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The Structure Properties of Even-Even $^{216-226}$Ra Isotopes

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ABSTRACT

The properties of even-even $^{216-226}$Ra isotopes were studied and its energy states calculated. To identify the properties of each isotope, the values of the first excited state $E2_1^+$ and the ratio of the second to the first excited state $R_{4/2} = E4_1^+/E2_1^+$ were adopted. The phenomenon of back (or up)-bending, the E-GOS curve, the relationship between successive excited states to those preceding them and the odd-even staggering were studied. The appropriate limit in the interacting boson model IBM-1, the IVBM model and the Bohr and Mottelson (BM) model were used to calculate the energy states for each isotope and compared the results with the experimental values. Due to the inaccuracy of some calculations in the models IVBM and BM, an adjustment term was added for each model and recalculated the energy states and the results were improved.

Keywords: BM, IBM band IVBM, $^{220-230}$Ra even-even isotopes.

الخصائص التركيبية لنظائر $^{216-226}$Ra الزوجية - الزوجية

الملخص

تم في هذا البحث دراسة خصائص نظائر الراديوم $^{216-226}$Ra الزوجية-الزوجية وحساب حالات الطاقة لها. ولتحديد خصائص كل نظير، تم دراسة مواقع حالات التهيج الأولى $E2_1^+$ والنسبة بين حالة التهيج الثانية إلى حالة التهيج الأولى $R_{4/2} = E4_1^+/E2_1^+$. وتم كذلك دراسة ظاهرة الانحناء الخلفي او العلوي ومنحني E-GOS والعلاقة بين حالات الطاقة المتتابعة الى الحالات التي تسبقها والتاريج في فروقات الطاقة بين الحالات ذات التماثل الموجب والحالات التالية لها ذات التماثل السالب. استخدم التحديد المناسب في انموذج البوزونات المتفاعلة IBM-1 وانموذج البوزونات المتفاعلة الاتجاهية IVBM وانموذج بور وموتلسون BM لحساب حالات الطاقة لهذه النظائر وقورنت مع القيم العملية. ولعدم دقة الحسابات لبعض الحالات في بعض النظائر باستخدام IVBM و BM، تم اضافة حد لكل انموذج ولوحظ تحسن كبير في النتائج.

الكلمات الدالة: أنموذج بور وموتلسون، أنموذج البوزونات المتفاعلة، أنموذج البوزونات الاتجاهية المتفاعلة، نظائر الراديوم $^{216-226}$Ra الزوجية-الزوجية.

INTRODUCTION

The even-even nuclei are characterized by certain energy states (Bohr and Mottelson, 1998). The first of these is the ground states band GSB, with even-angular momenta and positive parity states; ($I_1^+ = 0, 2, 4, 6, ...$). These nuclei show other bands, such as the beta-vibrational band which exhibit similar energy states to the GSB and the gamma-vibrational-band which has positive parity and successive even and odd angular moment; ($I_1^+ = 2, 3, 4, 5, ...$). Other states occur in these nuclei, such as negative parity states NPB with an odd-angular momentum; ($I_1^- = 1, 3, 5, ...$).

The first excited state $E2_1^+$ and the ratio of the second to the first excited state $R_{4/2} = E4_1^+/E2_1^+$ of the even-even nuclei provide an initial indication of the characteristics of the nucleus (Nomura, 2013), (Bonatsos and Skouras, 1991). The vibrational nuclei are characterized by $E2_1^+ \approx 500\ keV$ and $2 \leq R_{4/2} \leq 2.4$, while the gamma-soft nuclei are characterized by $E2_1^+ \approx 300\ keV$ and $2.4 < R_{4/2} \leq 3$, and the rotational nuclei are characterized by $E2_1^+ \approx 100\ keV$ and $3 < R_{4/2} \leq 3.3$.

