# الخصائص الجماعية لنظائر $W^{164-174}$ الزوجية – الزوجية

**ميسر فتحي فاضل*          عماد ممدوح احمد***

*قسم الفيزياء للعلوم الصرفة, كلية التربية, جامعة الموصل

**تاريخ الاستلام          تاريخ القبول**

**2018/11/29          2019/2/27**

**الملخص**

تم في هذا البحث حساب حالات الطاقة في الحزمة الارضية (GSB) وحزمة التماثل السالبة (NPB) لنظائر $^{164-174}W$ الزوجية–الزوجية. استُخدم انموذج بور–موتلسون (BM) وانموذج البوزونات المتفاعلة الاصدار الاول (IBM−1) وانموذج البوزونات الاتجاهية المتفاعلة (IVBM) لهذا الغرض. يتطلب انموذج (IBM−1) تحديد خاصية النواة لاستخدام المعادلات المناسبة في حسابات حالات الطاقة, لذلك استخدمت طرائق عدة لتحديد هذه الخصائص, فحالة التهيج الاولى $(E2_1^+)$ والنسبة بين حالة التهيج الثانية الى حالة التهيج الاولى$\left(R_{4/2} = E4_1^+/E2_1^+\right)$ تقدمان معلومات اولية عن خصائص النواة عند حالات التهيج الواطئة, وتقدم منحنيات (E−GOS) التي تمثل العلاقة بين طاقة كاما مقسومة على البرم $\left(E_\gamma/J\right)$ دالة للبرم$(J)$ للتعرف على شكل النواة, وتقدم النسبة بين حالات التهيج المختلفة الى الحالة التي تسبقها $r(J+2/J)$ معلومات عددية عن خاصية النواة عند كل حالة في (GSB) ولكل نواة,عند حالات التهيج المختلفة. كما تم استخدام ظاهرة التأرجح $\left(\Delta E_{1,\gamma}(J)\right)$ في فروقات الطاقة العملية بين حالات الحزمة الارضية وحالات الحزمة السالبة لمعرفة امكانية حصول تغير في طور النواة. لقد اظهرت هذه الطرائق الخصائص الانتقالية U(5)−O(6) للنظيرين $^{164}W$ و $^{166}W$, والخصائص العامة U(5)−O(6)−SU(3) للنظير $^{168}W$ والخصائص الانتقالية O(6)−SU(3) للنظيرين $^{170}W$, $^{172}W$ والخصائص الدورانية SU(3) للنظير $^{174}W$.

**الكلمات الدالة:** منحنيات E−GOS، النسبة بين طاقة حالتين متتابعتبن، التأرجح، انموذج البوزونات المتفاعلة الاصدار الاول، انموذج البوزونات الاتجاهية المتفاعلة.

# The Collective Properties of Even-Even $^{164-174}W$ Isotopes

**Muyasser Fatehi Fadil*, Imad Mamdouh Ahmed***

*Department of Physics, College of Education for Pure Science, University of Mosul.

**Received** 29/11/2018 **Accepted** 27/02/2019

## Abstract

Ground states bands (GSB) and negative parity band (NPB) of $^{164-174}W$ isotopes have been calculated. Bohr-Mottelson (BM), Interacting boson approximation-1(IBM-1) and Interacting vector boson model (IVBM) were used for this purpose. The principal excited state ($E2_1^+$) and the proportion of the second to the primary excited state ($R_{4/2} = E4_1^+/E2_1^+$) provide primary information about the properties of the nucleus. The ratio of the gamma energy, over spin $E_\gamma/J$ of each state as a function of the angular momentum $(J)$ (E-GOS), has been assessed to decide the ground states property of each nucleus. The ratio of energies of $(J+2)$ and $(J)$, states as a function of the angular momentum $(J)$, have been attracted to decide numerically the properties of the ground states band of all states and all nuclei. The $\Delta J = 1$ staggering between octupole band and ground state band is found to display a beat pattern as a function of angular momentum $(J)$. The methods which were used, showed the transitional U(5)-O(6) properties of $^{164-166}W$, the general properties that U(5)-O(6)-SU(3) of $^{168}W$, and the transitional properties O(6)-SU(3) of $^{170-172}W$; while $^{174}W$ showed the SU(3) properties.

