A Robust Spectral Three-Term Conjugate Gradient Algorithm for Solving Unconstrained Minimization Problems

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Received on: 19/12/2018 **Accepted on: 27/03/2019**

ABSTRACT

In this paper, we investigated a new improved Conjugate Gradient (CG) algorithm of a Three-Term type (TTCG) based on Dai and Liao procedure to improve the CG algorithm of (Hamoda, Rivaie, and Mamat / HRM). The new CG-algorithm satisfies both the conjugacy condition and the sufficient descent condition. The step-size of this TTCG-algorithm would be computed by accelerating the Wolfe-Powell line search technique. The proposed new TTCG algorithms have demonstrated their global affinity in certain specific circumstances given in this paper.

Keywords: Three-Term Conjugate Gradient, Scaling Parameter, Conjugacy Property, Search Directions, Large Dimensions, Wolfe-Powell Line Search.

خوارزمية كفؤة للتدرج المترافق الطيفي ذات الحدود الثلاث لحل المسائل التصغيرية غير المقيدة

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تاريخ استلام البحث: 2018/12/19 **تاريخ قبول البحث: 2019/03/27**

الملخص

في هذا البحث، تم التقصي في استحداث خوارزمية جديدة للتدرج المترافق ذات الحدود الثلاث (TTCG) بالاعتماد على خوارزمية التدرج المترافق (Dai & Liao, 2001) لتحسين خوارزمية HRMCG الخاصة بـ (Hamoda, Rivaie, and Mamat, 2016).

خوارزمية TTCG الجديدة تحقق شرطي الاقتران والانحدار الكافٍ وتم حساب حجم الخطوة المثلى في خوارزمية TTCG الجديدة من خلال تسريع تقنية خط البحث العائدة الى Wolfe-Powell. لقد أظهرت خوارزميات TTCG الجديدة المقترحة تقاربها الشامل في بعض الظروف المحددة الواردة في هذه الورقة مع الحصول على نتائج عددية مشجعة.

الكلمات المفتاحية: التدرج المترافق ذات الحدود الثلاث، معلمة القياس، خاصية الترافق، اتجاهات البحث، أبعاد كبيرة، خط بحث Wolfe-Powell .

1 . Introduction

To start by giving any issue of unconstrained optimization we need to know this issue as follows:

$$\min\{f(x): x \in R^n\} \tag{1}$$

Let us define the function as f from R^n to R is continuously differentiable function, and its gradient is denoted by $g(x) = \nabla f(x)$, these CG methods are known to be designed to solve the problem of type (1), specifically when the n dimension is very large due to the simplicity of repetition of the search, memory requirements are very low. The iterative formula for standard TTCG methods is given by:

$$x_{k+1} = x_k + \alpha_k d_k , \qquad k = 0,1,2 \ldots \tag{2}$$

The α_k amount of the step is always evident as a positive scalar and d_k is the search direction specified by:

$$d_{k+1} = \begin{cases} -g_{k+1} & if\ k = 0 \\ -g_{k+1} + \beta_k d_k & if\ k \geq 1 \end{cases} \tag{3}$$

Where β_k is scalar. The differences highlight CG methods in their speed and performance by specifying the numerical parameter β_k. This is a set of the famous versions of β_k as contained in their sources:

$$\beta_k^{HS} = \frac{g_{k+1}^T(g_{k+1} - g_k)}{(g_{k+1} - g_k)^T d_k} \qquad \beta_k^{FR} = \frac{g_{k+1}^T g_{k+1}}{g_k^T g_k} \qquad \beta_k^{PR} = \frac{g_{k+1}^T(g_{k+1} - g_k)}{g_k^T g_k}$$

$$\beta_k^{CD} = -\frac{g_{k+1}^T g_{k+1}}{d_k^T g_k} \qquad \beta_k^{LS} = -\frac{g_{k+1}^T(g_{k+1} - g_k)}{d_k^T g_k} \qquad \beta_k^{DY} = \frac{g_{k+1}^T g_{k+1}}{(g_{k+1} - g_k)^T d_k}$$

CG methods converge with the FR format worldwide but have poor numerical performance because of the behavior of the jamming along the repetition process. The PR and HS methods have good numerical performance but are not always close [11]. To clarify all of the $s_k = x_{k+1} - x_k$ and $y_k = g_{k+1} - g_k$; $\|.\|$ indicates the Euclidean norm. In the order given to those parameters, we will list their sources severally, HS (Hestenes Stiefel [17]), FR (Fletcher Reeves [25]), PR (Polak Ribière [2, 4]), CD (Conjugate Descent [26]), LS (Liu Storey [28]) and DY (Dai Yuan [30]) CG ways. Any methodology of gradient has to update the point by the line search used. The Wolfe-Powell (WWP) customary search terms square measure usually employed in CG ways. The search terms within the Weak Wolfe-Powell (WWP) line square measure as follows:

$$f(x_{k+1}) - f(x_k) \leq \delta \alpha_k g_k^T d_k \tag{4}$$

$$g_{k+1}^T d_k \geq \sigma g_k^T d_k \tag{5}$$

By this condition, d_k is a descent search direction such that $0 < \delta < \sigma < 1$. The Strong Wolfe-Powell (SWP) conditions defined in (4) and satisfies:

$$|g_{k+1}^T d_k| \leq \sigma g_k^T d_k \tag{6}$$

As a generalization of Strong Wolfe conditions, the absolute value opens in (6) with two disparities of inequality so that:

$$-\sigma g_k^T d_k \leq g_{k+1}^T d_k \leq \sigma g_k^T d_k$$

Moreover, from all of these previous conditions we obtain the characteristic of sufficient descent, namely:

$$g_k^T d_k \leq -c\|g_k\|^2 \tag{7}$$

where $c > 0$ is a positive constant.

In the last year, Wei and et. al. [32] introduced an odd plan of its predecessors from the formulas of the PR-CG methodology referred to as the WYL-CG methodology. Zhang recommended through his study a lot of and has improved WYL-CG normal methodology victimization the new CG, additionally it well-tried that the strategy of NPR-CG met the traditional demand ratios beneath conditions of line search SWP [15]. Finally, Zhang et al. planned another changed CG-method referred to as the MPR-CG methodology, wherever Dai and Wen [31] planned another changed version of NPR-CG methodology referred to as the DPR-CG methodology.

