A Geometric Construction of a (56,2)-Blocking Set in PG(2,19) and on Three Dimensional Linear $[325, 3, 307]_{19}$Griesmer Code

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ABSTRACT

In this paper we give a geometrical construction of a (56, 2)-blocking set in PG(2, 19) and We obtain a new (325,18)- arc and a new linear code $[325,3,307]_{19}$and apply the Grismer rule so that we prove it an optimal or non-optimal code, giving some examples of field 19 arcs Theorem (2.1)

Keywords: *Arc* **,** *Bounded Griesmer* **,** *double Blocking set* **,** *projection*$[n, k, d]_q$ *code. Projective Plane* **,** *Optimal Linear code.*

البناء الهندسي للمجاميع القالبية –(2 ,56) في (19 ,2)PG وفي الشفرات Griesmer $[325,3,307]_{19}$ الخطية ثلاثية الابعاد.

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الملخص

في هذا البحث سوف نعطي بناء هندسي للمجموعة القالبية – (56,2) في المستوي الاسقاطي$PG(2,19)$ ونحصل على قوس جديد–(325,18) وشفرة خطية $[325,3,307]_q$جديدة ونطبق قاعدة Grismer عليها لكي نبرهن بأنها شفرة مثلى او غير مثلى مع اعطاء بعض الامثلة على اقواس الحقل 19 مبرهنة (2.1) .الهدف من البحث الحصول على قوس جديد.

الكلمات المفتاحية: القوس، قيود(حدود)*Griesmer* ، المجموعة القالبية المزدوجة، الشفرة الاسقاطية$[n, k, d]_q$، المستوي الاسقاطي، الشفرة الخطية المثلى.

1 . Introduction

Give GF(q) a chance to indicate the Galois field of q components and V (3, q) be the vector space of column vectors of length three with sections in GF(q). Let PG(2, q) be the comparing projective plane. The purposes of PG(2, q) are the non-zero vectors of V (3, q) with the standard that X =(x1, x2, x3) and Y=(ƛx1,ƛx2,ƛx3) speak to a similar point, where ƛ ∈ GF(q)/{0}. The quantity of purposes of PG(2, q) is q^2+q+1.If the point P(X) is the proportionality class of the vector X, at that point we will state that X is a vector speaking to P(X). A subspace of measurement one is an arrangement of focuses the majority of whose speaking to vectors shape a subspace of measurement two of V (3, q).Such subspaces are called lines. The quantity of lines in PG(2, q) is q^2+q+1. There are q + 1[5][6].

Definition 1.1 . A (k, r)- circular segment is an arrangement of k purposes of a projective plane to such an extent that some r, however no r+1 of them, are collinear[3]

Definition 1.2 A (l, n)- blocking set S in PG(2, q) is an arrangement of l focuses to such an extent that each line of PG(2, q) crosses S in at any rate n focuses, and there is a line meeting S inaccurately n focuses Note that a (k, r)- circular segment is the supplement of a (q^2+q+1-k, q + 1 − r)- blocking set in a projective plane Let V (n, q) signify the vector space of all arranged n-tuples over .and alternately GF(q). A direct code C over GF(q) of length n and measurement k is a k-dimensional subspace of V (n, q). The vectors of C are called code words. The Hamming separation between two code words is characterized to be the quantity of facilitate puts in which they differ. The least separation of a code is the littlest of the separations between particular code words. Such a code is called a [n,k,d]_q-code if its minimum Hamming separation is d. A focal issue in coding hypothesis is that of streamlining one of the parameters n, k and d for given estimations of the other two and q-settled. One of the variants is[2][9]

Problem. Find nq(k, d), the littlest estimation of n for which there exists an$[n,k,d]_q$-code which accomplishes this esteem is called ideal. The outstanding lower destined for the capacity nq(k, d) is the accompanying Griesmer bound

$$\boldsymbol{nq(k,d) \geq gq(k,d) = \sum_{j=0}^{k-1} \left\lceil \frac{d}{q^j} \right\rceil \; and\ so\ are\ optimal.}$$

Now give some examples using Griesmer base to see the optimal code and the non-optimal code.

Example(1.4):- Let the code be linear is $[325,3,307]_{19}$

Solution:- by best Grismer $\boldsymbol{nq(k,d) \geq gq(k,d) = \sum_{j=0}^{k-1} \left\lceil \frac{d}{q^j} \right\rceil}$.

$$\boldsymbol{n = nq(k,d) = \sum_{j=0}^{3-1} \left\lceil \frac{307}{19^j} \right\rceil}$$

$$=\boldsymbol{\frac{307}{19^0} + \frac{307}{19^1} + \frac{307}{19^2}}$$

=307+16.1578947368+0.8504155125 ≅ 325 So that code is optimal.

Example(1.5):- Let the code be linear is $[145,3,133]_{13}$.

Solution:- by best Grismer $\boldsymbol{nq(k,d) \geq gq(k,d) = \sum_{j=0}^{k-1} \left\lceil \frac{d}{q^j} \right\rceil}$.

$$\boldsymbol{n = nq(k,d) = \sum_{j=0}^{3-1} \left\lceil \frac{133}{13^j} \right\rceil}$$

$$=\boldsymbol{\frac{133}{13^0} + \frac{133}{13^1} + \frac{133}{13^2}}$$

=133+10.23076923+0.786982248 ≅ 145

So that code is optimal.

