*i*D-Sets and Associated Separation Axioms

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الخلاصة

في هذا البحث، قدمنا المجاميع من النمط-iD و التي تعتمد على المجاميع المفتوحة من النمط-i [12]. حيث ناقشنا العلاقة بين هذه المجاميع و أنماط اخرى من المجاميع. بالإضافة إلى ذلك ناقشنا العلاقة بين بديهيات الفصل الخاصة بهذا النمط من المجاميع مع بعضها من ناحية وعلاقتها ببديهيات الفصل من النمط-i [1] من ناحية اخرى. ايضاً قدمنا الفضاء التبولوجي المتناظر من النمط-i و فضاء i-R_0 الضعيف. اهم النتائج التي حصلنا عليها إن الفضاء المتناظر من النمط-i الموسع تبولوجياً للمجاميع المفتوحة من النمط-i يكون فضاء i-T_1 إذا وفقط اذا كان فضاء i-T_0.

Abstract

In this paper, we introduce *iD*-sets which are depends on i-open sets [12]. Where we discussed the relationship between this sets and other types of sets, In addition, we discussed the relationship between the separation axioms for this type of sets with one hand and their relationship with i-separation axioms [1] on the other hand. Also we introduce i-symmetric typological space and the weakly i-R_0space. The most important results that we have obtained i-symmetric topological space which topologically extended for i-open sets will be i- T_1space if and only if it is i-T_0 Space.

1. Introduction

In 1982, Tong [14] introduced the notion of D-sets and used these sets to introduce a separation axioms D_1 which is strictly between T_0 and T_1. In 1975, Maheshwari and Prasad [9] introduce new separation axioms semi-T_0, semi T_1 and semi- T_2 by using semi-open sets due to Levine [8]. Borsan [2] and Caldas [3] introduced the notions of sD-sets and separation axioms sD_1 which is strictly between semi- T_0 and semi- T_1. In 1980, Maheshwari and Thakur [10] introduced α-T_2. In 1993, Maki, Devi and Balachandran [11] introduced α-T_0, α-T_1, by using α-open sets due to Caldas, Georgiou and Jafari introduced the notion of αD-sets and separation axioms αD_1 which is strictly between α- T_0 and α- T_1. Jardo in [6] introduced new separation axioms i- T_0 and i- $T_1$and Askander [1] introduced i- T_2 by using i-open sets. In this paper, we introduce *iD*-sets which are depends on i-open sets [12]. Where we discussed the relationship between this sets and other types of sets, In addition, we discussed the relationship between the separation axioms for this type of sets with one hand and their relationship with i-separation axioms [1] on the other hand. Also we introduce i-symmetric typological space and the weakly i-R_0 space. The most important results that we have obtained i-symmetric topological space which topologically extended for i-open sets will be i- T_1 space if and only if it is i-T_0 Space.

2. Preliminaries

Throughout this paper, by (X, τ) and (Y, σ) (or X and Y) we always mean topological spaces. A subset A of a topological space (X, τ) is called α-open [13] (res. Semi-open [8] and i-open [12]). If $A \subseteq int(cl(int(A)))$ (resp. $A \subseteq cl(int(A))$ and $A \subseteq cl(A \cap G)$, where $\exists G \in \tau$ and $G \neq X, \phi$). The complement of an α-open (resp. Semi-open and i-open) set is called α-closed [13] (resp. Semi-closed [4] and i-closed [12]). By $iO(X)$ (resp. $iC(X)$) we denote the family of all i-open (resp. i-closed) sets of X. The intersection of all i-closed subsets of X containing A is called the i-closure [12] of A, denoted by $cli(A)$. A subset A is i-closed if $A = cli(A)$. Recall that from [12] if (X, τ) is a topological space then X is called topologically extended for i-open sets (briefly, T. E. I.) if the family of all i-open sets of X is a topology on X.

Lemma 2.1 [6] Let (X, τ) be a topological space and $A \subseteq X$, then $cli(A)$ is i-closed, i.e. $cli\big(cli(A)\big) = cli(A)$.

