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On Differential Sandwich Results For Analytic Functions

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Abstract: In this paper , we obtain some subordination and superordination results involving the integral operator F_c^{δ}.Also,we get Differential sandwich results for classes of univalent functions in the unit disk.

Keywords: Analytic function, univalent function, differential subordination , superordination.

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1-Introduction :

Let H=H(U) be the class of analytic functions in the open unit disk $U = \{z \in \mathbb{C} : |z| < 1\}$. For n a positive number additionally $a \in \mathbb{C}$. Let H[a, n] be the subclass of H entailing of functions of the form:

$$f(z) = a + a_n z^n + a_{n+1} z^{n+1} + \ldots \quad (a \in \mathbb{C}). \qquad (1.1)$$

Too, let A be the subclass of H entailing of functions of the form:

$$f(z) = z + \sum_{k=2}^{\infty} a_k z^k . \qquad (1.2)$$

Let $f, g \in H$. The function f is said to be subordinate to g, or g is said to be subordinate to f, if there exists a Schwarz function w analytic in U with $w(0) = 0$ and $|w(z)| < 1$ ($z \in U$), to such an extent that $f(z) = g(w(z))$, In such a case we compose $f \prec g$ or $f(z) \prec g(z) (z \in U)$. If g is univalent function in U, then $f \prec g$ if and only if $f(0) = g(0)$ and $f(U) \subset g(U)$.

Let $p, h \in H$ and $\psi(r, \delta, t, z)$: $\mathbb{C}^3 \times U \to \mathbb{C}$. If p and $\psi(p(z), zp'(z), z^2p''(z); z)$ are univalent functions in U and if p fulfills the second-order differential superordination.

$$h(z) \prec \psi(p(z), zp'(z), z^2p''(z); z), \qquad (1.3)$$

then p is called a result of the differential superordination (1.3). (If f is subordinate to g, then g is superordinate to f) . An analytic function q is called a subordinant of (1.3) , if $q \prec p$ for very the functions p filling (1.3).

An univalent subordinant $\tilde{q}$ that fulfills $q \prec \tilde{q}$ for all the subordinants q of (1.3) is called the best subordinant. Miller and Mocanu [5] have gotten conditions on the functions h, q and ψ for which the accompanying ramifications holds:

$$h(z) \prec \psi(p(z), zp'(z), z^2p''(z); z) \to q(z) \prec p(z). \qquad (1.4)$$

For $f \in A$,Al-shaqsi [2] defined the following integral operator:

$$F_c^{\delta} f(z) = (1+c)^{\delta} \phi_{\delta}(c; z) * f(z) = \frac{(1-c)^{\delta}}{\Gamma(\delta)} \int_0^1 t^{c-1} (\log \frac{1}{t})^{\delta-1} f(tz)\, dt , (c > 0, \delta > 1 \text{ and } z \in U). \ (1.5)$$

We also note that the operator $F_c^{\delta} f(z)$ characterized by (1.5) can be communicated by the arrangement development as pursues:

$$F_c^{\delta} f(z) = z + \sum_{k=2}^{\infty} (\frac{1+c}{k+c})^{\delta} a_k z^k. \qquad (1.6)$$

In addition, from (1.6), it pursues that

$$z(F_c^{\delta+1} f(z))' = (c+1) F_c^{\delta} f(z) - c F_c^{\delta+1} f(z). \qquad (1.7)$$

Ali et al.[1] gotten adequate conditions for certain standardized scientific capacities to satisfy

$$q_1(z) \prec \frac{zf'(z)}{f(z)} \prec q_2(z),$$

where q_1 and q_2 are given univalent functions in U with $q_1(0) = q_2(0) = 1$. Additionally, Tuneski [9] acquired adequate conditions for starlikeness of f in relations of the amount $\frac{f''(z)f(z)}{(f'(z))^2}$ .Recently, Shanmugam et al.[7,8], Goyal et al .[4] also gotten sandwich consequences for certain classes of analytic functions.

The principle question of the present paper is to discover adequate conditions for certain standardized systematic capacities f to fulfill:

$$q_1(z) \prec (\frac{F_c^{\delta+1} f(z)}{z})^{\lambda} \prec q_2(z),$$

and

$$q_1(z) \prec (\frac{t\, F_c^{\delta+1} f(z) + (1-t)\, F_c^{\delta} f(z)}{z})^{\lambda} \prec q_2(z),$$

wherever q_1 and q_2 are known univalent functions in U with $q_1(0)= q_2(0)= 1$.