**Keywords:** E-GOS Curves, The ratio between the energies of $J$ and $J+2$, IBM-1, IVBM.

## Introduction

The properties of various even-even nuclei move with the amount of constituent protons and neutrons, what's more, relating the adjustments in the excitation of energy states [1,2]. Energy levels of even-even nuclei can be assembled into ground states band (GSB) with head state $k^\pi = 0^+$, $\beta$-band with head state $k^\pi = 0^+$, $\gamma$-band with head state $k^\pi = 2^+$ and negative parity band (NPB) with $J^\pi = 1^-, 3^-, 5^-, ...$ [3]. At some high angular momentum ($J$), the GSB and the NPB are interwoven forming a single octupole band with $J^\pi = 0^+, 1^-, 2^+, 3^-, ...$ [4-7]. Depending on an essential data, one can get the properties of the nucleus from the principal excited state which roughly measures up to 100, 300 and 500 keV, and the ratio of the second to the principle energy state ($R\,4/2 = E4_1^+/E2_1^+$) which obeys $3 < R4/2 \leq 3.3$ , $2.4 < R4/2 \leq 3$ and $2 \leq R4/2 \leq 2.4$ for rotational, $\gamma$-soft and vibration nuclei respectively [8]. The relation between the gamma energy over angular momentum $E_\gamma/J$ as a function of the angular momentum $J$ (E-GOS) curve is introduced by Regan and his colleagues [9];

they indicate a good information about the evolution that may occur in the yrast-line of the nuclei.

In octupole band of even-even nuclei, the energy levels with odd angular momentum and negative parity, interweave with the energy levels with even angular momentum and positive parity [11]. This behavior is known as the odd-even staggering; the examination and the understanding of this effect convey a definite data about the fine properties of the nucleus.

The proportion between the energies of $J+2$ and $J$ states $r\,(J+2/J)$ in GSB gives a numerical convenient sign of the property of the nucleus [10]. Applying this relationship to a set of different nuclei, the characteristics of each nucleus were determined by the numerical values of $r\,(J+2/J)$; which takes the limits: $0.1 \le r \le 0.35$ for the vibrational nuclei, $0.4 \le r \le 0.6$ for the transitional nuclei, and $0.6 \le r \le 1.0$ for the rotational nuclei.

Bohr and Mottelson (BM) showed a relation of the rotational energy $E$ of an axially symmetric nucleus as a function of $J(J+1)$ [3]. In interacting boson model IBM-1 examines, the different states schemes properties were all around approximation in terms of the U(6) unitary group. The group reduction scheme of the U(6) produces three limits that end in O(2) group. The possible limits are the vibration U(5), the $\gamma-soft$ O(6) and the rotation SU(3) [12]. Another phenomenological study, indicated that nuclei might have been an intrmediate structure: the U(5)-SU(3), U(5)-O(6) and SU(3)-O(6) limits or among the all three limits U(5)-O(6)-SU(3) [13]. The interacting vector boson model IVBM depends on two sorts of vector bosons; the proton p and the neutron n bosons, that comprise the energy levels in the nucleus. The IVBM was developed by Ganev *et al* [14]; it was used to describe the ground and octupole bands states, the excited of the nucleus. In the present work, the energy levels of the ground states and the negative parity band NPB of $^{164-174}W$ isotopes were calculated using BM, IBM-1 and IVBM and they were compared with their measured counterparts.

## Methodes of Calculations

The E-GOS strategy gives extraordinary knowledge about the advacement in the shape of even-even nuclei when the plot of $E_\gamma/J$ versus $J$ is set up. The vibration nuclei in the E-GOS curve drop fast from the most critical regard (≈250 keV/$\hbar$) at $2_1^+$ state to (0) at ($J\to\infty$). For $\gamma$-soft nucleus, the curve drops slow from the value (≈150 keV/$\hbar$) at $2_1^+$ state to the quartered of the principle energized state at ($J\to\infty$); for rotational nuclei the curve rises steadily from the most diminutive regard (≈50 keV/$\hbar$) at $2_1^+$ state to $4\hbar^2/2\vartheta$ at ($J\to\infty$) [9]. The relations between $E_\gamma/J$ vers $J$, for three types of the nuclei, are shown by [15,16]:

Vibrator: $$R=\frac{\hbar\omega}{J}\to 0 \text{ when } J\to\infty \quad (3)$$

$\gamma$-soft: $$R=\frac{E2_1^+}{4}\left(1+\frac{2}{J}\right)\to\frac{E2_1^+}{4} \quad \text{when } J\to\infty \quad (4)$$

Rotor: $$R=\frac{\hbar^2}{2\vartheta}\left(4-\frac{2}{J}\right)\to 4\frac{\hbar^2}{2\vartheta} \quad \text{when } J\to\infty \quad (5)$$

where $\vartheta$ is the moment of inertia.