Lemma 2.2 [6] For subsets, A, B of a topological space (X, τ), the following statement hold:

1) $A \subseteq cli(A)$,
2) If $A \subseteq B$, then $cli(A) \subseteq cli(B)$.

Lemma 2.3

1) Every open set in a topological space is an α-open set [13].

2) Every α-open set in a topological space is a semi-open set [5].
3) Every semi-open set in a topological space is an i-open set [6].

Definition 2.1 A subset A of a topological space (X, τ) is called:

a) D-set [14] if there are $U, V \in \tau$ such that $U \neq X$ and $A = U \backslash V$,
b) αD-set [14] if there are $U, V \in \alpha O(X)$ such that $U \neq X$ and $A = U \backslash V$,
c) sD-set [14] if there are $U, V \in SO(X)$ such that $U \neq X$ and $A = U \backslash V$.

Definition 2.2 A topological space (X, τ) is said to be:

a) i-T_0 [6] if for any distinct pair of points in X, there is an i-open set containing one of the points but not the other.
b) i-T_1 [6] if for any distinct pair of points x and y in X, there is an i-open set U in X containing x but not y and an i-open set V in X containing y but not x.
c) i-T_2 [1] if for any distinct pair of points x and y in X, there is an i-open sets U and Vin X containing x and y respectively, such that $U \cap V = \phi$.

3. *i*D-Sets and Associated Separation Axioms

Definition 3.1 A subset A of a topological space (X, τ) is called an i-deference (briefly, iD-set) if there are $U, V \in iO(X)$ such that $U \neq X$ and $A = U \backslash V$.

Remark 3.1 Every i-open set U different from X is an iD-set, if $A = U$ and $V = \phi$. But, the converse is false as the next example shows.

Example 3.1 Let (X, τ) be a topological space such that $X = \{a, b, c\}$ and $\tau = \{\phi, X, \{a, b\}\}$. Then $A = \{c\}$ is an iD-set but it is not i-open. In fact, since $iO(X) = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}\}$, then $U = \{a, c\} \neq X$ and $V = \{a\}$ are i-open sets in X. Since $A = U \backslash V = \{a, c\} \backslash \{a\} = \{c\}$ is an iD-set but it is not i-open set.

Theorem 3.1

a) Every D-set is an iD-set,
b) Every αD-set is an iD-set,
c) Every sD-set is an iD-set.

Proof: Clearly, since every open (α-open, semi-open) set is an i-open set.■

The converse of Theorem 3.1 is not true. Indeed

Example 3.2 Let X and τ be as in the Example 3.1, then $O(X) = \alpha O(X) = sO(X) = \{\phi, X, \{a, b\}\}$. Hence $A = \{c\}$ is an iD-set but it is not D-set, αD-set and sD-set.

Definition 3.2 A topological space (X,τ) is called iD_0 if for any pair of distinct points x and y of X there exists an iD-set of X containing x but not y or an iD-set of X containing y but not x.

Definition 3.3 A topological space (X,τ) is called iD_1 if for any pair of distinct points x and y of X there exists an iD-set $A$of X containing x but not y and an iD-set B of X containing y but not x.

Definition 3.4 A topological space (X,τ) is called iD_2 if for any pair of distinct points x and y of X there exists disjoint iD-sets A and B of X containing x and y respectively.

Remark 3.2

(i) If (X,τ) is i-T_k, then (X,τ) is i-T_{k-1}, $k = 1,2$.

(ii) If (X,τ) is i-T_k, then (X,τ) is i-D_k, $k = 0,1,2$.

(iii)If (X,τ) is i-D_k, then (X,τ) is i-D_{k-1}, $k = 1,2$.

Theorem 3.2 For a topological space (X,τ) the following statement hold:

(1) (X,τ) is i-D_0 if and only if it is i-T_0,

(2) (X,τ) is i-D_1 if and only if it is i-D_2.