Example(1.6):- Let the code be linear is $[336,3,317]_{19}$.

Solution:- by best Grismer $\boldsymbol{nq(k,d)} \geq \boldsymbol{gq(k,d)} = \sum_{j=0}^{k-1} \left\lceil \frac{d}{q^j} \right\rceil$.

$\boldsymbol{n = nq(k,d)} = \sum_{j=0}^{3-1} \left\lceil \frac{317}{19^j} \right\rceil$

$= \frac{317}{19^0} + \frac{317}{19^1} + \frac{317}{19^2}$

=317+16.68421053+0.87116343 $\leq \boldsymbol{336}$

So that code is non-optimal

Codes with parameters are called Griesmer codes. There exists a relationship between (n,r)-circular segments in PG(2,9) and [n,3,d]codes, given by the following hypothesis.[gq(k,d)]q

Theorem 1.7 There exists a projective $[n,3,d]_q$code if and only if there exists an (n, n−d)-arc in PG(2, q). In this paper we consider the case q =19 and the elements of GF(19) are denoted by0,1,2,3, 4,5,6,7, 8, 9,10 ,11,12,13,14,15,16,17,18.

2.The construction

It is obvious that in PG(2, q) (q is prime) three lines as a rule position frame a (3q, 2)- blocking set. The issue of finding a 2-blocking set with under 3q components had for since quite a while ago stayed unsolved as of not long ago Braun et al. [2] found the principal case of such a set. They developed the (57, 2)-blocking set in PG(2, 19),
comprising of the accompanying focuses on

The complement of the (57,2)-blocking set
{ (0, 1,5), (0, 1, 8), (0, 1, 11), (1, 0, 7), (1, 0,12), (1, 1,1), (1, 1, 4), (1, 2; 9), (1, 2,10)
(1, 2, 15), (1, 3,7),(1; 3, 17), (1, 4, 6), (1, 4, 18), (1, 5, 5), (1, 5,10), (1, 5, 14), (1,6, 2)
(1, 6, 3), (1, 6, 16), (1, 7, 8), (1, 7, 11),(1,7,13), (1, 8, 0), (1, 8, 5), (1, 9, 8), (1, 9, 11)
(1, 9, 16), (1,10, 8), (1, 10, 16), (1, 11, 0), (1, 1,15),(1, 11, 14), (1, 12, 8), (1, 12, 11)
(1, 12, 13), (1, 13, 2), (1, 13, 3), (1, 13, 17), (1, 14,9), (1, 14,10), (1, 14, 14),(1, 15, 1)
(1,15,6), (1,15, 18), (1, 16, 3), (1, 16, 7), (1, 16, 12), (1, 16, 17), (1, 17, 4), (1, 17, 9)
(1, 17, 10),(1, 17,13), (1, 17, 15), (1, 18, 1), (1, 18, 4)}

All the more unequivocally, this (57, 2)- blocking set emerges as the supplement of a (324, 18)- circular segment in PG(2, 19) which they built by the strategy displayed in their article [2] (see additionally [1]. In the following hypothesis we in demonstrate Lower bound,(324,18)- circular segment and Give us the accompanying:

1) Get new code $[325,3,307]_{19}$
2) Get a new $(325,18) - arc$

Theorem 2.1. There exists a (56, 2)- blocking set in PG(2, 19) and a (325,18)-curve Evidence. Consider the accompanying 74 points in PG(2,19)

Mi ={(1,1,4),(1,12,14)(1,14,2),(0,1,13)(1,9,13),(1,2,17),(1,16,16),(1,7,6),(1,3,11),(1,15,15) (1,16,9),(1,10,7),(1,8,0)(1,5,18),(1,13,8),(1,17,3),(1,0,10),(1,11,1)(1,6,12),(1,4,5)}

Qi ={(1,17,0),(0,1,17)(1,15,4),(1,5,5),(1,11,12),(1,3,9),(1,2,11),(1,1,13),(1,7,1),(1,18,17), (1,10,14),(1,14,6), (1,0,15),(1,13,8),(1,6,3),(1,9,16) (1,7,3)(1,4,7),(1,8,18),(1,12,10)}

Ni={(1,12,14),(1,17,16),(1, 5,15),(1,0,13)(1,11,6),(1,3,18),(1,8,1),(1,13,3),(1,7,12),(1,18,5), (1,15,0),(0,1,8), (1,9,9),(1,6,4),(1,16,8),(1,10,17),(1,4,7),(1,2,10),(1,8,18),(1,12,10)}

Pi={(1,1,14),(1,17, 0)(1, 9,7),(1,12,2)(1,13,13),(1,15,16) ,(1,14,5),(1,11,10),(1,8,15),(1,4,9) ,(1,5,1), (1, 3,17),(0,1,11), (1,18,11),(1,16,8),(1,10,18),(1,11,15),(1,6,12), (1,2,6),(1,7,4)}

The lines li: aix +biy+ ciz=0, (i=1, 2, 3, 4) are chosen with the goal that each line Li contains the point (ai, bi, ci), (i=1, 2, 3, 4).The focuses Mi (I = 1, 2, . . . , 20) have a place with the line L17:15x + 10y + 8z = 0, The focuses Ni (I = 1, 2, . . . , 20) have a place with the line L8:x+5y+16z = 0. The focuses Pi (I = 1, 2, . . . , 20) lie on hold L23 :52x+7y+8z = 0, and the focuses Qi (I =1, 2, . . . , 20) are the purposes of the line L24:105x +5y+12z = 0. The four lines meet pairwise at the focuses M1 =Q1,M2 = P2, N1 = P1, N2 = Q2, M6 = N6 and P11 = Q11, i.e. they are lines by and large position