The odd-even staggering (or $\Delta J=1$ staggering) determined by [10]:

$$\Delta E_{1,\gamma}(J)=\frac{1}{16}\left(6E_{1,\gamma}(J)-4E_{1,\gamma}(J-1)-4E_{1,\gamma}(J+1)+E_{1,\gamma}(J-2)+E_{1,\gamma}(J+2)\right) \quad (6)$$

Where $E_{1,\gamma}(J) = E(J+1) - E(J)$. The $\Delta E_{1,\gamma}(J)$ values show benefits of exchanging sign along various values of angular momentum. Odd-even staggering begins generally at high values and after that it is reduced to expanding angular momentum, and may reach a vanishing value ($\Delta E_{1,\gamma}(J) = 0$); the staggering starts raising again and afterward dropping once more; it gives a general picture of beats, at the point when the staggering ($\Delta E_{1,\gamma}(J) = 0$), a phase change in the nucleus occurs [6].

The proportion between the successive energy levels of $J+2$ and $J$ as a function of $J$, was developed to characterize the symmetry of the excited states of even-even nuclei which is shown by [10]:

$$r\left(\frac{J+2}{J}\right) = \left[\left(\frac{R(J+2)}{J}\right)_{exp.} - \frac{(J+2)}{J}\right] \times \frac{J(J+1)}{2(J+2)} \qquad (7)$$

where $\left(\frac{R(J+2)}{J}\right)$ is the measured energy values ratio between $J+2$ and $J$ states.

In BM model, the energy $E(J)$ of rotational nuclei can be expanded in power of $J(J+1)$; the GSB and the NPB levels were given by [3,6]:

$$E(J) = AJ(J+1) - BJ^2(J+1)^2 + CJ^3(J+1)^3 \qquad (8)$$

$$E(J) = E_\circ + AJ(J+1) - BJ^2(J+1)^2 + CJ^3(J+1)^3 \qquad (9)$$

where $E_0$ is the band head energy of the NPB. The values of A, B, and C parameters depend on the properties of the nucleus.

In IBM-1 the general Hamiltonian is given by [19]:

$$\mathrm{H} = \sum_{i=1}^{N} \varepsilon_i + \sum_{i<1}^{N} V_{i\,j} \qquad (10)$$

Where $\varepsilon$ is the intrinsic boson energy and $V_{i\,j}$ is the interaction strength between bosons $i$ and $j$, the multipole form the Hamiltonian is [18]:

$$H = \varepsilon\hat{n}_d + a_0\hat{p}^{+}\hat{p} + a_1\hat{L}.\hat{L} + a_2\hat{Q}.\hat{Q} + a_3\hat{T}_3.\hat{T}_3 + a_4\hat{T}_4.\hat{T}_4 \qquad (11)$$

where $a_0, a_1, a_2, a_3$ and $a_4$ are qualities of pairing, angular momentum, quadrupole, octupole and hexadecupole associated terms, respectively. The Hamiltonian as far as multipole development will in general decrease three structures to meet the necessities of the three symmetry restrains, the U(5), SU(3) and O(6). In U(5) limit, the viable parameter is $\varepsilon$, in the O(6) the overwhelming parameter is $a_0$and in SU(3) limit the successful parameter is $a_2$ For three types of nuclei, the eigenvalues is given by [17]:

U(5):$E(\varepsilon, n_d, \nu, L) = \varepsilon n_d + K_1 n_d(n_d+4) + K_4\nu(\nu+3) + K_5 L(L+1)$ (12)

O(6):$E(\sigma, \tau, L) = K_3[N(N+4) - \sigma(\sigma+4)] + K_4\tau(\tau+3) + K_5 L(L+1)$ (13)

SU(3):$E(\lambda, \mu, L) = K_2(\lambda^2 + \mu^2 + 3(\lambda+\mu) + \lambda\mu) + K_5 L(L+1)$ (14)

where $K_1, K_2, K_3, K_4$ and $K_5$ are other forms of the strength of the parameters. Numerous nuclei have properties between the U(5), O(6) and SU(3) limits and their eigenvalues for the yrast states are givin by [12]:

U(5)-O(6):$E(\varepsilon, n_d, \tau, L) = \varepsilon n_d + K_1 n_d(n_d+4) + K_4\tau(\tau+3) + K_5 L(L+1)$ (15)