Proof. (1) The **sufficiency** is stated in Remark 3.2(ii). To prove **necessity**, let (X,τ) be i-D_0. Then for each distinct pair $x,y \in X$, at least one of x,y, say x, belongs to an iD-set G where $y \notin G$. Let $G = U_1 \backslash U_2$ such that $U_1 \neq X$ and $U_1, U_2 \in iO(X)$. Then $x \in U1$. For $y \notin G$ we have two cases:

(a) $y \notin U_1$; (b) $y \in U_1$ and $y \in U_2$.

In case (a), $x \in U_1$ but $y \notin U_1$;

In case (b), $y \in U_2$ but $x \notin U_2$. Hence X is i-T_0.

(2) **Sufficiency.** Remark 3.2(iii). **necessity**, let (X,τ) be i-D_1. Then for each distinct pair $x,y \in X$, we have iD-set G_1, G_2 such that $x \in G_1, y \notin G_1$; $y \in G_2$, $x \notin G_2$. Let $G_1 = U_1 \backslash U_2$, $G_2 = U_3 \backslash U_4$. By $x \notin G_2$, it follows that either $x \notin U_3$ or $x \in U_3$ and $x \in U_4$. Now we consider two cases:

(1) $x \notin U_3$. By $y \notin G_1$ we have two sub cases:

(a) $y \notin U_1$. By $x \in U_1 \backslash U_2$, it follows that $x \in U_1 \backslash (U_2 \cup U_3)$ and by $y \in U_3 \backslash U_4$, we have $y \in U_3 \backslash (U_1 \cup U_4)$.

Hence, $U_1 \backslash (U_2 \cup U_3) \cap U_3 \backslash (U_1 \cup U_4) = \phi$.

(b) $y \in U_1$ and $y \in U_2$. We have $x \in U_1 \backslash U_2$, $y \in U_2$, $(U_1 \backslash U_2) \cap U_2 = \phi$.

(2) $x \in U_3$ and $x \in U_4$. We have $x \in U_3 \backslash U_4$, $y \in U_4$, $(U_3 \backslash U_4) \cap U_4 = \phi$.

Therefore X i-D_2.■

Corollary 3.1 if (X,τ) is i-D_1, then it is i-T_0.

Proof. By Remark 3.2 and Theorem 3.2.■

The converse of corollary 3.1 is not true. Indeed

Example 3.2 Let (X,τ) be a topological space such that $X=\{a,b\}$ and $\tau=\{\phi,X,\{a\}\}$. Clearly the space (X,τ) is i-T_0 but it is not i-D_1.

Theorem 3.3[6] A T. E. I. X is i-T_0 if and only if for each pair of distinct points x, y of X, $cli(\{x\})=cli(\{y\})$.

Theorem 3.4[6] A T. E. I. X is i-T_1 if and only if the singletons are i-closed sets.

Definition 3.5 A set U in a topological space (X,τ) is an i-neighborhood of a point x if U contains an i-open set V such that $x\in V$.

Definition 3.6 A point $x\in X$ which has X as the unique i-neighborhood is called an i-neat point. set U in a topological space (X,τ) is an i-neighborhood of a point x if U contains an i-open set V such that $x\in V$.

Theorem 3.5 For an i-T_0 topological space (X,τ) the following are equivalent:

(1) (X,τ) is i-D_1,

(2) (X,τ) has no i-neat point.

Proof. (1)→(2). Since (X,τ) is i-D_1, then each point x of X is contained in an iD-set $O=U\backslash V$ and thus in U. By definition $U\neq X$. This implies that x is not an i-neat point.

(2)→(1). If X is i-T_0, then for each pair of distinct points x and y of X, at least one of them, x (say) has an i-neighborhood U containing x and not y. Thus U which is different from X is an iD-set. If X has no i-neat point, then y is not an i-neat point. This means that there exists an i-neighborhood V of y such that $V\neq X$. Thus $y\in(V\backslash U)$ but not x and $V\backslash U$ is an iD-set. Hence X is iD_1.■

Remark 3.3 it is clear that an i-T_0 topological space (X,τ) is not iD_1 if and only if there is a unique i-neat point in X. It is unique because if x and y are both i-neat point in X, then at least one of them say x has an i-neighborhood U containing x but not y. But this is contradiction since $U\neq X$.

