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تحسين التباين للصور الرمادية

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الخلاصة

إن تحسين التباين (السطوع) هي خطوة مهمة في مجال معالجة الصور الرقمية وتمييز الأنماط والرؤية الحاسوبية , وان طريقة توزيع التباين هي أسلوب فعال جدا في هذا المجال . البحث ناقش استخدام خمسة طرائق لتحسين التباين وهي (HE) و (CLAHE) و (BBHE) و (DSIHE) و(RSIHE) حيث تم تطبيقها على ثمانية صور رقمية رمادية ضعيفة التباين , كذلك تم إجراء المقارنة بين هذه الطرق لبيان مقدار التحسين الحاصل على الصور من حيث سطوع و وضوح الصور وقد استخدم مقياسان لهذا الغرض: (PSNR) , و (AMBE) لقياس قوة الإشارة و معدل السطوع على التوالي. تمت برمجة البحث باستخدام لغة MATLAB Ver. 7.8 .

الكلمات المفتاحية: تحسين التباين , توزيع التباين.

Contrast Enhancement in Gray Level Images

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Abstract

Contrast enhancement is a very important step in image processing, pattern recognition and computer vision .Histogram Equalization (HE) is widely used for contrast Enhancement . This paper discussed five techniques of contrast enhancement, i.e. Histogram Equalization (HE), Contrast Limited Adaptive Histogram Equalization (CLAHE), Brightness preserving Bi_Histogram Equalization (BBHE), Dualistic Sub_Image Histogram Equalization (DSIHE) and Recursive Sub_Image Histogram equalization (RSIHE). Also it presented the comparison among the various techniques that show the image enhances the overall contrast and visibility of local details, by using two measures: Peak-Signal-to-Noise-Ratio (PSNR) and Absolute Mean Brightness Error (AMBE) to check the signal and brightness power respectively. This work is programmed by MATLAB VER.7.8.

Keywords: Histogram Equalization , Contrast Enhancement , HE , CLAHE , BBHE , DSIHE , RSIHE

1. Introduction

Contrast enhancement is an important area in image processing for both human and computer vision. It is widely used for medical image processing and as a pre-processing step in speech recognition, texture synthesis, and many other image/video processing applications[1] . There are several reasons for an image/video to have poor contrast: the poor quality of the used imaging device, lack of expertise of the operator, and the adverse external conditions at the time of acquisition. These effects result in under-utilization of the offered dynamic range. As a result, such images and videos may not reveal all the details in the captured scene, and may have a wasted-out and un-natural look. Contrast enhancement aims at eliminating these problems, in order to obtain a more visually pleasing or informative image or both[2].

Histogram is a statistical probability distribution of each gray level in a digital image. Histogram equalization (HE) is one of the most popular contrast enhancement techniques owing to its simplicity and effectiveness [3]. It is the most commonly used method in this aspect and this is due to its simplicity and its ability in giving better performance comparing to other types of images. HE transforms the gray levels of the image based on the probability distribution of the input gray levels.[4]

This research includes7sections. Section1 is the introduction.Section2 contains brief paragraphs of the previous works that are related with the title of the research. A number of enhancement techniques are displayed in section3. While section4 contains an explanation of the methods that are used in the research. Section5 lists the measures used for evaluating the images in the research. The part that includes the work steps, results tables, and the used images before and after enhancement, is displayed in section6. Finally, section7 briefly presents the conclusion of the research.

2.Previous Studies:

There are many researches attempted to enhance the contrast of gray level images by different contrast enhancement techniques, such as: Raijagopal and Santhi use the Bi-histogram equalization to preserve the brightness and enhance the local contrast of the gray scale original image with low contrast [5].

S.Singhuses a semi-automatic global contrast enhancement to increase or decrease the contrast from a threshold value for each pixel value of the Gray level image [6]. While, Turgay Celik uses 2DHE to enhance input gray and color image[7].

Finally, Zeng , Li and Ming use a new form of histogram equalization, in which the input x-ray image is divided into several equal-sized regions according to the intensities of gradient, then their corresponding statistical values of gray levels are modified respectively. After that, processed histogram of the whole image is obtained by the summation of all the weighted values of the divided regions[8].

3.Histogram Equalization Techniques:

The histogram-based equalization techniques are classified into two principle categories: Global and Local histogram equalization.

- Global Histogram Equalization (GHE) uses the histogram information of the entire input image in its transformation function. Though this global approach is suitable for overall techniques; it fails to preserve the local; bright features of the input image[5].
- Local Histogram Equalization (LHE)[9] tries to eliminate such problems. It uses a small window that slides through every pixel of the image sequentially. The block of pixels that are masked by the window is considered for HE. Then the gray level

mapping of the enhancement is done only for the center pixel of that window. Thus, it makes use of the local information remarkably.

Brightness preserving Bi-Histogram Equalization (BBHE)[10], Dualistic Sub-Image Histogram Equalization (DSIHE)[11], and Minimum Mean Brightness Error Bi-Histogram Equalization (MMBEBHE)[12] are the variant of HE based contrast enhancement.

- BBHE divides the input image histogram into two parts based on the mean brightness of the image and then each part is equalized independently. This method tries to overcome the problem of brightness preservation.
- DSIHE method uses the entropy value of histogram separation.
- MMBEBHE is an extension of BBHE method and provides maximal brightness preservation.

Though these methods can perform good contrast enhancement, they also cause more annoying side effects depending on the variation of gray level distribution in the histogram.

Recursive Mean-Separate Histogram Equalization (RMSHE)[13], and Recursive Sub-image Histogram Equalization (RSIHE)[14] are the improved versions of BBHE. However they are also not free from side effects.

Recursively Separated and Weighted Histogram Equalization (RSWHE) is presented in[15] to achieve sharpness in images.

Sub-Regions Histogram equalization (SRHE) proposed in[16] in which the image is partitioned, based on the smoothed intensity values, obtained by convolving the input image with a Gaussian filter, finally, a method called Mean Brightness Preserving Histogram Equalization (MBPHE): this technique could be split into two major terms called bisection-MBPHE and multi-sections MBPHE. Bisection process can preserve the illumination merely to a definite range. Multi-sections split the original image into sub-histograms, then all sub-histograms are then equalized independently[3].

More details can be found in [3],[5],[8],[17],[18] and [19].

4.Proposed Techniques:

In this research we used (8) poor-contrast Gray level images with formats (.JPG,.PNG);we processed these images using three techniques of contrast Enhancement. The first one depends on finding the histogram of the whole image as in HE and CLAHE, the second one divides the image into two sub-images and finds the histogram of each sub-image independently, as in(BBHE) and (DSIHE) methods, while the third technique divides the image into multi sub-images and finds the histogram of each one as in RSIHE method.

4.1. Histogram Equalization (HE):

Histogram Equalization is a contrast Enhancement technique in spatial domain in image processing using histogram of image .Histogram Equalization usually increases the global contrast of the processing image [20].

This method is useful for the image which is bright or dark [21].

Implementation:

Consider a discrete grayscale image {x} and let *ni* be the number of occurrences of gray level *i*. The probability of an occurrence of a pixel of level i in the image, as can be seen in eq.(1)

$$p_x(i) = p(x=i) = \frac{ni}{n}, \quad 0 \le i < L \qquad \text{.............(1)}$$

L is the total number of gray levels in the image (typically 256), n is the total number of pixels in the image, and $p_x(i)$ is in fact the image's histogram for pixel value i, normalized to [0,1].
Let us also define the cumulative distribution function (*cdf*) corresponding to ρ_χ as in eq.(2)

$$cdf_x(i) = \sum_{j=0}^{i} p_x(j) \qquad \text{.............(2)}$$

Means the summation of image's histogram for pixel value *j*, where *j* is changing from 0 to *i*. Which is also the image's accumulated normalized histogram.
We would like to create a transformation of the form y = T(x) to produce a new image {y}, with a flat histogram. Such an image would have a linearized *cdf* across the value range, eq.(3)

$$cdf_y(i) = iK \qquad \text{..............(3)}$$

for some constant K, the properties of the CDF allow us to perform such a transformation (see Inverse distributing function); it is defined as in eq.(4)

$$y = T(k) = cdf_x(k) \qquad \text{..............(4)}$$

where k is in the range [0,L). Notice that T maps the levels into the range [0,1], since we used a normalized histogram of {x}. In order to map the values back into their original range, the following simple transformation needs to be applied on the result, as in eq.(5):

$$\grave{y} = y.\,(\max\{x\} - \min\{x\}) + \min\{x\} \qquad \text{...........(5)}$$

More details about the equations can be found on [17] and [18].

4.2. Adaptive Histogram Equalization (AHE):

Adaptive histogram equalization (AHE) is an excellent contrast enhancement method of both natural images and medical and other initially non visual images. It is an extension to traditional Histogram Equalization technique. It enhances the contrast of images by transforming the values in the intensity of the image. It operates on small data regions (tiles), rather than the entire image. Each tile's contrast is enhanced so that the histogram of the output region approximately matches the specified histogram. The neighboring tiles are then combined using bilinear interpolation in order to eliminate artificially induced boundaries[22].

The contrast, especially in homogenous areas, can be limited in order to avoid amplifying the noise which might be presented in the image.

