17 New Existences linear $[n,3,d]_{19}$ Codes by Geometric Structure Method in PG(2,19)

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ABSTRACT

The purpose of this paper is to prove the existence of 17 new linear $[337,3,318]_{19}$, $[289,3,271]_{19}$, $[266,3,249]_{19}$, $[246,3,230]_{19}$, $[219,3,204]_{19}$, $[206,3,192]_{19}$, $[181,3,168]_{19}$, $[157,3,145]_{19}$, $[141,3,130]_{19}$, $[120,3,110]_{19}$, $[112,3,103]_{19}$, $[82,3,74]_{19}$, $[72,3,65]_{19}$, $[54,3,48]_{19}$, $[37,3,32]_{19}$, $[26,3,22]_{19}$, $[13,3,10]_{19}$ codes by geometric structure method in PG(2,19) .

Keywords: Linear code, $[n ,k ,d]_q$ codes, Finite geometry, (k ,r)-arc.

وجود 17شفرات خطية جديدة $[n,3,d]_{19}$بطريقة البناء الهندسي في PG(2,19)

ندى ياسين قاسم يحيى
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الملخص

الهدف من هذا البحث هو إثبات وجود 17 شفرة خطية جديدة $[337,3,318]_{19}$, $[289,3,271]_{19}$, $[266,3,249]_{19}$,$[246,3,230]_{19}$, $[219,3,204]_{19}$,$[206,3,192]_{19}$, $[181,3,168]_{19}$, $[157,3,145]_{19}$, $[141,3,130]_{19}$, $[120,3,110]_{19}$, $[112,3,103]_{19}$, $[82,3,74]_{19}$, $[72,3,65]_{19}$, $[54,3,48]_{19}$, $[37,3,32]_{19}$, $[26,3,22]_{19}$, $[13,3,10]_{19}$ بواسطة طريقة البناء (التركيب) الهندسي في المستوي الاسقاطي PG(2,19).

الكلمات المفتاحية: الشفرات الخطية، الشفرات [n,k ,d]q، الهندسة المنتهية ،القوس –(k ,r).

1. Introduction [1]

Let GF(q) denote the Galois field of q elements and V(3,q) be the vector space of row vectors of length three with entries in GF(q) . Let PG(2,q) be the corresponding projective plane. The points of PG(2,q) are the non-zero vectors of V(3,q) with the rule that $X= (x_1,x_2,x_3)$ and $Y= (\delta x_1, \delta x_2, \delta x_3)$ represent the same point , where $\delta \in GF(q)\setminus \{0\}$ .The number of points of PG(2,q) is q^2+q+1.,If the point P(X) is the equivalence class of the vector X, then we will say that X is a vector representing P(X) .

A subspace of dimension one is a set of points all of whose representing vectors form a subspace of dimension two of V(3,q) . Such subspaces are called lines. The number of lines in PG(2,q) is q^2+q+1. There are q+1 points on every line and q+1 lines through every point .

1.1 Definition " Double Blocking set " [5]

A double blocking set in a projective plane PG(2,q) is a set S of points with the property that every line contains at least two points of S .

1.2 Definition " A (k,r) –arc " [2]

A (k,r) –arc K in PG(2,q) is a set of k points with condition no line of the plane contains more then k points and there exists at least one line of the plane which contains k points .A (k,,r) –arc is called complete arc if is not contained in a (k+1,r)- arc .

1.3 Definition " The Linear [n,k,d]q codes " [4]

The linear codes [n,k,d]q in PG(2,q) where n is the length of codes and k is the dimension of codes , and minimum Hamming distance between the codes is called d over the Galois field GF(q) .

1.4 Definition " i-secant " [1]

A line L in PG(2,q) is an i-secant of a (k , r)-arc if $|L \cap K|=i$

1.5 Theorem 1: [4]

There exists linear [n,3,d] q codes if and only if there exists an (n ,n-d)-arc in PG(2,q)

2. The geometrical structure method in PG(2,19) .

Let A=(1,2,21,41) be the set of reference unit and reference points in PG(2,13) where 1=(1,0,0), 2=(0,1,0),21=(0,0,1) ,41=(1,1,1)
A is (4,2)-arc, since no three points of A are collinear,
[1,2]=[1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20]
[1,21]=[1,21,22,23,24,25,26,27,28,29,30,31,32,33,34,35,36,37,38,39]
[1,41]=[1,40,41,42,43,44,45,46,47,48,49,50,51,52,53,54,55,56,57,58]
[2,21]=[2,21,40,59,78,97,116,135,154,173,192,211,230,249,268,287,306,325,344,363]
[2,41]=[2,22,41,60,79,98,117,136,155,174,193,212,231,250,269,288,307,326,345,364]
[21,41]=[3,21,41,61,81,101,121,141,161,181,201,221,241,261,281,301,321,341,361 ,381]
The diagonal points of A are the points {3,22,40} where,$L_1 \cap L_6 = 3$; $L_2 \cap L_5 =22$; $L_3 \cap L_4 = 40$.
There are one hundred and one points of index zero for A , which are:
62,63,64,65,66,67,68,69,70,71,72,73,74,75,76,77,80,82,83,84,85,86,87,88,89,90,91,92,93,94,95,96,99,100,102,103,104,105,106,107,108,109,110,111,112,113,114,115,118,119,120,122,123,124,125,126,127,128,129,130,131,132,133,134,157,138,139,140,142,143,144,145,146,147,148,149,150,151,152,153,156,157,158,159,160,162,163,164,165,166,167,168,169,170,171,172,175,176,177,178,179,180,182,183,184,185,186,187,188,189,190,191,194,195,196,197,198,199,200,202,203,204,205,206,207,208,209,210,213,214,215,216,217,218,219,220,222,223,224,225,226,227,228,229,232,233,234,235,236,237,238,239,240,242,243,244,245,246,247,248,251,252,253,254,255,256,257,258,259,260,262,263,264,265,266,267,270,271,271,273,274,275,276,277,2782,79,280,282,283,284,285,286,289,290,291,292,293,294,295,296,297,298,299,300,302,303,304,305,308,309,310,311,

312,313,314,315,316,317,318,319,320,322,323,324,327,328,329,330,331,332,333,334,335,336,337,338,339,340,342,343,346,347,348,349,350,351,352,353,354,355,356,357,358,359,360,362,365,366,367,368,369,370,371,372,373,374,375,376,377,378,379,380
Hence ,A is incomplete (4,2)-arc .

3. The Conics in PG(2,19) Through the Reference and Unit Points

The general equation of the conic is :
$a_1x^2{}_1 + a_2x^2{}_2 + a_3x^2{}_3 + a_4x_1x_2 + a_5x_1x_3 + a_6x_2x_3 = 0$ ... (1)
By substituting the points of the arc A in (1), then:
1 = (1,0,0) implies that $a_1 = 0$, 2 = (0,1,0), then $a_2 = 0$, 21 = (0,0,1), then
$a_3 = 0$,41 = (1,1,1), then
$a_1 = a_2 = a_3 = 0$
$a_4 + a_5 + a_6 = 0$.
Hence, from equation (1)
$a_4 x_1 x_2 + a_5 x_1 x_3 + a_6 x_2 x_3 = 0$ … (2)
If $a_4 = 0$, then the conic is degenerated, therefore for $a_4 \neq 0$, similarly $a_5 \neq 0$
and $a_6 \neq 0$,
Dividing equation (2) by a4, one can get:
$x_1 x_2 + \alpha x_1 x_3 + \beta x_2 x_3 = 0$
where $\alpha = a_5/a_4$, $\beta = a_6/a_4$
then $\beta = -(1 + \alpha)$, since $1+\alpha+\beta=0$ (mod 13).
where $\alpha \neq 0$ and $\alpha \neq 12$, for if $\alpha = 0$ or $\alpha = 12$, then degenerated conics, thus α = 1, 2, 3, 4, 5, 6, 7, 8, 9,10,11,12,13,14,15,16,17 and can be written (2) as :
$x_1x_2 + \alpha x_1x_3 - (1 + \alpha)x_2x_3 = 0$ ... (3)
The equation and the points of the conics of PG(2,19) through the reference and unit points

1. If α= 1, then the equation of the conic
C_1 = $x_1x_2 + x_1x_3 + 17x_2x_3 = 0$,
the points of C_1:
{1,2,21,41,73,89,110,124,153,170,179,209,218,235,264,278,299,315,328,348} which is a complete (20,2)-arc, since there are no points of index zero.

2. If α= 2, then the equation of the conic C_2 = $x_1x_2 + 2x_1x_3 + 16x_2x_3 = 0$,
the points of C_2 :
{1,2,21,41,70,95,99,129,142,169,183,210,223,234,257,282,292,312,334,379}, which is a complete (20,2)-arc, since there are no points of index zero .

3. If α= 3, then the equation of the conicC_3 = $x_1x_2 + 3x_1x_3 + 15x_2x_3 = 0$,
the points of C_3 :
{1,2,21,41,72,80,102,128,144,172,188,195,217,244,256,276,297,322,355,380 }, which is a complete (20,2)-arc, since there are no points of index zero.

4. If α= 4, then the equation of the conic C_4 = $x_1x_2 + 4x_1x_3 + 14x_2x_3 = 0$,
the points of C_4:
{1,2,21,41,67,91,109,123,138,171,189,194,220,240,267,283,293,329,358,
374}, which is a complete (20,2)-arc, since there are no points of index zero.

5. If α= 5, then the equation of the conic C_5 = $x_1x_2 + 5x_1x_3 + 13x_2x_3 = 0$,
the points of C_5:
{1,2,21,41,77,85,106,119,140,167,184,204,215,246,251,285,320,335,359,371 }, which is a complete (20,2)-arc, since there are no points of index zero.

6. If α= 6, then the equation of the conicC_6 = $x_1x_2 + 6x_1x_3 + 12x_2x_3 = 0$,
the points of C_6:
{1,2,21,41,75,93,115,133,148,168,177,200,213,239,260,290,311,337,350,373 }, which is a complete (20,2)-arc, since there are no points of index zero.
7. If α= 7, then the equation of the conic C_7 = $x_1x_2 + 7x_1x_3 + 11x_2x_3 = 0$,
the points of C_7:
{1,2,21,41,65,88,112,132,146,158,176,206,228,237,277,305,308,338,356,368 }, which is a complete (20,2)-arc, since there are no points of index zero.
8. If α= 8, then the equation of the conic C_8 = $x_1x_2 + 8x_1x_3 + 10x_2x_3 = 0$,
the points of C_8:
{1,2,21,41,76,96,100,118,147,162,187,199,216,262,279,291,316,331,360, 378}, which is a complete (20,2)-arc, since there are no points of index zero.
9. If α= 9, then the equation of the conic C_9 = $x_1x_2 + 9x_1x_3 + 9x_2x_3 = 0$,
the points of C_9:
{1,2,21,41,66,90,103,125,139,156,190,197,245,252,286,303,317,339,352, 376}, which is a complete (20,2)-arc, since there are no points of index zero.
10. If α= 10, then the equation of the conicC_{10} = $x_1x_2 + 10x_1x_3 + 8x_2x_3 = 0$,
the points of C_{10}:
{1,2,21,41,64,82,111,126,151,163,180,226,243,255,280,295,324,342,346, 366}, which is a complete (20,2)-arc, since there are no points of index zero.
11. If α= 11, then the equation of the conicC_{11} = $x_1x_2 + 11x_1x_3 + 7x_2x_3 = 0$,
the points of C_{11}:
{1,2,21,41,74,86,104,134,137,165,205,214,236,266,284,296,310,330,354, 377}, which is a complete (20,2)-arc, since there are no points of index zero.
12. If α= 12, then the equation of the conicC_{12} = $x_1x_2 + 12x_1x_3 + 6x_2x_3 = 0$,
the points of C_{12}:
{1,2,21,41,69,92,105,131,152,182,203,229,242,265,274,294,309,327,349, 367}, which is a complete (20,2)-arc, since there are no points of index zero.
13. If α= 13, then the equation of the conicC_{13} = $x_1x_2 + 13x_1x_3 + 5x_2x_3 = 0$,
the points of C_{13}:
{1,2,21,41,71,83,107,122,157,191,196,227,238,258,275,302,323,336,357, 365}, which is a complete (20,2)-arc, since there are no points of index zero.
14. If α= 14, then the equation of the conicC_{14} = $x_1x_2 + 14x_1x_3 + 4x_2x_3 = 0$,
the points of C_{14}:
{1,2,21,41,68,84,113,149,159,175,202,222,248,253,271,304,319,333,351, 375}, which is a complete (20,2)-arc, since there are no points of index zero.
15. If α= 15, then the equation of the conicC_{15} = $x_1x_2 + 15x_1x_3 + 3x_2x_3 = 0$,
the points of C_{15}:
{1,2,21,41,62,87,120,145,166,186,198,225,247,254,270,298,314,340,362, 370}, which is a complete (20,2)-arc, since there are no points of index zero.
16. If α= 16, then the equation of the conicC_{16} = $x_1x_2 + 16x_1x_3 + 2x_2x_3 = 0$,
the points of C_{16}:
{1,2,21,41,63,108,130,150,160,185,208,219,232,259,273,300,313,343,347, 372}, which is a complete (20,2)-arc, since there are no points of index zero.
17. If α= 17, then the equation of the conic C_{17} = $x_1x_2 + 17x_1x_3 + x_2x_3 = 0$,
the points of C_{17}:
{1,2,21,41,94,114,127,143,164,178,207,224,233,263,272,289,318,332,353,369}, which

is a complete (20,2)-arc, since there are no points of index zero.