Definition 3.7 A topological space (X,τ) is i-symmetric if for every x and y in X, $x\in cli(\{y\})$ implies $y\in cli(\{x\})$.

Definition 3.8[6] A subset A of a topological space (X,τ) is called an i-generalized closed set (briefly, ig-closed) if $cli(A) \subseteq U$ whenever $A \subseteq U$ and U is an i-open set of X.

Lemma 3.1[6] Every i-closed set is ig-closed.

Theorem 3.6 A T. E. I. X is i-symmetric if and only if $\{x\}$ is ig-closed set for each $x \in X$.

Proof. Assume that $x \in cli(\{y\})$ but $y \notin cli(\{x\})$. This means that $[cli(\{x\})]^c$ contains y. Therefore the set $\{y\}$ is a subset of $[cli(\{x\})]^c$. This implies that $cli(\{y\})$ is a subset of $[cli(\{x\})]^c$. Now $[cli(\{x\})]^c$ contains x which is a contradiction.

Conversely, suppose that $\{x\} \subseteq E \in iO(X)$ but $cli(\{x\})$ is not a subset of E. This means that $cli(\{x\})$ and E^c are not disjoint. Let $y \in cli(\{x\}) \cap E^c$. We have $x \in cli(\{y\})$ which is a subset of E^c and $x \notin E$. But this is a contradiction.■

Corollary 3.2 If A T. E. I. X is i-T_1 space, then it is i-symmetric.

Proof. Let X be an i-T_1 space, then singleton sets are i-closed set (Theorem 3.4) and therefore ig-closed set (Lemma 3.1). By (Theorem 3.6) X is i-symmetric.■

Corollary 3.3 For a T. E. I. X the following are equivalent:

(1) X is i-T_0 and i-symmetric,

(2) X is i-T_1.

Proof. By Corollary 3.2 and Remark 3.2 (i) it suffices to prove only (1)→(2). Let $x \neq y$ and by i-T_0, we may assume that $x \in G_1 \subseteq \{y\}^c$ for some $G_1 \in iO(X)$. Then $x \notin cli(\{y\})$ and hence $y \notin cli(\{x\})$. There exists $G_2 \in iO(X)$ such that $y \in G_2 \subseteq \{x\}^c$. Hence X is i-T_1 space.■

Theorem 3.7 Let X is i-symmetric T. E. I. the following are equivalent:

(1) X is i-T_0,

(2) X is i-D_1,

(3) X is i-T_1.

The proof is straightforward and hence omitted.■

We recall the following:

Definition 3.9[12] A function $f:(X,\tau) \to (Y,\sigma)$ is i-irresolute if the inverse image of each i-open set is i-open.

Theorem 3.8 if $f:(X,\tau) \to (Y,\sigma)$ is an i-irresolute surjective function and E is an iD-set in Y, then the inverse image of E is an iD-set in X.

Proof. Let E be an iD-set in Y**,** then there are i-open sets $U_1$and U_2 in Y such that $E = U_1 \backslash U_2$ and $U_1 \neq Y$. By the irresoluteness of f, $f^{-1}(U_1)$ and $f^{-1}(U_2)$ are i-open sets in X.since $U_1 \neq Y$, we have $f^{-1}(U_1) \neq X$. Hence $f^{-1}(E) = f^{-1}(U_1) \backslash f^{-1}(U_2)$ is an iD-set in X.■

Theorem 3.9 if (Y, σ) is iD_1 and $f: (X, \tau) \to (Y, \sigma)$ is i-irresolute and bijective, then (X, τ) is iD_1 space.

Proof. Suppose that Y is iD_1 space. Let x and y be any pair of distinct points in X. Since f is injective and Y is iD_1, then there exist iD-sets $G_{f(x)}$and $G_{f(y)}$ of Y containing $f(x)$ and $f(y)$, respectively, such that $f(y) \notin G_{f(x)}$ and $f(x) \notin G_{f(y)}$. By Theorem 3.8, $f^{-1}(G_{f(x)})$and $f^{-1}(G_{f(y)})$ are iD-sets in X containing x and y, respectively, This implies that X is an iD_1 space.■

Definition 3.10 A function $f: (X, \tau) \to (Y, \sigma)$ is called always i-open if the image of each i-open set in X is i-open in Y.