Contrast Limited AHE (CLAHE) differs from the ordinary adaptive histogram equalization in its limiting contrast. This feature can also be applied to global histogram equalization, giving rise to contrast limited histogram equalization (CLHE), which is rarely used in practice. In the case of CLAHE, the limiting contrast procedure has to be applied to each neighborhood from which a transformation function is derived. CLAHE was developed[5] to prevent the over amplification of noise that the adaptive histogram equalization can give rise to[23].This is achieved by limiting the contrast enhancement of AHE. The contrast amplification in the vicinity of a given pixel value is given by the slope of the transformation function. This is proportional to the slope of the neighborhood cumulative distribution function (*cdf*) and therefore to the value of the histogram at that pixel value. CLAHE limits the amplification by clipping the histogram at a predefined value before computing the *cdf*. This limits the slope of the *cdf* and then of the transformation function. The value at which the histogram is clipped, the so-called clip limit, depends on the normalization of the histogram and on the size of the neighborhood region. Common values limit the resulting amplification to between 3 and 4 times the histogram mean value.

4.3.Brightness Preserving Bi-Histogram Equalization (BBHE):

BBHE first decomposes the input histogram H(X) into two sub-histograms $H_L(X)$ and $H_U(X)$ by using input mean X_m , where $H_L(X)$is associated with gray levels { X_0 , X_1, …, X_m} and $H_U(X)$ is associated with the gray levels {X_{m+1}, X_{m+2} ,…,X_{L-1}} . Then it performs conventional HE on $H_L(X)$ and $H_U(X)$ independently [24].

4.4 . Dualistic Sub Image Histogram Equalization (DSIHE):

DSIHE is similar to BBHE , except that the threshold of histogram segmentation is the median X_D of the input image . that is , the input histogram H(X) is partitioned into two sub-histograms $H_L(X)$and $H_U(X)$ by the input median X_D (not by input mean X_m as in BBHE method). each of $H_L(X)$ and $H_U(X)$ is then equalized independently as in BBHE[21].

4.5.Recursive Sub_Image Histogram Equalization (RSIHE):

RSIHE method is a generalization of DSIHE. It divides the histogram into multi sub-histograms rather than two sub-histograms as in BBHE and DSIHE.

In this method, histogram is divided on the basis of median values (gray level with cumulative probability density equal to 0.5) rather than the mean values. However utilization of median values shares the same number of pixels in each partition. Thereafter it divides the histogram into 2^r pieces of sub-histograms and it will preserve the brightness to maximum extent rather than other partitioning methods where r is recursion level and its value is defined by the user [25].

In this research the value of r is determined as 2, so the histogram is divided into 4 sub-histograms.

5. Image Quality Measuring Tools:

This section describes the image quality measuring tools which are used to evaluate the ability of the equalization techniques to maintain the mean brightness preserving (PSNR) and contrast enhancement (AMBE).

1- PSNR: Peak-Signal-Noise-Ratio.[9]

PSNR=$10\ log_{10}\frac{255^2}{MSE}$ ....(6)

Where:

MSE: the value of Mean Square Error

Greater the value of PSNR means better contrast enhancement of the image.

2- AMBE: Absolute differential gray level mean between the original image and the enhanced one.[9]

AMBE=$|X - Y|$ ………(7)

Where:

X: the original image.

Y: the enhanced image.

Lower the AMBE indicates the better brightness preservation of the image.

6.Proposed Work Results and Discussion:

6.1. Algorithm:

The algorithm of the research illustrated in the following steps:

1- Read a gray level image.
2- Find the histogram of the image.
3- Display the image and its histogram.
4- Equalize the image using HE ,CLAHE,BBHE, DSIHE and RSIHE techniques.
5- Find the histogram of the equalized image.
6- Display the equalized image and its histogram.
7- Implement PSNR and AMBE measures on the equalized image.
8- Compare the four techniques through the measures and the histogram of the output image, Find the better method.

We implement the steps illustrated on eight dim, foggy, unclear gray level images, named as: Pic1, Pic2, Pic3, Pic4,Pic5,Pic6,Pic7,Pic8 . To see the difference between the images, they are displayed (with their histogram) before and after the equalization.

6.2. Matlab Implementation:

- HE Function:

J=histeq(I) ; where I is a gray level image, J is the image after equalization.

- CLAHE Function:

J=adapthisteq(I)

- BBHE Function:

```
function y=bbhe(x)
sz=size of (x);
%% Decomposes the input histogram into two sub-histograms
%%lo & up depending on the l_mean value
l_mean=round(mean(x(:)));
lo=zeros(256,1);
up=zeros(256,1);
for i=1:sz(1)
for j=1:sz(2)
val=x(i,j);
if (val <=l_mean)
lo(val+1)=lo(val+1)+1;
else
up(val+1)=up(val+1)+1;
end
end
end
%% then uses the traditional HE method on each sub-histograms
```

- DSIHE Function:

The same function of BBHE except that it uses *median* matlab function instead of the *mean* function

- RSIHE Function:

The same function of DSIHE, except that recursion is used to divide the histogram into 4 sub-histograms, the number of iterations is determined as 2 (and this number can be changed by the programmer)

6.3. Flowchart:

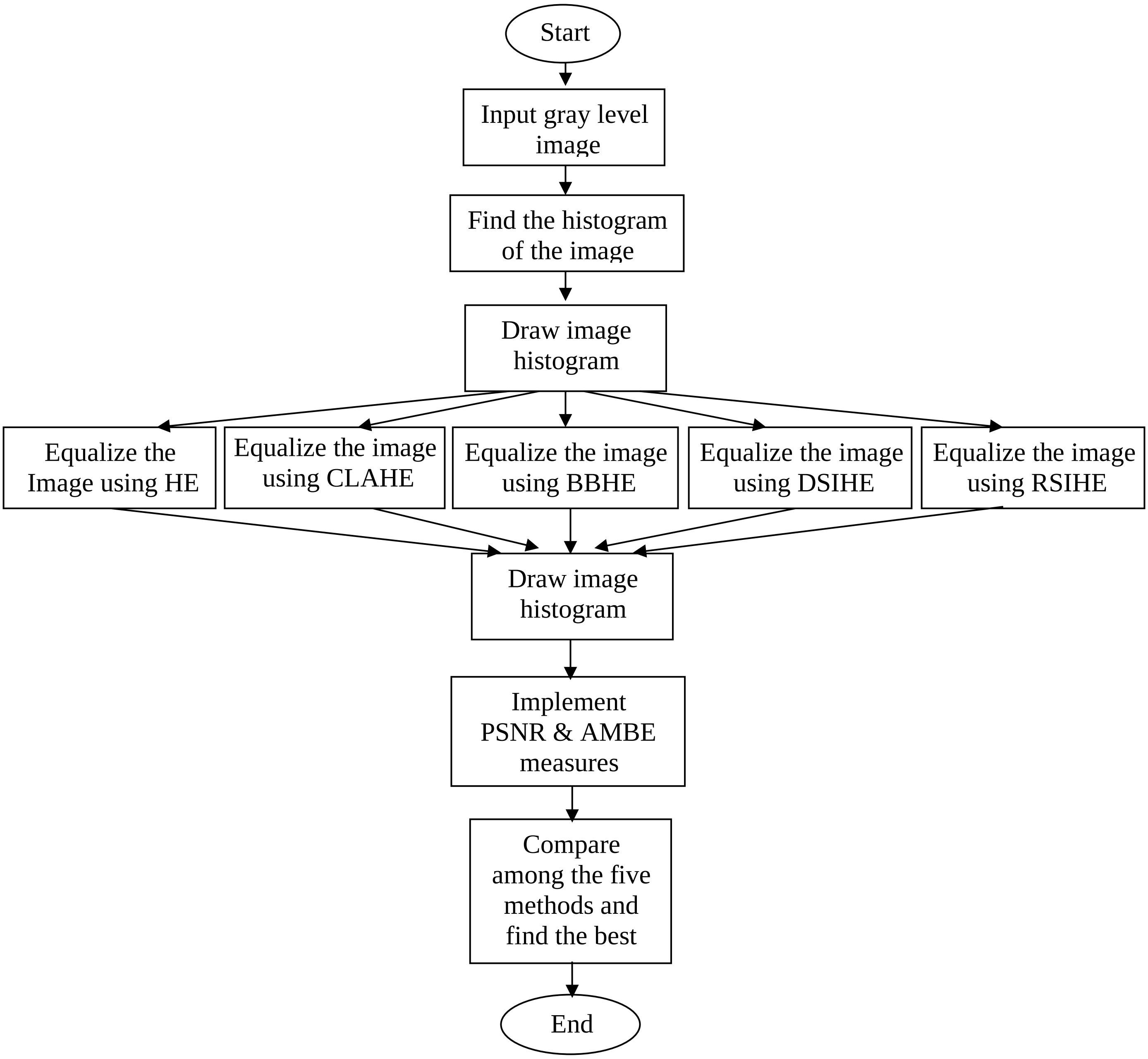
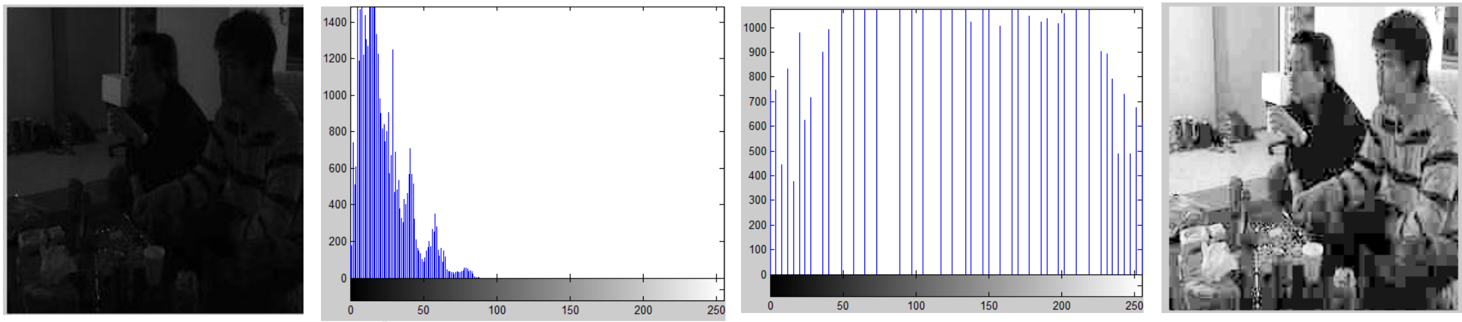