2-Preliminaries :

With the end goal to demonstrate our subordination and superordination result , we require the accompanying definition and lemmas.

Definition 2.1 [5] : Denote by Q the set of all functions f that are analytic and injective on $\bar{U} \setminus E(f)$, where

$$E(f) = \{\xi \in \partial U : \lim_{z \to \xi} f(z) = \infty \} \qquad (2.1)$$

and are such that $f'(\xi) \neq 0$ for $\xi \in \partial U \setminus E(f)$.

Lemma 2.1 [5] : Let q be univalent in the unit disk U and let θ and $\emptyset$be analytic in a domain D containing q(U) with $\emptyset(w) \neq 0$ when $w \in q(U)$. Set $Q(z) = zq'(z)\emptyset(q(z))$ and $h(z) = \theta(q(z)) + Q(z)$.

Suppose that

(i) Q(z) is starlike univalent in U,

(ii) $\mathrm{Re}\{\frac{zh'(z)}{Q(z)}\} > 0$ for $z \in U$.

If p is analytic in U with $p(0) = q(0), p(U) \subset D$ and

$$\theta(p(z)) + zp'(z)\emptyset(p(z)) \prec \theta(q(z)) + zq'(z)\emptyset(q(z)) \qquad (2.2)$$

then $p \prec q$ and q is the best dominant of (2.2).

Lemma 2.2 [6]: Let q be convex univalent in function in U and let $\alpha \in \mathbb{C}, \beta \in \mathbb{C} / \{0\}$ with

$$\mathrm{Re}\,(1 + \frac{zq''(z)}{q'(z)}) > \max(0, -\mathrm{Re}\left(\frac{\alpha}{\beta}\right)).$$

If p is analytic in U, and

$$\alpha p(z) + \beta zp'(z) \prec \alpha q(z) + \beta zq'(z), \qquad (2.3)$$

then $p \prec q$ and q is the best dominant of (2.3).

Lemma 2.3 [6]: Let q be convex univalent in U and let $\beta \in \mathbb{C}$, further assume that Re $(\beta) > 0$. If $P \in H[q(0)] \cap Q$ and $p(z) + \beta z p'(z)$ is univalent in U, then

$q(z) + \beta z q'(z) \prec p(z) + \beta z p'(z)$, (2.4)

which implies that $q \prec p$ and q is the best subordinant of (2.4).

Lemma 2.4 [3]: Let q be convex univalent in the unit disk U and let θ and ∅ be analytic in domain D containing q (U) . Suppose that

(i) Re $\left\{\frac{\theta'(q(z))}{\emptyset(q(z))}\right\} > o$ for $z \in U$,

(ii) $Q(z) = zq'(z)\emptyset(q(z))$ is starlike univalent in U.

If $p \in H[q(0),1] \cap Q$, with $p(U) \subset D$, $\theta(p(z)) + zp'(z)\,\emptyset p(z)$ is univalent in U and

$\theta(q(z)) + zq'(z)\emptyset(q(z)) \prec \theta(p(z)) + zp'(z)\emptyset(p(z))$, (2.5)

then $q \prec p$ and q is the best subordination of (2.5).

3- Subordination Consequences :

Theorem 3.1 : Let q be convex univalent function in U with $q(0) = 1$, $0 \neq \Psi \in \mathbb{C}$, $\lambda > 0$ also, assume that q satisfies:

$$\mathrm{Re}\left(1 + \frac{zq''(z)}{q'(z)}\right) > \max\left(0, -\mathrm{Re}\left(\frac{\lambda}{\Psi}\right)\right). \qquad (3.1)$$

If $f \in A$ satisfies the subordination

$$(1 - \Psi(c+1))\left(\frac{F_c^{\delta+1}f(z)}{z}\right)^{\lambda} + \Psi(c+1)\left(\frac{F_c^{\delta+1}f(z)}{z}\right)^{\lambda}\left(\frac{F_c^{\delta}f(z)}{F_c^{\delta+1}f(z)}\right) \prec q(z) + \frac{\Psi}{\lambda}zq'(z), \qquad (3.2)$$

then

$$\left(\frac{F_c^{\delta+1}f(z)}{z}\right)^{\lambda} \prec q(z), \qquad (3.3)$$

and q is the best dominant of (3.2).