U(5)-SU(3):$E(\varepsilon, \lambda, L) = \varepsilon n_d + K_2\left(\lambda^2 + 3(\lambda+\mu)\right) + K_5 L(L+1)$ (16)

O(6)-SU(3):$E(\tau, \lambda, L) = \varepsilon n_d + K_2\left(\lambda^2 + 3(\lambda+\mu)\right) + K_4\tau(\tau+3) + K_5 L(L+1)$ (17)

The eigenvalues for the GSB and NPB states in IVBM are given by [7]:

$$E(J) = \beta J(J+1) + \gamma J \qquad (18)$$

$$E(J) = \beta J(J+1) + (\gamma+\eta)J + \zeta \qquad (19)$$

The estimations of $\beta$ and $\gamma$ can be resolved from a fit to the positive ground states band while $\eta$ and $\zeta$ are assessed from the negative ones. Eqs. (18,19) demonstrate that the eigenstates of the GSB and NPB comprise of rotational $J(J+1)$ and vibration $J$ modes.

## Results and Discussion

The excitation energy states of a nucleus give a primary information about the property of the nucleus. In the present study, the first excited state ($E2_1^+$) and the ratios of the second excited state to the first excited state ($R_{4/2}$) of $^{164-174}W$ isotopes were investigated; the data were listed in table 1. The table shows that $^{164}W$ has a gamma-soft property, $^{166-170}W$ have a gamma-soft-rotational property and $^{172-174}W$ have a pure rotational property.

| Nucleus | $^{164}W$ | $^{166}W$ | $^{168}W$ | $^{170}W$ | $^{172}W$ | $^{174}W$ |
|---|---|---|---|---|---|---|
| $E2_1^+$ | 333 | 252 | 199 | 157 | 123 | 113 |
| $R = E4_1^+/E2_1^+$ | 2.47 | 2.678 | 2.82 | 2.949 | 3.065 | 3.15 |

Table 1. The measured values of $E2_1^+$ in (keV) and the ratios ($R_{4/2} = E4_1^+/E2_1^+$) of $^{164-174}W$ even-even isotopes [20].

Table.1 gives a primary information of the nuclei in the lower levels, but it does not give a complete information about the properties of the nucleus at their different excited states which may change. That is why the E-GOS of the measured $E_\gamma/J$ is drawn as a function for each isotope and shown in Fig.1

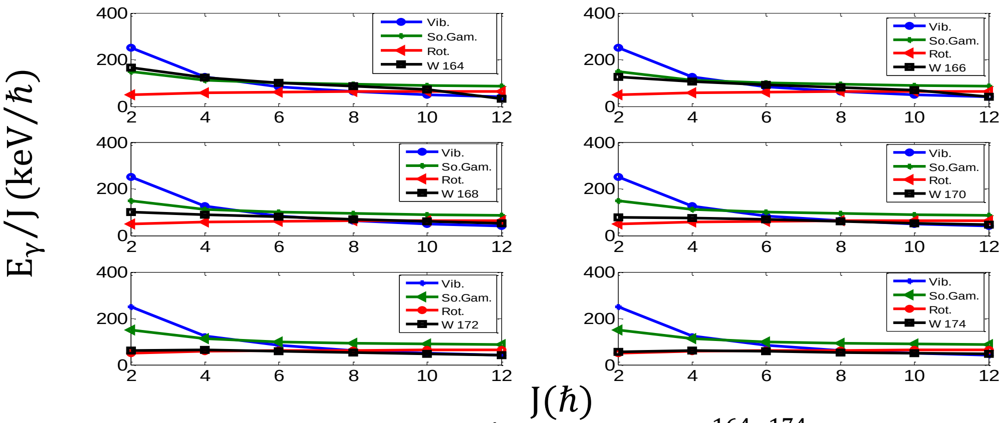


Fig 1. The E-GOS curves ($E_\gamma/J(keV/\hbar)\ Vs. J\ (\hbar)$) of $^{164-174}W$ isotopes, compared with the standard curves for U(5), SU(3) and O(6) limits

Fig. 1, shows that $^{164-172}W$ change their properties along their excited states while $^{174}W$ isotope has a pure rotational property.