Fig. 1
Shows Research Stages Flowchart

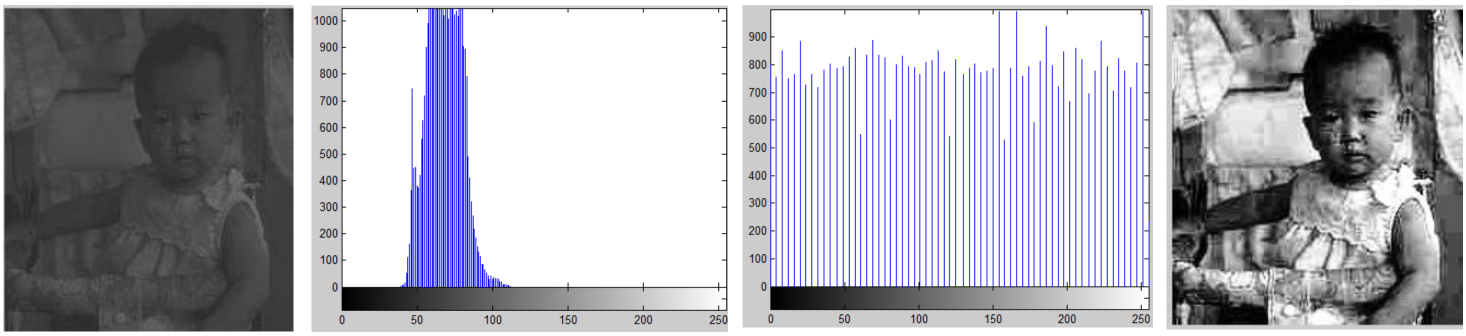
6.4. Results:

Here we present the results of all methods we used in our work , the tables of (PSNR) and (AMBE) measurements values and their diagrams .

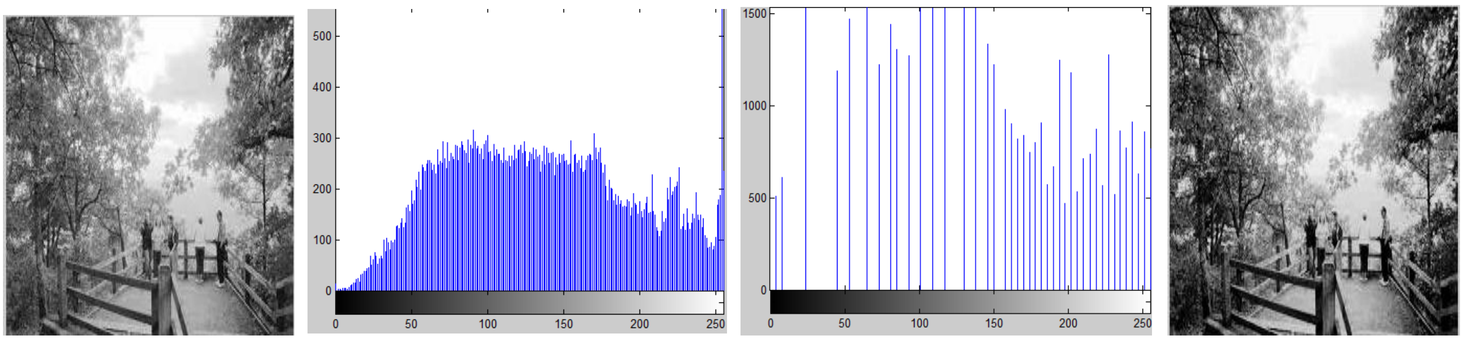
6.4.1.Result of (HE) Method:



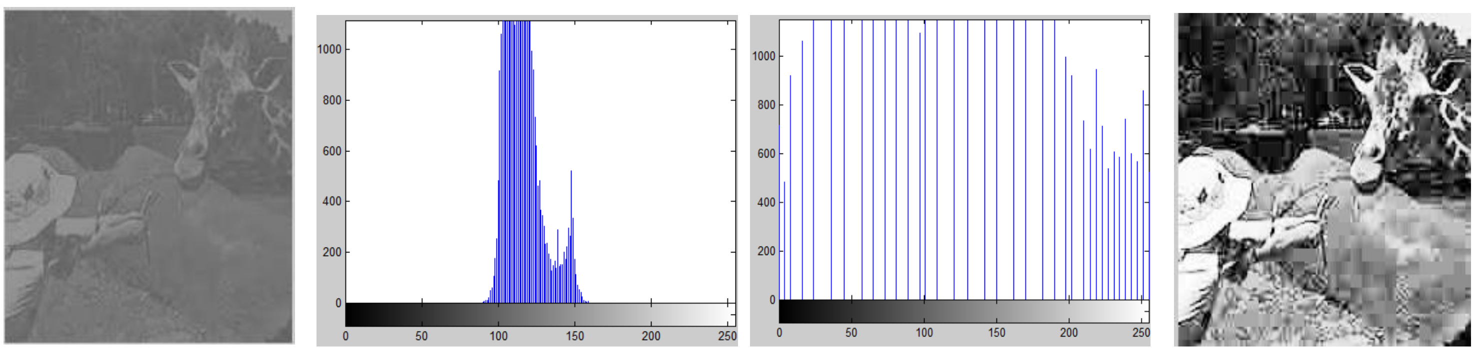
(a) (b) (c)



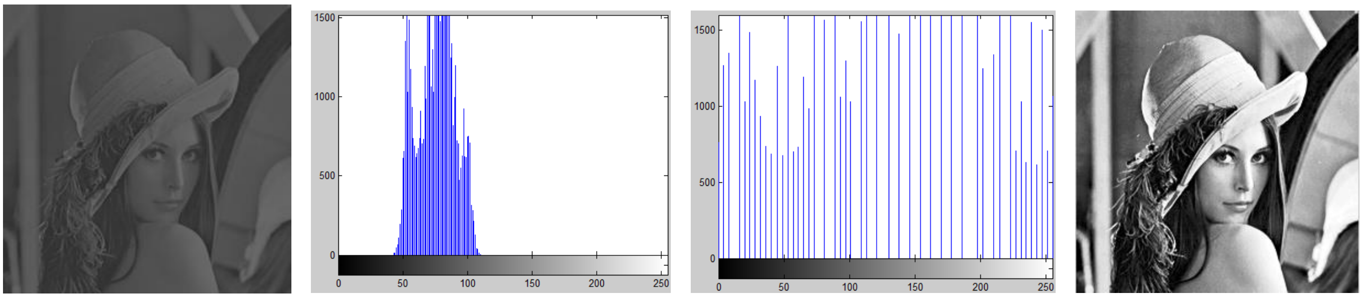
(a) (b) (c)



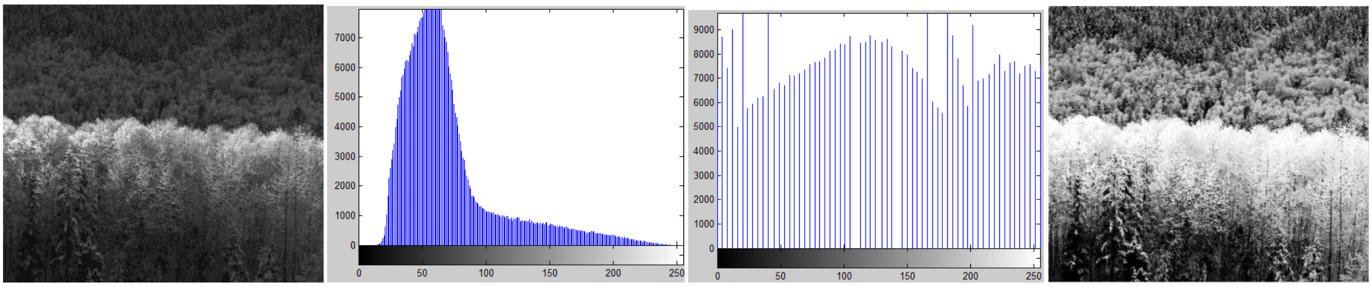
Pic.3 (a) (b) (c)



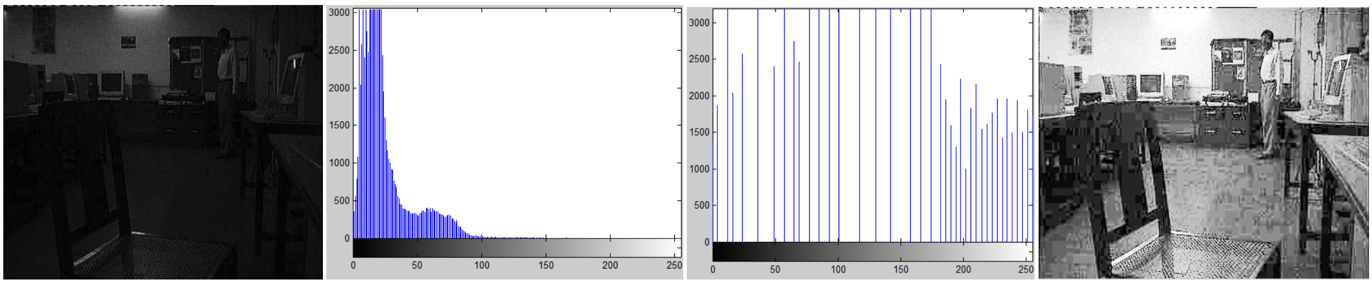
Pic.4 (a) (b) (c)