4. Existence of $[n,3,d]_{19}$ codes:

4.1 Existence of $[337,3,318]_{19}$ codes

We take one conic , and take π= PG(2,q) over Galois filed GF(q) contains 381 points and line, every line contains 20 points and every point there are 20 line, say C_1 , let K= π - C_1
{3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,22,23,24,25,26,27,28,29,3031,32,33,34,35,,36,37,38,39,40,42,43,44,45,46,47,48,49,50,51,52,53,54,55,56,57,58,59,60,61,62,63,64,65,66,67,68,69,70,71,72,74,75,76,77,78,79,80,81,82,83,84,85,86,87,88,90,91,92,93,94,95,96,97,98,99,100,101,102,103,104,105,106,107,108,109,111,112,113,114,115,116,117,118,119,120,121,122,123,125,126,127,128,129,130,131,132,133,134,135,136,137,138,139,140,141,142,143,144,145,146,147,148,149,150,151,152,154,155,156,157,158,159,160,161,162,163,164,165,166,167,168,169,171,172,173,174,175,176,177,178,180,181,182,183,184,185,186,187,188,189,190,191,192,193,194,195,196,197,198,199,200,201,202,203,204,205,206,207,208,210,211,212,213,214,215,216,217,219,220,221,222,223,224,225,226,227,228,229,230,231,232,233,234,235,236,237,238,239,240,241,242,243,244,245,246,247,248,249,250,251,252,254,255,256,257,258,259,260,261,262,263,265,266,267,268,269,270,271,272,273,274,275,276,277,279,280,281,282,283,284,285,286,287,288,289,290,291,292,293,294,295,296,297,298,300,301,302,303,304,305,306,307,308,309,310,311,312,313,314,316,317,318,319,320,321,322,324,325,326,327,329,330,331,332,333,334,335,336,337,338,339,340,341,342,343,344,345,346,347,349,350,351,352,353,354,355,356,357,358,359,360,361,362,363,364,365,366,367,368,369,370,370,372,373,374,375,376,377,378,379,380,381}.

The geometrical structure method must satisfy the following :

i. K intersects any line of π in at most 19 points .
ii. Every point not in K is on at least one 19-secant of K .

The point :
M=363,192,135,287,306,78,16,173,154,59,344,249,230,325,97,116,268,211,39,317,321,111,181,66,331,376,177,221Are eliminated from K to satisfy (1) . The points of index zero for 1,73,209 are added to K to satisfy (2) , then K_{19} = K∪ [1,73,209] / M
K_{19}=[1,3,4,5,6,7,8,9,10,11,12,13,14,15,17,18,19,20,22,23,24,25,26,27,28,29,30,31,32,33,34,35,36,37,38,40,42,43,44,45,46,47,48,49,50,51,52,53,54,55,56,57,58,60,61,62,63,64,65,67,68,69,70,71,72,73,74,75,76,77,79,80,81,82,83,84,85,86,87,88,90,91,92,93,94,95,96,98,99,100,101,102,103,104,105,106,107,108,109,112,113,114,115,117,118,119,120,121,122,123,125,126,127,128,129,130,131,132,133,134,136,137,138,139,140,141,142,143,144,145,146,147,148,149,150,151,152,155,156,157,158,159,160,161,162,163,164,165,166,167,168,169,171,172,174,175,176,178,180,182,183,184,185,186,187,188,189,190,191,193,194,195,196,197,198,199,200,201,202,203,204,205,206,207,208,209,210,212,213,214,215,216,217,219,220,222,223,224,225,226,227,228,229,231,232,233,234,235,236,237,238,239,240,241,242,243,244,245,246,247,248,250,251,252,254,255,256,257,258,259,260,261,262,263,265,266,267,269,270,271,272,273,274,275,276,277,279,280,281,282,283,284,285,286,288,289,290,291,292,293,294,295,296,297,298,300,301,302,303,304,305,307,308,309,310,311,312,313,314,316,318,319,320,322,324,326,327,329,330,332,333,334,335,336,337,338,339,340,341,342,343,345,346,347,349,350,351,352,353,354,355,356,357,358,359,360,361,362,364,365,366,367,368,369,370,370,372,373,374,375,377,378,379,380,381].Is a complete (155,13) –arc as shown in table (1) .Let $\beta_1 = \pi - k_{19}$

={2,21,39,41,59,66,78,89,16,97,110,111,116,124,135,153,154,170,173,177,179,181,192,211,218,221,230,249,253,264,278,287,299,306,317,321,328,331,344,348,363,376,325,315,268} is (44,1)-blocking set as shown in table (1) .$\boldsymbol{\beta 1}$ is of Redei -type contains the line $l1$ ={2,21,40,59,78,97,116,135,154,173,192,211,230,249,268,287,306,325,344363}/{40} and one point on each line through the point 40 which are non-collinear points 42,60,79,85,112,132,107,152,126,93,145,184,164,172,198,223,233,244,257 by theorem (1) ,there exists a projective $[337,3,318]_{19}$ code which is equivalent to the complete (337,19)-arc k_{19}

Table (1)

I	$K_{19} \cap$ Li	$B_1 \cap$ Li
1	40	2,21,40,59,78,97,116,135,154,173,192,211, 230,249,268,287,306,325,344363
2	1,22,23,24,25,26,27,28,29,30,31, 32,33,34,35,36,37,38	21,39
⋮	⋮	⋮
380	11,31,40,68,96,105,133,142,207, 216,244,281,290,318,327,355,364	170,179,253
381	20,22,40,77,95,113,131,149,167, 185,203,239,257,275,293,311,329,347,365	221

4.2 Existence of $[289,3,271]_{19}$ codes

We take two conic, say C_1 ,C_2,and let K= π - $C_1 \cup C_2$ {3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,22,23,24,25,26,27,28,29,30,31,32,33,34,35,36,37,38,39,40,42,43,44,45,46,47,48,49,50,51,52,53,54,55,56,57,58,59,60,61,62,63,64,65,66,67,68,69,71,72,74,75,76,77,78,79,80,81,82,83,84,85,86,87,88,90,91,92,93,94,96,97,98,100,101,102,103,104,105,106,107,108,109,111,112,113,114,115,116,117,118,119,120,121,122,123,125,126,127,128,130,131,132,133,134,135,136,137,138,139,140,141,143,144,145,146,147,148,149,150,151,152,154,155,156,157,158,159,160,161,162,163,164,165,166,167,168,171,172,173,174,175,176,177,178,180,181,182,184,185,186,187,188,189,190,191,192,193,194,195,196,197,198,199,200,201,202,203,204,205,206,207,208,211,212,213,214,215,216,217,219,220,221,222,224,225,226,227,228,229,230,231,232,233,236,237,238,239,240,241,242,243,244,245,246,247,248,249,250,251,252,253,254,255,256,258,259,260,261,262,263,265,266,267,268,269,270,271,272,273,274,275,276,277,279,280,281,283,284,285,286,287,288,289,290,291,293,294,295,296,297,298,300,301,302,303,304,305,306,307,308,309,310,311,313,314,316,317,318,319,320,321,322,323,324,325,326,327,329,330,331,332,333,335,336,337,338,339,340,341,342,343,344,345,346,347,349,350,351,352,353,354,355,356,357,358,359,360,361,362,363,364,365,366,367,368,369,370,371,372,373,374,375,376,377,378,380,381}.The geometrical Structure method must satisfy the following :

i. K intersects any line of π in at most 18 points .
ii. Every point not in K is on at least one 18-secant of K .

The point :
M=3,9,10,11,13,15,17,363,77,112,144,192,30,177,54,184,135,300,84,47,61,100,111,19

9,66,86,306,225,131,78,173,161,380,154,59,322,108,333,201,18,344,52,249,350,370,230,377,325,177,97,320,311,116,268,211,40,180,106 Are eliminated from K to satisfy (1) . The points of index zero for 70,209 are added to K to satisfy (2) , then K_{18} =K∪ [70,209] / M

K_{18}=[4,5,6,7,8,12,14,16,19,20,22,23,24,25,26,27,28,29,31,32,33,34,35,36,37,38,39,42,43,44,45,46,48,49,50,51,53,55,56,57,58,60,62,63,64,65,67,68,69,71,72,74,75,76,79,80,81,82,83,85,87,88,90,91,92,93,94,96,98,101,102,103,104,105,107,109,113,114,115,118,119,120,121,122,123,125,126,127,128,130,132,133,134,136,137,138,139,140,141,143,145,146,147,148,149,150,151,152,155,156,157,158,159,160,162,163,164,165,166,167,168,171,172,174,175,176,178,181,182,185,186,187,188,189,190,191,193,194,195,196,197,198,200,202,203,204,205,206,207,208,212,213,214,215,216,217,219,220,221,222,224,226,227,228,229,231,232,233,235,236,237,238,239,240,241,242,243,244,245,246,247,248,250,251,252,254,255,256,258,259,260,261,262,263,265,266,267,269,270,271,272,273,274,275,276,277,279,280,281,283,284,285,286,287,288,289,290,291,293,294,295,296,297,298,301,302,303,304,305,307,308,309,310,313,314,316,317,318,319,321,323,324,326,327,329,330,331,332,335,336,337,338,339,340,341,342,343,345,346,347,349,351,352,353,354,355,356,357,358,359,360,361,362,364,365,366,367,368,369,371,372,373,374,375,376,378,381].Is a complete (289,18) –arc as shown in table (2) .Let $\beta_2 = \pi - k_{18}$ ={1,2,3,8,9,10,11,13,15,17,18,21,30,40,41,47,52,54,59,61,66,73,77,78,84,86,89,95,97,99,100,106,108,110,111,112,116,117,124,129,131,135,142,144,153,154,161,169,170,173,177,179,180,183,184,192,199,210,211,218,223,225,230,234,249,253,257,264,268,278,282,292,299,300,306,311,312,315,320,322,325,328,333,334,344,348,350,363,370,377,379,380} is (92,2)-blocking set as shown in table (2) .by theorem (1) ,there exists a projective $[289,3,271]_{19}$ code which is equivalent to the complete (289,18)-arc k_{18}

Table (2)

I	K_{18}∩ Li	B_2 ∩ Li
1	287	2,21,40,59,78,97,116,135,154,173,192,211,230,249,268,306,325,344,363
2	22,23,24,25,26,27,28,29,31,32,33,34,35,36,37,38,39	1,21,30
⋮	⋮	⋮
380	31,68,96,105,133,207,216,244,281,290,318 327,355,364	11,40,142,170,179,253
381	20,22,113,149,167,185,203,221,239,275,293,329,347,365	40,77,95,131,257,311

4.3 Existence of $[266,3,249]_{19}$ codes

We take 3 conic, say C_1 ,C_2,C_3 and let

K= π - $C_1 \cup C_2 \cup C_3$

{3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,22,23,24,25,26,27,28,29,30,31,32,33,34,35,36,37,38,39,40,42,43,44,45,46,47,48,49,50,51,52,53,54,55,56,57,58,59,60,61,62,63,64,65,66,67,68,69,71,74,75,76,77,78,79,81,82,83,84,85,86,87,88,90,91,92,93,94,96,97,98,100,101,103,104,105,106,107,108,109,111,112,113,114,115,116,117,118,119,120,121,122,123,125,126,127,130,131,132,133,134,135,136,137,138,139,140,141,143,145,146,147,148,149,150,151,152,154,155,156,157,158,159,160,161,162,163,164,165,166,167,1

68,171,173,174,175,176,177,178,180,181,182,184,185,186,187,189,190,191,192,193,194,196,197,198,199,200,201,202,203,204,205,206,207,208,211,212,213,214,215,216,219,220,221,222,224,225,226,227,228,229,230,231,232,233,236,237,238,239,240,241,242,243,245,246,247,248,249,250,251,252,253,254,255,258,259,260,261,262,263,265,266,267,268,269,270,271,272,273,274,275,277,279,280,281,283,284,285,286,287,288,289,290,291,293,294,295,296,298,300,301,302,303,304,305,306,307,308,309,310,311,313,314,316,317,318,319,320,321,323,324,325,326,327,329,330,331,332,333,335,336,337,338,339,340,341,342,343,344,345,346,347,349,350,351,352,353,354,356,357,358,359,360,361,362,363,364,365,366,367,368,369,370,371,372,373,374,375,376,377,378,381}.

