lambda'_c(c_0) = \frac{a\lambda}{3\lambda^2 + 2a\lambda + d - ac} \bigg|_{c=0}$$

Substituting $\lambda_{3,4} = \pm i\sqrt{d}$, the real part and imaginary part of the $\lambda'_c(c_0)$ respectively are:

$$\text{Re}(\lambda'r_c(c_0)) = \frac{2a^2 d}{(-2d - ac)^2 + 4a^2 d} \neq 0, \text{Im}(\lambda'_c(c_0)) = \frac{(-2d - ac)a\sqrt{d}}{(-2d - ac)^2 + 4a^2 d} \neq 0.$$

Consequently, the system (3) displays a Hopf bifurcation at $O(0,0,0,0)$.

5 - Dynamical Behaviors of Hyperchaotic Pan System

In the following, we briefly describe some dynamical behaviors of system (2).

5.1-Symmetry:

Note that the invariance of the system (2) under the transformation $(x, y, z, u) \rightarrow (-x, -y, z, -u)$, i.e. under reflection in the z-axis. the symmetry persists for all values of the system parameters.

5.2- Dissipative of System (2):

The divergence of the new four dimensional system (2) is

$$div \vec{V} = \frac{\partial \dot{x}}{\partial x} + \frac{\partial \dot{y}}{\partial y} + \frac{\partial \dot{z}}{\partial z} + \frac{\partial \dot{u}}{\partial u} = -(a + b - d), \qquad \qquad ...(9)$$

Thus when $a + b - d > 0$ the new system(2) is dissipative, consequently all trajectories of the new system ultimately arrive at an attractor .

5.3- Equilibrium Points of System (2) :

System (2) has three equilibrium points $O(0,0,0,0)$, $p_{1,2}(\pm x_0, \pm y_0, z_0, \pm u_0)$

where $x_0 = \dfrac{d\sqrt{abc}}{\sqrt{d(ad-c)}}$, $y_0 = \dfrac{(ad-c)\sqrt{abc}}{a\sqrt{d(ad-c)}}$, $z_0 = c$, $u_0 = \dfrac{c\sqrt{abc}}{\sqrt{d(ad-c)}}$

when $d(ad-c) > 0$ while if $d(ad-c) \leq 0$ then, exist only one equilibrium point $O(0,0,0,0)$.

Theorem3: The solution of system (2) at the equilibrium point $O(0,0,0,0)$ when $a,b,d > 0$ is always unstable.

Proof: At the equilibrium point $O(0,0,0,0)$, system (2) is linearized, the Jacobian matrix is defined as :

$$J = \begin{bmatrix} -a & a & 0 & 1 \\ c & 0 & 0 & 0 \\ 0 & 0 & -b & 0 \\ 0 & 0 & 0 & d \end{bmatrix} \qquad \dots(10)$$

So, the eigenvalues are obtained as follows: $|J_0 - \lambda I| = 0$

$$\begin{vmatrix} -a-\lambda & a & 0 & 1 \\ c & -\lambda & 0 & 0 \\ 0 & 0 & -b-\lambda & 0 \\ 0 & 0 & 0 & d-\lambda \end{vmatrix} = 0 \quad \text{and} \quad (-b-\lambda)(d-\lambda)\begin{vmatrix} -a-\lambda & a \\ c & -\lambda \end{vmatrix} = 0$$

$\lambda_1 = -b$, $\lambda_2 = d$, $\lambda_{3,4} = \dfrac{-a \mp \sqrt{a^2 + 4ac}}{2}$, So $\lambda_2 > 0$ therefore system (2) is always unstable at the equilibrium point $O(0,0,0,0)$ when $a,b,d > 0$.

Proposition 2: The zero solution of system (2) has not purely imaginary roots if and only if $a,b,d > 0$, $c = \dfrac{d^2}{c+d} = c_0$ (c_0 critical value). In this case, the solutions of system (2) are $\lambda_1 = -b$ $\lambda_2 = \dfrac{cd}{c+d}$, $\lambda_{3,4} = \pm d$, therefore not satisfied one condition of Hopf bifurcation.

Due to the system is invariant under the transformation, so one only needs to consider the stability of anyone of the both. The stability of the system (2)at equilibrium point p_1 is analyzed in this paper.

Theorem4 : The solution of system (2)at the equilibrium point p_1 is
1) Asymptotically stable if

$$a > d - b \ , \ (a+b-d)\left[ab(1+\frac{cd}{ad-c}) - d(a+b)+c\right] > 2bc \ , \text{ and } c < 0$$

2) Unstable if $c > o$.