Several methods were introduced to identify the characteristics of even-even nuclei at higher excited states. The appearance of a back or up-bending is appropriate to determine the phase change of the nucleus (Wong, 1990). The relationship between the gamma-energy over spin (E_γ/I) as a function of the spin I presented a method to identify the characteristics of the nucleus (Regan *et al*., 2003). Another method was introduced to identify the characteristics of the nucleus in its various excited states by the ratio between the energy of the various excited states to the one that proceeds it $r\,(I+2/I)$ (Bonatsos and Skouras, 1991). By applying this relationship to a set of different nuclei, the characteristics of each nucleus were determined by the limits; $0.1 \leq r \leq 0.35$ for the vibrational nuclei, and $0.4 \leq r \leq 0.6$ for the transitional nuclei, while $0.6 \leq r \leq 1.0$ for the rotational nuclei.

Due to the absence of a uniform theory suitable for the study of different nuclei, different models were introduced to study these nuclei. Bohr and Mottelson introduced the collective model. The microscopic interacting bosons model IBM-1 calculation (Bonatsos *et al*., 2013), (Kumar and Gunye, 1985) is somehow in between the phenomenological and microscopic approaches. As a widely used model, the IBM-1 (Iachello and Arima, 1987) describes the quadrupole collective states of the medium and heavy nuclei and provides a unified description of collective nuclear states in terms of a system of interacting bosons. Different reductions of unitary group U(6) give three dynamical symmetry limits known as harmonic oscillator, deformed rotor and gamma-soft rotor, which are labeled as U(5), SU(3), and O(6), respectively (Arima and Iachello, 1976), (Arima and Iachello, 1978), (Arima and Iachello, 1979).

In the present study different methods were used to investigate the properties of the Radium isotopes $^{216\text{-}226}$Ra. These methods were the back (or up)-bending, the E-GOS, the ratios of the successive states to those proceeding them and the staggering phenomenon. The models, IBM-1, IVBM and BM, were used to calculate the GSB and NPB by fitting the parameters of the equations of each model to the experimental values which are then used to calculate many excited states of these isotopes and compared with the measured ones. Modified equations of IVBM and BM were suggested to improve the results for these two models.

CALCULATIONS

It was noted that for some nuclei, a dramatic change in the moment of inertia ϑ may occur by increasing the angular momentum I, causing a drop in the rotational energy of the γ-transition from a state with I to a state with $I-2$. This behavior causes back or up-bending in the value of the energy $\hbar\omega$ which is given by (Sorenson, 1973):

$$\hbar\omega = \frac{E_\gamma}{\sqrt{I(I+1)} - \sqrt{(I-2)(I-1)}} \qquad (1)$$

and the moment of inertia is given by (Wong, 1990):

$$2\vartheta/\hbar^2 = \frac{4I-2}{E(I)-E(I-2)} = \frac{4I-2}{E_\gamma} \qquad (2)$$

It is important to plot the ratio $E_\gamma/_I$ as a function of I (E-GOS) to identify the characteristics of the nucleus along its excited states (Regan *et al.*, 2003). The relation between $(R = E_\gamma/_I)$ and the angular momentum I for the three limits are given by (Bohr and Mottelson, 1953), (Scharff-Goldhaber and Weneser, 1955) as:

$$U(5):\ R = \frac{\hbar\omega}{I} \to 0 \ \ when \to \infty \tag{3}$$

$$O(6):\ R = \frac{E2_1^+}{4}\left(1 + \frac{2}{I}\right) \to \frac{E2_1^+}{4} \ \ when \to \infty \tag{4}$$

$$SU(3):\ R = \frac{\hbar^2}{2\vartheta}\left(4 - \frac{2}{I}\right) \to \frac{4\hbar^2}{2\vartheta} \ \ when\ I \to \infty \tag{5}$$

For a given GSB of a spin I, the following ratio was constructed to define the characteristics of the even-even nucleus at its excited states (Bonatsos and Skouras, 1991):

$$r\left(\frac{I+2)}{I}\right) = \left(R\left(\frac{I+2}{I}\right)_{exp} - \frac{(I+2)}{I}\right) \times \frac{I(I+1)}{2(I+2)} \tag{6}$$

where $R\left(\frac{I+2}{I}\right)_{exp}$ is the measured energy ratio between the two states $I+2$ and I.