Theorem 3.10 Let X be an arbitrary space, R an equivalence relation in X and $h: X \to X \backslash R$ the identification map. If $R \subseteq X \times X$ is i-closed in $X \times X$ and h is an always i-open map. Then $X \backslash R$ is i-T_2 .

Proof. Let $h(x)$, $h(y)$ be distinct members of $X \backslash R$. Since x and y are not related, $R \subseteq X \times X$ is i-closed in $X \times X$. There are i-open sets U and V such that $x \in U$, $y \in V$ and $U \times V \subseteq R^c$. Thus $h(U), h(V)$ are disjoint and also i-open in $X \backslash R$ since h is always i-open.■

Theorem 3.11 A topological space (X, τ) is iD_1 if for each pair of distinct points $x, y \in X$, there exists an i-irresolute surjective function $f: (X, \tau) \to (Y, \sigma)$, where (Y, σ) is an iD_1 space such that $f(x)$ and $f(y)$ are distinct.

Proof. Necessity. For every pair of distinct points of X, it suffices to take the identity function of X. Sufficiency, Let x and y be any pair of distinct points in X. By hypothesis, there exists i-irresolute surjective function f of a space (X, τ) onto a iD_1space (Y, σ) such that f$(x) \neq f(y)$. Therefore, there exists iD-sets $G_{f(x)}$ and $G_{f(y)}$ in Y such that $f(x) \in G_{f(x)}$ and $f(y) \in G_{f(y)}$. Since f is i-irresolute and surjective, by theorem 3.8, $f^{-1}(G_{f(x)})$and $f^{-1}(G_{f(y)})$ are iD-sets in X containing x and y, respectively. Therefore the space X is an iD_1 space.■

Lemma 3.2 Let (X, τ) be a topological space. If $A, B \in iO(X)$, then $A \times B \in iO(X)$.

Proof. Let $A, B \in iO(X)$, then $A \subseteq cl(A \cap G_1)$ and $B \subseteq cl(B \cap G_2)$ where $G_1, G_2 \in \tau$ and $G_1, G_2 \neq \phi, X$.
Now, $A \times B \subseteq cl(A \cap G_1) \times cl(B \cap G_2) = cl\big((A \cap G_1) \times (B \cap G_2)\big) =$
$cl\big((A \times B) \cap (A \times G_2) \cap (B \times G_1) \cap (G_1 \times G_2)\big) =$
$cl([(A \times B) \cap (G_1 \times G_2)] \cap [(A \times G_2) \cap (B \times G_1)]) \subseteq cl((A \times B) \cap (G_1 \times G_2)$
where $G_1 \times G_2$ is open set [8], and $G_1 \times G_2 \neq \phi, X$.
Hence, $A \times B \in iO(X)$.■

Theorem 3.12 Let X be a T. E. I., then the following properties are equivalent:

(1) X is i-T_2,

(2) Let $x \in X$. For each $y \neq x$, there exists an i-open set U such that $x \in U$ and $y \notin cli(U)$,

(3) For each $x \in X$, $\cap\{cli(U); U \in iO(X) \text{and } x \in U\} = \{x\}$,

(4) The diagonal $\Delta = \{(x, x); x \in X\}$ is i-closed in $X \times X$.

Proof. (1)→(2). Let $x \in X$ and $y \neq x$. Then there are disjoint i-open sets U and V such that $x \in U$ and $y \in V$. Clearly , V^c is i-closed set, $cli(U) \subseteq V^c$, $y \notin V^c$ and therefore $y \notin, cli(U)$.

(2)→(3). If $y \neq x$, then there exists an i-open set U such that $x \in U$ and $y \notin cli(U)$. So $y \notin \cap\{cli(U); U \in iO(X) \text{and } x \in U\}$.