The geometrical Structure method must satisfy the following :

i. K intersects any line of π in at most 17 points .
ii. Every point not in K is on at least one 17-secant of K .

The point :
M=40,22,61,10,79,363,225,100,192,320,135,350,77,177,287,30,66,306,112,54,184,131,15,84,377,199,3,333,78,130,9,13,173,161,106,154,92,180,59,8,344,11,39,111,249,370,230,20,325,171,117,50,47,97,247,373,250,113,5,186,268,181,211,222,190 Are eliminated from K to satisfy (1) . The points of index zero for 80,244 are added to K to satisfy (2) , then K_{17} =K∪ [80,244] / M

K_{17}=[4,6,7,12,14,16,17,18,19,23,24,25,26,27,28,29,31,32,33,34,35,36,37,38,42,43,44,45,46,48,49,51,52,53,55,56,57,58,60,62,63,64,65,67,68,69,71,74,75,76,80,81,82,83,84,85,87,88,90,91,93,94,96,98,101,103,104,105,107,108,109,114,115,116,118,119,120,121,122,123,125,126,127,132,133,134,136,137,138,139,140,141,143,145,146,147,148,149,150,151,152,155,156,157,158,159,160,162,163,164,165,166,167,168,174,175,176,178,182,185,187,189,191,193,194,196,197,198,200,201,202,203,204,205,206,207,208,212,213,214,215,216,219,220,221,224,226,227,228,229,231,232,233,236,237,238,239,240,241,242,243,244,245,246,248,251,252,253,254,255,258,259,260,261,262,263,265,266,267,269,270,271,272,273,274,275,277,279,280,281,283,284,285,286,288,289,290,291,293,294,295,296,298,300,301,302,303,304,305,307,308,309,310,311,313,314,316,317,318,319,321,323,324,326,327,329,330,331,332,335,336,337,338,339,340,341,342,343,345,346,347,349,351,352,353,354,356,357,358,359,360,361,362,364,365,366,367,368,369,371,372,374,375,376,378,381].Is a complete (266,17) –arc as shown in table (3) .Let $\beta_3 = \pi - k_{17}$
={1,2,3,5,8,9,10,11,13,15,20,21,22,30,39,40,41,47,50,54,59,61,66,70,72,73,77,78,79,86,89,92,95,97,99,100,102,106,110,111,112,113,117,124,128,129,130,131,135,142,144,153,154,161,169,170,171,172,173,177,179,180,181,183,184,186,188,190,192,195,199,209,210,211,217,218,222,223,225,230,234,235,247,249,250,256,257,264,268,276,278,282,287,292,297,299,306,312,315,320,322,325,328,333,334,344,348,350,355,363,370,373,377,379,380} is (115,3)-blocking set as shown in table (3) .

by theorem (1) ,there exists a projective $[266,3,249]_{19}$ code which is equivalent to the complete (266,17)-arc k_{17}

Table (3)

I	K_{17}∩ Li	B_3 ∩ Li
1	116	2,21,40,59,78,97,135,154,173,192,211,230,249,287,268,306,325,344,363
2	23,24,25,26,27,28,29,31,32,33,34,35,36,37,38	1,21,22,30,39
⋮	⋮	⋮

380	31,68,96,105,133,207,216,281,290,318,327,364,244,253	11,40,142,170,179,355
381	149,167,185,203,221,239,275,293,311,329,347,365	20,22,40,77,95,113,131,257

4.4 Existence of [246,3,230]19 codes

We take 4 conic, say C_1 ,C_2,C_3,C_4 and let

K= π - $C_1 \cup C_2 \cup C_3 \cup C_4$

{3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,22,23,24,25,26,27,28,29,30,31,32,33,34,35,36,37,38,39,40,42,43,44,45,46,47,48,49,50,51,52,53,54,55,56,57,58,59,60,61,62,63,64,65,66,68,69,71,74,75,76,77,78,79,81,82,83,84,85,86,87,88,90,92,93,94,96,97,98,100,101,103,104,105,106,107,108,111,112,113,114,115,116,117,118,119,120,121,122,125,126,127,130,131,132,133,134,135,136,137,139,140,141,143,145,146,147,148,149,150,151,152,154,155,156,157,158,159,160,161,162,163,164,165,166,167,168,173,174,175,176,177,178,180,181,182,184,185,186,187,190,191,192,193,196,197,198,199,200,201,202,203,204,205,206,207,208,211,212,213,214,215,216,219,221,222,224,225,226,227,228,229,230,231,232,233,236,237,238,239,241,242,243,245,246,247,248,249,250,251,252,253,254,255,258,259,260,261,262,263,265,266,268,269,270,271,272,273,274,275,277,279,280,281,284,285,286,287,288,289,290,291,294,295,296,298,300,301,302,303,304,305,306,307,308,309,310,311,313,314,316,317,318,319,320,321,323,324,325,326,327,330,331,332,333,335,336,337,338,339,340,341,342,343,344,345,346,347,349,350,351,352,353,354,356,357,359,360,361,362,363,364,365,366,367,368,369,370,371,372,373,375,376,377,378,381}.

The geometrical Structure method must satisfy the following :

i. K intersects any line of π in at most 16 points .
ii. Every point not in K is on at least one 16-secant of K .

The point :

M=40,59,22,30,61,161,3,5,140,79,363,66,112,181,186,100,177,47,192,39,320,10,184,60,135,350,131,106,130,225,86,287,15,306,326,333,78,117,222,173,54,199,154,7,9,180,147,344,11,113,377,249,160,230,77,115,8,325,111,50,81,247,378,116,250,373,268,211,20,Are eliminated from K to satisfy (1) . The points of index zero for 171,293 are added to K to satisfy (2) , then K_{16} =K$\cup$ [171,293] / M

K_{16}=[4,6,12,13,14,16,17,18,19,23,24,25,26,27,28,29,31,32,33,34,35,36,37,38,42,43,44,45,46,48,49,51,52,53,55,56,57,58,62,63,64,65,68,69,71,74,75,76,82,83,84,85,87,88,90,92,93,94,96,97,98,101,103,104,105,107,108,114,118,119,120,121,122,125,126,127,132,133,134,136,137,139,141,143,145,146,148,149,150,151,152,155,156,157,158,159,162,163,164,165,166,167,168,171,174,175,176,178,182,185,186,187,190,191,193,196,197,198,200,201,202,203,204,205,206,207,208,212,213,214,215,216,219,221,224,226,227,228,229,231,232,233,236,237,238,239,241,242,243,245,246,248,251,252,253,254,255,258,259,260,261,262,263,265,266,269,270,271,272,273,274,275,277,279,280,281,284,285,286,288,289,290,291,293,294,295,296,298,300,301,302,303,304,305,307,308,309,310,311,313,314,316,317,318,319,321,323,324,327,330,331,332,335,336,337,338,339,340,341,342,343,345,346,347,349,351,352,353,354,356,357,359,360,361,362,364,365,366,367,3

68,369,370,371,372,375,376,381].Is a complete (246,16) –arc as shown in table (4) .
Let $\beta_4 = \pi - k_{16}$
={1,2,3,5,7,8,9,10,11,13,15,20,21,22,30,39,40,41,47,50,54,59,60,61,66,67,70,72,73,77,78,79,80,81,86,89,91,95,99,100,102,106,109,110,111,112,113,115,116,117,123,124,128,129,130,131,135,138,140,142,144,147,153,154,160,161,169,170,172,173,177,179,180,181,183,184,186,188,189,192,194,195,199,209,210,211,217,218,220,222,223,225,230,234,235,240,244,247,249,250,256,257,264,267,268,276,278,282,283,287,292,297,299,306,312,315,320,322,325,326,328,329,333,334,344,348,350,355,358,363,373,374,377,378,379,380} is (135,4)-blocking set as shown in table (4) .by theorem (1) ,there exists a projective $[246,3,230]_{19}$ code which is equivalent to the complete (246,16)-arc k_{16}

Table (4)

I	$K_{16} \cap Li$	$B_4 \cap Li$
1	97	2,21,40,59,78,116,135,154,173,192,211,230,249,268,287,306,325,344,363
2	23,24,25,26,27,28,29,31,32,33,34,35,36,37,38	1,22,30,39,21
⋮	⋮	⋮
380	31,68,96,105,133,207,216,281,290,318,327,364,253	11,40,142,170,179,244,355
381	149,167,185,203,293,22,239,275,311,347,365	20,22,40,77,95,113,131,257,329

4.5 Existence of $[219,3,204]_{19}$ codes

We take 5 conic, say C_1 ,C_2,C_3,C_4 , C_5 and let
K= π - $C_1 \cup C_2 \cup C_3 \cup C_4 \cup C_5$
{3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,22,23,24,25,26,27,28,29,30,31,32,33,34,35,36,37,38,39,40,42,43,44,45,46,47,48,49,50,51,52,53,54,55,56,57,58,59,60,61,62,63,64,65,66,68,69,71,74,75,76,78,79,81,82,83,84,86,87,88,90,92,93,94,96,97,98,100,101,103,104,105,107,108,111,112,113,114,115,116,117,118,120,121,122,125,126,127,130,131,132,133,134,135,136,137,139,141,143,145,146,147,148,149,150,151,152,154,155,156,157,158,159,160,161,162,163,164,165,166,168,173,174,175,176,177,178,180,181,182,185,186,187,190,191,192,193,196,197,198,199,200,201,202,203,205,206,207,208,211,212,213,214,216,219,221,222,224,225,226,227,228,229,230,231,232,233,236,237,238,239,241,242,243,245,247,248,249,250,252,253,254,255,258,259,260,261,262,263,265,266,268,269,270,271,272,273,274,275,277,279,280,281,284,286,287,288,289,290,291,294,295,296,298,300,301,302,303,304,305,306,307,308,309,310,311,313,314,316,317,318,319,321,323,324,325,326,327,330,331,332,333,336,337,338,339,340,341,342,343,344,345,346,347,349,350,351,352,353,354,356,357,360,361,362,363,364,365,366,367,368,369,370,372,373,375,376,377,378,381}.
The geometrical Structure method must satisfy the following :

1. K intersects any line of π in at most 15 points .
2. Every point not in K is on at least one 15-secant of K .

The point :
M=59,78,22,30,39,61,81,161,3,5,10,197,60,79,363,20,111,112,66,181,247,57,86,131,100,186,166,177,180,47,192,347,225,377,135,130,271,115,287,54,15,190,306,199,333,222,9,173,11,381,154,7,356,101,147,121,90,344,8,249,120,230,113,174,325,69,97,18,116,

370,13,250,373,321,268,211,259,155,139,378

Are eliminated from K to satisfy (1) . The points of index zero for 251,359 are added to K to satisfy (2) , then K_{15} =K∪ [251,359] / M

K_{15}=[4,6,12,14,16,17,19,23,24,25,26,27,28,29,31,32,33,34,35,36,37,38,40,42,43,44,45,46,48,49,50,51,52,53,55,56,58,62,63,64,65,68,71,74,75,76,82,83,84,87,88,92,93,94,96,98,103,104,105,107,108,114,117,118,,122,125,126,127,132,133,134,136,137,141,143,145,146,148,149,150,151,152,156,157,158,159,160,162,163,164,165,168,175,176,178,182,185,187,191,193,196,198,200,201,202,203,205,206,207,208,212,213,214,216,219,221,224,226,227,228,229,231,232,233,236,237,238,239,241,242,243,245,248,252,253,254,255,258,260,261,262,263,265,266,269,270,272,273,274,275,277,279,280,281,284,286,288,289,290,291,294,295,296,298,300,301,302,303,304,305,307,308,309,310,311,313,314,316,317,318,319,323,324,326,327,330,331,332,336,337,338,339,340,341,342,343,345,346,349,350,351,352,353,354,357,360,361,362,364,365,366,367,368,369,372,375,376].