Odd-even staggering patterns between GSB and NPB have been investigated (Bonatsos *et al.*, 2000):

$$\Delta E_{1,\gamma}(I) = {}^1/_{16}\Big(6E_{1,\gamma}(I) - 4E_{1,\gamma}(I-1) - 4E_{1,\gamma}(I+1) + E_{1,\gamma}(I-2) + E_{1,\gamma}(I+2)\Big) \tag{7}$$

where $E_{1,\gamma}(I) = E(I+1) - E(I)$. All the levels are raised (or lowered) by an amount of energy with respect to the even levels. In other words, $\Delta I = 1$ staggering takes alternatively positive and negative of zigzag values and may reach zero, followed by other increases again.

To calculate the energy states of the used nuclei, the Bohr-Mottelson model (BM), interacting bosons model (IBM-1) and interacting vector bosons model (IVBM) were used. The BM model introduced an energy expansion of GSB and NPB in powers of $I(I+1)$ for the deformed nuclei, which are given by (Bohr and Mottelson, 1998), (Bonatsos *et al.*, 2000):

$$E(I) = AI(I+1) + BI^2(I+1)^2 + CI^3(I+1)^3 \tag{8}$$

$$E(I) = E_0 + AI(I+1) + BI^2(I+1)^2 + CI^3(I+1)^3 \tag{9}$$

where E_0 is the band head energy of the NPB, and the coefficients $A, B\ and\ C$ can be determined by fitting their equations with the available measured energy levels of GSB and NPB.

In IBM-1, the general Hamiltonian is given by (Scholten *et al.*, 1978):

$$H = \sum_{i=1}^{N} \varepsilon_i + \sum_{i<j}^{N} V_{ij} \tag{10}$$

where ε_i is the intrinsic boson energy and V_{ij} is the interaction strength between bosons *i* and *j*; the multipole form of the Hamiltonian is (Casten and Warner, 1988):

$$H = \varepsilon n_d + a_0 P.P + a_1 L.L + a_2 Q.Q + a_3 T_3.T_3 + a_4 T_4.T_4 \qquad (11)$$

where a_0, a_1, a_2, a_3 and a_4 are the strengths of pairing, angular momentum, quadrupole, octupole and hexadecupole interactions of each terms in the equation (11).

The Hamiltonian in terms of multipole expansion tends to reduce three forms to meet the requirements of the three symmetry limits; the vibrational U(5), the rotational SU(3) and the $\gamma-$soft O(6). In the U(5) limit, the effective parameter is ε, in the SU(3) limit, the dominating parameter is a_2 and in the O(6) limit, the predominate parameter is a_0. The eigenvalues of these three limits are (Casten and Warner, 1988):

$$U(5): \quad E = \varepsilon n_d + K_1 n_d(n_d+4) + K_4 v(v+3) + K_5 L(L+1) \qquad (12)$$

$$SU(3): \quad E = K_2(\lambda^2+\mu^2+3(\lambda+\mu)+\lambda\mu) + K_5 L(L+1) \qquad (13)$$

$$O(6): \quad E = K_3\big(N(N+4)-\sigma(\sigma+4)\big) + K_4 \tau(\tau+3) + K_5 L(L+1) \qquad (14)$$

where K_1, K_2, K_3, K_4 and K_5 are other forms of the strength parameters and N is the total boson number. Many nuclei have a transition property of two or three of the above limits.

