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Strongly b star(Sb*) – cleavability(splitability)

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ABSTRACT

A. Poongothai, R. Parimelazhagan[5] introduced some new type of seperation axioms and study some of their basic properties. Some implications between T_0, $T_1$and T_2 axioms are also obtained. In this paper we studied the concept of cleavability over these spaces: (sb*-T_0, sb*-T_1, sb*-T_2) as following:

1- If $\mathcal{P}$ is a class of topological spaces with certain properties and if X is cleavable over $\mathcal{P}$ then X∈ $\mathcal{P}$

2- If $\mathcal{P}$ is a class of topological spaces with certain properties and if Y is cleavable over $\mathcal{P}$ then Y∈ $\mathcal{P}$

MSC..

1. Introduction

In 1985 Arhangl' Skii [1] introduced different types of cleavability(originally named splitability) as following :
A topological space X is said to be cleavable over a class of spaces $\mathcal{P}$ if for $A \subset X$ there exists a continuous mapping $f: X \to Y \in \mathcal{P}$ such that $f^{-1}f(A) = A$, $f(X)$=Y. Throughout this paper, X and Y denote the topological spaces (X, τ) and (Y,σ) respectively , Let A be a subset of the space X. The interior and closure of a set A in X are denoted by int(A) and cl(A) respectively. The complement of A is denoted by (X-A) or A^c**.**

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3-Preliminaries

. In this section, we recall some definitions and results which are needed in this paper

Definition 3.1. [11]

A topological space X is called a T_0 - space if and only if it satisfies the following axiom of Kolmogorov. (T_0) If x and y are distinct points of X, then there exists an open set which contains one of them but not the other.

Definition 3.2. [11]

A topological space X is a T_1 -space if and only if it satisfies the following seperation axiom of Frechet. (T_1) If x and y are two distinct points of X, then there exists two open sets, one containing x but not y and the other containing y but not x.

Definition 3.3. [11]

A topological space X is said to be a T_2 - space or hausdorff space if and only if for every pair of distinct points x, y of X, there exists two disjoint open sets one containing x and the other containing y.

Definition3.4 [8]

A subset (X, τ) is said to be Sb*-closed set if $cl(int(A)) \subseteq U$, whenever $A \subseteq U$ and U is b-open in X. The complements of closed sets Sb*-closed set is Sb*- open sets .The family of all sb*-open sets of a space X is denoted by sb*O(X).

Theorem:3.1[5]

Let X be a topological space and A be a subset of X . Then A is Sb*open iff A contains a Sb* open neighbourhood of each of its points.

Definition3.5. [6]

A subset A of a topological space (X, τ) is called b-open set if $A \subseteq (cl(int(A)) \cup int(cl(A)))$. The complement of a b-open set is said to be b-closed. The family of all b-open subsets of a space X is denoted by BO(X)

Definition3.6 [11]

A map f:X ⟶Y is said to be Continuous function if f^{-1}(V) is closed in X for every closed set V in Y.

Definition 3.7

A map f: X⟶ Y is said to be Sb*-open map if the image of every open set in X is Sb*-open in Y.

Definition 3.8. [9]

Let X and Y be topological spaces. A map f: X ⟶ Y is called strongly b* - continuous (sb*- continuous) if the inverse image of every open set in Y is sb* - open in X.

Definition 3.9 [3]

Let X and Y be topological spaces. A map f : X ⟶ Y is called strongly b* -closed (briefly sb* - closed) map if the image of every closed set in X is sb*- closed in Y.

Definition 3.10

Let X and Y be topological spaces. A map f: $(X, \tau) \longrightarrow (Y, \sigma)$ is said to be sb* - Irresolute if the inverse image of every sb* - closed(respectively sb* - open) set in Y is sb* - closed (respectively sb* - open) set in X.

4- sb* – cleavability

Definition 4.1

A topological spaces X is said to be sb*- pointwise cleavable over a class of spaces $\mathcal{P}$. if for every point $x \in X$ there exists a sb*- continuous mapping $f : X \to Y \in \mathcal{P}$, such that $f^{-1} f(x)=\{x\}$.