To confirm the properties of these nuclei, the staggering between the GSB and NPB energy levels of $^{164-174}W$ isotopes have been drawn. Fig. 2. shows that the staggering of $^{164-174}W$ does not achieve a vanishing value ($\Delta E_{1,\gamma}(J) = 0$); it demonstrates that these nuclei do not change their modes, while the staggering of $^{164,166,168}W$ approach the vanishing esteems and expand again showing an adjustment in their properties.

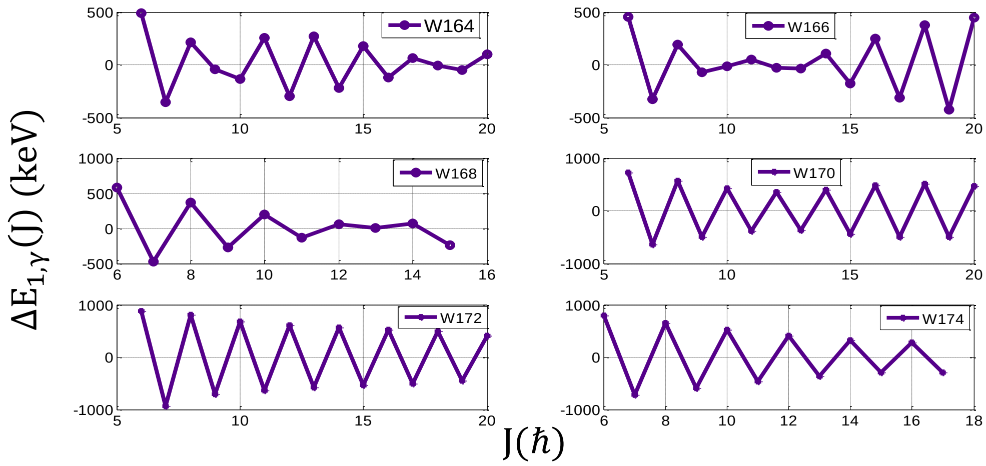


Fig. 2. The staggering factor $\Delta E_1,\gamma(J)(keV)\ Vs.J(\hbar)$ of $^{164-174}W$ isotopes

The first excited state $E2_1^+$, the ratio of the second energy level to the first energy level $R_{4/2}$, the E-GOS curves and the staggering of the examined nuclei give good information about the nuclei, but do not give the special property of each nuclei. In this manner, the connection between the various energized states and those proceeding them has been considered for each isotope along the yrast states. Fig. 5. exhibits that the estimation of $r(((J+2))\ /\ (J))$ for $^{164-166}W$ isotopes changes from the exceptional estimation of the gamma-soft passing the estimations of vibrational properties which implies that this isotope holds fast to these properties.

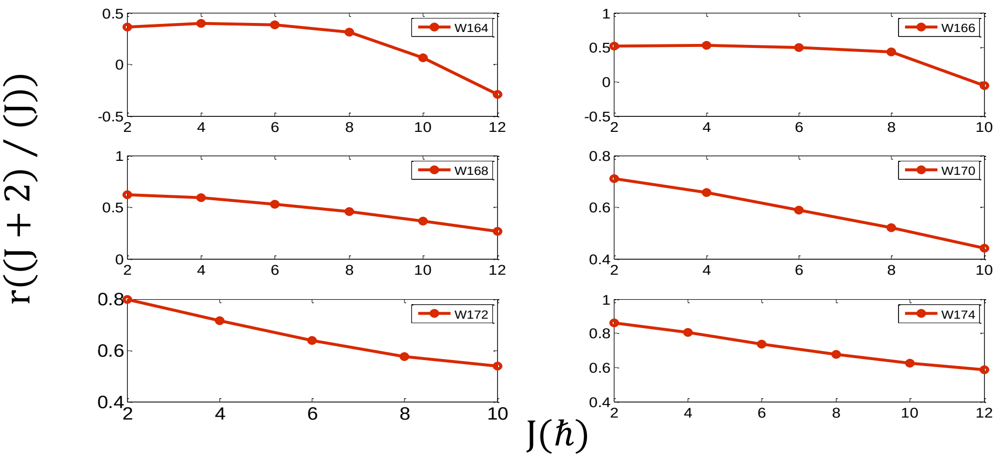


Fig. 3. The ratio $r((J+2)/J)$ Vs. $J(\hbar)$ of $^{164-174}W$ isotopes.

The $r(((J+2))\ /\ (J))$ values for $^{168}W$ change from the rotational properties passing the estimations of the γ-soft as far as possible which give the rotational-gamma-soft- vibrational properties of this isotope. The properties of alternate isotopes $^{170-172}W$ , change from the rotational to the gamma-soft cutoff points giving them these properties, while the properties of $^{174}W$ are pure rotational.