The eigenvalues in IVBM model for the GSB and NPB states are given by (Georgieva *et al.*, 1982):

$$E(I) = \beta I(I+1) + \gamma I \qquad (15)$$

$$E(I) = \beta I(I+1) + (\gamma+\eta)I + \zeta \qquad (16)$$

The values of β and γ can be determined from a fit to the positive GSB while η and ζ are estimated from the NPB.

RESULTS AND DISCUSSION

Radium nucleus has 88 protons, with 6 protons above the magic number 82 that gives close properties to the vibrational characteristics. The isotopes studied in this paper are $^{216\text{-}226}$Ra with 128 neutrons for ^{216}Ra which has 2 neutrons above the magic number (126) giving the magic properties for this isotope. The number of neutrons for ^{218}Ra is 130 which is still close to the magic number (126) and gives it the vibrational properties. The number of neutrons exceeds contiously by 2 for the rest $^{220\text{-}226}$Ra isotopes which give them an alternate properties from gamma-soft to the rotational.

The number of protons and neutrons is not enough to indicate properly the properties of nuclei along their excited states, so several methods were used to ensure these properties. It was found that the values of the first measured excited states $E2_1^+$ were equal to 688.2, 388.9, 178.4, 111.1, 84.3 and 67.6 keV, and the ratios of the second to the first measured excited states $E4_1^+/E2_1^+$ were 1.69, 1.9, 2.29, 2.7, 2.97 and 3.1 for $^{216\text{-}226}$Ra isotopes respectively. These values indicate that the $^{216\text{-}218}$Ra hve magic properties and ^{220}Ra has vibrational-gamma-soft properties. While the other isotopes $^{222\text{-}226}$Ra are close to the gamma-soft-rotational characteristics.

The back or up-bending curves, the E-GOS curves, the ratios of successive excited states to that proceeding it and the staggering phenomena are used to confirm the properties of these isotopes along their GSB. The relation between the moment of inertia $2\vartheta/\hbar^2$ and the rotational frequency $\hbar\omega$ of the photons emitted during the transition between the different states of $^{216\text{-}226}$Ra isotopes

was plotted in Fig. (1) which shows that there is a back-bending curvature that occurred in the $\hbar\omega$ value of the both isotope ^{216}Ra and ^{218}Ra, while no bending appered in $^{220\text{-}226}$Ra isotopes.

Fig. (2) shows the E-GOS curves, where the ^{216}Ra curve shows a magic shape curve, and the shape of ^{218}Ra curve was very close to the standard vibrational curve, while the ^{220}Ra curve close to the standard gamm-soft nuclei, and the shape of the $^{222\text{-}226}$Ra curves were close to the rotational-gamma-soft charecteristic.

Fig. (3) shows the relationship between the ratio r and the spin I. Some negative values of r occurred for ^{216}Ra which ensure the magic properties of this isotope. The relationship between r and I of the ^{218}Ra isotope showed values which coincide with the r values of the vibration limit U(5) with a single negative value that perturbed this properties. The r values for ^{220}Ra coincide with the r values of the vibrational nuclei, the values of r for ^{222}Ra are coincide with the gamma-soft nuclei and finally the values of r for ^{224,226}Ra are coincide with the r values of the gamma-soft-rotational nuclei.

Fig. (4) shows the apparent staggering in the differences between the energies of GSB and NPB of $^{218\text{-}226}$Ra, but not for ^{216}Ra due to the poor information of NPB for this isotope. The staggering approximated to zero at one point for ^{218}Ra but many points close to zero for the other isotopes $^{220\text{-}226}$Ra.