Proof: Now to find Jacobian matrix at p_1 we need the following transformation Under the linear transformation $(x, y, z, u) \to (X, Y, Z, U)$:

$$\left. \begin{aligned} x &= X + x_0 \\ y &= Y + y_0 \\ z &= Z + z_0 \\ u &= U + u_0 \end{aligned} \right\} \qquad \dots(11)$$

the system (2) becomes

$$\dot{X} = a(Y - X) + U$$

$$\dot{Y} = -\frac{d\sqrt{abc}}{\sqrt{d(ad-c)}} Z$$

$$\dot{Z} = \frac{(ad-c)\sqrt{abc}}{a\sqrt{d(ad-c)}} X + \frac{d\sqrt{abc}}{\sqrt{d(ad-c)}} Y - bZ \qquad \qquad \dots(12)$$

$$\dot{U} = -cX - \frac{d\sqrt{abc}}{\sqrt{d(ad-c)}} Z + dU$$

The equilibrium point p_1 of the system (2) is switched to the new equilibrium point $O'(0, 0, 0, 0)$ of the system (12) under the linear transformation,

The Jacobian matrix of the system (12) at $O'(0, 0, 0, 0)$ is:

$$J(O') = \begin{bmatrix} -a & a & 0 & 1 \\ 0 & 0 & -\dfrac{d\sqrt{abc}}{\sqrt{d(ad-c)}} & 0 \\ \dfrac{(ad-c)\sqrt{abc}}{a\sqrt{d(ad-c)}} & \dfrac{d\sqrt{abc}}{\sqrt{d(ad-c)}} & -b & 0 \\ -c & 0 & -\dfrac{d\sqrt{abc}}{\sqrt{d(ad-c)}} & d \end{bmatrix} \qquad \dots(13)$$

and the characteristic equation is :

$$\lambda^4 + (a+b-d)\lambda^3 + (ab(1+\frac{cd}{ad-c}) - d(a+b)+c)\lambda^2 + 2bc\lambda - 2abcd = 0 \qquad \dots(14)$$

Using Routh-Hurwitz criterion, the equation (14) has all roots with negative real parts if and only if the conditions are satisfied as follows:

$$A > 0, \quad AB - C > 0, \quad (AB-C)C - A^2 D > 0, \quad D > 0. \qquad \dots(15)$$

Consequently, conditions of Routh-Hurwitz are satisfied in (1) therefore, the is system (2) is asymptotically stable under the conditions:

$$a > d - b, \quad (a+b-d)\left[ab(1+\frac{cd}{ad-c}) - d(a+b)+c \right] > 2bc$$

while, when $c > 0$ then $D < 0$ Consequently, one of Routh-Hurwitz condition not satisfied, consequently, the system (2) is unstable under the condition $c > 0$, the proof is completed.

Corollary 1: State u is not effective on the stability in system (2) when $u = a_{14}$ while is effective on the stability system(3) when $u = \pm a_{14}$ or $u = -a_{24}$,

Corollary 2: we can generate another one hyperchaotic Pan system which has the same roots of system (2) as following:

$$\left. \begin{array}{l} \dot{x} = a(y-x) \\ \dot{y} = cx - xz + u \\ \dot{z} = xy - bz \\ \dot{u} = -xz + du \end{array} \right\} \qquad \dots(16)$$

Also we can generate other three modified hyperchaotic Pan system which have different characteristic equation of system (3) as following:

$$\left.\begin{array}{lll} \dot{x} = a(y-x) & \dot{x} = a(y-x)+u & \dot{x} = a(y-x)-u \\ \dot{y} = cx - xz - u & \dot{y} = cx - xz & \dot{y} = cx - xz \\ \dot{z} = xy - bz & \dot{z} = xy - bz & \dot{z} = xy - bz \\ \dot{u} = -dy & \dot{u} = -dy & \dot{u} = -dy \end{array}\right\} \qquad \ldots(17)$$

Proof: the eigenvalues of linearized system (16) are obtained as follows:

$$\left| J_0 - \lambda I \right| = 0 \quad , \quad \begin{vmatrix} -a-\lambda & a & 0 & 0 \\ c & -\lambda & 0 & \pm 1 \\ 0 & 0 & -b-\lambda & 0 \\ 0 & 0 & 0 & d-\lambda \end{vmatrix} = 0$$

$$\lambda_1 = -b \ , \quad \lambda_2 = d \ , \quad \lambda_{3,4} = \frac{-a \mp \sqrt{a^2 + 4ac}}{2}$$

while, the characteristic equation in systems (17) yield the following form respectively

$$\lambda^3 + a\lambda^2 - (d-ca)\lambda - ad = 0 \quad , \lambda^3 + a\lambda^2 - ac\lambda + cd = 0, \qquad \lambda^3 + a\lambda^2 - ac\lambda - cd = 0 \, \text{w}$$