Computing programs were composed utilizing MATLAB 2013a to calculate the energy levels of the GSB utilizing IBM-1, BM and IVBM models. While BM and IVBM strategies were used to calculate the energy levels of NPB. The quantity of bosons and the best estimations of the parameters which gives the best fitting among hypothetical and estimated yrast energy levels are illustrated in Table 2. The best fitting parameters of NPB of BM and IVBM are shown in Table 3.

Tables 2. IBM-1, IVBM and BM parameters in (keV) used to calculate the yrast energy levels for $^{164-174}W$ even-even isotopes.

| Nucl | IBM-1 | | | | | | IVBM | | BM | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | N | $\mathcal{E}$ | $k_1$ | $k_2$ | $k_4$ | $k_5$ | $\beta$ | $\gamma$ | A | B* $10^{-2}$ | C* $10^{-4}$ |
| $^{164}W$ | 8 | 483 | …. | …… | …. | 2.25 | 2.25 | 242 | 40 | 15.5 | 2.4 |
| $^{166}W$ | 9 | 631 | …. | ….. | …. | 2.21 | 2.21 | 209 | 35 | 13.3 | 2 |
| $^{168}W$ | 10 | 290 | 0.68 | 0.01 | 0.04 | -0.04 | 3.2 | 169 | 29 | 9.8 | 1.4 |
| $^{170}W$ | 11 | ..…. | …. | 74 | …… | -199 | 6 | 117 | 23 | 5.6 | 0. 63 |
| $^{172}W$ | 12 | ….. | …. | 40 | …… | -87.4 | 8.1 | 68 | 19 | 4.2 | 0. 64 |
| $^{174}W$ | 13 | ….. | …. | 8.9 | ….. | 22.8 | 9.34 | 57 | 18 | 3.8 | 0. 38 |

Table 3. IVBM and BM parameters in (keV) used to calculate the negative parity band energy levels for $^{164-174}W$ even-even isotopes.

| Nucleus | IVBM | | BM | | | |
|---|---|---|---|---|---|---|
| | η | $\zeta * 10^2$ | $E_o$ | A | B | C∗ $10^{-4}$ |
| $^{164}W$ | -84.9 | 9.35 | 1758 | 0.37 | -0.11 | -3 |
| $^{166}W$ | -44.1 | 6.61 | 1587 | 2.95 | -0.063 | -1.64 |
| $^{168}W$ | -31.38 | 7.113 | 1536 | 1.79 | -0.072 | -1.85 |
| $^{170}W$ | -38.8 | 9.246 | 1517 | 3.26 | -0.044 | -0.9 |
| $^{172}W$ | -39 | 11.187 | 1762 | -1.667 | -0.069 | -1.2 |
| $^{174}W$ | -79.6 | 4.08 | 1401 | -0.043 | -0.05 | -1 |

The measured energy levels of the yrast states, and the calculated ones of IBM-1, IVBM and BM are appeared in table 4. It is Obvious that the calculated energy states are in agreement with the experimental ones for all $^{164-174}W$ isotopes and for all states. The BM and IBM-1 calculations are superior than the IVBM. In the $^{164,166}W$ the BM estimation is the best.

Table 4. The experimental and calculated energy levels (keV) of the ground states band of the even-even $^{164-174}W$ isotopes.