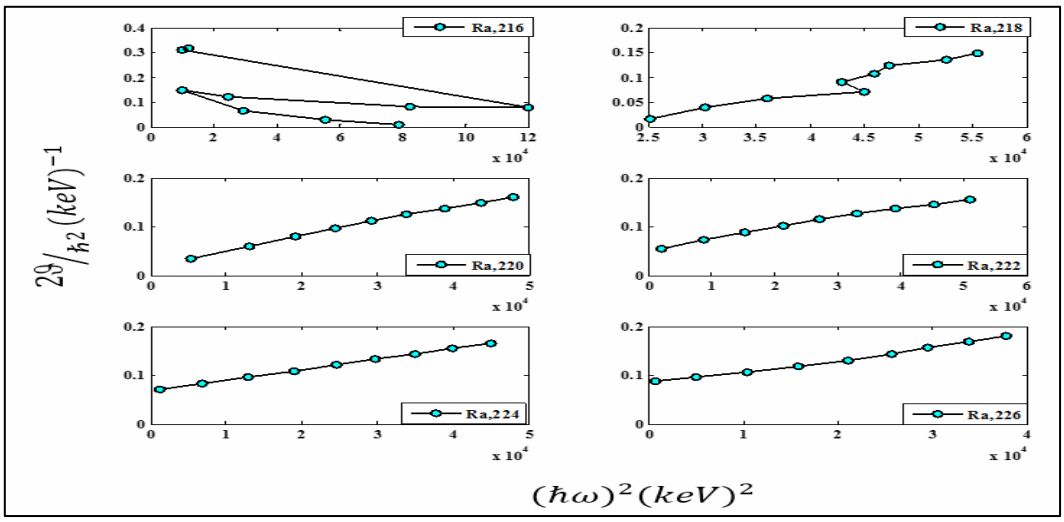

Fig. 1: The Back (or Up)-Bending of $^{216\text{-}226}$Ra Even-Even Isotopes

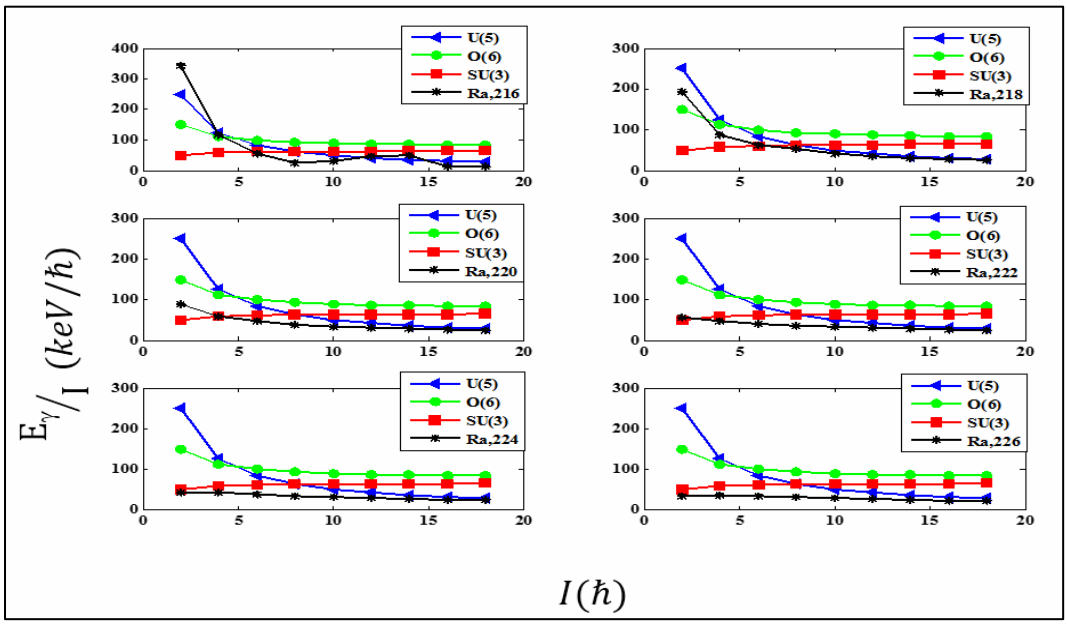