$P1: Y = 0$	$P2: X = 0$
$p3: x + y = 0$	$p4: x + 2y = 0$
$p5: x + 3y = 0$	$p6: x + 4y = 0$
$p7: x + 5y = 0$	$p8: x + 6y = 0$
$p9: y + 7y = 0$	$p10: x + 8y = 0$
$p11: x + 9y = 0$	$p12: x + 10y = 0$
$p13: x + 11y = 0$	$p14: x + 12y = 0$
$p15: x + 13y = 0$	$p16: x + 14y = 0$
$p17: y + 15y = 0$	$p18: x + 16y = 0$
$p19: x + 17y = 0$	$p20: x + 18y = 0$

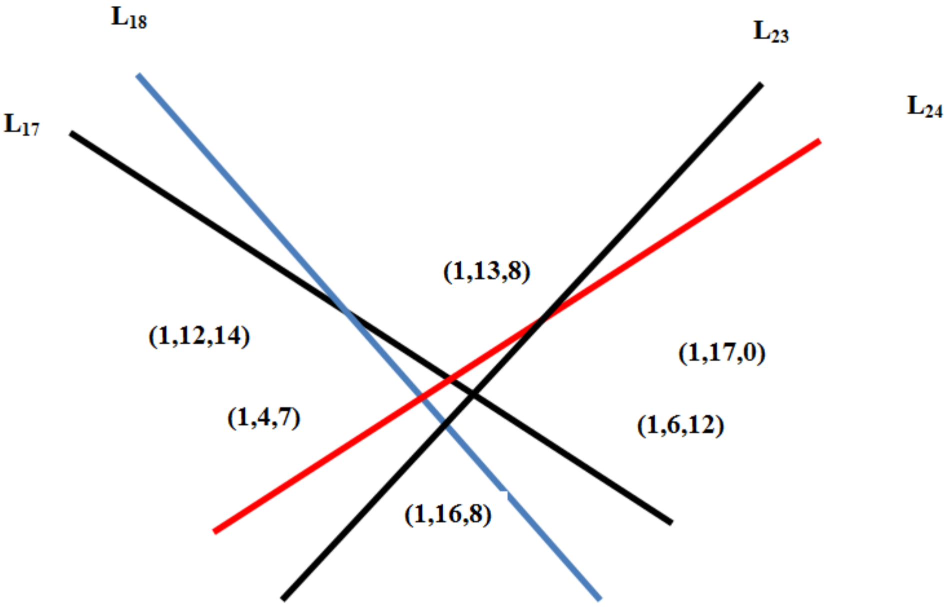
The watchful investigation of the lines L17, L18, L23 and L24 demonstrates that each fourfold (on account of I = 6, 11—each triple, and on account of I = 1, 2—each match) of focuses Mi , Ni , Pi , Qi (I = 1, 2 . . . , 20) has a place with one of the20 lines pi. Presently given us a chance to set the accompanying undertaking: Remove22 focuses from the set L17 ∪ L18∪ L23∪ L24, so that:

a) There is no line in PG(2, 19) which is unique in relation to li and which)
contains four of the expelled focuses
b) The lines that contain three of the evacuated focuses meet at most four new)
focuses A1, A2, A3,
c) The lines that contain only two of the evacuated focuses don't go through the
.crossing point focuses M1,M2, N1, N2,M6 and P11
The conditions (a)– (c) will ensure that including the focuses A1, A2, A3,A4 to the arrangement of outstanding purposes of the lines ,we will acquire a 2-blocking set without any than 56. Clearly we ought not expel any focuses from the quadruples Mi , Ni , Pi , Qi , I = 1, 2; generally, the lines p2: x = 0 and p1: y = 0 will move toward becoming 1-or 0-secants. Correspondingly, it isn't alluring to expel any focuses from the quadruples Mi , Ni , Pi , Qi , I = 6, 11, on the grounds that expelling a crossing point purpose of the lines Li will either not diminish enough the quantity of focuses, or the lines p5: x + 3y = 0 or p12: x + 10y = 0 will move toward becoming 1-or 0-secants.We require to abserve the accompanying guideline: on the off chance that we have officially expelled two points from a fourfold Mi , Ni , Pi , Qi , we ought to leave the staying two in the set. Else, one of the lines pi will turn into a 1-or 0-secant.Let us take out the accompanying 22: from the lineL17– M1, M4, M5, M8,M11,M12,M14,M20 from the lineL18– N2, N6, N11, N12,N14,N17,N16, from the lineL23– P1, P8, P14, P17 and from the lineL24– Q2, Q3, Q22

Now we select four lines intersecting six points and lines are L_{17},L_{18},L_{23},L_{24} such that

$L_{17} \cap L_{18} = 18 = (1,12,14)$
$L_{17} \cap L_{23} = 356 = (1,6,12)$
$L_{17} \cap L_{24} = 284 = (1,13,8)$
$L_{18} \cap L_{23} = 290 = (1,10,14)$
$L_{18} \cap L_{24} = 357 = (1,6,8)$
$L_{23} \cap L_{24} = 24 = (1,17,0)$