| $^{164}W$ | | | | | $^{166}W$ | | | |
|---|---|---|---|---|---|---|---|---|
| $J_1^+$ | $E_{exp.}$ | $E_{cal.}$ | | | $E_{exp.}$ | $E_{cal.}$ | | |
| | | IBM-1 | IVBM | BM | | IBM-1 | IVBM | BM |
| 2 | 333 | 497 | 497 | 235 | 252 | 432 | 432 | 205 |
| 4 | 824 | 1011 | 1011 | 742 | 675 | 881 | 881 | 659 |
| 6 | 1432 | 1544 | 1544 | 1428 | 1226 | 1348 | 1348 | 1251 |
| 8 | 2118 | 2095 | 2095 | 2173 | 1865 | 1833 | 1833 | 1909 |
| 10 | 2833 | 2663 | 2663 | 2857 | 2551 | 2336 | 2336 | 2516 |
| 12 | 3441 | 3250 | 3250 | 3404 | 3031 | 2856 | 2856 | 3005 |
| 14 | 3834 | 3854 | 3854 | 3838 | 3356 | 3394 | 3394 | 3388 |
| 16 | 4343 | 4473 | 4473 | 4346 | 3821 | 3949 | 3949 | 3812 |
| $^{168}W$ | | | | | $^{170}W$ | | | |
| $J_1^+$ | $E_{exp.}$ | $E_{cal.}$ | | | $E_{exp.}$ | $E_{cal.}$ | | |
| | | IBM-1 | IVBM | BM | | IBM-1 | IVBM | BM |
| 2 | 199 | 357 | 357 | 171 | 157 | 269 | 269 | 135 |
| 4 | 562 | 740 | 740 | 543 | 463 | 586 | 586 | 436 |
| 6 | 1042 | 1148 | 1148 | 1058 | 876 | 949 | 949 | 867 |
| 8 | 1600 | 1582 | 1582 | 1635 | 1364 | 1359 | 1359 | 1380 |
| 10 | 2202 | 2042 | 2042 | 2192 | 1902 | 1816 | 1816 | 1922 |
| 12 | 2817 | 2527 | 2527 | 2669 | 2465 | 2321 | 2321 | 2443 |
| 14 | 3419 | 3038 | 3038 | 3057 | 2911 | 2872 | 2872 | 2915 |
| 16 | 4003 | 3574 | 3574 | 3435 | 3344 | 3470 | 3470 | 3345 |
| $^{172}W$ | | | | | $^{174}W$ | | | |
| $J_1^+$ | $E_{exp.}$ | $E_{cal.}$ | | | $E_{exp.}$ | $E_{cal.}$ | | |
| | | IBM-1 | IVBM | BM | | IBM-1 | IVBM | BM |
| 2 | 123 | 185 | 185 | 111 | 113 | 170 | 170 | 106 |
| 4 | 377 | 435 | 435 | 358 | 356 | 415 | 415 | 345 |
| 6 | 728 | 751 | 751 | 716 | 706 | 734 | 734 | 698 |
| 8 | 1147 | 1131 | 1131 | 1152 | 1139 | 1128 | 1128 | 1141 |
| 10 | 1617 | 1577 | 1577 | 1633 | 1638 | 1597 | 1597 | 1647 |
| 12 | 2130 | 2088 | 2088 | 2136 | 2189 | 2141 | 2141 | 2194 |
| 14 | 2680 | 2663 | 2663 | 2664 | 2785 | 2760 | 2760 | 2774 |
| 16 | 3256 | 3304 | 3304 | 3261 | 3397 | 3453 | 3453 | 3401 |

Table 5. The experimental and calculated energy levels (keV) of the negative parity band for $^{164-174}W$ even-even isotopes.

| $^{164}W$ | | | | $^{166}W$ | | | $^{168}W$ | | |
|---|---|---|---|---|---|---|---|---|---|
| $J^-$ | $E_{exp.}$ | $E_{cal.}$ | | $E_{exp.}$ | $E_{cal.}$ | | $E_{exp.}$ | $E_{cal.}$ | |
| | | IVBM | BM | | IVBM | BM | | IVBM | BM |
| 5 | 1758 | 1788 | 1865 | 1587 | 1553 | 1729 | 1536 | 1495 | 1650 |
| 7 | 2181 | 2161 | 2090 | 1928 | 1941 | 1924 | 1834 | 1854 | 1830 |
| 9 | 2632 | 2552 | 2517 | 2337 | 2346 | 2250 | 2213 | 2238 | 2147 |
| 11 | 2906 | 2960 | 3162 | 2743 | 2769 | 2711 | 2628 | 2648 | 2604 |
| 13 | 3326 | 3387 | 3935 | 3174 | 3210 | 3247 | 3073 | 3083 | 3135 |
| 15 | 3877 | 3832 | 4573 | 3720 | 3669 | 3698 | 3577 | 3544 | 3559 |
| $^{170}W$ | | | | $^{172}W$ | | | $^{174}W$ | | |
| $J^-$ | $E_{exp.}$ | $E_{cal.}$ | | $E_{exp.}$ | $E_{cal.}$ | | $E_{exp.}$ | Ecal. | |
| | | IVBM | BM | | IVBM | BM | | IVBM | BM |
| 5 | 1517 | 1421 | 1652 | | | | 1401 | 1288 | 1443 |
| 7 | 1792 | 1808 | 1823 | 1762 | 1776 | 1863 | 1676 | 1671 | 1541 |
| 9 | 2154 | 2213 | 2104 | 2106 | 2111 | 2079 | 1999 | 2072 | 1742 |
| 11 | 2578 | 2637 | 2512 | 2519 | 2508 | 2453 | 2396 | 2491 | 2072 |
| 13 | 3036 | 3078 | 3033 | 2992 | 2971 | 2977 | 2862 | 2929 | 2532 |
| 15 | 3538 | 3537 | 3602 | 3511 | 3499 | 3571 | 3388 | 3384 | 3075 |
| 17 | 4095 | 4015 | 4071 | 4067 | 4092 | 4046 | 3969 | 3857 | 3578 |