Proof : Characterize the capacity p by

$$p(z) = \left(\frac{F_c^{\delta+1}f(z)}{z}\right)^{\lambda}. \qquad (3.4)$$

Differentiating (3.4) with admiration to z logarithmically, we get

$$\frac{zp'(z)}{p(z)} = \lambda\left(\frac{z(F_c^{\delta+1}f(z))'}{F_c^{\delta+1}f(z)} - 1\right). \qquad (3.5)$$

Presently , in perspective of (1.7), we get the accompanying subordination

$$\frac{zp'(z)}{p(z)} = \lambda\left(c\left(\frac{F_c^{\delta}f(z)}{F_c^{\delta+1}f(z)} - 1\right) + \left(\frac{F_c^{\delta}f(z)}{F_c^{\delta+1}f(z)} - 1\right)\right),$$

therefore $\frac{zp'(z)}{\lambda} = \left(\frac{F_c^{\delta+1}f(z)}{z}\right)^{\lambda}\left(c\left(\frac{F_c^{\delta}f(z)}{F_c^{\delta+1}f(z)} - 1\right) + \left(\frac{F_c^{\delta}f(z)}{F_c^{\delta+1}f(z)} - 1\right)\right)$.

The subordination (3.2) from the speculation moves toward becoming

$$p(z) + \frac{\Psi}{\lambda}zp'(z) \prec q(z) + \frac{\Psi}{\lambda}zq'(z).$$

An request of Lemma 2.2 with $\beta = \frac{\Psi}{\lambda}$ and $\alpha = 1$, we get (3,3)

Putting $q(z) = \left(\frac{1+z}{1-z}\right)$ in Theorem 3.1 ,we get the following

Corollary 3.1 : Let $0 \neq \Psi \in \mathbb{C}$, $\lambda > 0$ also
Re $\{1 + \frac{2z}{1-z}\} > \max\{0, -\mathrm{Re}(\frac{\lambda}{\Psi})\}$.
If $f \in A$ satisfies the subordination

$$(1 - \Psi(c+1))\left(\frac{F_c^{\delta+1}f(z)}{z}\right)^{\lambda} + \Psi(c+1)\left(\frac{F_c^{\delta+1}f(z)}{z}\right)^{\lambda}\left(\frac{F_c^{\delta}f(z)}{F_c^{\delta+1}f(z)}\right) \prec \left(\frac{1 - z^2 + 2\frac{\Psi}{\lambda}z}{(1-z)^2}\right),$$

then

$$\left(\frac{F_c^{\delta+1}f(z)}{z}\right)^{\lambda} \prec \left(\frac{1+z}{1-z}\right),$$

and $q(z) = \left(\frac{1+z}{1-z}\right)$ is the best dominant.

Theorem 3.2 : Let q be convex univalent function in U with $q(0) = 1, q(z) \neq 0 (z \in U)$ furthermore, accept that q fulfills

$$\mathrm{Re}\left(1 - \frac{\lambda}{\Psi} + \frac{zq''(z)}{q'(z)}\right) > 0 , \qquad (3.6)$$

where $\Psi \in \mathbb{C}/\{0\}, \lambda > 0$ and $z \in U$.

Supposing that $-\Psi zq'(z)$ is starlike univalent function in U, if $f \in A$ fulfills:

$$\emptyset(\lambda, \delta, c, \Psi; z) \prec \lambda q(z) - \Psi zq'(z), \qquad (3.7)$$

where $\emptyset(\lambda, \delta, c, \Psi; z) =$

$$\lambda\left(\frac{tF_c^{\delta+1}f(z) + (1-t)F_c^{\delta}f(z)}{z}\right)^{\lambda} - \lambda\Psi\left(\frac{tF_c^{\delta+1}f(z) + (1-t)F_c^{\delta}f(z)}{z}\right)^{\lambda}\left(\frac{tF_c^{\delta}f(z) + (1-t)F_c^{\delta-1}f(z)}{tF_c^{\delta+1}f(z) + (1-t)F_c^{\delta}f(z)} - 1\right), \qquad (3.8)$$

then

$$\left(\frac{tF_c^{\delta+1}f(z) + (1-t)F_c^{\delta}f(z)}{z}\right)^{\lambda} \prec q(z), \qquad (3.9)$$

and q(z)is the best dominant of (3.7).