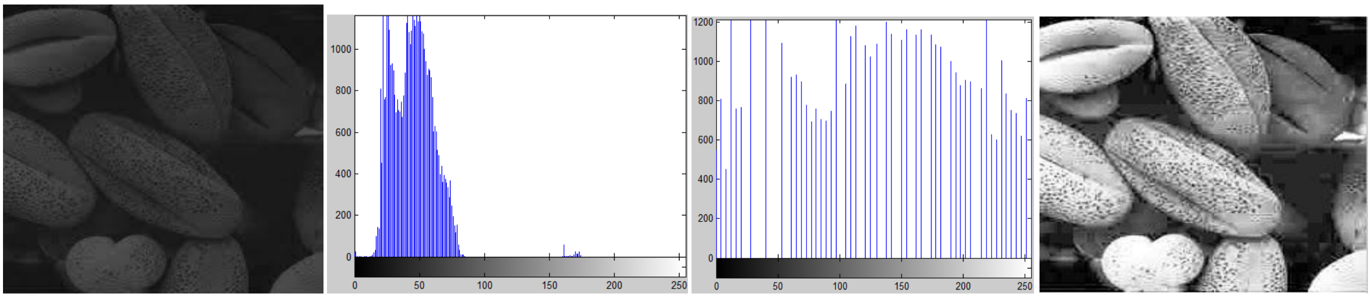
Pic.5 (a) (b) (c)



Pic.6 (a) (b) (c)



Pic.7 (a) (b) (c)



Pic.8 (a) (b) (c)

Fig.1

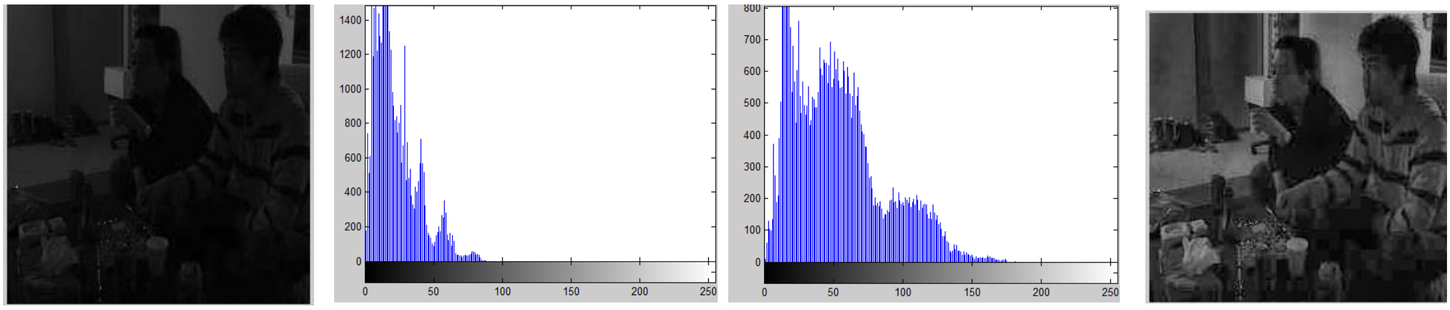
HE implemented on Pic.1, Pic.2, Pic.3, Pic.4, Pic.5, Pic.6, Pic.7 and Pic.8

(a)The histogram of the original image

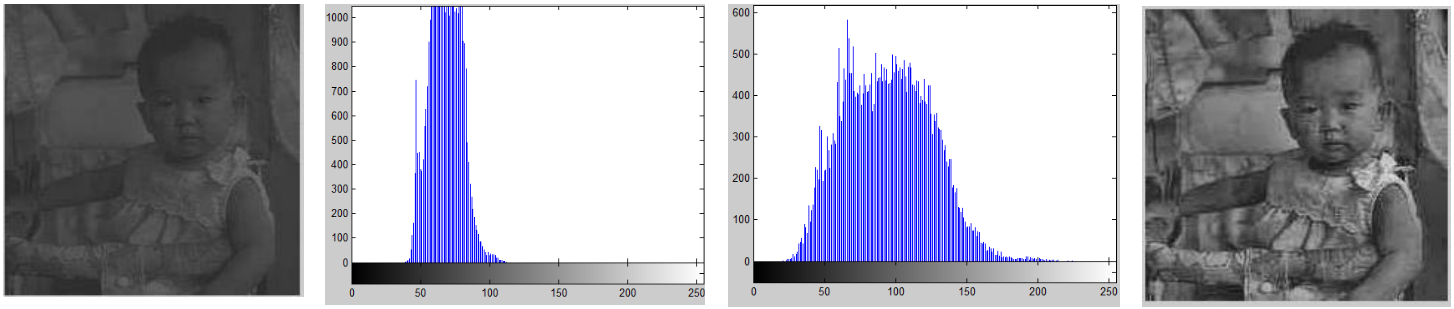
(b) The histogram of the equalized image

(c) The equalized image

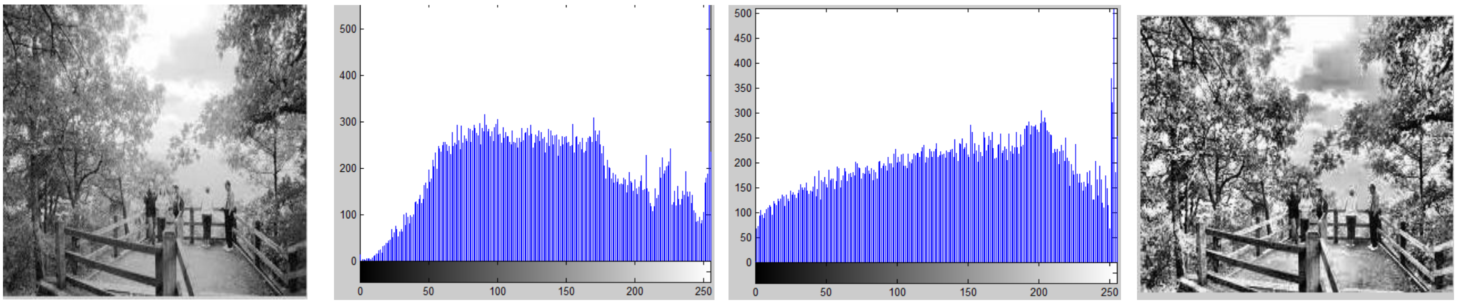
6.4.2.Result of (CLAHE) Method :



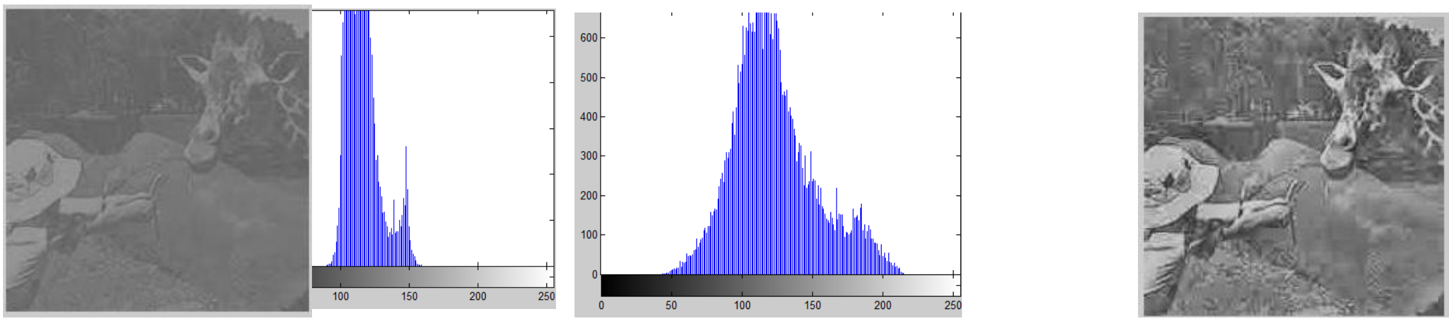
Pic.1 (a) (b) (c)



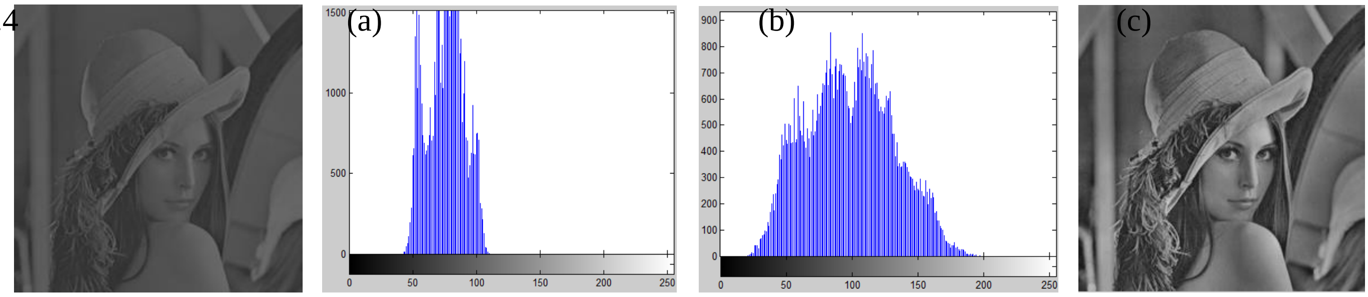
Pic.2 (a) (b) (c)