Is a complete (219,15) –arc as shown in table (5) .

Let $\beta_5 = \pi - k_{15}$

={1,2,3,5,7,8,9,10,11,13,15,18,20,21,22,30,39,41,47,57,54,59,60,61,66,67,69,70,72,73,77,78,79,80,81,85,86,89,90,91,95,97,99,100,101,102,106,109,110,111,112,113,115,116,119,120,121,123,124,128,129,130,131,135,138,139,140,142,144,147,153,154,155,161,166,167,169,170,171,172,173,174,177,179,180,181,183,184,186,188,189,190,192,194,195,197,199,204,209,210,211,215,217,218,220,222,223,225,230,234,235,240,244,246,247,249,250,256,257,259,264,267,268,271,276,278,282,283,285,287,292,293,297,299,306,312,315,320,322,321,325,328,329,333,334,335,344,347,348,355,356,358,363,370,371,373,374,377,378,379,380,381} is (162,15)-blocking set as shown in table (5) .

by theorem (1) ,there exists a projective $[219,3,204]_{19}$ code which is equivalent to the complete (219,15)-arc k_{15}

Table (5)

I	K_{15}∩ Li	B_5 ∩ Li
1	40	2,21,59,78,97,116,135,154,173,192,211,230,249,268,287,306,325,344,363
2	23,24,25,26,27,28,29,31,32,33,34,35,36,37,38	1,21,22,30,39
⋮	⋮	⋮
380	31,68,96,105,133,207,216,281,290,318,327,364,253	11,40,142,170,179,244,355
381	149,185,203,221,239,275,311,365	20,22,40,77,95,113,131,167,257,293,347,329

4.6 Existence of $[206,3,192]_{19}$ codes

We take 6 conic, say C_1 ,C_2,C_3,C_4 ,C_5 ,C_6 and let

K= π - $C_1 \cup C_2 \cup C_3 \cup C_4 \cup C_5 \cup C_6$

{3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,22,23,24,25,26,27,28,29,30,31,32,33,34,35,36,37,38,39,40,42,43,44,45,46,47,48,49,50,51,52,53,54,55,56,57,58,59,60,61,62,63,64,65,66,68,69,71,74,76,78,79,81,82,83,84,86,87,88,90,92,94,96,97,98,100,101,103,104,105,107,108,111,112,113,114,116,117,118,120,121,122,125,126,127,130,131,132,134,135,136,137,139,141,143,145,146,147,149,150,151,152,154,155,156,157,158,159,160,161,162,163,164,165,166,173,174,175,176,178,180,181,182,185,186,187,190,191,192,19

3,196,197,198,199,201,202,203,205,206,207,208,211,212,214,216,219,221,222,224,225,226,227,228,229,230,231,232,233,236,237,238,241,242,243,245,247,248,249,250,252,253,254,255,258,259,261,262,263,265,266,268,269,270,271,272,273,274,275,277,279,280,281,284,286,287,288,289,291,294,295,296,298,300,301,302,303,304,305,306,307,308,309,310,313,314,316,317,318,319,321,323,324,325,326,327,330,331,332,333,336,338,339,340,341,342,343,344,345,346,347,349,351,352,353,354,356,357,360,361,362,363,364,365,366,367,368,369,370,372,375,376,377,378,381}.

The geometrical Structure method must satisfy the following :

1. K intersects any line of π in at most 14 points .
2. Every point not in K is on at least one 14-secant of K .

The point :

M=40,59,78,97,22,25,30,39,3,61,81,101,5,8,9,197,60,79,117,363,11,90,112,181,225,86,191,131,100,186,155,13,180,47,192,347,139,15,10,159,135,247,287,300,161,190,377,87,29,306,54,199,178,333,66,174,147,173,222,113,154,69,255,344,31,249,120,230,325,141,50,116,321,250,268,52,378Are eliminated from K to satisfy (1) . The points of index zero for 188,373 are added to K to satisfy (2) , then

K_{14} =K∪ [188,373] / M

K_{14}=[4,6,7,12,14,16,17,18,19,20,23,24,26,27,28,32,33,34,35,36,37,38,42,43,44,45,46,48,49,51,53,55,56,57,58,62,63,64,65,68,71,74,76,82,83,84,88,92,94,96,98,103,104,105,107,108,111,114,118,121,122,125,126,127,130,132,134,136,137,143,145,146,149,150,151,152,156,157,158,160,162,163,164,165,166,175,176,182,185,187,188,193,196,198,201,202,203,205,206,207,208,211,212,214,216,219,221,224,226,227,228,229,231,232,233,236,237,238,241,242,243,245,248,252,253,254,258,259,261,262,263,265,266,269,270,271,272,273,274,275,277,279,280,281,284,286,288,289,291,294,295,296,298,301,302,303,304,305,307,308,309,310,313,314,316,317,318,319,323,324,326,327,330,331,332,336,338,339,340,341,342,343,345,346,349,351,352,353,354,356,357,360,361,362,364,365,366,367,368,369,370,372,373,375,376,381].Is a complete (206,14) –arc as shown in table (6) .Let $\beta_6 = \pi - k_{14}$ ={1,2,3,5,8,9,10,11,13,15,21,22,25,29,30,31,39,40,41,47,50,52,54,59,60,61,66,67,69,70,72,73,75,77,78,79,80,81,85,86,87,89,90,91,93,95,97,99,100,101,102,106,109,110,112,113,115,116,117,119,120,123,124,128,129,131,133,135,138,139,140,141,142,144,147,148,153,154,155,159,161,167,168,169,170,171,172,173,174,177,178,179,180,181,183,184,186,189,190,191,192,194,195,197,199,200,204,209,210,213,215,217,218,220,222,223,225,230,234,235239,240,244,246,247,249,250,251,255,256,257,260,264,267,268,276,278,282,283,285,287,290,292,293,297,299,300,306,311,312,315,320,321,322,325,328,329,333,334,335,337,344,347,348,350,355,358,359,363,371,374,377,378,379,380} is (175,14)-blocking set as shown in table (5) .by theorem (1) ,there exists a projective $[206,3,192]_{19}$ code which is equivalent to the complete (204,14)-arc k_{14}

Table (6)

I	K_{14}∩ Li	B_6 ∩ Li
1	211	2,21,40,59,78,97,116,135,173,192,230,249,268,287,306,154,325,344,363
2	23,24,26,27,28,32,33,34,35,36,37,38	1,22,25,29,30,21,31,39
⋮	⋮	⋮
38	68,96,105,207,216,281,318,32	11,31,40,133,142,170,179,244,290,355

0	7,364,253	
381	20,95,149,185,203,221,275,365	40,22,77,113,131,167,239,257,293,311,329,347

4.7 Existence of $[181,3,168]_{19}$ codes

We take 7 conic, say C_1, C_2, C_3, C_4, C_5, C_6, C_7 and let

$K = \pi - C_1 \cup C_2 \cup C_3 \cup C_4 \cup C_5 \cup C_6 \cup C_7$

{3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,22,23,24,25,26,27,28,29,30,31,32,33,34,35,36,37,38,39,40,42,43,44,45,46,47,48,49,50,51,52,53,54,55,56,57,58,59,60,61,62,63,64,66,68,69,71,74,76,78,79,81,82,83,84,86,87,90,92,94,96,97,98,100,101,103,104,105,107,108,111,113,114,116,117,118,120,121,122,125,126,127,130,131,134,135,136,137,139,141,143,145,147,149,150,151,152,154,155,156,157,159,160,161,162,163,164,165,166,173,174,175,178,180,181,182,185,186,187,190,191,192,193,196,197,198,199,201,202,203,205,207,208,211,212,214,216,219,221,222,224,225,226,227,229,230,231,232,233,236,238,241,242,243,245,247,248,249,250,252,253,254,255,258,259,261,262,263,265,266,268,269,270,271,272,273,274,275,279,280,281,284,286,287,288,289,291,294,295,296,298,300,301,302,303,304,306,307,309,310,313,314,316,317,318,319,321,323,324,325,326,327,330,331,332,333,336,339,340,341,342,343,344,345,346,347,349,351,352,353,354,357,360,361,362,363,364,365,366,367,369,370,372,375,376,377,378,381}.

The geometrical Structure method must satisfy the following :

1. K intersects any line of π in at most 13 points .
2. Every point not in K is on at least one 13-secant of K .

The point :

M=40,59,78,97,116,22,79,90,103,25,30,31,39,31,6,81,161,181,5,7,9,10,197,60,250,174,363,333,247,86,131,100,186,166,13,192,347,180,225,139,135,130,287,69,47,377,15,190,242,306,199,178,111,66,117,54,121,173,222,259,154,378,52,101,20,191,381,11,150,187,159,74,8,249,120,352,147,230,325,137,141,370,268,211,50,87Are eliminated from K to satisfy (1) . The points of index zero for 209,210 are added to K to satisfy (2) , then $K_{13} = K \cup [209,210] / M$

K_{13}=[4,6,12,14,16,17,18,19,23,24,26,27,28,29,32,33,34,35,36,37,38,42,43,44,45,46,48,49,51,53,55,56,57,58,62,63,64,68,71,76,82,83,84,92,94,96,98,104,105,107,108,113,114,118,122,125,126,127,134,136,143,145,149,151,152,155,156,157,160,162,163,164,165,175,182,185,193,196,198,201,202,203,205,207,208,209,210,212,214,216,219,221,224,226,227,229,231,232,233,236,238,241,243,245,248,252,253,254,255,258,261,262,263,265,266,269,270,271,272,273,274,275,279,280,281,284,286,288,289,291,294,295,296,298,300,301,302,303,304,307,309,310,313,314,316,317,318,319,321,323,324,326,327,330,331,332,336,339,340,341,342,343,344,345,346,349,351,353,354,357,360,361,362,364,365,366,367,369,372,375,376].