Some Important Updates for Recent Different CG-Methods

Researchers	**Formulas**
WYL-CG	$\beta_k^{WYL} = \dfrac{g_{k+1}^T\left(g_{k+1} - \frac{\|g_{k+1}\|}{\|g_k\|} g_k\right)}{\|g_k\|^2}$
NPR-CG	$\beta_k^{NPR} = \dfrac{\|g_{k+1}\|^2 - \frac{\|g_{k+1}\|}{\|g_k\|}\lvert g_{k+1}^T g_k\rvert}{\|g_k\|^2}$
MPR-CG	$\beta_k^{MPR} = \dfrac{\delta\,\|g_{k+1}\|^2}{\|g_k\|^2 + \lvert g_k^T d_k\rvert}$
DPR-CG	$\beta_k^{DPR} = \dfrac{\|g_{k+1}\|^2 - \frac{\|g_{k+1}\|}{\|g_k\|}\lvert g_{k+1}^T g_k\rvert}{\mu\lvert g_{k+1}^T d_k\rvert + \|g_k\|^2}$

The researchers (Hamoda, Rivaie and Mamat) in [15] have developed another new CG-type formula for the formulas mentioned in the above table by changing the compound based on the convex structure method to obtain better results compared to the previous formulas by suggesting the following formula:

$$\beta_k^{HRM} = \frac{g_{k+1}^T\left(g_{k+1} - \frac{\|g_{k+1}\|}{\|g_k\|} g_k\right)}{u\|g_k\|^2 + (1-u)\|s_k\|^2} = \frac{a}{b} \tag{8}$$

Then from the results that obtained by [15], the quantity of the parameter u=0.9 i.e. $u \in (0,1)$ and from β_k^{HRM} given in equation (8), the search direction of this formula was given in the following gradient:

$$d_{k+1} = -g_{k+1} + \beta_{k_k}^{HRM} d_k \tag{9}$$

1.1 Spectral CG-Methods.

As a necessary and important type of gradient methods to solve the problem (1) is the method of spectral gradient (SCG), which was originally developed by Barzilai and Borwein [9]. In 1988, Raydan [19] added SCG method to the problems of unrestricted improvement on a large scale. The idea of this method depends mainly on the use of gradient trends only in each line search in order to ensure the regression strategy by multiplying the first limit by a parameter derived from one of the known derivation methods or by the parameter β_k as in the first idea of this method, Good global. So this method is superior to the CG method developed in many problems but when you devise the exact line of search with both methods you will revert to the traditional method. The first to give the first idea of this method was Birgin and Martinez [6] proposed three types of SCG methods. So that the direction d_k of this method is defined as follows:

$$d_{k+1} = -\phi_k g_{k+1} + \beta_k d_k \tag{10}$$

Variable parameter β_k formulas vary in these ways:

$$\beta_k^1 = \frac{(\phi_k y_k - s_k)^T g_{k+1}}{s_k^T y_k} \qquad \beta_k^2 = \frac{\phi_k y_k^T g_{k+1}}{\alpha_k \phi_k g_k^T g_k} \qquad \beta_k^3 = \frac{\phi_k g_{k+1}^T g_{k+1}}{\alpha_k \phi_k g_k^T g_k} \tag{11}$$

The gradient formula varies according to the parameter ϕ_k calculated by the following:

$$\phi_k = \frac{s_k^T s_k}{s_k^T y_k} \tag{12}$$

Note in the numerical results that these methods give good effectiveness, but they may not be able to guarantee the generation of d_k descent directions if the use of inexact-line searches as when using the search Armijo or Wolfe as a type of inexact-line searches, in the case of general functions is not guaranteed so each algorithm is proven to be effective in these properties theoretically and practically through numerical results. To ensure descent property, Dixon [14] and Al-Baali [18] suggested to use the steepest descent direction $-g_k$ instead of d_k shown in (3). Recently noted, there are many modified CG methods studied. Liu et al. [10] make an adjustment to the CD-method so that the direction that is always generated is the descent direction and d_k is determined by the following:

$$d_{k+1} = \begin{cases} -g_{k+1}, & if \quad k = 1 \\ -\phi_k g_{k+1} + \beta_k d_k, & if \quad k \geq 2 \end{cases} \tag{13}$$

where β_k is specified by the following

$$\beta_k = \begin{cases} \beta_k^{CD}, & if \ g_{k+1}^T d_k \leq 0, \\ 0, & else, \end{cases} \tag{14}$$

and

$$\phi_k = 1 - \frac{g_{k+1}^T d_k}{g_k^T d_k} \tag{15}$$

They demonstrate that this method can ensure that descent directions are generated and are globally convergent.

This paper is divided as follows: In Section 2, we evaluate the new form of the CG-three-term method using the spectral gradient method with derivation of θ_k^{SBi} , $i = 1,2,3$. In Section 3, we give some proof to prove sufficient properties of global proportions and convergence using standard derivation θ_k^{SBi} of Section 2. Finally, in Section 4, the task of this part is the good numerical results of the new algorithm. The new CG-algorithms under standard conditions gave new search directions which have better performance than the two-dimensional algorithms for solving problems of large scale dimensions, i.e. n = 1000, .., 10000, for total of thirty-eight standard non-linear test functions.

2. New CG- Algorithm (Modified HRM).

Through what researchers (Zhang, Zhou and Li [12, 13]) have prompt in recent years, three CG strategies prompt a way that forever satisfies the case of sufficient descent condition (7), freelance of the techniques of line search. Here, during this work, we've superimposed another extra term for the search direction (9) to become a scaled three-term search direction to enhance the number of iterations calculated and therefore the number of conniving the objective functions and their gradients. Will be rewritten by a three-term form:

$$d_{k+1}^{new} = -\phi_k g_{k+1} + \beta_k^{HRM} s_k - \theta_k^{SBi} y_k \quad (16)$$

The parameter ϕ_k can be calculated, normally, from (12) and the new proposed search direction defined in (16), gives more efficiency and more stable CG-algorithm to reach the optimal minimum point compared with the original two-term search direction defined in (9). Also, the new proposed parameter used in (16) has good theoretical background compared to the theories derived from the both PR-CG and HRM –CG methods.

2.1 Derivation of The New Parameters θ_k^{SBi}.

To derive a formula θ_k^{SBi}, depending on the classic conjugacy property given in the following equation:

$$y_k^T d_{k+1} = 0 \quad (17)$$

The first conditional condition, in this approach, gives a method of approximation of the search direction by deleting one term through derivation and as a generalization of this formula by placing the inexact line search (ILS) condition of the formula. We obtain the conditional condition given by Perry [1] as in the following formula:

$$y_k^T d_{k+1} = -s_k^T g_{k+1} \quad (18)$$

A. Derivation of The New Parameter θ_k^{SB1} .

From (16), we use the direction

$$d_{k+1}^{new} = -\emptyset_k g_{k+1} + \beta_k^{HRM} s_k - \theta_k^{SB1} y_k \quad , \emptyset_k = \frac{s_k^T s_k}{s_k^T y_k}$$

$$d_{k+1}^{new} = -\emptyset_k g_{k+1} + \frac{g_{k+1}^T\left(g_{k+1} - \frac{\|g_{k+1}\|}{\|g_k\|} g_k\right)}{u\|g_k\|^2 + (1-u)\|s_k\|^2} s_k - \theta_k^{SB1} y_k \quad (19)$$