Pic.3 (a) (b) (c)



Pic.4 (a) (b) (c)



Pic.5 (a) (b) (c)

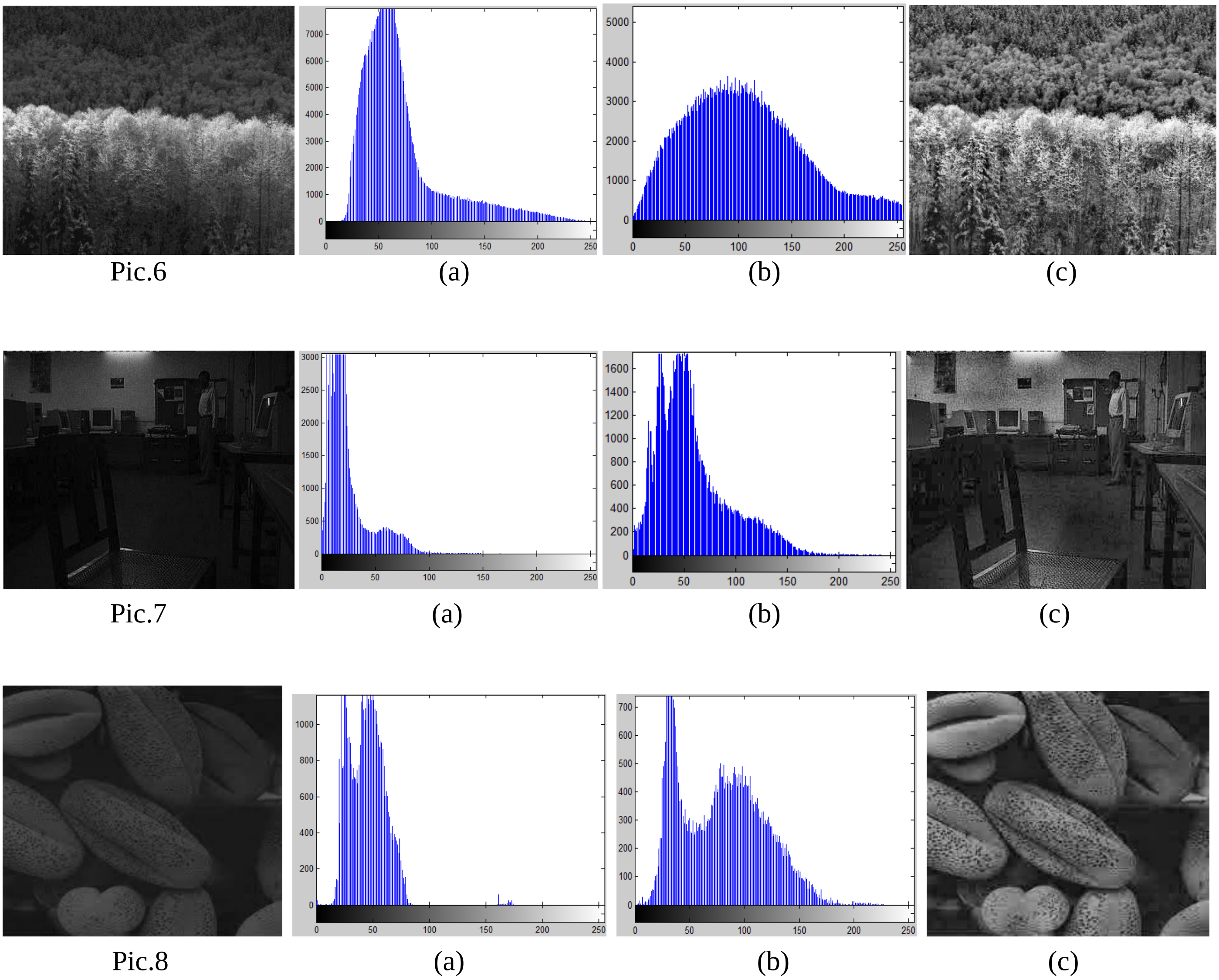
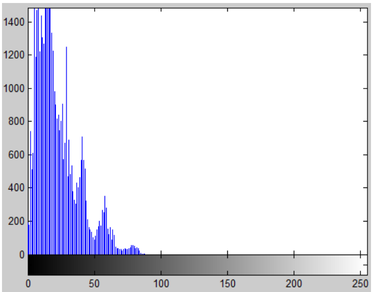


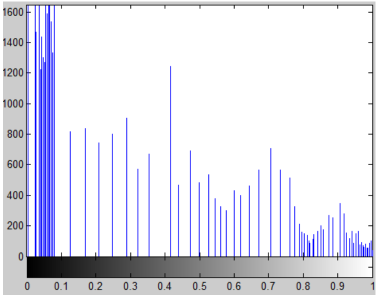
Fig.2
CLAHE implemented on Pic.1, Pic.2, Pic.3, Pic.4, Pic.5, Pic.6, Pic.7 and Pic.8
(a) The histogram of the original image
(b) The histogram of the equalized image
(c) The equalized image

6.4.3.Results of (BBHE) Method:


Pic.1



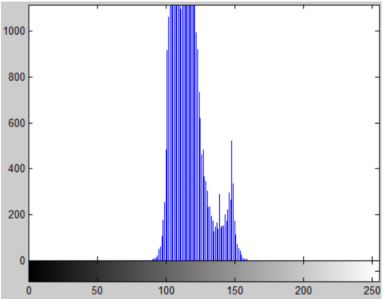
(a)



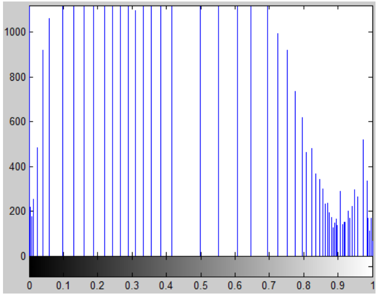
(b)


(c)


Pic.2



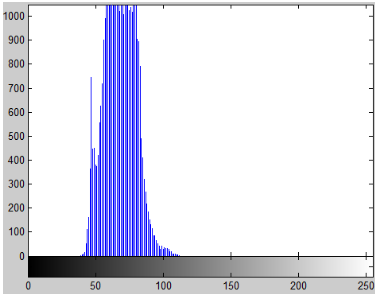
(a)



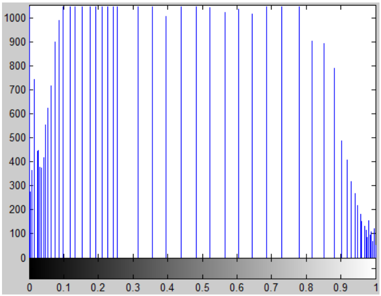
(b)


(c)


Pic.3



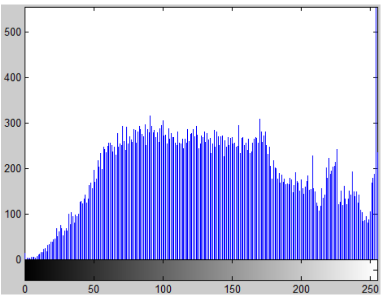
(a)



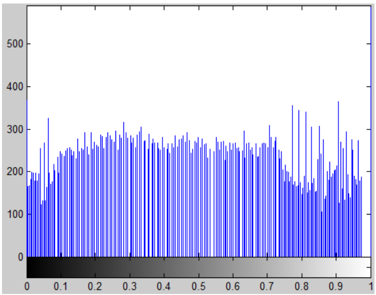
(b)


(c)


Pic.4

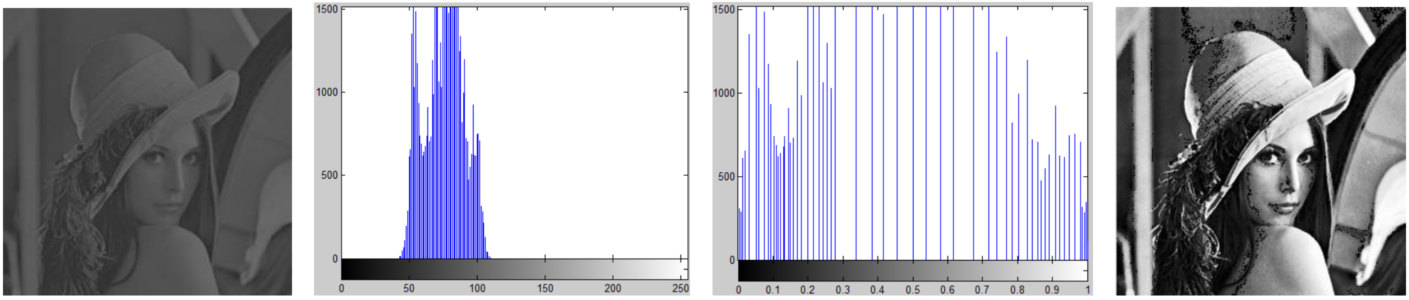


(a)