Is a complete (181,13) –arc as shown in table (7) .Let $\beta_7 = \pi - k_{13}$ ={1,2,3,5,7,8,9,10,11,13,15,20,21,22,25,30,31,39,40,41,47,50,52,54,59,60,61,65,66,67,69,70,72,73,74,75,77,78,79,80,81,85,86,87,88,89,90,91,93,95,97,99,100,101,102,103,106,109,110,111,112,115,116,117,119,120,121,123,124,128,129,130,131,132,133,135,137,138,139,140,141,142,144,146,147,148,150,153,154,158,159,161,166,167,168,170,171,172,173,174,176,177,178,179,180,181,183,184,186,187,188,189,190,191,192,194,195,197,199,200,204,206,211,213,215,217,218,220,222,223,225,228,230,234,235,237,239,240,242,244,246,247,249,250,251,253,256,257,259,260,264,267,268,276,277,278,282,283,285,287,290,292,293,297,299,305,306,308,311,312,315,320,322,325,328,329,333,334,335,337,338,347,348,350,352,355,356,358,359,363,368,370,371,373,374,377,378,379,3

80,381} is (200,13)-blocking set as shown in table (7) .by theorem (1) ,there exists a projective $[181,3,168]_{19}$ code which is equivalent to the complete (181,13)-arc k_{13}

Table (7)

I	$K_{13} \cap$ Li	$B_7 \cap$ Li
1	344	2,21,40,59,78,97,116,145,173,192,211,230,249,268,287,135,306,325,363
2	23,24,26,27,28,29,32,33,34,35,36,37,38	1,21,22,25,30,31,39
⋮	⋮	⋮
380	68,96,105,207,216,281,318,327,364,253	11,31,40,133,142,170,179,244,290,355
381	113,149,185,203,221,275,365	20,22,40,77,167,95,131,293,239,257,311,329,347

4.8 Existence of $[157,3,145]_{19}$ codes

We take 8 conic, say C_1 ,C_2,C_3,C_4 ,C_5 ,C_6 ,C_7 , C_8 and let

$K= \pi - C_1 \cup C_2 \cup C_3 \cup C_4 \cup C_5 \cup C_6 \cup C_7 \cup C_8$

{3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,22,23,24,25,26,27,28,29,30,31,32,33,34,35,36,37,38,39,40,42,43,44,45,46,47,48,49,50,51,52,53,54,55,56,57,58,59,60,61,62,63,64,66,68,69,71,74,78,79,81,82,83,84,86,87,90,92,94,97,98,101,103,104,105,107,108,111,113,114,116,117,120,121,122,125,126,127,130,131,134,135,136,137,139,141,143,145,149,150,151,152,154,155,156,157,159,160,161,163,164,165,166,173,174,175,178,180,181,182,185,186,190,191,192,193,196,197,198,201,202,203,205,207,208,211,212,214,219,221,222,224,225,226,227,229,230,231,232,233,236,238,241,242,243,245,247,248,249,250,252,253,254,255,258,259,261,263,265,266,268,269,270,271,272,273,274,275,280,281,284,286,287,288,289,294,295,296,298,300,301,302,303,304,306,307,309,310,313,314,317,318,319,321,323,324,325,326,327,330,332,333,336,339,340,341,342,343,344,345,346,347,349,351,352,353,354,357,361,362,363,364,365,366,367,369,370,372,375,376,377,381}.

The geometrical Structure method must satisfy the following :

i. K intersects any line of π in at most 12 points .
ii. Every point not in K is on at least one 12-secant of K .

The point :

M=40,59,78,97,116,135,22,25,30,31,29,39,16,3,5,7,9,10,61,81,101,161,181,8,6,15,178,197,60,79,117,174,250,20,363,66,130,86,186,131,13,166,192,180,191,14,300,333,225,121,54,287,69,47,19,190,87,306,377,111,18,139,381,259,173,90,11,201,222,114,154,52,247,347,370,301,344,159,303,343,249,120,339,152,230,310,311,325,107,50,141,271,12,56,231Are eliminated from K to satisfy (1) . The points of index zero for 311,312 are added to K to satisfy (2) , then K_{12} =K∪ [311,312] / M

K_{12}=[4,17,23,24,26,27,28,32,33,34,35,36,37,38,42,43,44,45,46,48,49,51,53,55,57,58,62,63,64,68,71,74,82,83,84,92,94,98,103,104,105,108,113,122,125,126,127,134,136,137,143,145,149,150,151,155,156,157,160,163,164,165,175,182,185,193,196,198,202,203,205,207,208,211,212,214,219,221,224,226,227,229,232,233,236,238,241,242,243,245,248,252,253,254,255,258,261,263,265,266,268,269,270,272,273,274,275,280,281,284,286,288,289,294,295,296,298,302,304,307,309,311,312,313,314,317,318,319,321,323,324,326,327,330,332,336,340,341,342,345,346,349,351,352,353,354,357,361,362,364,365,36

6,367,369,372,375,376].Is a complete (157,12) –arc as shown in table (8) .Let $\beta_8 = \pi - k_{12}$
={1,2,3,5,6,7,8,9,10,11,12,13,14,15,16,18,19,20,21,22,25,29,30,31,39,40,41,47,50,52,54,56,59,60,61,65,66,67,69,70,72,73,75,76,77,78,79,80,81,85,86,87,88,89,90,91,93,95,96,97,99,100,101,102,106,107,109,110,111,112,114,115,116,117,118,119,120,121,123,124,128,129,130,131,132,133,135,138,139,140,141,142,144,146,147,148,152,153,154,158,159,161,162,166,167,168,169,170,171,172,173,174,176,177,178,179,180,181,183,184,186,187,188,189,190,191,192,194,195,197,199,200,201,204,206,209,210,211,213,215,216,217,218,220,222,223,225,228,230,231,234,235,237,239,240,244,246,247,249,250,251,256,257,259,260,262,264,267,271,276,277,278,279,282,283,285,287,290,291,292,293,297,299,300,301,303,305,306,308,310,315,316,320,322,325,328,329,331,333,334,335,337,338,339,343,344,347,348,350,355,356,358,359,360,363,368,370,371,373,374,377,378,379,380,381} is (200,12)-blocking set as shown in table (8) .
by theorem (1) ,there exists a projective $[157,3,145]_{19}$ code which is equivalent to the complete (157,12)-arc k_{12}

Table (8)

I	$K_{12} \cap$ Li	$B_8 \cap$ Li
1	268	2,21,40,59,78,97,116,135,154,173,192,211,249,287,306,325,344,363,230
2	23,24,26,27,28,32,33,34,35,36,37,38	1,21,22,25,30,31,29,39
⋮	⋮	⋮
380	68,105,207,281,318,327,364,253	11,31,40,96,133,142,170,179,216,244,290,355
381	113,149,185,311,203,221,275,365	20,22,40,77,95,131,167,239,257,293,347,329

4.9 Existence of $[141,3,130]_{19}$ codes

We take 9 conic, say C_1 ,C_2,C_3,C_4 ,C_5 ,C_6 ,C_7 , C_8 , C_9 and let
K= π - $C_1 \cup C_2 \cup C_3 \cup C_4 \cup C_5 \cup C_6 \cup C_7 \cup C_8 \cup C_9$
{3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,22,23,24,25,26,27,28,29,30,31,32,33,34,35,36,37,38,39,40,42,43,44,45,46,47,48,49,50,51,52,53,54,55,56,57,58,59,60,61,62,63,64,68,69,71,74,78,79,81,82,83,84,86,87,92,94,97,98,101,104,105,107,108,111,113,114,116,117,120,121,122,126,127,130,131,134,135,136,137,141,143,145,149,150,151,152,154,155,157,159,160,161,163,164,165,166,173,174,175,178,180,181,182,185,186,191,192,193,196,198,201,202,203,205,207,208,211,212,214,219,221,222,224,225,226,227,229,230,231,232,233,236,238,241,242,243,247,248,249,250,253,254,255,258,259,261,263,265,266,268,269,270,271,272,273,274,275,280,281,284,287,288,289,294,295,296,298,300,301,302,304,306,307,309,310,313,314,318,319,321,323,324,325,326,327,330,332,333,336,340,341,342,343,344,345,346,347,349,351,353,354,357,361,362,363,364,365,366,367,369,370,372,375,377,381}.
The geometrical Structure method must satisfy the following :

i. K intersects any line of π in at most 11 points .
ii. Every point not in K is on at least one 11-secant of K .

The point :
M=40,59,78,97,116,135,20,154,22,25,30,31,39,35,3,61,81,101,141,161,181,5,7,9,10,12,14,159,178,121,60,79,117,174,250,155,363,333,160,225,191,86,107,186,166,300,192,180,130,52,15,16,54,13,38,287,69,47,19,151,87,306,377,347,50,111,18,222,255,104,173,

247,131,259,11,4,344,150,159,344,343,8,120,152,370,230,301,114,325,17,207,268,211 Are eliminated from K to satisfy (1) . The points of index zero for 251,252 are added to K to satisfy (2) , then K_{11} =K∪ [251,252] / M

K_{11}=[6,23,24,26,27,28,32,33,34,36,37,38,42,43,44,45,46,48,49,51,53,55,57,62,63,64,68 ,71,74,82,83,84,92,94,98,105,108,113,122,126,127,134,136,137,143,145,149,157,163,1 64,165,175,182,185,193,196,198,201,202,203,205,208,212,214,219,221,224,226,227,22 9,231,232,233,236,238,241,242,243,248,249,251,252,253,254,258,261,263,265,266,269 ,270,271,272,273,274,275,280,281,284,288,289,294,295,296,298,302,304,307,309,310, 313,314,318,319,321,323,324,326,327,330,332,336,340,341,342,345,346,349,351,353,3 54,357,361,362,364,365,366,367,369,372,375].Is a complete (141,11) –arc as shown in table (9) .Let $\beta_9 = \pi - k_{11}$
={1,2,3,4,5,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,25,29,30,31,35,39,40,41,47,5 0,52,54,56,58,59,60,61,65,66,67,69,70,72,73,75,76,77,78,79,80,81,85,86,87,88,89,90,91 ,93,95,96,97,99,100,101,102,103,104,106,107,109,110,111,112,114,115,116,117,118,11 9,120,121,123,124,125,128,129,130,130,132,133,135,138,139,140,141,142,144,146,147 ,148,150,151,152,153,154,155,156,158,159,160,161,162,166,167,168,169,170,171,172, 173,174,176,177,178,179,180,181,183,184,186,187,188,189,190,191,192,194,195,197,1 99,200,204,206,207,209,210,211,213,215,216,217,218,220,222,223,225,228,230,234,23 5,237,239,240,244,245,246,247,250,255,256,257,259,260,262,264,267,268,276,277,278 ,279,282,283,285,286,287,290,291,292,293,297,299,300,301,303,305,306,308,311,312, 315,316,317,320,322,325,328,329,331,333,334,335,337,338,339,343,344,347,348,350,3 52,355,356,358,359,360,363,368,370,371,373,374,376,377,378,379,380,381} is (240,11)-blocking set as shown in table (9) .

by theorem (1) ,there exists a projective $[141,3,130]_{19}$ code which is equivalent to the complete (141,11)-arc k_{11}

Table (9)

I	K_{11}∩ Li	B_9 ∩ Li
1	249	2,21,40,59,78,97,116,135,154,173,192,211,230,268, 287,306,344,325,363
2	23,24,26,27,28,32,33,34,3 6,37,38	1,21,25,29,30,31,35,39,22
⋮	⋮	⋮
38 0	68,105,281,318,327,364,2 53	11,31,40,96,133,142,170,179,207,216,244,290,355
38 1	113,149,185,203,221,275, 365	20,22,40,77,131,167,239,257,95,239,311,329,347

4.10 Existence of $[120,3,110]_{19}$ codes

We take 10 conic , say C_1 ,C_2,C_3,C_4 ,C_5 ,C_6 ,C_7 , C_8 , C_9 , C_{10} and let K= π - $C_1 \cup C_2 \cup C_3 \cup C_4 \cup C_5 \cup C_6 \cup C_7 \cup C_8 \cup C_9 \cup C_{10}$
{3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,22,23,24,25,26,27,28,29,30,31,32,33,34 ,35,36,37,38,39,40,42,43,44,45,46,47,48,49,50,51,52,53,54,55,56,57,58,59,60,61,62,63, 68,69,71,74,78,79,81,83,84,86,87,92,94,97,98,101,104,105,107,108,113,114,116,117,12 0,121,122,127,130,131,134,135,136,137,141,143,145,149,150,152,154,155,157,159,160 ,161,164,165,166,173,174,175,178,181,182,185,186,191,192,193,196,198,201,202,203, 205,207,208,211,212,214,219,221,222,224,225,227,229,230,231,232,233,236,238,241,2 42,247,248,249,250,253,254,258,259,261,263,265,266,268,269,270,271,272,273,274,27

5,281,284,287,288,289,294,296,298,300,301,302,304,306,307,309,310,313,314,318,319,321,323,325,326,327,330,332,333,336,340,341,343,344,345,347,349,351,353,354,357,361,362,363,364,365,367,369,370,372,375,377,381}.The geometrical Structure method must satisfy the following :

i. K intersects any line of π in at most 10 points .
ii. Every point not in K is on at least one 10-secant of K .