Multiplying (19) $by\ y_k$:

$$y_k^T d_{k+1}^{new} = -\emptyset_k y_k^T g_{k+1} + \frac{g_{k+1}^T\left(g_{k+1} - \frac{\|g_{k+1}\|}{\|g_k\|} g_k\right)}{u\|g_k\|^2 + (1-u)\|s_k\|^2} y_k^T s_k - \theta_k^{SB1} y_k^T y_k \quad (20)$$

Using (17) and substituting β_k^{HRM} from the formula:

$$\beta_k^{HRM} = \frac{a}{b} \quad (21)$$

Then we get:

$$0 = -\emptyset_k y_k^T g_{k+1} + \frac{a}{b} y_k^T s_k - \theta_k^{SB1} \|y_k\|^2$$

Hence,

$$\theta_k^{SB1} = \frac{\frac{a}{b} y_k^T s_k - \emptyset_k y_k^T g_{k+1}}{\|y_k\|^2}$$

$$\theta_k^{SB1} = \frac{a y_k^T s_k - b\emptyset_k y_k^T g_{k+1}}{b\|y_k\|^2} \quad (22)$$

B. Derivation of The New Parameter θ_k^{SB2} .

From (18), (21) and (16) we use the direction:

$$d_{k+1}^{new} = -\emptyset_k g_{k+1} + \beta_k^{HRM} s_k - \theta_k^{SB2} y_k$$

$$d_{k+1}^{new} = -\emptyset_k g_{k+1} + \frac{g_{k+1}^T\left(g_{k+1} - \frac{\|g_{k+1}\|}{\|g_k\|} g_k\right)}{u\|g_k\|^2 + (1-u)\|s_k\|^2} s_k - \theta_k^{SB2} y_k \qquad (23)$$

Multiplying (23) by y_k:

$$y_k^T d_{k+1}^{new} = -\emptyset_k y_k^T g_{k+1} + \frac{g_{k+1}^T\left(g_{k+1} - \frac{\|g_{k+1}\|}{\|g_k\|} g_k\right)}{u\|g_k\|^2 + (1-u)\|s_k\|^2} y_k^T s_k - \theta_k^{SB2} y_k^T y_k$$

$$-s_k^T g_{k+1} = -\emptyset_k y_k^T g_{k+1} + \frac{a}{b} y_k^T s_k - \theta_k^{SB2}\|y_k\|^2$$

$$\theta_k^{SB2} = \frac{\frac{a}{b} y_k^T s_k + s_k^T g_{k+1} - \emptyset_k y_k^T g_{k+1}}{\|y_k\|^2}$$

Then

$$\theta_k^{SB2} = \frac{a\, y_k^T s_k + b s_k^T g_{k+1} - b\emptyset_k y_k^T g_{k+1}}{b\|y_k\|^2} \qquad (24)$$

C. Derivation of The New Parameter $\boldsymbol{\theta_k^{SB3}}$.

Now, Use the conjugacy condition defined in [29] to derive θ_k^{SB3}

$$y_k^T d_{k+1}^{new} = -t s_k^T g_{k+1}$$

Rewrite the search direction as:

$$d_{k+1}^{new} = -\emptyset_k g_{k+1} + \beta_k^{HRM} s_k - \theta_k^{SB3} y_k$$

$$d_{k+1}^{new} = -\emptyset_k g_{k+1} + \frac{g_{k+1}^T\left(g_{k+1} - \frac{\|g_{k+1}\|}{\|g_k\|} g_k\right)}{u\|g_k\|^2 + (1-u)\|s_k\|^2} s_k - \theta_k^{SB3} y_k \qquad (25)$$

From (21) and multiplying (25) by y_k to get:

$$y_k^T d_{k+1}^{new} = -\emptyset_k y_k^T g_{k+1} + \frac{a}{b} y_k^T s_k - \theta_k^{SB3} y_k^T y_k$$

$$-t s_k^T g_{k+1} = -\emptyset_k y_k^T g_{k+1} + \frac{a}{b} y_k^T s_k - \theta_k^{SB3}\|y_k\|^2$$

$$\theta_k^{SB3} = \frac{\frac{a}{b} y_k^T s_k + t s_k^T g_{k+1} - \emptyset_k y_k^T g_{k+1}}{\|y_k\|^2}$$

Therefore,

$$\theta_k^{SB3} = \frac{a\, y_k^T s_k + b t s_k^T g_{k+1} - b\emptyset_k y_k^T g_{k+1}}{b\|y_k\|^2} \qquad (26)$$

Now the new proposed three new CG-formulas have been derived.

Below we can write the outlines of the proposed TTCG-algorithm in general using the most comprehensive formula, namely, the third version, defined in (26). To compare its numerical results, we compute the percentage performance of our new TTCG-algorithm against the numerical results of (PR and HRM) CG-algorithms.

2.2 Different (Two and Three) Terms CG-Algorithms.

2.2.1 Algorithm PR-CG [4].

Step 1: Let the initial value variable $x_0 \in R^n, \in \geq 0$ and initial direction $d_0 = -g_0$; if $\|g_0\| \leq \in$ then stop.
Step 2: Evaluate step size α_k by Wolfe line search technique from (4) and (5).
Step 3: Update the variable $x_{k+1}=x_k+\alpha_k d_k$; if (stop criteria) $\|g_{k+1}\| < \in$ then stop.
Step 4: Compute the parameter $\beta_k^{PR} = \frac{g_{k+1}^T(g_{k+1}-g_k)}{g_k^T g_k}$
and generate the new search direction $d_{k+1} = -g_k + \beta_k^{PR} d_k$
Step 5: Set k=k+1 and go to step 2.

2.2.2 Algorithm HRM-CG [15].

Step 1: Initial input $x_0 \in R^n, \in \geq 0$ and first direction is $d_0 = -g_0$; if $\|g_0\| \leq \in$ then stop.
Step 2: Evaluate α_k by Wolfe line search technique from (4) and (5).
Step 3: Let $x_{k+1}=x_k+\alpha_k d_k$, if $\|g_{k+1}\| < \in$ then stop.
Step 4: Set u=0.9 and compute β_k from (8) to generate d_{k+1} from (9).
Step 5: Set k=k+1 go to step 2.

2.3 An Accelerated Scheme of Wolfe Line Search Parameter.

In the first a part of this paper, we tend to mention the concept of standard CG strategies, showing us that analysis trends might not be gradated. To boost the performance of those strategies, search trends supported second order data square measure based mostly. Jorge Nocedal [3] gave some observations indicating that the lengths of steps within the CG strategies of one may be larger or smaller than one looking on however the matter is measured. Calculations show that the latter is a lot of victorious [7].