Definition 4.2

A topological spaces X is said to be sb* Irresolute - pointwise cleavable over a class of spaces $\mathcal{P}$. if for every point $x \in X$ there exists a sb* - Irresolute - continuous mapping $f : X \to Y \in \mathcal{P}$, such that $f^{-1} f(x)=\{x\}$.

Definition 4.3

By a sb*- -open(closed) pointwise cleavable ,we mean that the sb*-(Irresolute) continuous function f: $X \to Y \in \mathcal{P}$ is an bijective and open(closed) respectively

Definition 4.4.[5]

A topological space X is said to be sb*- T_0 if for every pair of distinct points x and y of X, there exists a sb*-open set G such that x∈G and y∉G or y∈G and x ∉G.

Proposition 4.1

Let X be a sb* - irresolute pointwise cleavable over a class of sb*-T_0 spaces $\mathcal{P}$, then X ∈ $\mathcal{P}$.

Proof:

Let $x \in X$, then there exists sb*T_0- space Y and sb* irresolute a continuous mapping $f : X \to Y \in \mathcal{P}$, such that $f^{-1}f(x)=\{x\}$. This implies that for every $y \in X$ with $x \neq y$,we have $f(x) \neq f(y)$ since Y is a sb* T_0-space, so there exists a sb*-open set G in Y contains one of the two points but not the other. let $f(x) \in G$, $f(y) \notin G$, then $f^{-1}f(x) \in f^{-1}(G)$, $f^{-1}f(y) \notin f^{-1}(G)$. This implies that $x \in f^{-1}(G)$ and $y \notin f^{-1}(G)$, since f is a sb*irresolute a continuous, so $f^{-1}(G)$ is a sb*-open set in X. Therefore X is a sb* T_0 - space .

Theorem 4.1.[5]

Every subspace of a sb*-T_0. space is sb*-T_0.

Proof:

Let (Y, t^*) be a subspace of a space X where t^* is the relative topology of τ on Y. Let y_1, y_2 be two distinct points of Y , as $Y \subseteq X$, y_1 and y_2 are distinct points of X and there exists a sb*-open set G such that $y_1 \in G$ but $y_2 \notin G$ since X is sb*-T_0. Then G⋂Y is a sb*-open set in (Y, t^*) which contains y_1 but does not contain y_2. Hence (Y, t^*) is a sb*-T_0 space

Proposition 4.2

Let X be a sb* T_0–space is a sb* - irresolute pointwise cleavable over a class spaces $\boldsymbol{\mathcal{P}}$, then $\boldsymbol{Y} \in \boldsymbol{\mathcal{P}}$.

Proof:

Let $y \in Y$, then there exists an sb*-irresolute continuous mapping $f: X \to Y \in \mathcal{P}$ such that $f^{-1}f\{f^{-1}(y)\}=f^{-1}(y)$,
This implies that for every $x \in Y$ with $y \neq x$,we have $f^{-1}(x) \neq f^{-1}(y)$
since X is a sb*-T_0 -space , so there exists a sb*-open sets U contains one of the two points but not the other .Let $f^{-1}(y) \in U$ and $f^{-1}(x) \notin U$, then $ff^{-1}(y) \in f(U)$ and $ff^{-1}(x) \notin f(U)$. This implies that $y \in f(U)$ and $x \notin f(U)$ .Therefore Y is sb*-T_0 –space **,**then $Y \in \mathcal{P}$.

Definition 4.5.[5] A space X is said to be sb*- T_1 if for every pair of distinct points x and y in X, there exist sb* - open sets U and V such that x ∈ U but y∉U and y∈ V but x∉V.

Proposition 4.3

Let $\boldsymbol{X}$ be a sb* - irresolute pointwise cleavable over a class of sb*-T_1 spaces $\mathcal{P}$, then X ∈ $\mathcal{P}$.