The point :
M=40,59,78,97,116,135,154,173,22,25,29,30,31,33,35,39,3,61,81,101,121,141,161,181,5,9,10,12,14,16,20,60,79,117,155,174,250,231,363,160,130,247,225,87,191,86,107,4,186,300,259,166,13,192,69,15,54,178,47,19,134,104,58,375,330,232,377,74,306,50,222,347,150,11,136,201,52,301,381,344,159,307,249,120,152,108,370,230,18,114,340,325,207,17,270,268,211,40,310 Are eliminated from K to satisfy (1) . The points of index zero for 65,66 are added to K to satisfy (2) , then K_{10} =K∪ [65,66] / M
K_{10}=[6,7,8,23,24,26,27,28,32,34,36,37,38,42,43,45,46,48,49,51,53,55,56,57,62,63,65,66,68,71,83,84,92,94,98,105,113,122,127,131,137,143,145,149,157,164,165,175,182,185,193,196,198,202,203,205,208,212,214,219,221,224,227,229,233,236,238,241,242,248,253,254,258,261,263,265,266,269,271,272,273,274,275,281,284,287,288,289,294,296,298,302,304,309,313,314,318,319,321,323,326,327,332,333,336,341,343,345,349,351,353,354,357,361,362,364,365,367,369,372].Is a complete (120,10) –arc as shown in table (10) .Let $\beta_{10} = \pi - k_{10}$
={1,2,3,4,5,9,10,11,12,13,14,15,16,17,18,19,20,21,22,25,29,30,31,33,35,39,40,41,44,47,50,52,54,58,59,60,61,64,67,69,70,72,73,74,75,76,77,78,79,80,81,82,85,86,87,88,89,90,91,93,95,96,97,99,100,101,102,103,104,106,107,108,109,110,111,112,114,115,116,117,118,119,120,121,123,124,125,126,128,129,130,132,133,134,135,136,138,139,140,141,142,144,146,147,148,150,151,152,153,154,155,156,158,159,160,161,162,163,166,167,168,169,170,171,172,173,174,176,177,178,179,180,181,183,184,186,187,188,189,190,191,192,194,195,197,199,200,201,204,206,207,209,210,211,213,215,216,217,218,220,222,223,225,226,228,230,231,232,234,235,237,239,240,243,244,245,246,247,249,250,251,252,255,256,257,259,260,262,264,267,268,270,276,277,278,279,280,282,283,285,286,290,291,292,293,295,297,299,300,301,303,305,306,307,308,310,311,312,315,316,317,320,322,324,325,328,329,330,331,334,335,337,338,339,340,342,344,346,347,348,350,352,355,356,358,359,360,363,366,368,370,371,373,374,375,376,377,378,379,380,381} is (261,11)-blocking set as shown in table (10) .by theorem (1) ,there exists a projective $[120,3,110]_{19}$ code which is equivalent to the complete (120,10)-arc k_{10}

Table (10)

I	K_{10}∩ Li	B_{10} ∩ Li
1	287	2,21,40,59,78,97,116,135,154,173,192,211,230,249,268,306,325,344,363
2	23,24,26,27,28,32,34,36,37,38	1,21,22,25,29,30,31,33,35,39
⁞	⁞	⁞
380	68,105,281,318,327,364,253	11,31,40,96,133,142,170,179,207,216,244,290,355
381	113,149,185,203,221,275,365	20,22,40,77,95,131,167,239,257,293,311,329,347

4.11 Existence of $[112,3,103]_{19}$ codes

We take 11 conic , say C_1 ,C_2,C_3,C_4 ,C_5 ,C_6 ,C_7 , C_8 , C_9 , C_{10} , C_{11} and let K= π - $C_1 \cup C_2 \cup C_3 \cup C_4 \cup C_5 \cup C_6 \cup C_7 \cup C_8 \cup C_9 \cup C_{10} \cup C_{11}$
{3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,22,23,24,25,26,27,28,29,30,31,32,33,34,35,36,37,38,39,40,42,43,44,45,46,47,48,49,50,51,52,53,54,55,56,57,58,59,60,61,62,63,68,69,71,78,79,81,83,84,87,92,94,97,98,101,105,107,108,113,114,116,117,120,121,122,127,130,131,135,136,141,143,145,149,150,152,154,155,157,159,160,161,164,166,173,174,175,178,181,182,185,186,191,192,193,196,198,201,202,203,207,208,211,212,219,221,222,224,225,227,229,230,231,232,233,238,241,242,247,248,249,250,253,254,258,259,261,263,265,268,269,270,271,272,273,274,275,281,287,288,289,294,298,300,301,302,304,306,307,309,313,314,318,319,321,323,325,326,327,332,333,336,340,341,343,344,345,347,349,351,353,357,361,362,363,364,365,367,369,370,372,375,381}.

The geometrical Structure method must satisfy the following :

1. K intersects any line of π in at most 9 points .
2. Every point not in K is on at least one 9-secant of K .

The point :
M=40,59,78,97,116,135,154,173,192,22,25,29,30,31,33,35,39,36,3,61,81,101,121,5,6,7,10,9,12,14,16,41,161,181,381,60,79,117,155,174,250,307,363,333,160,130,20,191,107,44,4,300,207,166,136,47,186,178,52,114,341,87,6,15,247,113,287,69,54,58,306,18,159,347,222,150,259,271,50,11,201,225,344,343,249,289,108,19,370,230,268,211Are eliminated from K to satisfy (1) . The points of index zero for 216,217 are added to K to satisfy (2) , then K_9 =K∪ [216,217] / M
K_9=[8,13,17,23,24,26,27,28,32,34,37,38,42,43,45,46,48,49,51,53,55,56,57,62,63,68,71,83,84,92,94,98,105,120,122,127,131,143,145,149,152,157,164,175,182,185,193,196,198,202,203,208,212,216,217,219,221,224,227,229,231,232,233,238,241,242,248,253,254,258,261,263,265,269,270,272,273,274,275,281,288,294,298,301,302,304,309,313,314,318,319,321,323,325,326,327,332,336,340,345,349,351,353,357,361,362,364,365,367,369,372,375].Is a complete (112,9) –arc as shown in table (11) .Let $\beta_{11} = \pi - k_9$ ={1,2,3,4,5,6,7,9,10,11,12,14,15,16,18,19,20,21,22,25,29,30,31,33,35,36,39,40,41,44,47,49,50,52,54,56,58,59,60,61,64,65,66,67,69,70,72,73,74,75,76,77,78,79,80,81,82,85,86,87,88,89,90,91,93,95,96,97,99,100,101,102,103,104,106,107,108,109,110,111,112,113,114,115,116,117,118,119,121,123,124,125,126,128,129,130,132,133,134,135,136,137,138,139,140,141,142,144,146,147,148,150,151,153,154,155,156,158,159,160,161,162,163,165,166,167,168,169,170,171,172,173,174,176,177,178,179,180,181,183,184,186,187,188,189,190,191,192,194,195,197,199,200,201,204,205,206,207,209,210,211,213,214,215,,218,220,222,223,225,226,228,230,234,235,236,237,239,240,243,244,245,246,247,249,250,251,252,254,255,256,257,259,260,262,264,266,267,268,271,276,277,278,279,280,282,283,284,285,286,287,289,290,291,292,293,295,296,297,299,300,303,305,306,307,308,310,311,312,315,316,317,320,322,324,328,329,330,331,333,334,335,337,338,339,341,342,343,344,346,347,348,350,352,354,355,356,358,359,360,363,366,368,370,371,373,374,376,377,378,379,380,381} is (269,9)-blocking set as shown in table (11) .by theorem (1) ,there exists a projective $[112,3,103]_{19}$ code which is equivalent to the complete (112,9)-arc k_9

Table (11)

I	$K_9 \cap$ Li	$B_{11} \cap$ Li
1	325	2,21,40,59,78,97,116,135,154,173,192,211,230,249,268,287,306,344,363
2	23,24,26,27,28,32,34,37	1,21,22,25,29,30,31,33,35,36,39

	,38	
⋮	⋮	⋮
380	68,105,281,318,327,364,216,253	11,31,40,96,133,142,170,179,207,244,290,355
381	149,185,131,203,221,275	20,22,40,77,95,113,167,239,257,293,311,329,347,365

4.12 Existence of $[82,3,74]_{19}$ codes

We take 12 conic , say C_1 ,C_2,C_3,C_4 ,C_5 ,C_6 ,C_7 , C_8 , C_9 , C_{10} , C_{11} ,C_{12} and let K= π - $C_1 \cup C_2 \cup C_3 \cup C_4 \cup C_5 \cup C_6 \cup C_7 \cup C_8 \cup C_9 \cup C_{10} \cup C_{11} \cup C_{12}$
{3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,22,23,24,25,26,27,28,29,30,31,32,33,34,35,36,37,38,39,40,42,43,44,45,46,47,48,49,50,51,52,53,54,55,56,57,58,59,60,61,62,63,68,71,78,79,81,83,84,87,94,97,98,101,107,108,113,114,116,117,120,121,122,127,130,135,136,141,143,145,149,150,154,155,157,159,160,161,164,166,173,174,175,178,181,185,186,191,192,193,196,198,201,202,207,208,211,212,219,221,222,224,225,227,230,231,232,233,238,241,247,248,249,250,253,254,258,259,261,263,268,269,270,271,272,273,275,281,287,288,289,298,300,301,302,304,306,307,313,314,318,319,321,323,325,326,332,333,336,340,341,343,344,345,347,351,353,357,361,362,363,364,365,369,370,372,375,381}.

The geometrical Structure method must satisfy the following :

- K intersects any line of π in at most 8 points .
- Every point not in K is on at least one 8-secant of K .

The point : M=40,59,78,97,116,135,154,173,192,211,22,24,25,29,30,31,33,35,36,39,3,61,81,101,121,161,181,201,301,381,4,5,6,7,8,10,12,14,20,60,79,117,136,155,174,250,307,269,363,17,130,160,9,54,44,107,191,300,259,207,166,178,225,16,15,50,108,87,52,114,289,186,58,56,287,19,47,340,122,150,347,120,222,113,159,18,247,333,271,302,344,343,249,141,370,26,49,230,325,34,270,268,336,45,37.Are eliminated from K to satisfy (1) .The points of index zero for 200,300 are added to K to satisfy (2), then K_8 =K∪ [200,300] / M

K_8=[11,13,23,27,28,32,38,42,43,46,48,51,53,55,57,62,63,68,71,83,84,94,98,127,143,145,149,157,164,175,185,193,196,198,200,202,208,212,219,221,224,227,231,232,233,238,241,248,253,254,258,261,263,272,273,275,281,288,298,300,304,306,313,314,318,319,321,323,326,332,341,345,351,353,357,361,362,364,365,369,372,375].Is a complete (82,8) –arc as shown in table (12) .Let $\beta_{12} = \pi - k_8$
={1,2,3,4,5,6,7,8,9,10,12,14,15,16,17,18,19,20,21,22,24,25,26,29,30,31,33,34,35,36,37,39,40,41,44,45,47,49,50,52,54,56,58,59,60,61,64,65,66,67,69,70,72,73,74,75,76,77,78,79,80,81,82,85,86,87,88,89,90,91,92,93,95,96,97,99,100,101,102,103,104,105,106,107,108,109,110,111,112,113,114,115,116,117,118,119,121,122,123,124,125,126,128,129,130,131,132,133,134,135,136,137,138,139,140,141,142,144,146,147,148,150,151,152,153,154,155,156,158,159,160,161,162,163,165,166,167,168,169,170,171,172,173,174,176,177,178,179,180,181,183,184,186,187,188,189,190,191,192,194,195,197,199,201,203,204,205,206,207,209,210,211,213,214,215,216,217,218,220,222,223,225,226,228,229,230,234,235,236,237,239,240,242,243,244,245,246,247,249,250,251,252,255,256,257,259,260,262,264,265,266,267,268,269,270,271,274,276,277,278,279,280,282,283,284,285,286,287,289,290,291,292,293,294,295,296,297,299,301,302,303,305,306,307,308,309,310,311,312,315,316,317,320,322,324,325,327,328,329,330,331,333,334,335,336,337,338,339,340,342,343,344,346,347,348,349,350,352,354,355,356,358,359,360,363,366,367