Here, we tend to talk to the acceleration theme in Andrei's [22]; primarily, this step length is changed during a multiplying way to improve the reduction of function values on the frequencies [20, 24].

2.4 Outlines of the New Spectral TTCG-Algorithm (NEW).

Step 1: Initial input point $x_0 \in R^n, \in \geq 0$; first search direction $d_0 = -g_0$; if $\|g_0\| \leq \in$ then stop.
Step 2: Determine α_k by using the Strong Wolfe-Powell technique (4) and (6).
Step 3: Evaluate new point by $z_k=x_k + \alpha_k d_k$, $g_z = \nabla f(z)$ and $y_k = g_k - g_z$
Step 4: Compute $\overline{a_k} = \alpha_k g_k^T d_k$ and $\overline{b_k} = -\alpha_k y_k^T d_k$
Step 5: Acceleration scheme. If $\overline{b_k} > 0$ and the evaluate $\varepsilon_k = -\frac{\overline{a_k}}{\overline{b_k}}$
and update the points as $x_{k+1} = x_k + \varepsilon_k \alpha_k d_k$,
else update the points as $x_{k+1} = x_k + \alpha_k d_k$
Step 6: Evaluate θ_k^{SBi} where i=1,2,3 , such that (SB1) is defined in (16); (SB2) is defined in (18) and (SB3) is defined in (20).
Step 7: Attend parameter β_k^{HRM} from (8) and compute $\phi_k = \frac{s_k^T s_k}{s_k^T y_k}$
Step 8: Update search direction $d_{k+1}^{new} = -\phi_k g_{k+1} + \beta_k^{HRM} s_k - \theta_k^{SBi} y_k$
Step 9: Use Powell restarting criterion [16];
If $|g_{k+1}^T g_k| > 0.2\|g_{k+1}\|^2$, then set $d_{k+1} = -g_{k+1}$
Step 10: Update the iteration k=k+1 go to **Step 2**.

3. Full Study of the Sufficient Descent and Global Convergence Properties.

Let us consider the foremost vital assumptions accustomed demonstrate the potency of algorithms employed in previous studies of the related to gradient ways, see [20] for example:

3.1 Assumption A.

Let f(x) is delimited from below on the extent set $S = \{x \in R^n, f(x) \leq f(x_0)\}$, where x_0 is that the place to begin.

3.2 Assumption B.

In some neighborhood N of S, the objective function is unceasingly differentiated, and its gradient Lipchitz continuous, that is, there exists a continuing $0 < L < \infty$ such that

$$\|g(x) - g(y)\| \leq L\|x - y\| \ \forall x, y \in N. \tag{27}$$

Now we derive the sufficient condition of three term from θ_k^{SBi}, such that i=1,2,3.

3.3 New Theorem1.

In the TTCG- Algorithm we generate a sequence of duplicates $\{x_k\}$ from (2) and $\{d_k\}$ in (16), assuming that α_k is determined by the strong Wolfe-Powell line search (4)-(6) and $0 < t < 1$. If $0 < \theta_k < 1$ from (22)-(28), then the new proposed three term search direction given by (16) is a sufficient descent direction where:

$$c_1 = s_k^T y_k \left[\left(\frac{0.8\bar{\delta}}{0.4\delta^3 + 0.6\delta\|s_k\|^2}\right)\left(\frac{1.2\bar{\delta}^2}{\|y_k\|^2} - 1\right)\right] \tag{28}$$

$$c_2 = s_k^T y_k \left[\left(\frac{0.8\bar{\delta}}{0.4\delta^3 + 0.6\delta\|s_k\|^2}\right)\left(\frac{1.2\bar{\delta}^2}{\|y_k\|^2} - 1\right) + \frac{1.2\,t}{\|y_k\|^2}\right] \tag{29}$$

Proof:

Case I: Use $\boldsymbol{\theta_k^{SB1}}$ defined in (22) in (16) and multiply this equation by g_{k+1} to get:

$$d_{k+1}^T g_{k+1} = -\emptyset_k\|g_{k+1}\|^2 + \beta_k^{HRM} s_k^T g_{k+1} - \theta_k^{SB1} y_k^T g_{k+1}$$

Use an inexact line searches, in (4) and (6) ,to get :

$$s_k^T g_{k+1} = s_k^T g_{k+1} - s_k^T g_k + s_k^T g_k$$

$$= s_k^T(g_{k+1} - g_k) + s_k^T g_k = s_k^T y_k + s_k^T g_k < s_k^T y_k$$

Hence,

$$d_{k+1}^T g_{k+1} \leq -\emptyset_k\|g_{k+1}\|^2 + \beta_k^{HRM} s_k^T y_k - \left(\frac{\beta_k^{HRM} s_k^T y_k - \emptyset_k y_k^T g_{k+1}}{\|y_k\|^2}\right) y_k^T g_{k+1}$$

furthermore,

$$d_{k+1}^T g_{k+1} \leq -\emptyset_k\|g_{k+1}\|^2 + \beta_k^{HRM} s_k^T y_k - \frac{\beta_k^{HRM} s_k^T y_k y_k^T g_{k+1}}{\|y_k\|^2} + \frac{\emptyset_k\|y_k\|^2\|g_{k+1}\|^2}{\|y_k\|^2}$$

$$d_{k+1}^T g_{k+1} \leq -\emptyset_k\|g_{k+1}\|^2 + \beta_k^{HRM} s_k^T y_k \left(1 - \frac{y_k^T g_{k+1}}{\|y_k\|^2}\right) + \emptyset_k\|g_{k+1}\|^2$$

From Powell restarting condition $|g_{k+1}^T g_k| > 0.2\|g_{k+1}\|^2$ we have

$$-0.2\|g_{k+1}\|^2 > g_{k+1}^T g_k > 0.2\|g_{k+1}\|^2$$

Which implies that

$$y_k^T g_{k+1} = (g_{k+1} - g_k)^T g_{k+1} > \|g_{k+1}\|^2 + 0.2\|g_{k+1}\|^2 = 1.2\|g_{k+1}\|^2$$

$$d_{k+1}^T g_{k+1} \leq \beta_k^{HRM} s_k^T y_k \left(1 - \frac{y_k^T g_{k+1}}{\|y_k\|^2}\right)$$

$$d_{k+1}^T g_{k+1} \le \frac{g_{k+1}^T\left(g_{k+1}-\frac{\|g_{k+1}\|}{\|g_k\|}g_k\right)}{u\|g_k\|^2+(1-u)\|s_k\|^2} s_k^T y_k\left(1-\frac{1.2\|g_{k+1}\|^2}{\|y_k\|^2}\right)$$