So the six common points are the sequence points $[18,356,284,290,357,24]$
Now we draw the intersection points and show the intersection



The set of removed points

$$A=\left\{\begin{matrix}(1,1,4),(0,1,13),(1,9,13),(1,7,6),(1,16,9),(1,10,7),(1,5,18),(1,4,5)\\(1,17,16),(1,,14,11),(1,18,5)\\(1,15,10),(1,9,9),(1,1,2),(1,2,10)(1,1,14)\\(1,11,10),(1,18,11),(1,11,15),(0,1,17),(1,15,4),(1,12,10)\end{matrix}\right\}$$

is a(22,8)-arc in PG(2,19) and has the following secant distribution:

$T_0 = 111$, $T_2 = 83$, $T_1 = 162$, $T_3 = 22$, $T_4 = 1$, $= 1$, $T_8 = 1, T_7 = 1$

condition (a) is satisfied. The forty-eight 2-secants of A are

$X + 5Y = 0$	X + 14y+= 0	$X + 2Y + 10Z = 0$
$X + 3Y + 15Z = 0$	$X + 11Y + 5Z = 0$	$X + 16Z = 0$
$X + 7Y + 5Z = 0$	$X + 12Y + 15Z = 0$	$X + 9Y + 18Z = 0$
$X + Y + 11Z = 0$	$Y + 3Z = 0$	$X + 14Y + 15Z = 0$
$X + 18Y + 18Z = 0$	$X + 8Y + 4Z = 0$	$X + 13Y + 15Z = 0$
$X + 2Y + 12Z = 0$	$X + 11Y + 11Z = 0$	$X + 6Y + 13Z = 0$
$X + 2Z = 0$	$X + 5Y + 2Z = 0$	$X + 13Y + 12Z = 0$
$X + 17Y + 11Z = 0$	$X + 8Y + 5Z = 0$	$X + 9Y + Z = 0$
$X + 7Y = 0$	$X + 8Y + 13Z = 0$	$X + 18Y + 6Z = 0$
$X + Y + 12Z = 0$	$X + 1Y + 13Z = 0$	$X + 18Y + 10Z = 0$
$X + 9Y + 3Z = 0$	$X + 17Y + 9Z = 0$	$X + 3Y + 5Z = 0$
$X + 8Y + 7Z = 0$	$X + 3Y + Z = 0$	$X + 18Y + 14Z = 0$
$X + 9Y + 2Z = 0$	$Y + 16Z = 0$	$X + 10Y + 8Z = 0$
$X + 16Y + 7Z = 0$	$Y + 8Z = 0$	$X + 12Y + 7Z = 0$
$X + 9Y + 3Z = 0$	$X + 10Y + 9Z = 0$	$X + 2Z = 0$
$X + 15Y + 12Z = 0$	$X + 13Y + 7Z = 0$	$X + 15Y + 8Z = 0$
$X + 12Y + 18Z = 0$	$X + 18Y + 6Z = 0$	$X + 6Y + Z = 0$
$X + 17Y + 15Z = 0$	$X + 12Y + Z = 0$	$X + 16Y + 18Z = 0$
$X + 12Y + 8Z = 0$	$X + 6Y + Z = 0$	$X + 16Y + 18Z = 0$
$X + 5Y + 7Z = 0$	$X + 4Y + 17Z = 0$	$X + 13Y + 4Z = 0$
$X + 6Y = 0$	$X + 11Y + 13Z = 0$	$X + 5Y + 9Z = 0$

$X+13Y+11Z=0$ $X+8Y+10Z=0$ $X+7Y+5Z=0$
$X+10Y+3Z=0$ $X+5Y+8Z=0$ $X+15Y+7Z=0$
$X+4Y+16Z=0$ $X+9Y+12Z=0$ $X+7Y+7Z=0$
$X+3Y+2Z=0$ $X+17Y+2Z=0$ $X+9Y+18Z=0$
$X+18Y+4Z=0$ $X+11Y+Z=0$ $X+7Y+10Z=0$
$X+11Y+2Z=0$ $X+7Y+14Z=0$ $X+5Y+9Z=0$
$X+6Y+14Z=0$ $X+18Y+10Z=0$

It is anything but difficult to check now that none of them contains a crossing point purpose of lines li; therefore condition (c) is fulfilled .Let us take a gander at condition (b). The 3-secants of An, i.e. the lines not the same as li , with the end goal that each contains three of the evacuated focuses, are