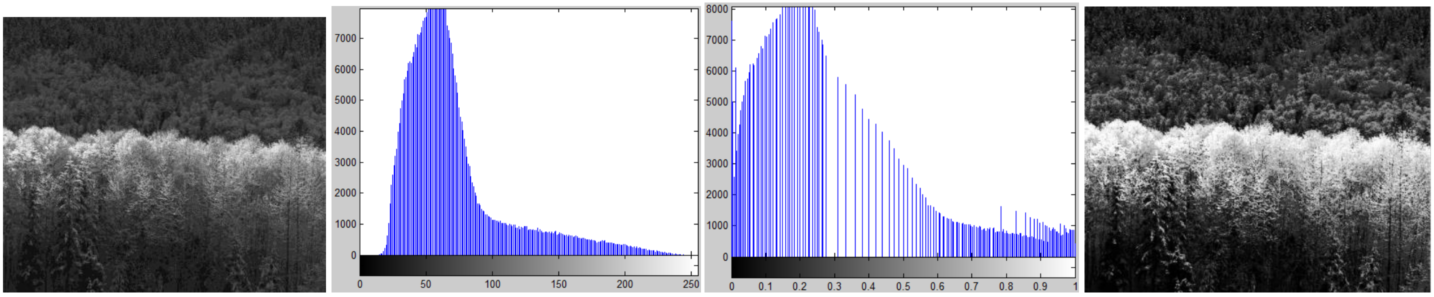


(b)

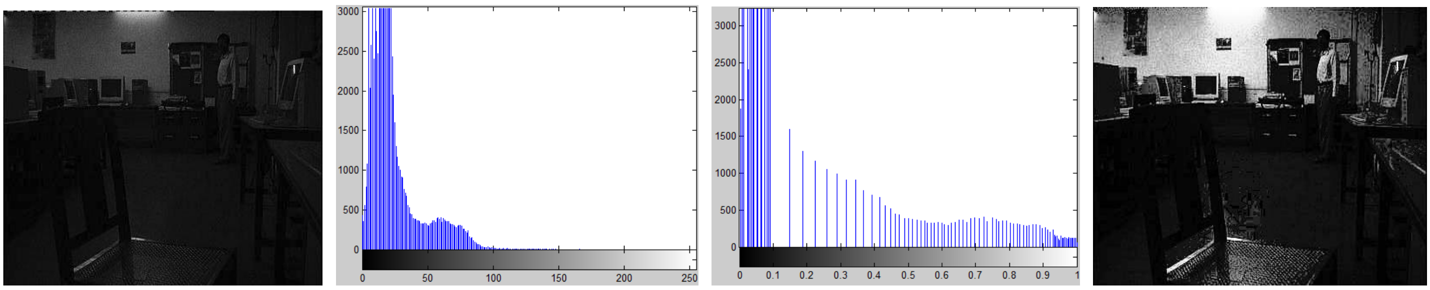

(c)



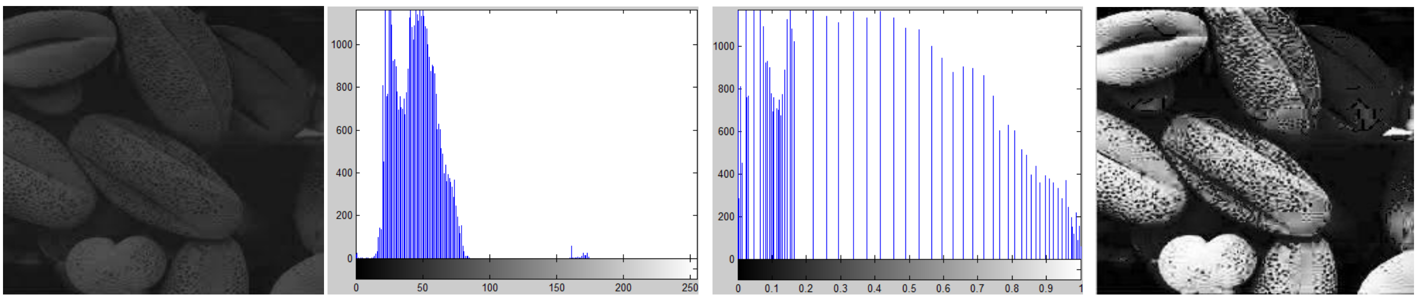
Pic.5 (a) (b) (c)



Pic.6 (a) (b) (c)



Pic.7 (a) (b) (c)

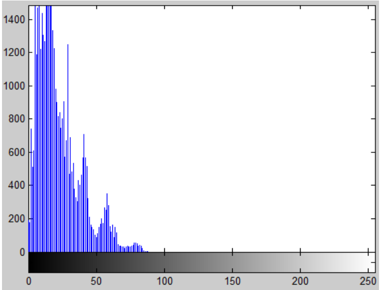


Pic.8 (a) (b) (c)

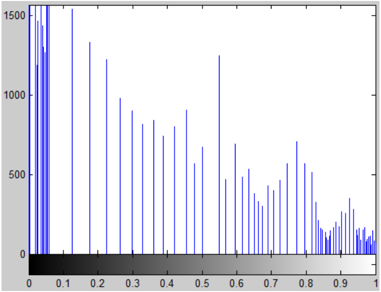
Fig.3
BBHE implemented on Pic.1, Pic.2, Pic.3, Pic.4, Pic.5, Pic.6, Pic.7 and Pic.8
(a)The histogram of the original image
(b)The histogram of the equalized image
(c) The equalized image

6.4.4. Results of (DSIHE) Method:


Pic.1

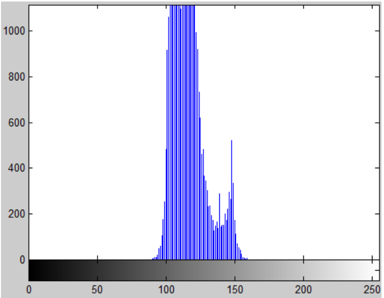


(a)

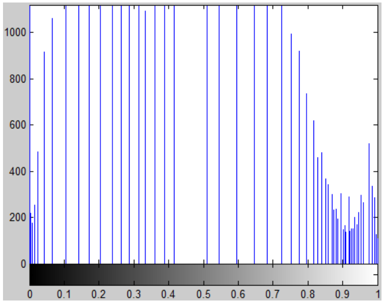

(b)


(c)


Pic.2



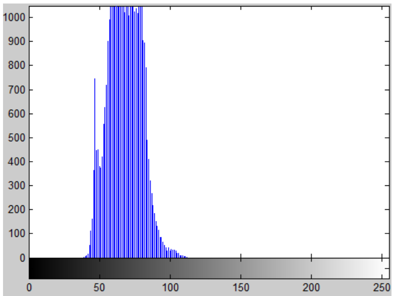
(a)



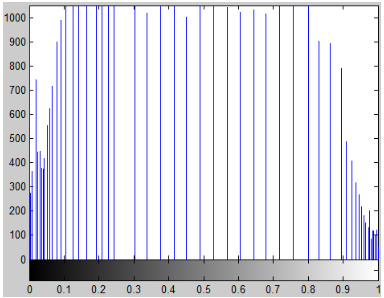
(b)


(c)


Pic.3



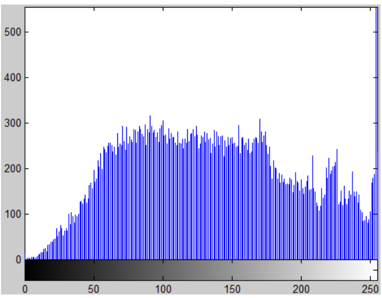
(a)

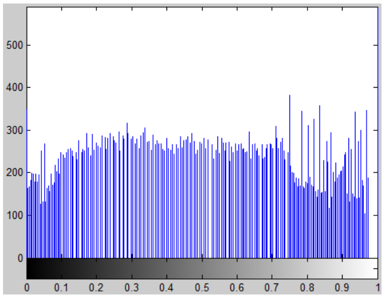


(b)


(c)

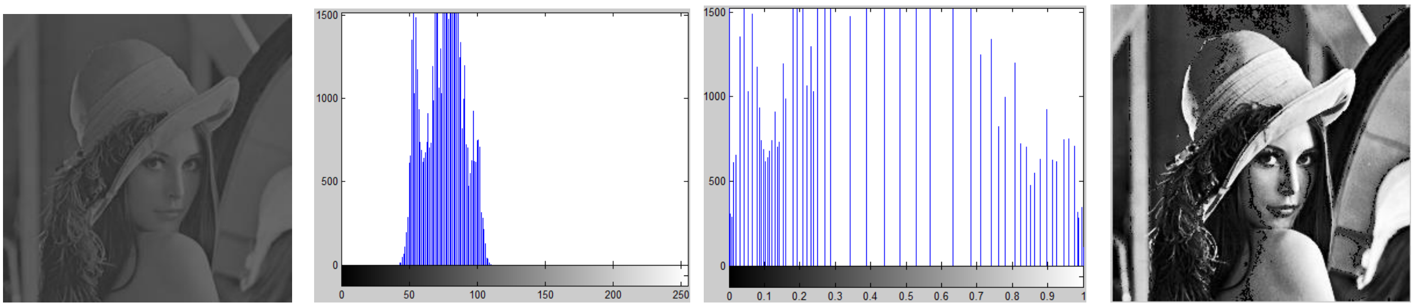

Pic.4


(a)

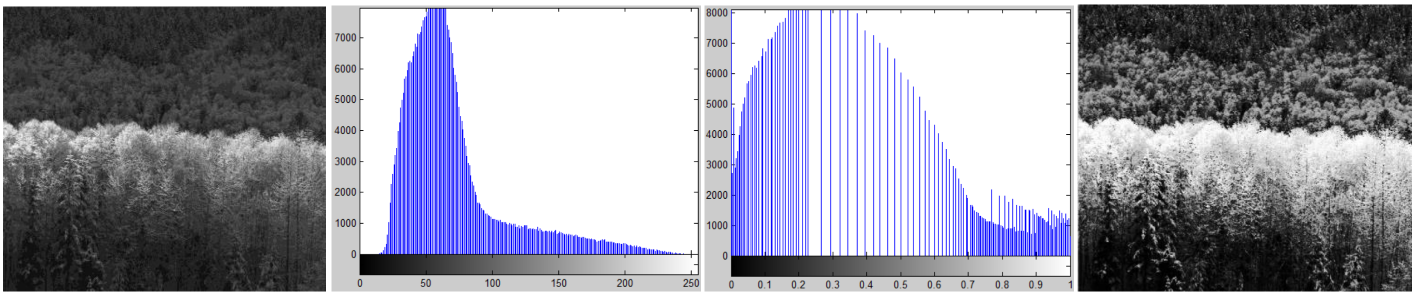


(b)