$$d_{k+1}^T g_{k+1} \le \frac{\|g_{k+1}\|^2-\frac{\|g_{k+1}\|}{\|g_k\|}g_{k+1}^T g_k}{u\|g_k\|^2+(1-u)\|s_k\|^2} s_k^T y_k\left(1-\frac{1.2\|g_{k+1}\|^2}{\|y_k\|^2}\right)$$

$$d_{k+1}^T g_{k+1} \le \frac{\|g_k\|\|g_{k+1}\|^2-0.2\|g_{k+1}\|^3}{u\|g_k\|^3+(1-u)\|s_k\|^2\|g_k\|} s_k^T y_k\left(1-\frac{1.2\|g_{k+1}\|^2}{\|y_k\|^2}\right)$$

$$d_{k+1}^T g_{k+1} \le -\|g_{k+1}\|^2 s_k^T y_k\left(\frac{\|g_k\|-0.2\|g_{k+1}\|}{u\|g_k\|^3+(1-u)\|s_k\|^2\|g_k\|}\right)\left(\frac{1.2\|g_{k+1}\|^2}{\|y_k\|^2}-1\right)$$

Where $\delta \le \|g_k\| \le \overline{\delta}$ and $u = 0.9$

$$d_{k+1}^T g_{k+1} \le -\|g_{k+1}\|^2\left(\frac{0.8\overline{\delta}}{0.4\delta^3+0.6\delta\|s_k\|^2}\right)\left(\frac{1.2\overline{\delta}^2}{\|y_k\|^2}-1\right)s_k^T y_k$$

Then

$$c_1 = \left(\frac{0.8\overline{\delta}}{0.4\delta^3+0.6\delta\|s_k\|^2}\right)\left(\frac{1.2\overline{\delta}^2}{\|y_k\|^2}-1\right)s_k^T y_k > 0$$

Since $s_k^T y_k > 0$ and small and $\|s_k\|^2 > 0$ for these reason we can get the sufficient condition of three term from $0 < \theta_k^{SB1} < 1$, then

$$d_{k+1}^T g_{k+1} \le -c_1\|g_{k+1}\|^2 \qquad (30)$$

Case II: Again we use the same procedure to proof that the new search direction generated by θ_k^{SB3} (we can consider a general form of θ_k^{SB2} when use t=0.8) is sufficient descent direction too.

$$d_{k+1}^T g_{k+1} = -\emptyset_k\|g_{k+1}\|^2 + \beta_k^{HRM} s_k^T g_{k+1} - \theta_k^{SB3} y_k^T g_{k+1}$$

Considering ($s_k^T g_{k+1} < s_k^t y_k$) then we get:

$$d_{k+1}^T g_{k+1} \le -\emptyset_k\|g_{k+1}\|^2 + \beta_k^{HRM} s_k^T y_k - \left(\frac{\beta_k^{HRM} s_k^T y_k + t s_k^T g_{k+1} - \emptyset_k y_k^T g_{k+1}}{\|y_k\|^2}\right) y_k^T g_{k+1}$$

$$d_{k+1}^T g_{k+1} \le -\emptyset_k\|g_{k+1}\|^2 + \beta_k^{HRM} s_k^T y_k - \frac{\beta_k^{HRM} s_k^T y_k y_k^T g_{k+1} + t s_k^T g_{k+1} y_k^T g_{k+1} - \emptyset_k (y_k^T g_{k+1})^2}{\|y_k\|^2}$$

$$d_{k+1}^T g_{k+1} \le -\emptyset_k\|g_{k+1}\|^2 + \beta_k^{HRM} s_k^T y_k\left(1-\frac{y_k^T g_{k+1}}{\|y_k\|^2}\right) - \frac{t s_k^T y_k y_k^T g_{k+1}}{\|y_k\|^2} + \emptyset_k\|g_{k+1}\|^2$$

From Powell restart condition we conclude ($y_k^T g_{k+1} > 1.2\|g_{k+1}\|^2$) and then:

$$d_{k+1}^T g_{k+1} \le \beta_k^{HRM} s_k^T y_k\left(1-\frac{1.2\|g_{k+1}\|^2}{\|y_k\|^2}\right) - \frac{t s_k^T y_k y_k^T g_{k+1}}{\|y_k\|^2}$$

$$d_{k+1}^T g_{k+1} \le \frac{g_{k+1}^T\left(g_{k+1}-\frac{\|g_{k+1}\|}{\|g_k\|}g_k\right)}{u\|g_k\|^2+(1-u)\|s_k\|^2}\left(1-\frac{1.2\|g_{k+1}\|^2}{\|y_k\|^2}\right)s_k^T y_k - \frac{1.2\, t s_k^T y_k}{\|y_k\|^2}\|g_{k+1}\|^2$$

$$d_{k+1}^T g_{k+1} \le \frac{\|g_k\|\|g_{k+1}\|^2 - 0.2\|g_{k+1}\|^3}{u\|g_k\|^3 + (1-u)\|s_k\|^2\|g_k\|}\left(1 - \frac{1.2\|g_{k+1}\|^2}{\|y_k\|^2}\right)s_k^T y_k - \frac{1.2\, t s_k^T y_k}{\|y_k\|^2}\|g_{k+1}\|^2$$

$$d_{k+1}^T g_{k+1} \le -\|g_{k+1}\|^2 s_k^T y_k \left[\left(\frac{\|g_k\| - 0.2\|g_{k+1}\|}{u\|g_k\|^3 + (1-u)\|s_k\|^2\|g_k\|}\right)\left(\frac{1.2\|g_{k+1}\|^2}{\|y_k\|^2} - 1\right) + \frac{1.2\, t}{\|y_k\|^2}\right]$$

where $\delta \le \|g_k\| \le \bar{\delta}$ and $u = 0.9$, implies :

$$d_{k+1}^T g_{k+1} \le -\|g_{k+1}\|^2 s_k^T y_k \left[\left(\frac{\bar{\delta} - 0.2\bar{\delta}}{0.9\delta^3 + 0.6\delta\|s_k\|^2}\right)\left(\frac{1.2\bar{\delta}^2}{\|y_k\|^2} - 1\right) + \frac{1.2\, t}{\|y_k\|^2}\right]$$

$$d_{k+1}^T g_{k+1} \le -\|g_{k+1}\|^2 s_k^T y_k \left[\left(\frac{0.8\bar{\delta}}{0.4\delta^3 + 0.6\delta\|s_k\|^2}\right)\left(\frac{1.2\bar{\delta}^2}{\|y_k\|^2} - 1\right) + \frac{1.2\, t}{\|y_k\|^2}\right]$$

Then

$$c_2 = s_k^T y_k \left[\left(\frac{0.8\bar{\delta}}{0.4\delta^3 + 0.6\delta\|s_k\|^2}\right)\left(\frac{1.2\bar{\delta}^2}{\|y_k\|^2} - 1\right) + \frac{1.2\, t}{\|y_k\|^2}\right] > 0$$

Since $s_k^T y_k > 0$ and small and $\|s_k\|^2 > 0$ this implies that we can get the sufficient condition of three term from $0 < \theta_k^{SB2} < 1$.