Fig. 2: The E-GOS of $^{216\text{-}226}$Ra Even-Even Isotopes

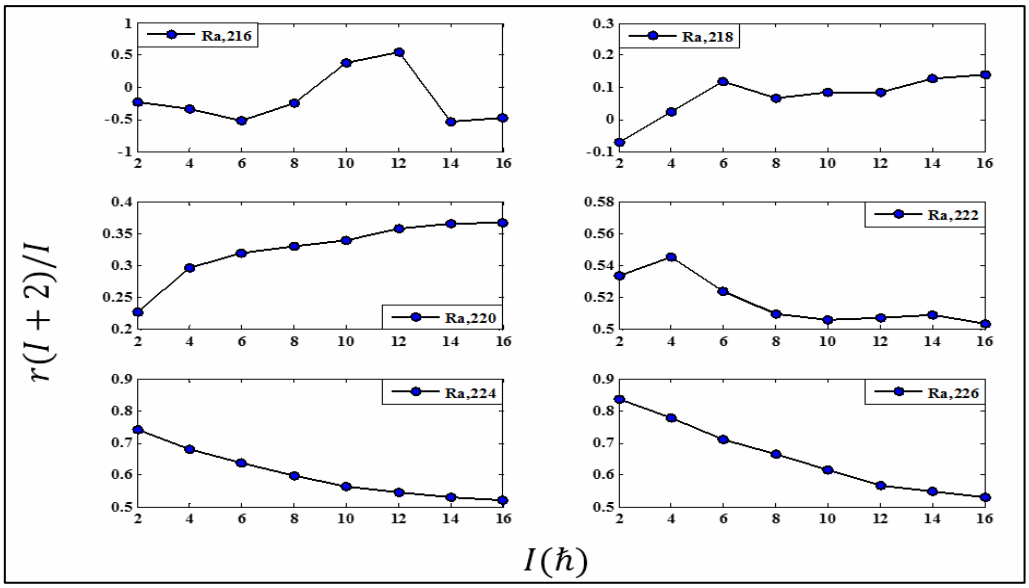

Fig. 3: The ratio r(I+2)/I as a function of I for $^{216\text{-}226}$Ra Even-Even Isotopes