## Conclusions

Taking everything into account: the value of the first energy level, the ratio of the second energy level to the first energy level, the E-GOS curves and the proportion r($(J+2)/(J)$)of the GSB and NPB for $^{164-174}W$ isotopes, ensured that $^{164-166}W$ isotopes have a vibrational-γ-soft properties; while $^{168}W$ isotope has a general properties which are vibrational-gamma-soft-rotational, and $^{170-172}W$ which have transitional properties which are gamma-soft-rotational, and $^{174}W$ has pure rotational properties. The BM, IBM-1 and IVBM that are demonstrated have been utilized to compute the energy levels of the GSB for $^{164-174}W$ isotopes. The NPB of $^{164-174}W$ isotopes are determined using the IVBM and BM models, the registered energy levels are in agreement with the experimental values. We have explained that the relation between the GSB and the NPB, in the examined isotopes, display the staggering and the vanishing value of it, $\Delta E_{1,\gamma}(J) = 0$, occur for $^{164-168}W$.

## References


[1] Iachello F. and Talmi I.,Mod.Phys.59,339(1987).

[2] Greiner W. and Maruhn J.A.,"Nuclear Models",Vol.II, Springer-Verlag Heidelberg,P.375(1996).

[3] Bohr A and Mottelson B.R.,"Nuclear Structure",Vol.II,World Scientific Publishing,P.748(1998).

[4] Raymond K.S. and Sood P.C., Phys. Rev. C 34,2362, (1986).

[5] Phillips W.R., *et al*., Phys. Rev. Lett. 57,3257 (1986).

[6] Minkov N., Drenska S.B., Raychev P.P., Roussev R.P., and Dennis Bonatsos,"Ground–γ band mixing and odd-even staggering in heavy deformed nuclei",Phys. Rev. C 61, 064301 – Published 27 April 2000.
[7] Ganev H., *et al*., Phys. Rev. C 69, 014305-7 (2004).
[8] Krane K.S., "Introductory Nuclear Physics", John Willey and Sons, 1987.
[9] Regan P.H., *et al*, "Signature for Vibrational evolution along the yrast line", phys.rev. Lett.,90(15): 152502(1-4) (2003).
[10] Bonatsos D. and Skoures L.D., "Successive energy ratio in medium and heavy mass nuclei as indicators of different kinds of collectivity, Phys.Rev. C43, 952R-956R,(1991).
[11] Khalaf A.M. and Ismail A.M., "Structure shape evolution in lanthanide and actinide nuclei", Progress in physics, Vol.2, (2013).
[12] Iachello F. and Arima A., "The Interacting Boson Model", Cambridge, (1987).
[13] Iachello F., "Dynamics Symmetries at The Critical Point", Phys. Rev. Lett. **85,** No.17, (2000).
[14] Ganev H.G., Garistov V.P. and Georgieva A.I.,"Description of the ground and octupole bands in the simplistic extension of the interacting vector boson model",Phys.Rev.C 69,014305 (2004).
[15] Scharff-Goldhaber G. and Weneser J.,"System of even-even nuclei", Phys.Rev.98,212,(1955).
[16] Bohr A. and Mottelson B.R., "Rotational states in even-even nclei", Phys.Rev. 90, 717 (1953).
[17] Scholten A.," The Interacting Boson Approximation Model And Applications", Rijks Universiteit Te Groningen, (1980).
[18] Iachello F., Abrahams K., Allart K. and Dieperink A.E.L., (Plenum Press, New York, 1981).
[19] Arias J.M., Alonso C.E., Vitturj A., Garcia-Ramos J.E., Dnkelsky J. and Frank A., "The U(5)-O(6) transition in the interacting boson model and the E(5) critical point symmetry",Phys.Rev. C68,(2003).
[20] ENSDF, http:// www.nndc.bnl.gov/ensdf (Nuclear data sheet) (2009).