Proof: Express the function p by

$$p(z) = \left(\frac{tF_c^{\delta+1}f(z) + (1-t)F_c^{\delta}f(z)}{z}\right)^{\lambda}, \qquad (3.10)$$

by setting :

$\theta(w) = \lambda w$ and $\emptyset(w) = -\Psi, w \neq 0$.

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We see that $\theta(w)$ is analytic in $\mathbb{C}$, $\emptyset(w)$ is analytic in $\mathbb{C}/\{0\}$ and so on $\emptyset(w) \neq 0, w \in \mathbb{C}^*$.
Too, we get

$$Q(z) = zq'(z)\emptyset q(z) = -\Psi zq'(z),$$

and

$$h(z) = \theta q(z) + Q(z) = \lambda q(z) - \Psi zq'(z).$$

It is clear that Q(z)is starlike univalent in U ,

$$\operatorname{Re}\left\{\frac{zh'(z)}{Q(z)}\right\} = \operatorname{Re}\left\{1 - \frac{\lambda}{\Psi} + \frac{zq''(z)}{q'(z)}\right\} > 0.$$

By a straightforword computation , we obtain$\lambda p(z) - \Psi zp'(z) = \emptyset(\lambda, \delta, c, \Psi; z)$, (3.11)
where $\emptyset(\lambda, \delta, c, \Psi; z)$ is given by (3.8).
From (3.7) and (3.11), we have
$\lambda p(z) - \Psi z\, p'(z) \prec \lambda q(z) - \Psi z\, q'(z)$.
(3.12)
So , by Lemma 2.1, we become $p(z) \prec q(z)$. By using (3.10) , we get the result .
Putting $q(z) = \frac{1+Az}{1+Bz}$ $(-1 \leq B < A \leq 1)$ in Theorem 3.2 , we obtain the next corollary :

Corollary3.2 : Let $-1 \leq B < A \leq 1$ while

$$\operatorname{Re}\left\{1 - \frac{\lambda}{\Psi} + \frac{z2B}{(1 + Bz)}\right\} > 0,$$

where $\Psi \in \mathbb{C}/\{0\}$ and $z \in U$, if $f \in A$ contents
$\emptyset(\lambda, \delta, c, \Psi; z) \prec \left(\lambda\left(\frac{1+Az}{1+Bz}\right) - \Psi z \frac{A-B}{(1+Bz)^2}\right)$,
and $\emptyset(\lambda, \delta, c, \Psi; z)$ is given by (3.8),

$$\left(\frac{tF_c^{\delta+1}f(z) + (1-t)F_c^{\delta}f(z)}{z}\right)^{\lambda} \prec \frac{1 + Az}{1 + Bz}$$

while $q(z) = \frac{1+Az}{1+Bz}$ is the best dominant.

4-Superordination Consequences :

Theorem 4.1: Let q be convex univalent function in U with $q(0) = 1, \lambda > 0$ and $\operatorname{Re}\{\Psi\} > 0$. Let $f \in A$ satisfies

$$\left(\frac{F_c^{\delta+1}f(z)}{z}\right)^{\lambda} \in H[q(0), 1] \cap Q,$$

and

$$(1 - \Psi(c + 1))\left(\frac{F_c^{\delta+1}f(z)}{z}\right)^{\lambda} + \Psi(c + 1)\left(\frac{F_c^{\delta+1}f(z)}{z}\right)^{\lambda}\left(\frac{F_c^{\delta}f(z)}{F_c^{\delta+1}f(z)}\right),$$

exist univalent in U . If

$$q(z) + \frac{\Psi}{\lambda} zq'(z) \prec (1 - \Psi(c + 1))\left(\frac{F_c^{\delta+1}f(z)}{z}\right)^{\lambda} + \Psi(c + 1)\left(\frac{F_c^{\delta+1}f(z)}{z}\right)^{\lambda}\left(\frac{F_c^{\delta}f(z)}{F_c^{\delta+1}f(z)}\right), \qquad (4.1)$$

then

$$q(z) \prec \left(\frac{F_c^{\delta+1}f(z)}{z}\right)^{\lambda}, \qquad (4.2)$$

and q is the best subordinant of (4.1).