Then

$$d_{k+1}^T g_{k+1} \le -c_2\|g_{k+1}\|^2 \qquad (31)$$

Looking at revealed analysis papers during this field, has to mention some vital lemma and a crucial theorem for any CG methodology uses robust Wolfe line search. The subsequent general results hold.

3.4 Lemma [8, 13].

Consider a general CG-method, and suppose that $0 < \delta \le \|g_k\| \le \bar{\delta}$ holds. We call a method has **Lemma 3.2** if there exist two constants b>1 and p>0 such that for all k, $|\beta_k| \le b$ and

$$\|s_k\| \le p \Rightarrow |\beta_k| \le \frac{1}{2b} \qquad (32)$$

3.5 Lemma [7] (Zoutendijk Condition).

Suppose that assumptions A and B hold. Consider any CG-type method in the form of $x_{k+1} = x_k + \alpha_k d_k$ where d_k is a descent direction and α_k satisfies the Wolfe-Powell line search conditions in (4)-(6) . Then we have that:

$$\sum_{k\ge 0} \frac{(g_k^T d_k)^2}{\|d_k\|^2} < +\infty \qquad (33)$$

3.6 Lemma [8].

Let Assumptions A and B hold and think about any CG technique outlined by (2) and (3), wherever d_k may be a descent direction and α_k is obtained by the strong Wolfe line search. If

$$\sum_{k\geq 1}\frac{1}{\|d_k\|^2}=\infty \quad \Rightarrow \liminf_{k\to\infty}\|g_k\|=0 \tag{34}$$

Now we have to prove the convergence of our proposed three-term CG-Algorithm after they have shown helpful and good properties as in the following theorem:

3.7 New Theorem2.

Suppose that the Assumptions A and B hold. Think about the TTCG-Algorithm outlined by (2) and (16), wherever d_{k+1} is a sufficient descent direction if for $k \geq 0$, $\|s_k\|$ tend to zero and also there exists δ and $\bar{\delta}$ such that $(0 < \delta \leq \|g_k\| \leq \bar{\delta})$ and the function f is a general function with Lipchitz condition, then:

$\liminf_{k\to\infty}\|g_k\| = 0.$

Proof:

Let us now take the more general form of θ_k^{SB3} defined in (26) and use them within the formula in the equation (16) as in:

$$d_{k+1} = -\emptyset_k g_{k+1} + \beta_k^{HRM} s_k - \theta_k^{SB3} y_k$$

$$\|d_{k+1}\| = \emptyset_k\|g_{k+1}\| + |\beta_k^{HRM}|\|s_k\| + |\theta_k^{SB3}|\|y_k\|$$

Where,

$$\beta_k^{HRM} = \frac{\|g_{k+1}\|^2 - \frac{\|g_{k+1}\|}{\|g_k\|}g_{k+1}^T g_k}{u\|g_k\|^2 + (1-u)\|s_k\|^2}$$

$$|\beta_k^{HRM}| \leq \frac{\|g_k\|\|g_{k+1}\|^2}{u\|g_k\|^3 + (1-u)\|s_k\|^2\|g_k\|} + \frac{\|g_{k+1}\|^2\|g_k\|}{u\|g_k\|^2 + (1-u)\|s_k\|^2\|g_k\|}$$

Let $\|s_k\| = D$ and then we get:

$$|\beta_k^{HRM}| \leq \frac{\bar{\delta}^3}{u\delta^3 + (1-u)\delta D^2} + \frac{\bar{\delta}^3}{u\delta^3 + (1-u)\delta D^2}$$

$$|\beta_k^{HRM}| \leq \frac{2\bar{\delta}^3}{u\delta^3 + (1-u)\delta D^2} = \mathrm{E}$$

Doing the same procedure for θ_k^{SB3} yields

$$\theta_k^{SB3} = \frac{\beta_k^{HRM}\, y_k^T s_k + t s_k^T g_{k+1} - \emptyset_k y_k^T g_{k+1}}{\|y_k\|^2}$$

$$|\theta_k^{SB3}| \leq \frac{|\beta_k^{HRM}|\,\|y_k\|\|s_k\|}{\|y_k\|^2} + \frac{|t|\|s_k\|\|g_{k+1}\|}{\|y_k\|^2} + \frac{|\emptyset_k|\|y_k\|\|g_{k+1}\|}{\|y_k\|^2}$$

, where

$$\emptyset_k = \frac{\|s_k\|^2}{s_k^T y_k} = \frac{D^2}{l\|s_k^T s_k\|} = \frac{D^2}{lD^2} = \frac{1}{l}$$

From Lipchitz condition $\|y_k\| \leq l\|s_k\|$ implies that:

$$|\theta_k^{SB3}| \leq \frac{|\beta_k^{HRM}|l\|s_k\|^2}{l^2\|s_k\|^2} + \frac{|t|\|s_k\|\bar{\delta}}{l^2\|s_k\|^2} + \frac{l\|s_k\|\bar{\delta}}{l^2\|s_k\|^2} * (\frac{1}{l})$$

$$|\theta_k^{SB3}| \leq \frac{1}{l}\left(E + \frac{t\bar{\delta}}{lD} + \frac{\bar{\delta}}{lD}\right)$$

Then added the above result to get the following :

$$\|d_{k+1}\| \leq \frac{\|g_{k+1}\|}{l} + |\beta_k^{HRM}|\|s_k\| + |\theta_k^{SB3}|\|y_k\|$$

$$\|d_{k+1}\| \le \frac{\|g_{k+1}\|}{l} + ED + \frac{1}{l}\left(E + \frac{t\bar{\delta}}{lD} + \frac{\bar{\delta}}{lD}\right) lD$$

$$\|d_{k+1}\| \le \frac{\|g_{k+1}\|}{l} + ED + ED + \frac{t\bar{\delta}}{l} + \frac{\bar{\delta}}{l}$$

$$\|d_{k+1}\| \le \frac{\bar{\delta}}{l} + 2ED + \frac{t\bar{\delta}}{l} + \frac{\bar{\delta}}{l}$$

$$\|d_{k+1}\| \le 2ED + \frac{t\bar{\delta}}{l} + \frac{2\bar{\delta}}{l}$$

Therefore,

$$0 < \sum_{k=0}^{\infty} \frac{(g_k^T d_k)^2}{\|d_k\|^2} < \infty$$

$$\sum_{k=0}^{\infty} \frac{\|g_k\|^4}{\|d_k\|^2} \le \sum_{k=0}^{\infty} \frac{1}{c^2} \frac{(g_k^t d_k)^2}{\|d_k\|^2} < \infty$$

Then $\lim_{k\to\infty} \inf \|g_k\| = 0$.