,368,370,371,373,374,376,377,378,379,380,381} is (299,8)-blocking set as shown in table (12) .by theorem (1) ,there exists a projective $[82,3,74]_{19}$ code which is equivalent to the complete (82,8)-arc k_8

Table (12)

I	$K_8 \cap$ Li	$B_{12} \cap$ Li
1	306	2,21,40,59,78,97,116,135,154,173,192,211,230,249,268,287,325,344,363
2	23,27,28,32,38	1,21,22,24,25,26,29,30,31,33,34,35,36,37,39
⋮	⋮	⋮
380	11,68,281,318,364,253	31,40,96,105,133,142,170,179,207,216,244,290,327,355
381	149,185,221,275,365	20,22,40,77,95,113,131,347,329,167,203,239,257,293,311

4.13 Existence of $[72,3,65]_{19}$ codes

We take 13 conic , say C_1 ,C_2,C_3,C_4 ,C_5 ,C_6 ,C_7 , C_8 , C_9 , C_{10} , C_{11} ,C_{12}, C_{13} and let $K= \pi - C_1 \cup C_2 \cup C_3 \cup C_4 \cup C_5 \cup C_6 \cup C_7 \cup C_8 \cup C_9 \cup C_{10} \cup C_{11} \cup C_{12} \cup C_{13}$ {3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,22,23,24,25,26,27,28,29,30,31,32,33,34,35,36,37,38,39,40,42,43,44,45,46,47,48,49,50,51,52,53,54,55,56,57,58,59,60,61,62,63,68,78,79,81,84,87,94,97,98,101,108,113,114,116,117,120,121,127,130,135,136,141,143,145,149,150,154,155,159,160,161,164,166,173,174,175,178,181,185,186,192,193,198,201,202,207,208,211,212,219,221,222,224,225,230,231,232,233,241,247,248,249,250,253,254,259,261,263,268,269,270,271,272,273,281,287,288,289,298,300,301,304,306,307,313,314,318,319,321,325,326,332,333,340,341,343,344,345,347,351,353,361,362,363,364,369,370,372,375,381}.The geometrical Structure method must satisfy the following :

1. K intersects any line of π in at most 7 points .
2. Every point not in K is on at least one 7-secant of K .

The point :
M=40,59,78,97,116,135,154,173,211,230,22,24,25,26,27,29,30,31,33,35,39,249,61,3,81,101,121,141,161,181,201,301,381,4,5,6,7,8,10,12,60,79,117,136,155,174,14,16,20,250,269,307,345,363,178,17,36,130,136,333,247,225,9,54,44,340,186,13,166,207,259,300,347,222,150,164,15,50,108,52,87,114,289,287,306,120,18,58,271,11,344,325,343,268,45,47,49,175,46.Are eliminated from K to satisfy (1) . The points of index zero for 112,312 are added to K to satisfy (2) , then K_7 =K∪ [112,312] / M
K_7=[19,23,28,32,34,37,38,42,43,48,51,53,55,56,57,62,63,68,84,94,98,112,113,127,143,145,149,159,185,192,193,198,202,208,212,219,221,224,231,232,233,241,248,253,254,261,263,270,272,273,281,288,298,304,312,313,314,318,319,321,326,332,341,351,353,361,362,364,369,370,372,375].Is a complete (72,7) –arc as shown in table (13) .Let $\beta_{13} = \pi - k_7$
={1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,20,21,22,24,25,26,27,29,30,31,33,35,36,39,40,41,44,45,46,47,49,50,52,54,58,59,60,61,64,65,66,67,69,70,71,72,73,74,75,76,77,78,79,80,81,82,83,85,86,87,88,89,90,91,92,93,95,96,97,99,100,101,102,103,104,105,106,107,108,109,110,111,114,115,116,117,118,119,121,122,123,124,125,126,128,129,130,131,132,133,134,135,136,137,138,139,140,141,142,144,146,147,148,150,151,152,153,154,155,156,157,158,160,161,162,163,164,165,166,167,168,169,170,171,172,173,174,175,176,177,178,179,180,181,182,183,184,186,187,188,189,190,191,192,194,195,196,197,

199,200,201,203,204,205,206,207,209,210,211,213,214,215,216,217,218,220,222,223,225,226,227,228,229,230,234,235,236,237,238,239,240,242,243,244,245,246,247,249,250,251,252,255,256,257,258,259,260,262,264,265,266,267,268,269,271,274,275,276,277,278,279,280,282,283,284,285,286,287,289,290,291,292,293,294,295,296,297,299,301,302,303,305,306,307,308,309,310,311,315,316,317,320,322,323,324,325,327,328,329,330,331,333,334,335,336,337,338,339,340,342,343,344,345,346,347,348,349,350,352,354,355,356,357,358,359,360,363,365,366,367,368,371,373,374,376,377,378,379,380,381 } is (309,7)-blocking set as shown in table (13).by theorem (1) ,there exists a projective $[72,3,65]_{19}$ code which is equivalent to the complete (72,7)-arc k_7

Table (13)

I	$K_7 \cap Li$	$B_{13} \cap Li$
1	192	2,21,40,59,78,97,116,135,154,173,211,230,249,268,287,306,325,344,363
2	23,28,32,34,37	1,21,22,24,25,26,27,29,30,31,33,35,36,38,39
⋮	⋮	⋮
380	68,281,318,364,253	11,31,40,96,105,133,142,170,179,207,216,244,290,327,355
381	113,149,185,221	20,22,40,77,95,131,167,203,239,347,365,257,275,293,311,329

4.14 Existence of $[54,3,48]_{19}$ codes

We take 14 conic , say C_1 ,C_2,C_3,C_4 ,C_5 ,C_6 ,C_7 , C_8 , C_9 , C_{10} , C_{11} ,C_{12}, C_{13} , C_{14} and let K= π - $C_1 \cup C_2 \cup C_3 \cup C_4 \cup C_5 \cup C_6 \cup C_7 \cup C_8 \cup C_9 \cup C_{10} \cup C_{11} \cup C_{12} \cup C_{13} \cup C_{14}$ {3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,22,23,24,25,26,27,28,29,30,31,32,33,34,35,36,37,38,39,40,42,43,44,45,46,47,48,49,50,51,52,53,54,55,56,57,58,59,60,61,62,63,78,79,81,87,94,97,98,101,108,114,116,117,120,121,127,130,135,136,141,143,145,150,154,155,160,161,164,166,173,174,178,181,185,186,192,193,198,201,207,208,211,212,219,221,224,225,230,231,232,233,241,247,249,250,254,259,261,263,268,269,270,272,273,281,287,288,289,298,300,301,306,307,313,314,318,321,325,326,332,340,341,343,344,345,347,353,361,362,363,364,369,370,372,381}.The geometrical Structure method must satisfy the following :

i. K intersects any line of π in at most 6 points .
ii. Every point not in K is on at least one 6-secant of K .

The point :
M=40,59,78,97,116,154,173,192,211,230,249,268,22,24,25,26,27,28,29,30,31,33,35,39,3,61,81,101,121,141,161,181,201,301,381,341,4,5,6,7,8,9,10,12,14,16,20,60,79,117,136,155,174,250,269,307,345,231,363,160,130,36,247,186,340,300,259,207,166,47,225,178,164,150,108,343,289,114,87,127,370,58,49,56,287,306,19,344,120,13,11,325,270,48,44,45,45,50,52,54,37 Are eliminated from K to satisfy (1) . The points of index zero for 171,271 are added to K to satisfy (2) , then K_6 =K∪ [171,271] / M
K_6=[15,17,18,23,32,34,38,42,43,51,53,55,57,62,63,94,98,135,143,145,171,185,193,198,208,212,219,221,224,232,233,241,254,261,263,271,272,273,281,288,298,313,314,318,321,326,332,347,353,361,362,364,369,372].Is a complete (54,6) –arc as shown in table (14) .Let $\beta_{14} = \pi - k_6$
={1,2,3,4,5,6,7,8,9,10,11,12,13,14,16,19,20,21,22,24,25,26,27,28,29,30,31,33,35,36,37,39,40,41,44,45,46,47,48,49,50,52,54,56,58,59,60,61,64,65,66,67,68,69,70,71,72,73,74,7

5,76,77,78,79,80,81,82,83,84,85,86,87,88,89,90,91,92,93,95,96,97,99,100,101,102,103,104,105,106,107,108,109,110,111,112,113,114,115,116,117,118,119,120,121,122,123,124,125,126,127,128,129,130,131,132,133,134,136,137,138,139,140,141,142,144,146,147,148,149,150,151,152,153,154,155,156,157,158,159,160,161,162,163,164,165,166,167,168,169,170,172,173,174,175,176,177,178,179,180,181,182,183,184,186,187,188,189,190,191,192,194,195,196,197,199,200,201,202,203,204,205,206,207,209,210,211,213,214,215,216,217,218,220,222,223,225,226,227,228,229,230,231,234,235,236,237,238,239,240,242,243,244,245,246,247,248,249,250,251,252,253,255,256,257,258,259,260,262,264,265,266,267,268,269,270,274,275,276,277,278,279,280,282,283,284,285,286,287,289,290,291,292,293,294,295,296,297,299,301,302,303,304,305,306,307,308,309,310,311,312,315,316,317,319,320,322,323,324,325,327,328,329,330,331,333,334,335,336,337,338,339,340,341,342,343,344,345,346,348,349,350,351,352,354,355,356,357,358,359,360,363,365,366,367,368,370,371,373,374,376,377,378,379,380,381} is (327,6)-blocking set as shown in table (14) .

by theorem (1) ,there exists a projective $[54,3,48]_{19}$ code which is equivalent to the complete (54,6)-arc k_6

Table (14)

I	$K_6 \cap Li$	$B_{14} \cap Li$
1	135	2,21,40,59,78,97,116,154,173,192,211,230,249,268,287,306,325,344,363
2	23,34,38	1,21,22,24,25,26,27,28,29,30,31,32,33,35,36,37,39
⋮	⋮	⋮
380	281,318,364	11,31,40,68,96,105,133,142,170,179,207,216,244,253,290,327,355
381	185,221	20,22,40,77,95,113,131,149,167,95,347,329,203,239,257,275,293,311

4.15 Existence of $[37,3,32]_{19}$ codes

We take 15 conic, say C_1 ,C_2,C_3,C_4 ,C_5 ,C_6 ,C_7 , C_8 , C_9 , C_{10} , C_{11} ,C_{12}, C_{13} , C_{14}, C_{15} and let

$K= \pi - C_1 \cup C_2 \cup C_3 \cup C_4 \cup C_5 \cup C_6 \cup C_7 \cup C_8 \cup C_9 \cup C_{10} \cup C_{11} \cup C_{12} \cup C_{13} \cup C_{14} \cup C_{15}$

{3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,22,23,24,25,26,27,28,29,30,31,32,33,34,35,36,37,38,39,40,42,43,44,45,46,47,48,49,50,51,52,53,54,55,56,57,58,59,60,61,63,78,79,81,94,97,98,101,108,114,116,117,121,127,130,135,136,141,143,150,154,155,160,161,164,173,174,178,181,185,192,193,201,207,208,211,212,219,221,224,230,231,232,233,241,249,250,259,261,263,268,269,272,273,281,287,288,289,300,301,306,307,313,318,321,325,326,332,341,343,344,345,347,353,361,363,364,369,372,381}.