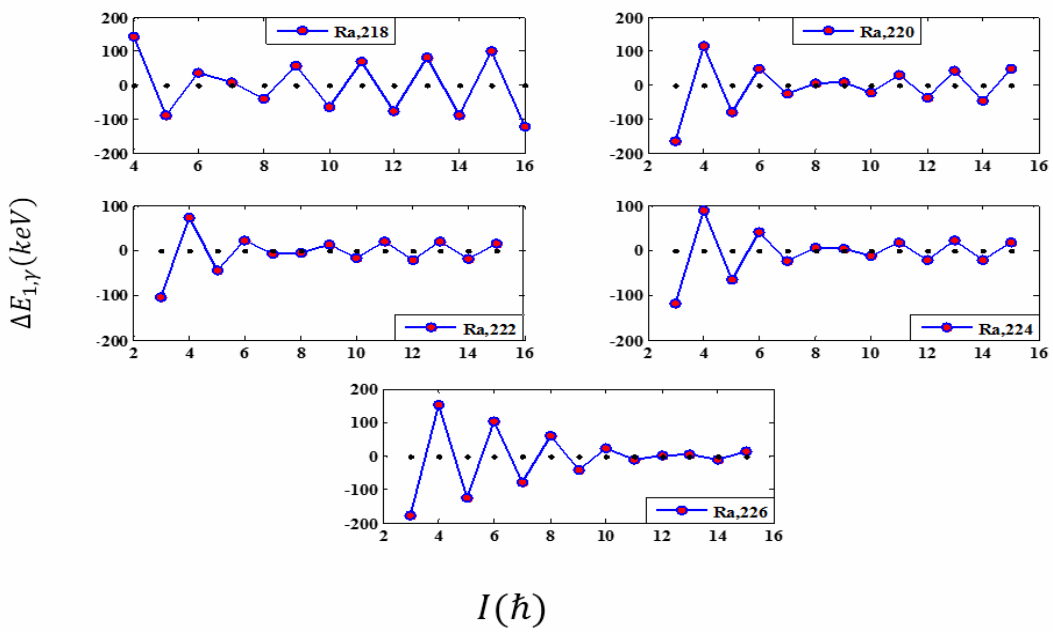

Fig. 4: The staggering of $^{216\text{-}226}$Ra Even-Even Isotopes

The IBM-1, IVBM, and BM models were used to calculate the energy states in the ground states band (GSB), while IVBM and BM models were used to calculate the energy states in the negative parity band (NPB) of the $^{216\text{-}226}$Ra isotopes. The BM model is perfect for calculating the energy states of rotational nuclei but it fails to calculate the energy states in other nuclei. To make the BM model successful for not rotational nuclei, a vibrational limit term was added to improve the calculations. A significant improvement was observed in the calculations for non-rotational nuclei, and the modified equation of BM is given by:

$$E(I) = A^* I(I+1) + B^* I^2(I+1)^2 + C^* I^3(I+1)^3 + D^* I \qquad (17)$$

There are two terms in the IVBM model; one represents the rotational movement and the other represents the vibrational movement of the nuclei. Because of the perturbation which is caused by the effects of the gamma-soft characteristics, a third term was added for the isotopes which are affected by this characteristic, and the following equation illustrates this.

$$E(I) = \beta I(I+1) + \gamma I + \delta \frac{(I+2)}{4} \qquad (18)$$

Computing programs were written using MATLAB 6.5 (Imad and Al-Jubbori, 2009), to calculate the energy levels of GSB and NPB for $^{216\text{-}226}Ra$ isotopes using BM, IBM-1, and IVBM models. The number of bosons and the best values of the parameters which give the best fitting between theoretical and experimental energy levels of the above isotopes are shown in (Table 1). It is clear from (Table 1), that the $^{216\text{-}220}$Ra have the U(5) parameters ε which decreases with increasing neutron number, while the other isotopes $^{222\text{-}226}$Ra have no ε value, since they are far from the vibrational properties. The A value of BM model decreases from 32.76 for ^{216}Ra to 10.59 for ^{226}Ra, and its value for ^{216}Ra is very far from the value of $\hbar^2/_{2\eta}$ which is equal 114.7 for this isotope. But the values of A close to the $\hbar^2/_{2\eta}$ by increasing neutron number (n) and its value for ^{226}Ra is equal 10.95 which is very close to $\hbar^2/_{2\eta}$ which is equal 11.3 for this isotope, and this is because the increasing the rotational properties for these isotopes by increasing (n).

(Table 1) shows that there are modification parameters for both the BM and IVBM, due to the lack of the vibration parameter in the BM model and there is a lack of gamma-soft parameter in IVBM model. It is noticed from (Table 1) the negative value of A^* for ^{216}Ra and the small values of this parameter for ^{218,220}Ra due to the pure vibrational properties for these isotopes, while the A^* for $^{222\text{-}224}$Ra close from $\hbar^2/_{2\eta}$ specially for ^{224}Ra which refers to the rotational properties for these isotopes. The BM.M not use for ^{226}Ra since this nucleus is a pure rotational one. The D^* parameter of ^{216}Ra is equal 436.6 keV which is big, because the magic properties of this nucleus. The D^* parameters for other isotopes decreas continuously by increasing neutron numbers. The IVBM.M was used for ^{216}Ra where the β^* parameter is deleted and **a** big value of δ^* are proper for the magic properties, but the negative values of δ^* for $^{222\text{-}226}$Ra refer to the reduction of the vibrational properties for these isotopes.