Therefore, when $0 < \emptyset_k < 1$, so we are able to prove that the new TTCG algorithm is global convergence, we are able to additionally say $g_k = 0$ for a few k or (34) holds. It will be discovered, it's from the info of the new Theorem1 theory, the direction d_{k+1} satisfied the sufficient descent condition severally of the line search.

3.8 Theorem [8].

Assume that Assumptions A and B hold. Contemplate the strategy (2)-(3) with the subsequent 3 :

(i) $\beta_k \ge 0$ for all k;

(ii) The line search satisfies the Zoutendijk condition, and the sufficient descent conditions (30) and (31);

(iii) Lemma 3.4 holds.

All these conditions provide us with a transparent and comprehensive convergence of the quality CG-algorithm, then $\lim_{k\to\infty} \inf \|g_k\| = 0$.

After all this proof, the projected new TTCG formula with three versions features a convergent global characteristic by satisfying the wants of the Zoutendijk theorem and therefore the line search satisfy the robust Wolfe–Powell condition then from Gilbert and Nocedal in [8] these methodology has the global convergence property.

4. Numerical Experiments.

In this section, we glance at the tables and graphs of some computations to check the relative performance of the mathematical implementation of packaging, human resource management and projected new standards (TTCG Algorithm) (NEW) to a collection of thirty eight any old nonlinear issues. Its details will be found in [21, 24, 27]. We have chosen completely different test functions for each free and tiny scale optimization problem. Every one tested with completely different variables between one and 10,000. We have a tendency to test these issues mistreatment the Wolfe Powell technique.

These CG-Algorithm were enforced by mistreatment Visual FORTRAN version 2002, we've applied the Wolfe-Powell line search technique. All of the numerical experiments were run on identical laptop with associate Intel(R), Core TM, i7-3612QM

(2.10GHZ) CPU, 6GB of RAM , associated mistreatment Windows seven as an operating system.

In order to assess the responsible-ness of the new projected scaled three-term CG-Algorithm, we tend to test this algorithmic rule against the well-known classical and changed ways of the PR and HRM CG-Algorithms mistreatment constant issues, and assumed that the most effective algorithmic rule ought to need fewer range of iterations, NOI; number of function and gradient evaluations NOFG; less time needed to induce the answer purpose processor and also the total work outlined by, TOTAL WORK = NOI + NOFG + TIME. All of those CG-Algorithms terminated once:

$\|g_k\| < 10^{-6}$.

Numerical results were comparatively compared with the processor time, NOFG and number of iterations (NOI). The performance results square measure shown in Figures (1) and (2) severally, employing a performance profile introduced by Dolan and More [5].

Table(1). Comparison between NEW against (PR & HRM) CG-Algorithms
For the Total of (38) different test functions with dimensions n= 1000, 2000, … 10000

Test Problems	PR (1969) NOI/NOFG/TIME	HRM (2016) NOI/NOFG/TIME	New (2018) NOI/NOFG/TIME
1-Freudenstien & Roth- (CUTE)	1803/3356/0.43	5518/7177/0.27	103/434/0.09
2-Trigonometric	362/695/0.31	372/741/0.14	475/1365/2.29
3-Extended White & Holst	5715/7744/0.25	9227/10886/0.39	348/1151/0.20
4-Extended Beale	498/1217/0.04	487/1202/0.03	125/365/0.07
5-Penalty	274/690/0.01	271/685/0.02	142/565/0.11
6-Raydan 2	106/319/0.03	106/319/0.03	30/100/0.07
7-Generalized Tri-diagonal 1	286/596/0.02	291/604/0.02	250/875/0.20
8- Extended Three Expo Terms	193/426/0.06	205/440/0.07	60/170/0.21
9- Generalized Tri-diagonal 2	3920/5163/0.26	4500/5752/0.29	633/1688/0.45
10- Diagonal 4	345/918/0.02	269/750/0.02	20/60/0.02
11- Diagonal 5	78/284/0.06	78/284/0.05	20/70/0.10
12- Extended Himmelblau	200/503/0.01	208/512/0.02	60/190/0.03
13- Extended PSC1	194/453/0.06	218/478/0.06	70/230/0.26
14- Extended Wood (CUTE)	6149/10914/0.29	10010/11636/0.41	1770/5136/0.98
15- Extended EP1	54/294/0.01	54/294/0.02	19/58/0.02
16-ARWHEAD (CUTE)	1336/1946/0.08	2409/3138/0.18	40/176/0.02
17-NONDIA (CUTE)	5623/6604/0.28	9454/10602/0.44	42/127/0.01
18- DIXMAANA (CUTE)	132/354/0.01	132/354/0.02	53/159/0.08
19- DIXMAANB (CUTE)	144/351/0.02	144/351/0.02	70/201/0.14
20- DIXMAANC	140/373/0.01	140/373/0.02	70/220/0.12

(CUTE)			
21-EDENSCH (CUTE)	321/686/0.02	1296/1674/0.07	260/962/0.28
22-LIARWHD (CUTE)	1397/2816/0.07	3759/5577/0.23	195/648/0.13
23-DIAGONAL 6	106/319/0.02	106/319/0.02	30/100/0.07
24-ENGVAL1 (CUTE)	304/672/0.02	328/701/0.02	261/1228/0.22
25-DENSCHNA (CUTE)	257/495/0.04	256/500/0.04	80/240/0.14
26-DENSCHNC (CUTE)	1399/1887/0.14	1434/1935/0.18	130/390/0.33
27-DENSCHNB (CUTE)	143/394/0.00	143/394/0.01	60/190/0.03
28- Extended Block-Diagonal BD2	495/1041/0.09	1428/1936/0.19	80/240/0.14
29-Generalized quartic GQ1	144/399/0.03	142/395/0.01	71/266/0.06
30-DIAGONAL 7	108/329/0.10	108/329/0.02	40/140/0.09
31-DIAGONAL 8	100/305/0.10	100/305/0.03	30/100/0.10
32-Full Hessian	109/408/0.02	109/408/0.04	20/70/0.06
33-SINCOS	194/453/0.08	218/478/0.05	70/230/0.26
34-Generalized quartic GQ2	496/1011/0.02	2424/2926/0.13	369/981/0.23
35-ARGLINB (CUTE)	10/30/0.00	10/30/0.00	23/164/0.08
36-FLETCHCR (CUTE)	626/1247/0.03	1436/1985/0.05	228/702/0.14
37-HIMMELBG (CUTE)	80/100/0.01	80/100/0.01	60/140/0.07
38-HIMMELBH (CUTE)	180/420/0.01	170/420/0.01	60/190/0.03
Total	**34021/56212/3.06**	**57640/76990/3.63**	**6467/20321/7.93**
Total Work= NOI + NOFG + Time	**90236.06**	**134633.63**	**26795.93**

In the beginning we used the three new versions of the new proposed scaled TTCG- algorithms in our implementation to obtain numerical results but later we noticed that the last version is the best one among these three version formulas and therefore within the numerical result Tables, we will list the CG-Algorithm of the third version only which is more important to highlight the numerical results and efficiency of the three CG-Algorithms.