$g1=x+y+4z=0$, $N_7,M_{10},P_3\in g1$
$g2=0x+y+13z=0$, $M_{11},Q_{18},P_4\in g2$
$g3=1x+9y+13z=0$, $M_9,N_{11},P_{13}\in g3$
$g4=x+7y+6z=0$, $M_{13},Q_6,N_5\in g4$
$g5=x+16y+9z=0$, $M_4,N_{15},P_{16}\in g5$
$g6=x+10y+7z=0$, $Q_1,P_2,N_9\in g6$
$g7=x+5y+18z=0$, $Q_7,P_{18},M_{19}\in g7$
$g8=x+4y+5z=0$, $Q_{13},P_8,N_8\in g8$
$g9=x+17y+16z=0$, $M_9,N_4,P_3\in g9$
$g10=x+14y+11z=0$, $N_1,M_2,P_{19}\in g10$
$g11=x+18y+5z=0$, $P_{14},Q_{13},N_{18}\in g11$
$g12=x+15y+0z=0$, $Q_4,P_{11},N_3\in g12$
$g13=x+9y+9z=0$, $M_{15},N_7,P_8\in g13$
$g14=x+y+2z=0$, $Q_{18},P_{12},N_{19}\in g14$
$g15=x+2y+10z=0$, $M_{19},N_9,P_{18}\in g15$
$g16=x+y+14z=0$, $M_2,N_1,P_8\in g16$
$g17=x+11y+10z=0$, $Q_7,P_1,M_{20}\in g17$
$g18=x+18y+11z=0$, $M_{15},N_{11},P_{20}\in g18$
$g19=x+11y+15z=0$, $M_{19},N_7,Q_2\in g19$
$g20=0x+y+17z=0$, $Q_{15},P_{17},N_{16}\in g20$
$g21=x+15y+4z=0$, $Q_{14},P_{12},N_5\in g21$
$g22=x+12y+10z=0$, $M_2,N_1,P_9\in g22$

Each line giintersects some line $\boldsymbol{li}$ at a point not in the set A. Indeed**:**
$g1\cap l2=(1,14,11)$, $g2\cap l1=(1,10,7)$
$g3\cap l3=(1,11,10)$, $g4\cap l4=(1,1,13)$
$g5\cap l2=(1,17,16)$, $g6\cap l1=(1,9,13)$
$g7\cap l4=(0,1,17)$, $g8\cap l3=(1,2,6)$
$g9\cap l1=(0,1,13)$, $g10\cap l4=(1,12,10)$
$g11\cap l1=(1,4,5)$, $g12\cap l1=(1,7,9)$
$g13\cap l4=(1,15,4)$, $g14\cap l2=(1,0,13)$
$g15\cap l2=(1,6,11)$, $g16\cap l4=(1,13,6)$
$g17\cap l2=(1,2,5)$, $g18\cap l4=(1,9,16)$
$g19\cap l3=(1,9,9)$, $g20\cap l2=(1,3,18)$
$g21\cap l2=(1,18,6)$, $g22\cap l1=(1,10,7)$

Furthermore, the lines giintersect one another in quadruples at the points $(1,12,10), (1,11,15), (1,3,2), (1,1,1)$).
More preciseely

$g1 \cap g3 \cap g5 \cap g7 \cap g16 \cap g18 \cap g21 = (1,12,10)$
$g6 \cap g9 \cap g12 \cap g13 \cap g14 \cap g15 \cap g21 \cap g22 = (1,11,15)$
$g2 \cap g7 \cap g8 = (1,3,2)$
$g4 \cap g14 \cap g17 \cap g20 = (1,1,1)$

Therefore, $(1,12,10), (1,11,15), (1,3,2), (1,1,1)$ are the points A1,A2,A3,A4.
Adding these four points to the rest 52 points, we obtain the set

$$B = \left\{ \begin{array}{c} (1,12,14), \\ (1,14,2), (1,2,17), (1,18,16), (1,3,11), (1,15,15), (1,8,0), (1,17,3), (1,0,10) \\ (1,11,1) \\ (1,5,15) \\ (1,0,13), (1,11,6), (1,3,18), (1,8,1), (1,13,3), (1,7,12), (0,1,8), (1,6,4), (1,10,17) \\ (1,9,7), (1,12,2), (1,13,13), (1,15,16), (1,14,5), (1,8,15), (1,4,9), (1,5,1), (1,3,17), \\ (0,1,11) \\ (1,10,18) \\ (1,11,12) \\ (1,6,2)(1,7,4)(1,5,5)(1,3,9)(1,2,11)(1,1,13)(1,7,1) \\ (1,18,17)(1,10,14)(1,14,6)(1,0,15)(1,6,3)(1,9,16)(1,7,3)(1,8,18)(1,13,8)(1,6,12) \\ (1,16,8)(1,4,7)(1,17,0) \\ ,(1,12,10), (1,11,15), (1,3,2), (1,1,1) \end{array} \right\}$$

The supplement of the set B is a (325, 18)- bend
.
It pursues now by Theorem 1.7 that there exists a $[325,3,307]_{19}$ Griesmer code