ith $\lambda_1 = -b$

$$\dot{x} = a(y-x)$$
$$\dot{y} = cx - xz - u$$
$$\dot{z} = xy - bz$$
$$\dot{u} = -dy$$

$$\dot{x} = a(y-x)+u$$
$$\dot{y} = cx - xz$$
$$\dot{z} = xy - bz$$
$$\dot{u} = -dy$$

$$\dot{x} = a(y-x)-u$$
$$\dot{y} = cx - xz$$
$$\dot{z} = xy - bz$$
$$\dot{u} = -dy$$

$$\dot{x} = a(y-x)$$
$$\dot{y} = cx - xz + u$$
$$\dot{z} = xy - bz$$
$$\dot{u} = -dy$$

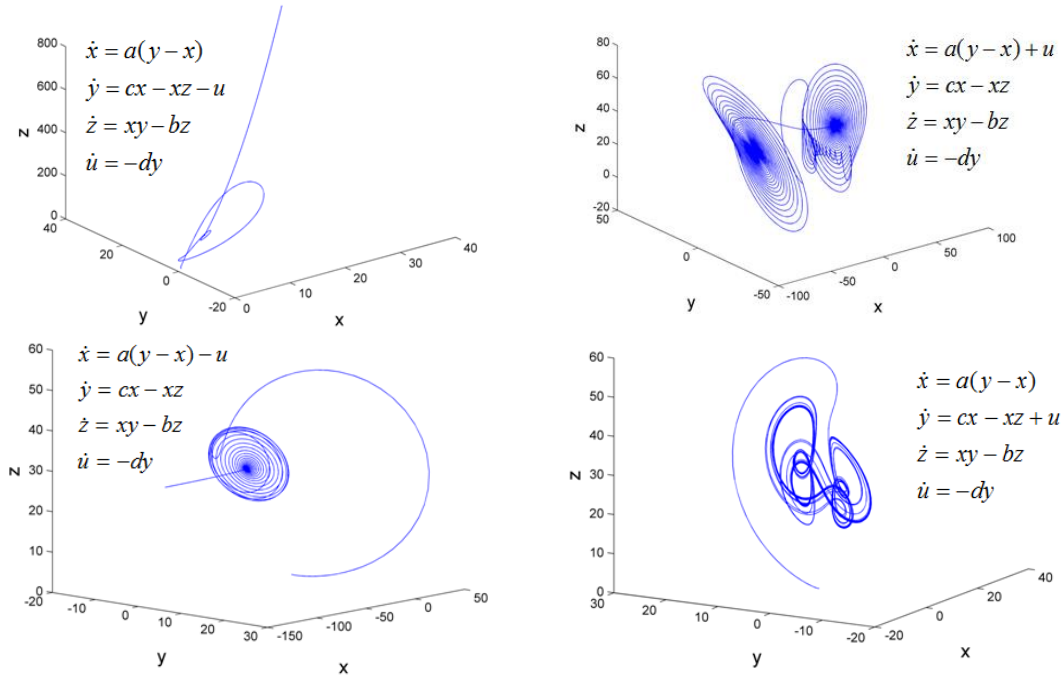**Fig. 7. The attractors of systems(17) with system(3) when a =50, b=2 , c=30 and d=10**

We explain the difference between the two systems by using of the following table

	hyperchaotic Pan system	A new modified hyperchaotic Pan system
Equation	$\dot{x} = a(y-x)+u$ $\dot{y} = cx - xz$ $\dot{z} = xy - bz$ $\dot{u} = -xz + du$	$\dot{x} = a(y-x)$ $\dot{y} = cx - xz + u$ $\dot{z} = xy - bz$ $\dot{u} = -dy$
Equilibrium	$0(0,0,0,0)$, $p_{1,2}(\pm x_0, \pm y_0, z_0, \pm u_0)$	$0(0,0,0,0)$
Stability	unstable always	asymptotically stable when $c < 0$ unstable when $c > 0$
Bifurcation	not Hopf bifurcation	Hopf bifurcation when $c = 0$
Effective u	not effective u on the system	effective u on the system
Dissipative	dissipative when $a+b-d > 0$	dissipative when $a+b > 0$
Proposes in	2011	2012

6- Conclusions:

In this paper a new four dimensional hyperchaotic system called a new modified hyperchaotic Pan system was presented. and some properties and dynamic behaviors for this system have been investigated, we studied the effective location and sign for state u of the system. We conclued that the effect of state u was different from system to another, where the state u was effective of a new system while was not effective on hyperchaotic pan system.

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