Table(2). Percentage Performance of (PR) against (NEW)

TOOLS	PR	NEW
NOI	**100%**	**19.0%**
NOFG	**100%**	**36.2%**

Total Work	**100%**	**29.7%**

Table(3). Percentage Performance of (HRM) against (NEW)

TOOLS	**HRM**	**NEW**
NOI	**100%**	**11.2%**
NOFG	**100%**	**26.4%**
Total Work	**100%**	**19.9%**

In Tables (2 & 3), the efficiency of the (NEW) TTCG-Algorithm was compared. The new algorithm gave the best numerical results according to our used 38-test problems.

Table(4). No. of best test problems
(NEW) against (PR)

Tools	**No. of New best functions**	**No. of PR best functions**	**NEW 100%**	**PR 100%**
NOI	**36**	**2**	**94.7%**	**5.3%**
NOFG	**32**	**6**	**84.2%**	**15.8%**
TIME	**5**	**31**	**13.2%**	**81.6%**

Table (5). No. of best test problems
(NEW) against (HRM)

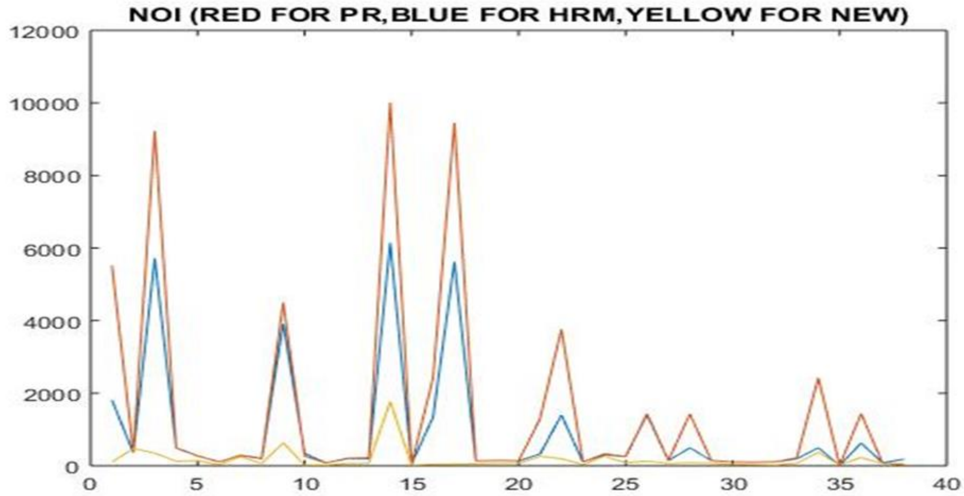
Tools	**No. of New best functions**	**No. of HRM best functions**	**NEW 100%**	**HRM 100%**
NOI	**36**	**2**	**94.7%**	**5.3%**
NOFG	**33**	**5**	**86.8%**	**13.2%**
TIME	**6**	**30**	**15.8%**	**78.9%**

If we tend to calculate the number of the best take a look at functions once PR is compared with NEW in step with NOI, NOFG and C.P.U. in Table(4), we tend to see that the NEW rule is actually higher than PR, and if we tend to calculate the amount of best take a look at functions once HRM is compared with NEW in NOI, NOFG and C.P.U. in Table(5) , show that the NEW rule is that the higher than HRM, that the NEW rule is that the most strong rule in step with the number of iteration , NOFG and TIME underneath the accelerated Wolfe-Powell line search:

Figure(1) shows the performance file for all measures in CG-Algorithms with the required NOI.

Figure(2) shows the performance file for all measures in CG-Algorithms with the required NOFG.

Accurate examination of all shapes indicates that the bottom curve represents the PR CG-Algorithm. Therefore, this CG-Algorithm has less performance. Also, HRM incorporates a slightly higher performance than PR. Finally, the exact check shows that the, NEW technique has the best performance by exploitation each tools; NOI and NOFG.



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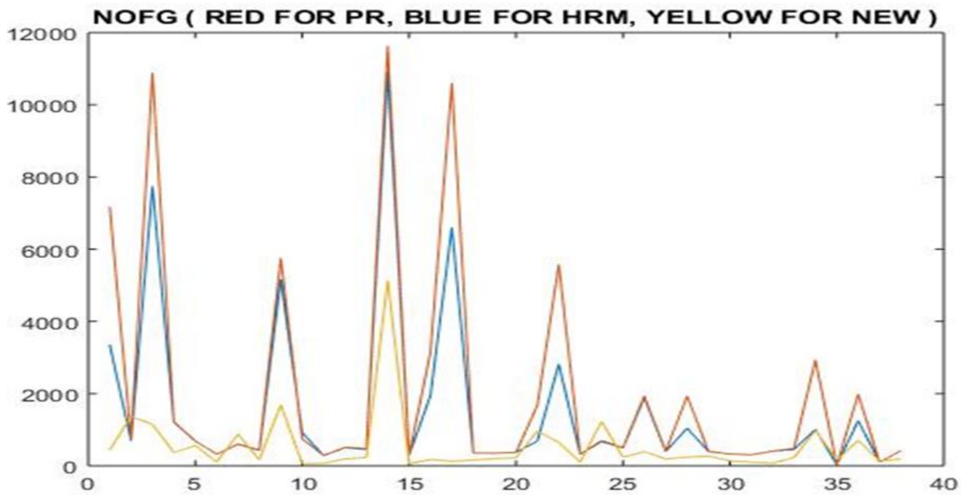
Figure(1): Performance profile relative to the NOI



Figure(2): Performance profile relative to the NOFG

5. Conclusions.

In this paper, we've investigated a replacement scaled TTCG-Algorithm within three completely different new versions for resolving a collection of unconstrained non-linear minimization issues each on paper and by experimentation. The last version of this TTCG-Algorithm, namely SB3, is that the sturdy one. This is often compared numerically against the 2 accepted CG-Algorithms (PR and HRM). The numerical and theoretical studies during this paper show that the state of decent condition will be derived and converged globally if the Wolfe-Powell line search was used. Numerical results for 38 check issues, in general, and for this hand-picked kind of numerical examples, show that the NEW-Algorithm is the sturdy and effective one.

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