M18(2,19)={(1,0,0),(0,1,0),(0,0,1),(1,0,2),(1,5,2),(1,5,8),(1,6,13)
(1,4,16),(1,3,14),(1,17,15),(1,7,7),(1,15,12),(1,4,5),(1,2,10),(1,1,4)
(1,3,15),(1,12,10),(1,1,14),(0,1,17),(1,0,16),(1,3,2),(1,5,18),(1,14,15)
(1,7,5),(1,2,16),(1,3,8),(1,6,1),(1,16,15),(1,7,0),(0,1,7),(1,0,17),(1,7,18)
(1,9,8),(1,6,18),(1,9,18),(1,15,4),(1,12,11),(1,13,6),(1,8,11),(1,13,11)
(1,13,0),(0,1,13),(1,11,2),(1,5,0),(0,1,5),(1,0,4),(1,2,12),(1,4,10),(1,1,6)
(1,8,10),(1,1,10),(1,1,3),(1,16,18),(1,9,13),(1,17,5),(1,14,11),(1,4,8)
(1,18,18),(1,9,12),(1,4,0),(0,1,4),(1,0,14),(1,17,2),(1,15,11),(1,13,10)
(1,1,15),(1,7,9),(1,18,14),(1,17,4),(1,12,16),(1,3,0),(0,1,3),(1,0,18)
(1,9,2),(1,5,9),(1,9,10),(1,1,11),(1,13,15),(1,13,11),(1,18,8),(1,6,15)
(1,7,6),(1,10,6),(1,8,6),(1,8,9),(1,18,14),(1,11,10),(1,11,13),(1,11,9)
(1,18,10),(1,1,1),(1,10,12),(1,4,4),(1,12,12),(1,4,12),(1,4,18),(1,9,0)
(0,1,9),(1,0,1),(1,10,2),(1,5,14),(1,17,11),(1,13,11),(1,17,14),(1,17,6)
(1,8,5),(1,2,18),(1,9,1),(1,10,16),(1,3,16),(1,16,1),(1,10,10),(1,1,12)
(1,4,6),(1,10,15),(1, 7,15),(1,7,13),(1,11,3),(1,16,7),(1,15,14),(1,17,10)
(1,1,0),(0,1,1),(1,4,2),(1,5,3),(1,16,6),(1,8,16),(1,3,7),(1,15,9),(1,18,6)
(1,8,13),(1,11,14),(1,17,18),(1,9,3),(1,16,13),(1,11,7),(1,10,5),(1,6,9)
(1,2,3),(1,13,16),(1,3,3),(1,16,12),(1,14,7),(1,15,3),(1,16,4),(1,17,8)
(1,18,15),(1,7,14),(1,17,7),(1,1,17),(1,14,16),(1,3,6),(1,8,7),(1,15,8)
(1,6,16),(1,3,1),(1,10,13),(1,11,17),(1,14,4),(1,12,18),(1,9,15),(1, 7,8)

(1,6,6),(1,8,12),(1,4,15),(1,7,11),(1,13,17),(1,14,13),(1,11,4),(1,12,1)
(1,8,14),(1,10,8),(1,6,5),(1,2,14),(1,17,17),(1,14,12),(1,4,1),(1,10,4)
(1,12,8),(1,6,17),(1,14,10),(1,1,16),(1,3,5),(1,2,8),(1,6,14),(1,17,9),
(1,18,4)(1,12,9),(1,18,9),(1,18,3),(1,16,5),(1,2,15),(1,7,16),(1,3,4)
(1,12,0),(0,1,12),(0,1,6),(1,8,2),(1,5,4),(1,12,5),(1,2,7),(1,15,13)
(1,11,15),(1,16,0),(0,1,16),(1,0, 6),(1,2,2),(1,5,12),(1,4,3),(1,16,9)
(1,18,5),(1,2,0),(0,1,2),(1,0,7),(1,15,2),(1,17,1),(1,10,1),(1,10,7)
(1,15,0),(0,1,15),(1,0,9),(1,18,2),(1,5,16),(1,0,8),(1,6,2),(1,5,13)
(1,11,0),(1,7,2),(1,5,18),(1,9,9),(1,18,12),(1,4,17),(1,3,13),(1,11,16)
(1,14,1),(1,10,9),(1,18,11),(1,12,17),(1,14,18),(1,9,14),(1,16,3),(1,16,11)
(1,13,1),(1,3,10),(1,1,5),(1,2,4),(1,12,7),(1,15,11),(1,15,7),(1,6,7)
(1,15,18),(1,9,4),(1,12,15),(1,7, 10),(1,1,9),(1,18,1),(1,10,11),(1,13,18)
(1,9,5),(1,2,1),(1,10,3),(1,16,10),(1,4,11),(1,1,18),(1,9,11),(1,13,5)
(1,2,9),(1,18,0),(0,1,18),(1,0,11),(1,13,2),(1,5,10),(1,1,7),(1,15,17)
(1,14,3),(1,16,17),(1,14,17),(1,14,8),(1,6,10),(1,1,8),(1,6,0),(0,1,6)
(1,1,2),(1,5,7),(1,15,1),(1,10,0),(0,1,10),(1,0,3),(1,16,2),(1,5,6)
(1,8,4),(1,12,3),(1,16,4),(1,12,4),(1,12,13),(1,14,9),(1,18,7),(1,11,11)
(1,15,6),(1,8,8),(1,15,5),(1,2, 13),(1,11,5),(1,2,5),(1,9,17),(1,14,14)
(1,17,12),(1,4,13),(1,11,8),(1,6,11),(1,13,4),(1,12,6),(1,8,3),(1,16,16)
(1,3,12),(1,4,14),(1,17,13),(1,11,18),(1,9,6),(1,8,17),(1,14,0),(0,1,14)
(1,13,12),(1,17,16)}.