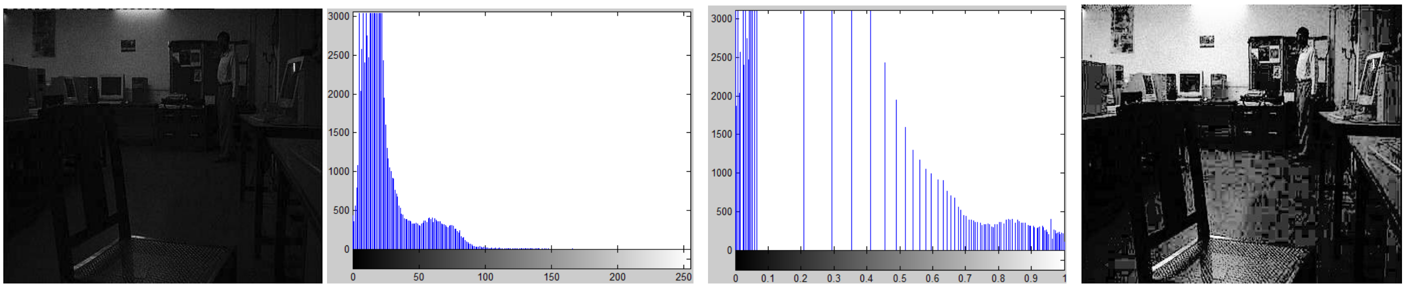

(c)



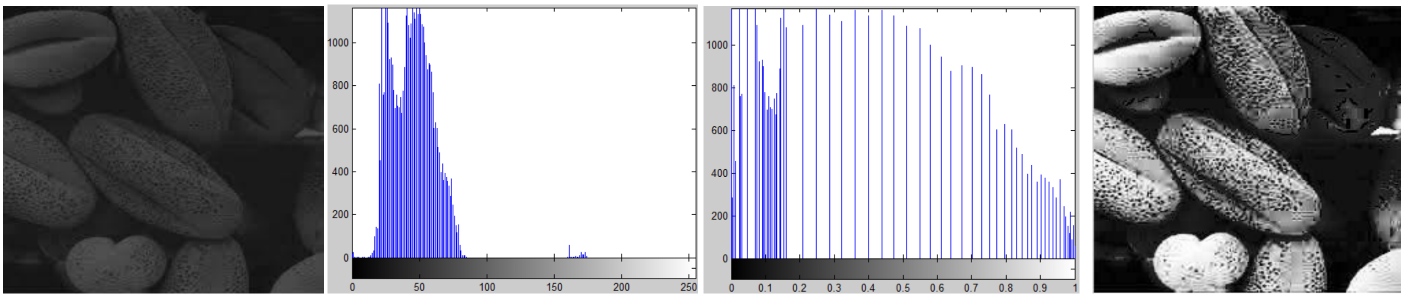
Pic.5 (a) (b) (c)



Pic.6 (a) (b) (c)



Pic.7 (a) (b) (c)

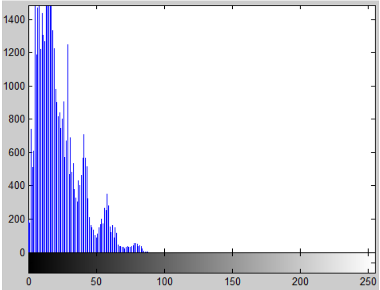


Pic.8 (a) (b) (c)

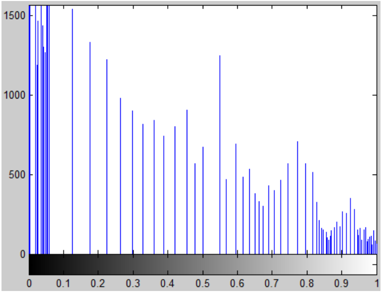
Fig.4
DSIHE implemented on Pic.1, Pic.2, Pic.3, Pic.4, Pic.5, Pic.6, Pic.7 and Pic.8
(a)The histogram of the original image
(b)The histogram of the equalized image
(c)The equalized image

6.4.5. Result of RSIHE Method:


Pic. 1



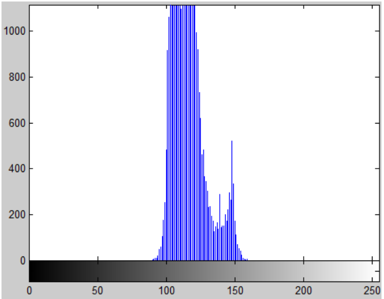
(a)



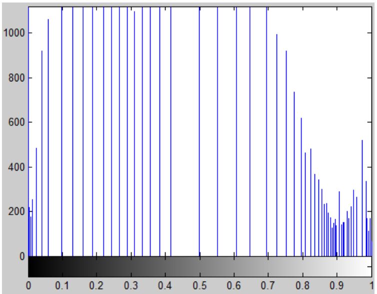
(b)


(c)


Pic.2



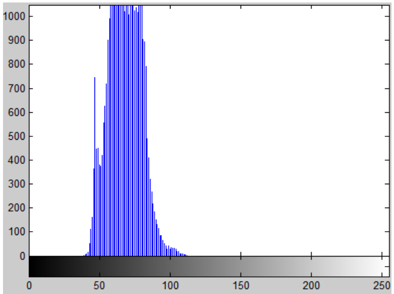
(a)



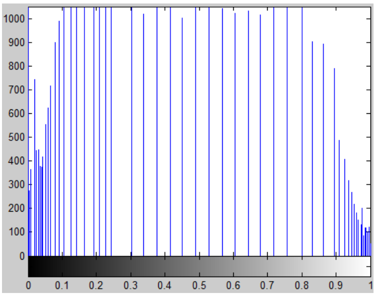
(b)


(c)


Pic.3



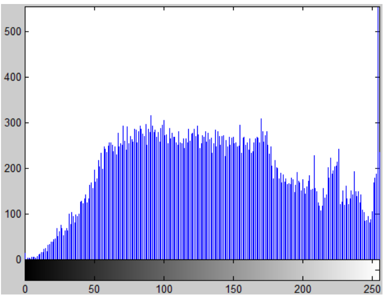
(a)



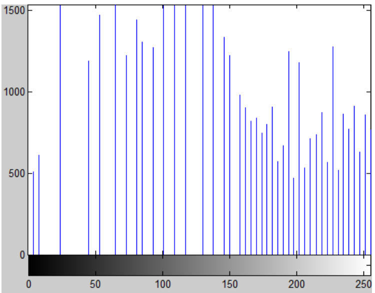
(b)


(c)


Pic.4

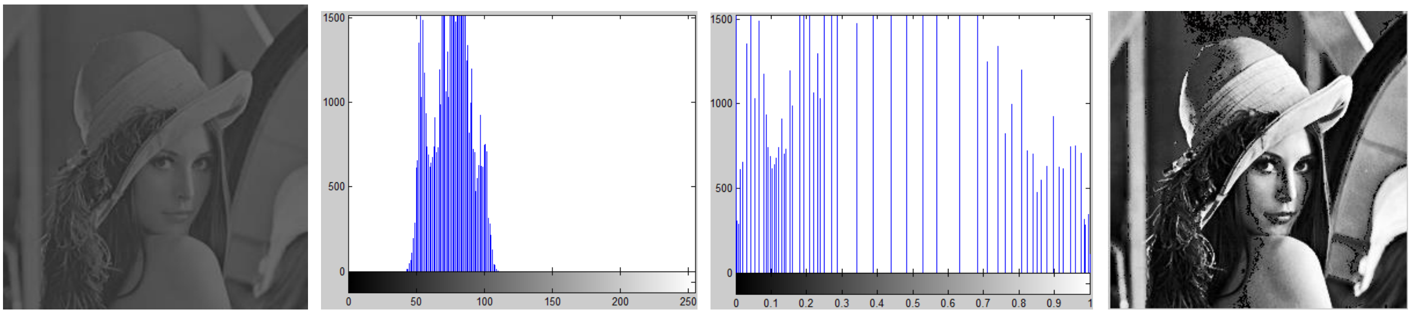


(a)

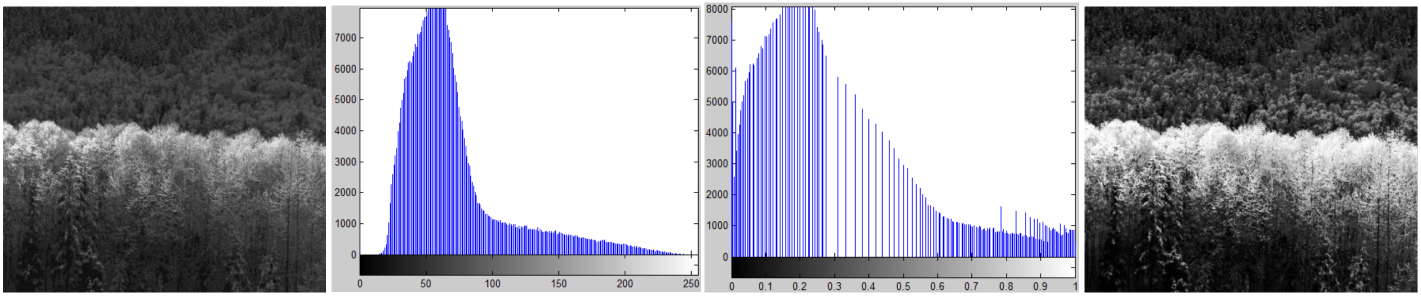


(b)