Proof: Express the function p by

$$p(z) = \left(\frac{F_c^{\delta+1}f(z)}{z}\right)^{\lambda}. \qquad (4.3)$$

Differentiating (4.3) with respect to z logarithmically , we get

$$\frac{zp'(z)}{p(z)} = \lambda\left(\frac{z(F_c^{\delta+1}f(z))'}{F_c^{\delta+1}f(z)} - 1\right).$$

(4.4)
After some computations and using (1.7) , from (4.4), we obtain

$$(1 - \Psi(c + 1))\left(\frac{F_c^{\delta+1}f(z)}{z}\right)^{\lambda} + \Psi(c + 1)\left(\frac{F_c^{\delta+1}f(z)}{z}\right)^{\lambda}\left(\frac{F_c^{\delta}f(z)}{F_c^{\delta+1}f(z)}\right)$$

$$= p(z) + \frac{\Psi}{\lambda} zp'(z),$$

and now , by using Lemma 2.3, we get the desired result .
Putting $q(z) = \frac{1+z}{1-z}$ in Theorem 4.1 , we acquire the accompanying corollary :

Corollary 4.1: Let $\lambda > 0$ and $\operatorname{Re}\{\Psi\} > 0$. If $f \in A$ satisfies:

$$\left(\frac{F_c^{\delta+1}f(z)}{z}\right)^{\lambda} \in H[q(0), 1] \cap Q,$$

and

$$(1 - \Psi(c + 1))\left(\frac{F_c^{\delta+1}f(z)}{z}\right)^{\lambda} + \Psi(c + 1)\left(\frac{F_c^{\delta+1}f(z)}{z}\right)^{\lambda}\left(\frac{F_c^{\delta}f(z)}{F_c^{\delta+1}f(z)}\right),$$

be univalent in U . If

$$\left(\frac{1 - z^2 + 2\frac{\Psi}{\lambda}z}{(1-z)^2}\right) \prec (1 - \Psi(c + 1))\left(\frac{F_c^{\delta+1}f(z)}{z}\right)^{\lambda} + \Psi(c + 1)\left(\frac{F_c^{\delta+1}f(z)}{z}\right)^{\lambda}\left(\frac{F_c^{\delta}f(z)}{F_c^{\delta+1}f(z)}\right),$$

then

$$\left(\frac{1 + z}{1 - z}\right) \prec \left(\frac{F_c^{\delta+1}f(z)}{z}\right)^{\lambda},$$

and $q(z) = \frac{1+z}{1-z}$ is the best subordinant.

Theorem 4.2: Let q be convex univalent function in U with $q(0) = 1$, also, accept that q fulfills

$$\operatorname{Re}\left\{\frac{-\lambda q'(z)}{\Psi}\right\} > 0, \qquad (4.5)$$

where $\eta \in \mathbb{C}/\{0\}$ and $z \in U$.
Assume that $-\Psi zq'(z)$ is starlike univalent function in U , let $f \in A$ satisfies

$$\left(\frac{tF_c^{\delta+1}f(z) + (1-t)F_c^{\delta}f(z)}{z}\right) \in H[q(0), 1] \cap Q,$$

and $\emptyset(\lambda, \delta, c, \Psi; z)$ is univalent function in U , where $\emptyset(\lambda, \delta, c, \Psi; z)$ is given by (3.8). If

$$\lambda q(z) - \Psi zq'(z) \prec \emptyset(\lambda, \delta, c, \Psi; z), \qquad (4.6)$$

then

$$q(z) \prec \left(\frac{tF_c^{\delta+1}f(z) + (1-t)F_c^{\delta}f(z)}{z}\right)^{\lambda}. \qquad (4.7)$$

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and q is the best subordinant of (4.6).

Proof: Express the function p by

$$p(z) = \left(\frac{tF_c^{\delta+1}f(z)+(1-t)F_c^{\delta}f(z)}{z}\right)^{\lambda}, \tag{4.8}$$

by setting

$\theta(w) = \lambda w$ and $\emptyset(w) = -\Psi$, $w \neq 0$,

we see that $\theta(w)$ is analytic in $\mathbb{C}$, $\emptyset(w)$ is analytic in $\mathbb{C}^*$ and that $\emptyset(w) \neq 0$, $w \in \mathbb{C}^*$. Too , we get

$Q(z) = zq'(z)\emptyset q(z) = -\Psi zq'(z)$.