The geometrical Structure method must satisfy the following :

1. K intersects any line of π in at most 5 points .
2. Every point not in K is on at least one 5-secant of K .

The point :

M=40,59,78,97,116,135,154,192,211,230,249,268,306,22,24,25,27,28,29,30,31,33,35,36,38,39,18,3,61,81,101,121,141,161,181,201,301,341,361,381,4,5,6,7,8,10,12,14,16,20,9,11,60,79,117,136,155,174,250,269,307,345,231,193,363,130,300,259,207,347,50,150,164,108,114,26,212,48,160,343,47,54,17,127,232,49,37,58,45,173,289,32,344,43,325,15,34,42,46,52,56,241 Are eliminated from K to satisfy (1) . The points of index zero for 102,240 are added to K to satisfy (2) , then $K_5 = K \cup [102,240] / M$

K_5=[13,19,23,44,51,53,55,57,63,94,98,102,143,178,185,208,219,221,224,233,240,261,263,272,273,281,287,288,313,318,321,326,332,353,364,369,372]Is a complete (37,5) – arc as shown in table (15) .Let $\beta_{15} = \pi - k_5$ ={1,2,3,4,5,6,7,8,9,10,11,12,14,16,20,21,22,24,25,26,27,28,29,30,31,32,33,34,35,36,37,39,40,41,42,43,45,46,47,48,49,50,52,54,56,58,59,60,61,62,64,65,66,67,68,69,70,71,72,73,74,75,76,77,78,79,80,81,82,83,84,85,86,87,88,89,90,91,92,93,95,96,97,99,100,101,103,104,105,106,107,108,109,110,111,112,113,114,115,116,117,118,119,120,121,122,123,124,125,126,127,128,129,130,131,132,133,134,135,136,137,138,139,140,141,142,144,145,146,147,148,149,150,151,152,153,154,155,156,157,158,159,160,161,162,163,164,165,166,167,168,169,170,171,172,173,174,175,176,177,179,180,181,182,183,184,186,187,188,189,190,191,192,193,194,195,196,197,198,199,200,201,202,203,204,205,206,207,209,210,211,212,213,214,215,216,217,218,220,222,223,225,226,227,228,229,230,231,232,234,235,236,237,238,239,241,242,243,244,245,246,247,248,249,250,251,252,253,254,255,256,257,258,259,260,262,264,265,266,267,268,269,270,271,274,275,276,277,278,279,280,282,283,284,285,286,289,290,291,292,293,294,295,296,297,298,299,300,301,302,303,304,305,306,307,308,309,310,311,312,314,315,316,317,319,320,322,323,324,325,327,328,329,330,331,333,334,335,336,337,338,339,340,341,342,343,344,345,346,347,348,349,350,351,352,354,355,356,357,358,359,360,361,362,363,365,366,367,368,370,371,373,374,376,377,378,379,380,381} is (344,5)-blocking set as shown in table (15) .by theorem (1) ,there exists a projective $[37,3,32]_{19}$ code which is equivalent to the complete (37,5)-arc k_5

Table (15)

I	$K_5 \cap$ Li	$B_{15} \cap$ Li
1	287	2,21,40,59,78,97,116,135,173,192,211,230,249,268,306,154,325,344,363
2	23	1,21,22,24,25,26,27,28,29,30,31,32,33,34,35,36,37,38,39
⋮	⋮	⋮
380	281,318,364	11,31,40,68,96,105,133,142,170,179,207,216,244,253,290,327,355
381	185,221	20,22,40,77,95,113,131,149,167,203,239,293,257,275,311,329,347,365

4.14 Existence of $[37,3,32]_{19}$ codes

We take 16 conic, say C_1 ,C_2,C_3,C_4 ,C_5 ,C_6 ,C_7 , C_8 , C_9 , C_{10} , C_{11} ,C_{12}, C_{13} , C_{14}, C_{15} , C_{16} and let

K= π - $C_1 \cup C_2 \cup C_3 \cup C_4 \cup C_5 \cup C_6 \cup C_7 \cup C_8 \cup C_9 \cup C_{10} \cup C_{11} \cup C_{12} \cup C_{13} \cup C_{14} \cup C_{15} \cup C_{16}$

{3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,22,23,24,25,26,27,28,29,30,31,32,33,34,35,36,37,38,39,40,42,43,44,45,46,47,48,49,50,51,52,53,54,55,56,57,58,59,60,61,78,79,81,94,97,98,101,114,116,117,121,127,135,136,141,143,154,155,161,164,173,174,178,181,192,193,201,207,211,212,221,224,230,231,233,241,249,250,261,263,268,269,272,281,287,288,289,301,306,307,318,321,325,326,332,341,344,345,353,361,363,364,369,381}.The geometrical Structure method must satisfy the following :

i. K intersects any line of π in at most 4 points .
ii. Every point not in K is on at least one 4-secant of K .

The point :
M=40,59,78,97,116,135,154,325,192,211,230,249,268,287,22,24,25,26,27,28,29,30,31,33,35,36,38,39,3,61,81,101,121,141,161,181,201,301,341,381,321,221,4,5,6,7,8,9,10,12

,14,16,18,20,11,60,79,117,136,155,174,193,212,250,269,307,345,231,363,289,19,45,13, 37,50,47,51,127,207,48,306,56,32,369,344,178,44,46,49,52,54,56,58,42,43,34,144,164. Are eliminated from K to satisfy (1) . The points of index zero for 195,265 are added to K to satisfy (2) , then K_4 =K∪ [195,265] / M

K_4=[15,17,23,53,55,57,94,98,143,173,195,224,233,241,261,263,265,272,281,288,318,326,332,353,361,364].Is a complete (26,4) –arc as shown in table (16) .Let $\beta_{16} = \pi - k_4$ ={1,2,3,4,5,6,7,8,9,10,11,12,14,16,18,19,20,21,22,24,25,26,27,28,29,30,31,32,33,34,35, 36,37,38,39,40,41,42,43,44,45,46,47,48,49,50,51,52,54,56,58,59,60,61,62,63,64,65,66,67,68,69,70,71,72,73,74,75,76,77,78,79,80,81,82,83,84,85,86,87,88,89,90,91,92,93,95,96,97,99,100,101,102,103,104,105,106,107,108,109,110,111,112,113,114,115,116,117,118,119,120,121,122,123,124,125,126,127,128,129,130,131,132,133,134,135,136,137,138,139,140,141,142,144,145,146,147,148,149,150,151,152,153,154,155,156,157,158,159, 160,161,162,163,164,165,166,167,168,169,170,171,172,174,175,176,177,178,179,180,181,182,183,184,185,186,187,188,189,190,191,192,193,194,196,197,198,199,200,201,202,203,204,205,206,207,208,209,210,211,212,213,214,215,216,217,218,220,221,222,223,225,226,227,228,229,230,231,232,234,235,236,237,238,239,240,242,243,244,245,246, 247,248,249,250,251,252,253,254,255,256,257,258,259,260,262,264,266,267,268,269,270,271,273,274,275,276,277,278,279,280,282,283,284,285,286,287,289,290,291,292,293,294,295,296,297,298,299,300,301,302,303,304,305,306,307,308,309,310,311,312,313,314,315,316,317,319,320,321,322,323,324,325,327,328,329,330,331,333,334,335,336, 337,338,339,340,341,342,343,344,345,346,347,348,349,350,351,352,354,355,356,357,358,359,360,362,363,365,366,367,368,370,371,372,373,374,375,376,377,378,379,380,381} is (355,4)-blocking set as shown in table (16) .by theorem (1) ,there exists a projective $[26,3,22]_{19}$ code which is equivalent to the complete (26,4)-arc k_4

Table (16)

I	K_4∩ Li	B_{16} ∩ Li
1	173	2,21,40,59,78,97,116,135,154,192,211,230,249,268,287,306,325,344,363
2	23	1,21,22,24,25,26,27,28,29,30,31,32,33,35,36,38,39,34,37
⋮	⋮	⋮
380	281,318	11,31,40,68,96,105,133,142,170,179,207,216,244,253,290,327,355, 364
381	Ø	20,22,40,77,95,113,131,149,167,185,203,221,239,257,295,275,311, 329,347,365

4.17 Existence of $[13,3,10]_{19}$ codes

We take 17 conic, say C_1 ,C_2,C_3,C_4 ,C_5 ,C_6 ,C_7 , C_8 , C_9 , C_{10} , C_{11} ,C_{12}, C_{13} , C_{14}, C_{15} , C_{16} and C_{17} let

K= π - C_1 ∪ C_2 ∪ C_3 ∪ C_4 ∪ C_5 ∪ C_6 ∪ C_7 ∪ C_8 ∪ C_9 ∪ C_{10} ∪ C_{11} ∪ C_{12}∪ C_{13} ∪ C_{14} ∪ C_{15} ∪ C_{16} ∪ C_{17}

{3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,22,23,24,25,26,27,28,29,30,31,32,33,34,35,36,37,38,39,40,42,43,44,45,46,47,48,49,50,51,52,53,54,55,56,57,58,59,60,61,78,79, 81,97,98,101,116,117,121,135,136,141,154,155,161,173,174,181,192,193,201,211,212, 221,230,231,241,249,250,261,268,269,281,287,288,301,306,307,321,325,326,341,344,345,361,363,364,381}.The geometrical Structure method must satisfy the following :

i. K intersects any line of π in at most 3 points .
ii. Every point not in K is on at least one 3-secant of K .

The point :
M=40,59,78,97,116,135,154,173,192,230,249,268,287,306,325,22,24,25,26,27,28,29,30,31,33,35,36,38,39,3,61,81,101,121,141,161,181,201,301,341,361,381,261,241,4,5,6,7,8,9,10,11,12,13,14,15,16,20,60,79,98,117,136,155,174,212,193,231,250,269,307,345,363,47,17,19,18,173,37,344,32,321,43,44,45,46,48,49,50,23,52,54,56,58,51Are eliminated from K to satisfy (1) . The points of index zero for 162,202 are added to K to satisfy (2) , then K_3 =K∪ [162,202] / M

K_3= [34,42,53,55,57,162,202,211,221,281,288,326,364].Is a complete (13,3) –arc as shown in table (17) .Let $\beta_{17} = \pi - k_3$ ={1,2,3,4,5,6,7,8,9,10,11,12,14,15,16,17,18,19,20,21,22,23,24,25,26,27,28,29,30,31,32,33,35,36,37,38,39,40,41,43,44,45,46,47,48,49,50,51,52,54,56,58,59,60,61,62,63,64,65,66,67,68,69,70,71,72,73,74,75,76,77,78,79,80,81,82,83,84,85,86,87,88,89,90,91,92,93,94,95,96,97,98,99,100,101,102,103,104,105,106,107,108,109,110,111,112,113,114,115,116,117,118,119,120,121,122,123,124,125,126,127,128,129,130,131,132,133,134,135,136,137,138,139,140,141,142,143,144,145,146,147,148,149,150,151,152,153,154,155,156,157,158,159,160,161,163,164,165,166,167,168,169,170,171,172,173,174,175,176,177,178,179,180,181,182,183,184,185,186,187,188,189,190,191,192,193,194,196,197,198,199,200,201,203,204,205,206,207,208,209,210,212,213,214,215,216,217,218,220,222,223,224,225,226,227,228,229,230,231,232,233,234,235,236,237,238,239,240,241,242,243,244,245,246,247,248,249,250,251,252,253,254,255,256,257,258,259,260,261,262,263,264,265,266,267,268,269,270,271,273,274,275,276,277,278,279,280,282,283,284,285,286,287,289,290,291,292,293,294,295,296,297,298,299,300,301,302,303,304,305,306,307,308,309,310,311,312,313,314,315,316,317,318,319,320,321,322,323,324,325,327,328,329,330,331,332,333,334,335,336,337,338,339,340,341,342,343,344,345,346,347,348,349,350,351,352,353,354,355,356,357,358,359,360,361,362,363,365,366,367,368,370,371,372,373,374,375,376,377,378,379,380,381} is (368,3)-blocking set as shown in table (17) .by theorem (1) ,there exists a projective $[13,3,10]_{19}$ code which is equivalent to the complete (13,3)-arc k_3

Table (17)

I	K_3∩ Li	B_{17} ∩ Li
1	211	2,21,40,59,78,97,116,135,154,173,193,230,249,268,287,306,325,344,363
2	Ø	1,21,22,23,24,25,26,27,28,29,30,31,32,33,34,35,36,37,38,39
⋮	⋮	⋮
380	281,364	11,31,40,68,96,105,133,142,170,179,207,216,244,253,290,318,327,355
381	Ø	20,22,40,77,95,113,131,149,167,185,203,221,239,257,275,293,311,329,347,365

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