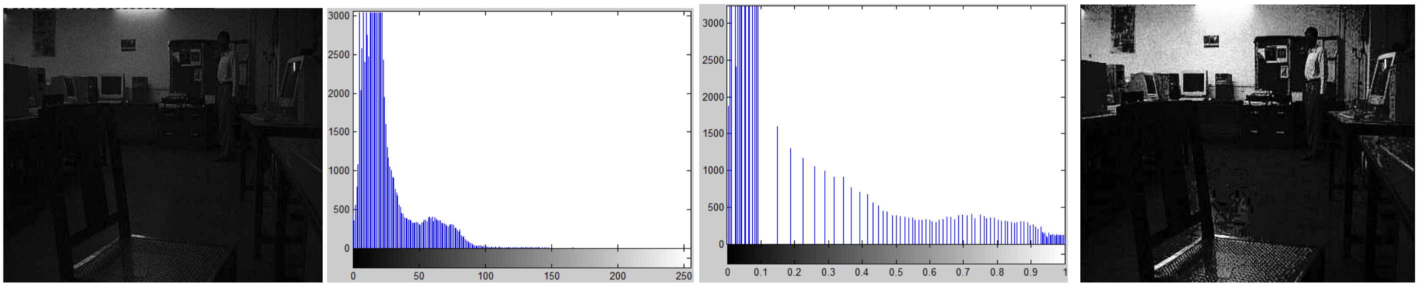

(c)



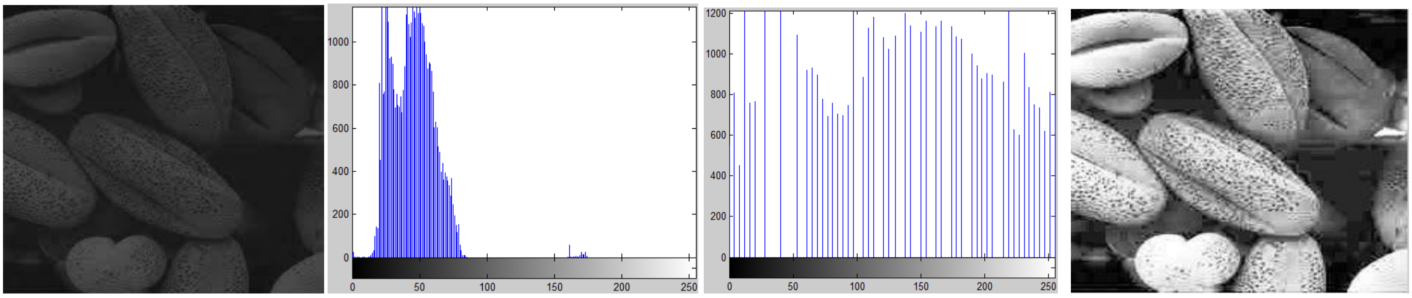
Pic.5 (a) (b) (c)



Pic.6 (a) (b) (c)



Pic.7 (a) (b) (c)



Pic.8 (a) (b) (c)

Fig.5
RSIHE implemented on Pic.1, Pic.2, Pic.3, Pic.4, Pic.5, Pic.6, Pic.7 and Pic.8
(a)The histogram of the original image
(b)The histogram of the equalized image
(c)The equalized image

Table (1)
indicates the AMBE values of all
image contrast enhancement methods

Test image	HE	CLAHE	BBHE	DSIHE	RSIHE
Pic.1	56.7791	16.2905	58.143	50.261	30.743
Pic.2	70.958	32.5792	59.336	60.965	36.932
Pic.3	56.3954	15.6309	78.115	59.799	40.354
Pic.4	71.6850	28.341	85.793	80.69	70.685
Pic.5	64.791	24.347	40.347	59.336	40.022
Pic.6	60.818	38.348	76.951	74.739	53.523
Pic.7	80.062	35.223	46.111	60.965	38.924
Pic.8	85.621	34.726	83.939	60.162	58.204

Table (2)
indicates PSNR values of all
image contrast enhancement methods

Test image	HE	CLAHE	BBHE	DSIHE	RSIHE
Pic.1	11.622	22.116	19.833	19.801	20.762
Pic.2	6.3419	16.459	6.643	8.642	10.622
Pic.3	11.946	21.841	11.456	13.437	18.242
Pic.4	9.2983	17.403	7.642	9.643	15.223
Pic.5	10.082	18.339	18.339	12.418	14.562
Pic.6	10.965	14.869	9.947	9.944	10.226
Pic.7	6.384	16.023	18.102	18.09	18.257
Pic.8	7.921	15.631	14.743	14.717	15.231

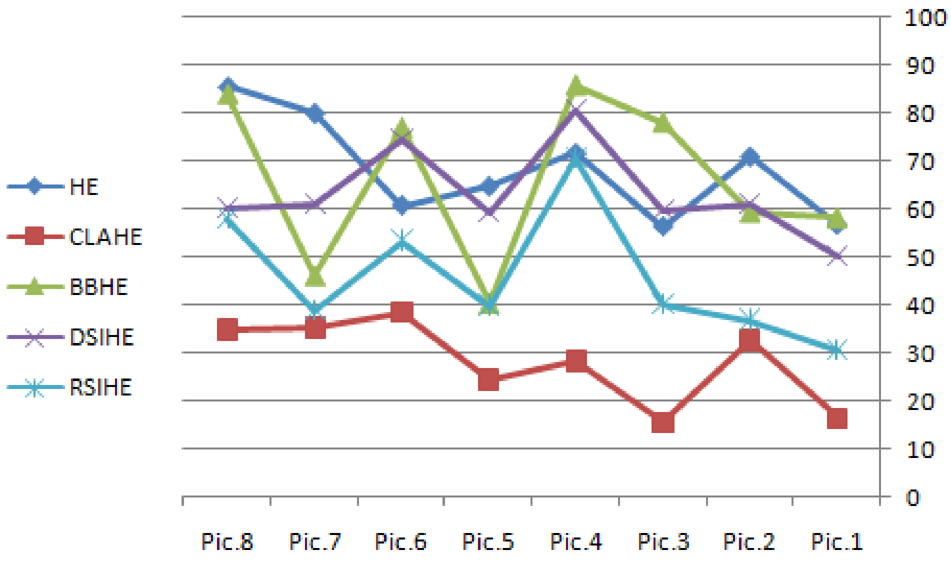


Fig.5
A diagram indicates the use of AMBE values

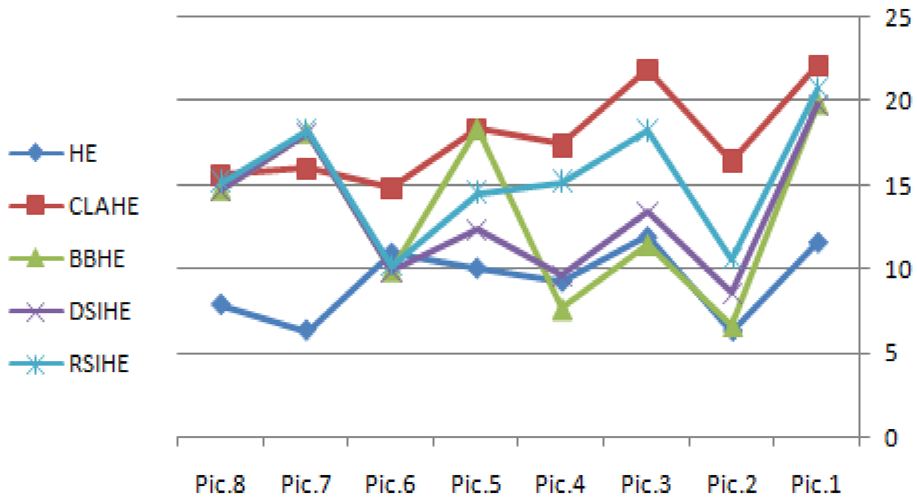


Fig. 6
A diagram indicates the use of PSNR values

6.5. Discussion:

- Assessment of brightness perservation :
  The results, shown in table(1), presentthe AMBE(eq.7) values of various methods discussed in this work. AMBE describes the brightness error, so the lower AMBE value indicates better brightness preservation. Depending on the observation of table (1) we see that CLAHE is best brightness perservation.
- Assessment of contrast enhancement:
  The results shown in table(2) present the PSNR values (eq.8) values of various methods discussed in this work**.** PSNR describes the image quality, the greater the PSNR the better image quality is. Based on the observation of table (2), we see that CLAHE has the greater PSNR values of the approximately all used images. So CLAHE is the best contrast enhancement.
- In RSIHE, the histogram is divided into 4 sub-histograms (as the recursion level is defined as 2). The method gives better enhancement than DSIHE, but the enhancement performance will be very less if the recursion level is large.

7. Conclusion:

This work presents a study of different histogram equalization based images enhancement methods. After the observation of the experimental results of brightness perservation, we observed that the brightness perservation is not handled well by HE, BBHE, DSIHE and RSIHE, but it can be handled properly by CLAHE which offers better brightness perservation and better contrast enhancement compared to other methods.

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