It is clear that $Q(z)$ is starlike univalent function in U ,

$$\mathrm{Re}\left\{\frac{\theta'(q(z))}{\emptyset(q(z))}\right\} = \mathrm{Re}\left\{\frac{-\lambda q'(z)}{\Psi}\right\} > 0 .$$

By a straightforword computation ,we obtain

$$\emptyset(\lambda,\delta,c,\Psi;z) = \lambda\, p(z) - \Psi z p'(z), \tag{4.9}$$

where $\emptyset(\lambda,\delta,c,\Psi;z)$ is given by (3.8).

From (4.6) and (4.9) , we have

$$\lambda q(z) - \Psi zq'(z) \prec \lambda p(z) - \Psi p'(z) \quad . \tag{4.10}$$

So , by Lemma 2.4, we become $q(z) \prec p(z)$. By using (4.8), we get the outcome.

5-Sandwich Consequences :

Concluding the consequences of differential subordination and superordination we arrive at the next "sandwich consequence".

Theorem 5.1 : Let q_1 be convex univalent function in U with $q_1(0)=1$, Re $\{\Psi\} > 0$ and let q_2 be univalent in U , $q_2(0)=1$ and fulfills (3,1), let $f \in A$ satisfies :

$$\left(\frac{F_c^{\delta+1}f(z)}{z}\right)^{\lambda} \in H[1,1] \cap Q ,$$

and

$$(1-\Psi(c+1))\left(\frac{F_c^{\delta+1}f(z)}{z}\right)^{\lambda} + \Psi(c+1)\left(\frac{F_c^{\delta+1}f(z)}{z}\right)^{\lambda}\left(\frac{F_c^{\delta}f(z)}{F_c^{\delta+1}f(z)}\right),$$

be univalent in U . If

$$q_1(z) + \frac{\Psi}{\lambda} zq'_1(z) \prec (1-\Psi(c+1))\left(\frac{F_c^{\delta+1}f(z)}{z}\right)^{\lambda} + \Psi(c+1)\left(\frac{F_c^{\delta+1}f(z)}{z}\right)^{\lambda}\left(\frac{F_c^{\delta}f(z)}{F_c^{\delta+1}f(z)}\right) \prec q_2(z) + \frac{\Psi}{\lambda} z\, q'_2(z), \text{then}$$

$$q_1(z) \prec \left(\frac{F_c^{\delta+1}f(z)}{z}\right)^{\lambda} \prec q_2(z),$$

and q_1 and q_2 are correspondingly , the best subordinant and the best dominant .

Theorem 5.2: Let q_1 be convex univalent function in U with $q_1(0)=1$, and fulfills (4.5), let q_2 be univalent function in U $q_2(0)=1$, satisfies (3.6), let $f \in A$ satisfies

$$\left(\frac{tF_c^{\delta+1}f(z)+(1-t)F_c^{\delta}f(z)}{z}\right)^{\lambda} \in H[1,1] \cap Q.$$

And $\emptyset(\lambda,\delta,c,\Psi;z)$ is univalent in U . Where $\emptyset(\lambda,\delta,c,\Psi;z)$ is given by (3.8) . If $\lambda q_1(z) - \Psi zq'_1(z) \prec \emptyset(\lambda,\delta,c,\Psi;z) \prec \lambda q_2(z) - \Psi zq'_2(z)$ then

$$q_1(z) \prec \left(\frac{tF_c^{\delta+1}f(z)+(1-t)F_c^{\delta}f(z)}{z}\right)^{\lambda} \prec q_2(z).$$

In addition q_1 and q_2 are correspondingly , the best subordinant and the best dominant .

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نتائج الساندوج التفاضلية للدوال التحليلية

وقاص غالب عطشان سارة عبدالحميد جواد
قسم الرياضيات ، كلية علوم الحاسوب وتكنولوجيا المعلومات ، جامعة القادسية - االعراق

الملخص:

في هذا البحث، نحصل على بعض نتائج التبعية والتبعية العليا باستخدام المشغل التكاملي F_c^δ.ايضا، وحصلنا على نتائج الساندوج التفاضلية لصنف من الدوال احادية التكافؤ في قرص الوحدة .