Proof:

Let $x \in X$, then there exists a sb ∗ -T_1-space Y and a sb* - irresolute- continuous mapping $f: X \to Y \in \mathcal{P}$ such that $f^{-1}f(x) =\{x\}$,$f^{-1}f(x)=\{x\}$ .This implies mapping $f: X \to Y \in \mathcal{P}$ such that $f^{-1}f(x) =\{x\}$, $f^{-1}f(x)=\{x\}$ .This implies that for every $y \in X$ with $x \neq y$, we have $f(x) \neq f(y)$. Since Y is sb*-T_1space , so there exist two sb*-- open sets U and V such that $f(x) \in U, f(y) \notin U$ and $f(y) \in V$, $f(x) \notin V$, then $f^{-1}f(x) \in f^{-1}(U)$, $f^{-1}f(y) \notin f^{-1}(U)$ and $f^{-1}f(y) \in f^{-1}f(V)$, $f^{-1}f(x) \notin f^{-1}(V)$. This implies that $x \in f^{-1}(U)$, $y \notin f^{-1}(U)$ and $y \in f^{-1}(V)$, $x \notin f^{1-}(V)$ .By a sb* - irresolute - continuity of f , $f^{-1}(U), f^{-1}(V)$ are sb*- open sub sets in X . Then X ∈ $\mathcal{P}$

Proposition 4.4

Let $\boldsymbol{X}$ be a sb* - pointwise cleavable over a class of T_1 - spaces $\mathcal{P}$, then X is sb*- T_1- space

Proof:

Let $x \in X$, then there exists a T_1- space Y and a sb*- continuous mapping $f: X \to Y \in \mathcal{P}$ such that $f^{-1}f(x) =\{x\}$,$f^{-1}f(x)=\{x\}$ .This implies mapping $f: X \to Y \in \mathcal{P}$ such that $f^{-1}f(x) =\{x\}$, $f^{-1}f(x)=\{x\}$. This implies that for every $x^* \in X$ with $x \neq x^*$, we have $f(x) \neq f(x^*)$. Since Y is T_1 -space , so there exist two open sets G and H such that $f(x) \in G$, $f(x^*) \notin G$ and $f(x^*) \in H$, $f(x) \notin H$, then $f^{-1}f(x) \in f^{-1}(G)$, $f^{-1}f(x^*) \notin f^{-1}(G)$ and $f^{-1}f(x^*) \in f^{-1}f(H)$, $f^{-1}f(x) \notin f^{-1}(H)$. This implies that $x \in f^{-1}(H)$, $x^* \notin f^{-1}(G)$ and $x^* \in f^{-1}(H)$, $x \notin f^{1-}(H)$ .By a sb* - continuity of f then $f^{-1}(G), f^{-1}(H)$ are sb*- open sub sets in X . Thus X is sb*- T_1- space, then X ∈ $\mathcal{P}$

Proposition 4.5

Let X be sb* T_1–space is an sb* - open pointwise cleavable over a class of spaces $\mathcal{P}$, then $Y \in \mathcal{P}$.

Proof:

Let $y \in Y$, then there exists a sb* T_1-space X and sb* - open continuous mapping $f : X \to Y \in \mathcal{P}$, such that $ff^{-1}\{f^{-1}(y)\} = f^{-1}(y)$. This implies that for every $x \in Y$ with $y \neq x$, we have $f^{-1}(y) \neq f^{-1}(x)$. Since X is sb* T_1-space, so there exist two sb* -open sets V and W such that $f^{-1}(y) \in V, f^{-1}(x) \notin V$ and $f^{-1}(x) \in W$, $f^{-1}(y) \notin W$. Then $ff^{-1}(y) \in f(V)$, $ff^{-1}(x) \notin f(V)$ and $ff^{-1}(x) \in f(W)$, $ff^{-1}(y) \notin f(W)$. This implies that $y \in f(V)$, $x \notin f(V)$ and $x \in f(W)$, $y \notin f(W)$, since f is a sb* open , so $f(V)$, $f(W)$ are open sb* sets of Y . Therefore $Y \in \mathcal{P}$.

Definition 4.6[5].

A space X is said to be sb*- T_2 if for every pair of distinct points x and y in X, there are disjoint sb*- open sets U and V in X containing x and y respectively

Theorem 4.2.[5] Every sb*- T_2 space is sb*- T_1.