Table (1)
Projection Level Points in PG(2,19)

i	pi	i	pi	i	pi	i	pi	i	pi
1	(1,0,0)	29	(1,14,15)	57	(0,1,5)	85	(1,17,2)	113	(1,8,9)
2	(0,1,0)	30	(1,7,5)	58	(1,0,4)	86	(1,15,11)	114	(1,18,14)
3	(0,0,1)	31	(1,2,16)	59	(1,12,2)	87	(1,13,10)	115	(1,11,10)
4	(1,0,2)	32	(1,3,8)	60	(1,5,5)	88	(1,1,15)	116	(1,1,13)
5	(1,5,2)	33	(1,6,1)	61	(1,2,12)	89	(1,7,9)	117	(1,11,13)
6	(1,5,8)	34	(1,10,5)	62	(1,4,10)	90	(1,18,14)	118	(1,11,9)
7	(1,6,13)	35	(1,2,3)	63	(1,1,6)	91	(1,17,4)	119	(1,18,10)
8	(1,11,11)	36	(1,16,15)	64	(1,8,10)	92	(1,12,16)	120	(1,1,1)
9	(1,13,12)	37	(1,7,0)	65	(1,1,10)	93	(1,3,0)	121	(1,10,12)
10	(1,4,16)	38	(0,1,7)	66	(1,1,3)	94	(0,1,3)	122	(1,4,4)
11	(1,3,14)	39	(1,0,17)	67	(1,16,18)	95	(1,0,18)	123	(1,12,12)
12	(1,17,15)	40	(1,14,2)	68	(1,9,13)	96	(1,9,2)	124	(1,4,12)
13	(1,7,7)	41	(1,5,15)	69	(1,11,6)	97	(1,5,9)	125	(1,4,18)
14	(1,15,12)	42	(1,7,18)	70	(1,8,14)	98	(1,18,16)	126	(1,9,0)
15	(1,4,5)	43	(1,9,8)	71	(1,17,5)	99	(1,3,18)	127	(0,1,9)
16	(1,2,10)	44	(1,6,18)	72	(1,2,17)	100	(1,9,10)	128	(1,0,1)
17	(1,1,4)	45	(1,9,18)	73	(1,14,11)	101	(1,1,11)	129	(1,10,2)
18	(1,12,14)	46	(1,9,7)	74	(1,13,13)	102	(1,13,15)	130	(1,5,14)

19	(1,17,16)	47	(1,15,4)	75	(1,11,12)	103	(1,7,17)	131	(1,17,11)
20	(1,3,15)	48	(1,12,11)	76	(1,4,8)	104	(1,14,5)	132	(1,13,14)
21	(1,7,4)	49	(1,13,6)	77	(1,6,7)	105	(1,2,11)	133	(1,17,14)
22	(1,12,10)	50	(1,8,11)	78	(1,15,16)	106	(1,13,11)	134	(1,17,6)
23	(1,1,14)	51	(1,13,11)	79	(1,3,9)	107	(1,18,8)	135	(1,8,5)
24	(1,17,0)	52	(1,13,0)	80	(1,18,18)	108	(1,6,15)	136	(1,2,18)
25	(0,1,17)	53	(0,1,13)	81	(1,9,12)	109	(1,7,6)	137	(1,9,1)
26	(1,0,16)	54	(1,0,13)	82	(1,4,0)	110	(1,8,1)	138	(1,10,16)
27	(1,3,2)	55	(1,11,2)	83	(0,1,4)	111	(1,10,6)	139	(1,3,13)
28	(1,5,18)	56	(1,5,0)	84	(1,0,14)	112	(1,8,6)	140	(1,11,16)