(3)→(4). We will prove that Δ^c is i-open. Let $(x, y) \notin \Delta$. Then $y \neq x$ and since $\cap\{cli(U); U \in iO(X) \text{and } x \in U\} = \{x\}$, there is some $U \in iO(X)$ where $x \in U$ and $y \notin cli(U)$. Since $U \cap \big(cli(U)\big)^c = \phi$, $U \times \big(cli(U)\big)^c$ is an i-open set such that $(x, y) \in U \times \big(cli(U)\big)^c \subseteq \Delta^c$.

(4)→(1). If $y \neq x$, then $(x, y) \notin \Delta$ and thus there exists i-open sets U and V such that $(x, y) \in U \times V$ and $U \times V \cap \Delta = \phi$. Clearly, for the i-open sets U and V we have, $x \in U$ and $y \in V$ and $U \cap V = \phi$.■

4. Weakly i-R_0 spaces

Definition 4.1 Let A be a subset of a topological space (X, τ). The i-kernel of A, denoted by i-ker(A) is defined to be the set i-ker(A) $= \cap\{O \in iO(X); A \subseteq O)\}$.

Lemma 4.1 Let X be a T. E. I. and $A \subseteq X$, $x \in X$. Then i-ker(A) = $\{x \in X; cli(\{x\}) \cap A \neq \phi\}$.

Proof. Let $x \in$ i-ker(A) and suppose $cli(\{x\}) \cap A = \phi$. Hence $x \notin [cli(\{x\})]^c$ which is an i-open set containing A. This is impossible, since $x \in$ i-ker(A). Consequently, $cli(\{x\}) \cap A \neq \phi$. Next, let

$cli(\{x\}) \cap A \neq \phi$ and suppose that $x \notin i$-ker(A). Then, there exists an i-open G containing A and $\notin G$. Let $y \in cli(\{x\}) \cap A$. Hence, G is an i-neighborhood of y which $\notin G$. By this contradiction $x \in i$-ker(A).■

5. Definition 4.2 A topological space (X, τ) is said to be Weakly i-R_0 if $\bigcap_{x\in X} cli(\{x\}) = \phi$.

Theorem 4.1 A topological space (X, τ) is weakly i-R_0 if and only if i-ker$(\{x\}) \neq X$ for every $x \in X$.

Proof. Suppose that the space (X, τ) is weakly i-R_0. Assume that there is a point y in X such that i-ker$(\{y\}) = X$. Then $y \notin O$ which O is proper i-open set of X. This implies that $y \in \bigcap_{x\in X} cli(\{x\})$. But this is a contradiction. Now assume that i-ker$(\{x\}) \neq X$ for every $x \in X$. If there exists a point y in X such that $y \in \bigcap_{x\in X} cli(\{x\})$, then every i-open set containing y must contain every point of X. This implies that the space X is the unique i-open set containing y. Hence, i-ker$(\{y\}) = X$ which is a contradiction. Therefore (X, τ) is weakly i-R_0.■

Definition 4.3 A function $f: (X, \tau) \to (Y, \sigma)$ is called always i-closed if the image of each i-closed set of X is i-closed in Y.

Theorem 4.2 if $f: (X, \tau) \to (Y, \sigma)$ is an bijective always i-closed function and X is weakly i-R_0, then Y is weakly i-R_0.

Proof. Straightforward.■

Theorem 4.3 If the topological space X is weakly i-R_0 and Y is any topological space, then the product $X \times Y$ is weakly i-R_0.

Proof. By showing that $\bigcap_{(x,y)\in X\times Y} cli(\{(x,y)\}) = \phi$ we are done. We have: $\bigcap_{(x,y)\in X\times Y} cli(\{(x,y)\}) \subseteq \bigcap_{(x,y)\in X\times Y} (cli(\{x\} \times cli(\{y\})) =$ $\bigcap_{x\in X} cli(\{x\}) \times \bigcap_{y\in Y} cli(\{y\}) \subseteq \phi \times Y = \phi$.■

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