Proof:

Let X be a sb*- T_2 space. Let x and y be two distinct points in X. Since X is sb*- T_2, there exist disjoint sb*-open sets U and V such that $x \in U$ and $y \in V$. Since U and V are disjoint, $x \in U$ but $y \notin U$ and $y \in V$ but $x \notin V$. Hence X is sb*- T_1.

Proposition 4.6

Let X be sb*- T_2 - space is a sb* - open pointwise cleavable over a class of spaces $\mathcal{P}$, then $Y \in \mathcal{P}$.

Proof:

Let $y_1 \in Y$, then there exists a sb*- T_2- space X and a sb* open continuous mapping $f: X \to Y \in \mathcal{P}$ such that $f^{-1}f(f^{-1}(y)) = f^{-1}(y)$. This implies that for every $y_2 \in Y$, with $y_1 \neq y_2$, we have $f^{-1}(y_1) \neq f^{-1}(y_2)$,so there exist x_1 , x_2 in X ,such that $x_1 = f^{-1}(y_1)$, $x_2 = f^{-1}(y_2)$ with $x_1 \neq x_2$, Since X is sb*- T_2 , so there exist two sb* open sets G , H Such that $f^{-1}(y_1) \in G$, $f^{-1}(y_2) \in H$ and $G \bigcap H = \emptyset$, then $ff^{-1}(y_1) \in f(G)$, $ff^{-1}(y_2) \in f(H)$. Since f is sb* open , then $f(G)$, $f(H)$ are sb* open sets of Y and $y_1 \in f(G)$, $y_2 \in f(H)$ and $f(G) \bigcap f(H) = f(G \bigcap H) = f(\emptyset) = \emptyset$. Then $Y \in \mathcal{P}$.

Proposition 4.7

Let X be sb* - open pointwise cleavable over a class of sb*- T_2–spaces $\mathcal{P}$, then $X \in \mathcal{P}$

Proof:

Let $x \in X$, then there exists a sb*- T_2 space Y and a sb*- continuous mapping $f: X \to Y \in \mathcal{P}$ such that $f^{-1}f(x) = \{x\}$. This implies that for every $y \in Y$ with $x \neq y$,we have $f(x) \neq f(y)$. Since Y is sb*- T_2, so there exist two sb* open sets U and V such that $f(x) \in U$, $f(y) \in V$ and $U \bigcap V = \emptyset$, then $f^{-1}f(x) \in f^{-1}(U)$, $f^{-1}f(y) \in f^{-1}(V)$, this implies that $x \in f^{-1}(U)$, $y \in f^{-1}(V)$, since f is sb*- continuous , so $f^{-1}(U)$, $f^{-1}(V)$ are sb* open sets of X and $f^{-1}(U) \bigcap f^{-1}(V) = f^{-1}(U \bigcap V) = f^{-1}(\emptyset) = \emptyset$.

thus $X \in \mathcal{P}$.

5-conclusion:

In this paper we have studied and proved these cases:

1) If $\mathcal{P}$ is a class of (sb*- T_0 , sb*- T_1) spaces with certain properties and if X is a sb* - irresolute pointwise cleavable over $\mathcal{P}$, then $X \in \mathcal{P}$,also if $\mathcal{P}$ is a class of (sb*- T_0‘ sb*- T_1) spaces with certain properties and if Y is a sb* - irresolute pointwise cleavable over $\mathcal{P}$, then $Y \in \mathcal{P}$.

2) If $\mathcal{P}$ is a class of (sb*- T_1 ‘ sb*- T_2) spaces with certain properties and if X is point wise sb^* – cleavable over $\mathcal{P}$, then $X \in \mathcal{P}$, also If $\mathcal{P}$ is a class of sb^*-T_1 spaces with certain properties and if X is a sb* - irresolute pointwise cleavable over $\mathcal{P}$, then $X \in \mathcal{P}$.

3) If $\mathcal{P}$ is a class of (sb^*-T_1 ، sb^*-T_2) spaces with certain properties and if Y is point wise $sb*$ cleavable over $\mathcal{P}$, then $Y \in \mathcal{P}$.

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