	pi	i	pi	i	pi	i	pi	i	pi
141	(1,3,16)	176	(1,13,16)	211	(1,10,8)	246	(1,0,6)	281	(1,14,1)
142	(1,3,11)	177	(1,3,3)	212	(1,6,5)	247	(1,2,2)	282	(1,10,9)
143	(1,13,3)	178	(1,16,12)	213	(1,2,14)	248	(1,5,12)	283	(1,18,11)
144	(1,16,1)	179	(1,4,9)	214	(1,17,17)	249	(1,4,3)	284	(1,13,8)
145	(1,10,10)	180	(1,18,17)	215	((1,14,12)	250	(1,16,9)	285	(1,6,4)
146	(1,1,12)	181	(1,14,7)	216	(1,4,1)	251	(1,18,5)	286	(1,12,17)
147	(1,4,6)	182	(1,15,3)	217	(1,10,4)	252	((1,2,0)	287	(1,14,18)
148	(1,8,15)	183	(1,16,14)	218	(1,12,8)	253	(0,1,2)	288	(1,9,14)
149	(1,7,1)	184	(1,17,8)	219	((1,6,17)	254	(1,0,7)	289	(1,17,3)
150	(1,10,15)	185	(1,6,9)	220	(1,14,10)	255	(1,15,2)	290	(1,16,8)
151	(1,7,15)	186	(1,18,15)	212	(1,1,16)	256	(1,5,1)	291	(1,6,3)
152	(1,7,13)	187	(1,7,14)	222	(1,3,5)	257	(1,10,14)	292	(1,16,3)
153	(1,11,3)	188	(1,17,7)	223	(1,2,8)	258	((1,17,1)	293	(1,16,11)
154	(1,16,7)	189	(1,15,10)	224	(1,6,14)	259	(1,10,1)	294	(1,13,1)
155	(1,15,14)	190	(1,1,17)	225	(1,17,9)	260	(1,10,7)	295	(1,10,18)
156	(1,17,10)	191	(1,14,16)	226	(1,18,4)	261	(1,15,0)	296	(1,9,16)
157	(1,1,0)	192	(1,3,6)	227	((1,12,9)	262	(0,1,15)	297	(1,3,10)
158	(0,1,1)	193	(1,8,7)	228	(1,18,9)	263	(1,0,9)	298	(1,1,5)
159	(1,0,12)	194	(1,15,8)	229	(1,18,3)	264	(1,18,2)	299	(1,2,4)
160	(1,4,2)	195	(1,6,16)	230	(1,16,5)	265	(1,5,16)	300	(1,12,7)
161	(1,5,3)	196	(1,3,1)	231	(1,2,15)	266	(1,3,17)	301	(1,15,11)
162	(1,16,6)	197	(1,10,13)	232	(1,7,16)	267	(1,14,6)	302	(1,13,7)
163	(1,8,16)	198	(1,11,17)	233	(1,3,4)	268	(1,8,0)	303	(1,15,7)
164	(1,3,7)	199	(1,14,4)	234	(1,12,0)	269	(0,1,8)	304	(1,15,18)
165	(1,15,9)	200	(1,12,18)	235	(0,1,12)	270	(1,0,8)	305	(1,9,4)
166	(1,18,6)	201	(1,9,15)	236	(1,0,6)	271	(1,6,2)	306	(1,12,15)
167	(1,8,13)	202	(1,7,8)	237	(1,8,2)	272	(1,5,13)	307	(1,7,10)
168	(1,11,14)	203	(1,6,6)	238	(1,5,4)	273	(1,11,0)	308	(1,1,9)
169	(1,17,18)	204	(1,8,12)	239	(1,12,5)	274	(0,1,11)	309	(1,18,1)
170	(1,9,3)	205	(1,4,15)	240	(1,2,7)	275	(1,0,15)	310	(1,10,11)
171	(1,16,13)	206	(1,7,11)	241	(1,15,13)	276	(1,7,2)	311	(1,13,18)
172	(1,11,7)	207	(1,13,17)	242	(1,11,15)	277	(1,5,18)	312	(1,9,5)
173	(1,15,15)	208	(1,14,13)	243	(1,7,3)	278	(1,9,9)	313	(1,2,1)
174	(1,7,12)	209	(1,11,4)	244	(1,16,0)	279	(1,18,12)	314	(1,10,3)
175	(1,4,11)	210	(1,12,1)	245	(0,1,16)	280	(1,4,17)	315	(1,16,10)

i	pi	i	pi	i	pi	i	pi	i	pi
316	(1,1,18)	330	(1,14,8)	344	(1,5,6)	358	(1,15,5)	372	(1,8,3)
317	(1,9,11)	331	(1,6,10)	345	(1,8,4)	359	(1,2,13)	373	(1,16,16)
318	(1,13,5)	332	(1,1,8)	346	(1,12,3)	360	(1,11,5)	374	(1,3,12)
319	(1,2,9)	333	(1,6,8)	347	(1,16,4)	361	(1,2,5)	375	(1,4,14)
320	(1,18,0)	334	(1,6,0)	348	(1,12,4)	362	(1,2,6)	376	(1,17,3)
321	(0,1,18)	335	(0,1,6)	349	(1,12,13)	363	(1,8,18)	377	(1,11,18)
322	(1,0,11)	336	(1,0,10)	350	(1,11,1)	364	(1,9,17)	378	(1,9,6)
323	(1,13,2)	337	(1,1,2)	351	(1,10,17)	365	(1,14,14)	379	(1,8,17)
324	(1,5,10)	338	(1,5,7)	352	(1,14,9)	366	(1,17,12)	380	(1,14,0)
325	(1,1,7)	339	(1,15,1)	353	(1,18, 7)	367	(1,4,13)	381	(0,1,14)
326	(1,15,17)	340	(1,10,0)	354	(1,15,6)	368	(1,11,8)		
327	(1,14,3)	341	(0,1,10)	355	(1,8,8)	369	(1,6,11)		
328	(1,16,17)	342	(1,0,3)	356	(1,6,12)	370	(1,13,4)		
329	(1,14,17)	343	(1,16,2)	357	(1,4, 7)	371	(1,12,6)		

Table (2)
Projection Level Lines in PG(2,19)

L_1	1,2,24,37,52,56,82,93,126,157,234,244,252,261,268,273,320,334,340,380
L_2	2,3,25,38,53,57,83,94,127,158,235,245,253,262,289,274,321,335,341,381
L_3	3,4,26,39,54,58,84,95,128,159,236,246,254,263,290,275,322,336,342,1
L_4	4,5,27,40,55,59,84,96,129,160,237,247,255,264,291,276,323,337,343,2
⋮	⋮
L_{381}	381,1,23,36,51,55,81,92,125,156,233,243,251,260,267,272,319,333,379

Table(3) The bounds of linear codes :

r \ q	11	13	16	17	19
2	12	14	18	18	20
3	21	23	28	28-33	31-39
4	32	38-40	52	48-52	52-58
5	43-45	49-53	65	61-69	68-77
6	56	64-66	78-82	79-86	86-96
7	67	79	93-97	95-103	105-115
8	78	92	120	114-120	126-134
9	89-90	105	129-131	137	147-153
10	100-102	118-119	142-148	154	172
11		132-133	159-164	166-171	191

12		143-147	180-181	183-189	204-210
13			195-199	205-207	225-230
14			210-214	221-225	243-250
15			231	239-243	265-270
16				256-261	286-290
17					305-310
18					324-330

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