(Table 2) shows the measured (Wu, 2007; Askor and Balraj, 2006; Brown and Tuli, 2011, Sukhjeet *et al*., 2011; Sukhjeet and Balraj, 2015; Akovali, 1996) and the calculated energy levels in the GSB for $^{216\text{-}226}$Ra, which show that the calculations of IBM-1 are the best compared with IVBM and BM for $^{216\text{-}224}$Ra. The error in the BM calculations are equal to 72.1%, 57.3%, 39.7%, 24.1% and 13.7% of the first excited states for $^{216\text{-}224}$Ra respectively and less error for the other states. The calculations were improved well by using the modified BM.M, while the BM calculations of ^{226}Ra was the best since this isotope is a pure rotational nucleus. The error in IVBM calculations of $^{216,222\text{-}226}$Ra equal to 28.4, -12.9, 17.1 and -27 of the first excited states respectively and less error for the other states. Howe ever these calculations were improved well using the IVBM.M, while the IVBM calculations are good and equal to IBM-1 calculations for ^{218,220}Ra.

Table 1: IBM-1, BM, BM.M, IVBM and IVBM.M parameters of GSB in keV for $^{216-226}$Ra isotopes

Isotopes	IBM-1						BM		
	N	ε	K1	K2	K4	K5	A	B(10^{-2})	C(10^{-5})
^{216}Ra	4	323.64	5.543	4.016	3.39	-1.26	32.76	-12.07	16.75
^{218}Ra	5	315.92	7.986	-------	5.52	-1.83	28.29	-10.38	15.65
^{220}Ra	6	110.57	7.472	-------	7.25	1.53	18.34	-6.54	11.88
^{222}Ra	7	-------	-------	-0.177	46.56	-5.86	14.28	-3.84	6.59
^{224}Ra	8	-------	-------	-0.142	35.29	-2.70	12.29	-2.73	4.32
^{226}Ra	9	-------	-------	-0.135	32.18	-2.57	10.59	-1.96	2.79
Isotopes	BM.M				IVBM		IVBM.M		
	A^*	B^* (10^{-2})	C^* (10^{-5})	D^*	β	γ	β^*	γ^*	δ^*
^{216}Ra	-32.6	13.28	-21.44	436.6	-2.28	253.1	------	55.012	564.5
^{218}Ra	1.43	0.045	-0.048	179.5	1.55	178.8	------	-------	-------
^{220}Ra	6.67	-0.885	1.205	70.18	5.21	77.42	------	-------	-------
^{222}Ra	11.87	-2.499	3.935	42.12	6.93	41.91	5.778	79.316	-84.45
^{224}Ra	11.12	-2.076	3.023	20.52	7.29	27.52	6.119	65.626	-86.04
^{226}Ra	------	-------	-------	------	6.47	23.69	5.472	65.727	-101.9

Table 2:The measured (Wu, 2007; Askor and Balraj, 2006; Brown and Tuli, 2011, Sukhjeet *et al*., 2011; Sukhjeet and Balraj, 2015; Akovali, 1996)**and calculated energy levels in keV of GSB $^{216-226}$Ra isotopes**

^{216}Ra										
I_i^+		2	4	6	8	10	12	14	16	18
$E_{exp.}$(keV)		688.2	1164.1	1507.6	1711.14	2026.04	2600	3292.7	3491.65	3712.15
$E_{cal.}$ keV	BM	192.3	608.3	1175.5	1795.8	2366.6	2810.2	3109.9	3354.9	3791.9
	Δ%	72.1	47.8	22.0	-4.9	-16.8	-8.1	5.6	3.9	-2.2
	BM.M	682.4	1146.1	1469.4	1755.1	2103.3	2574.2	3141.4	3635	3674.6
	Δ%	0.8	1.6	2.5	-2.6	-3.8	1.0	4.6	-4.1	1.0
	IBM-1	710.8	1075.9	1448.8	1829.4	2217.9	2614	3018	3429.7	3849.1
	Δ%	-3.3	7.6	3.9	-6.9	-9.5	-0.5	8.3	1.8	-3.7
	IVBM	492.5	966.8	1422.8	1860.6	2280.2	2681.6	3064.8	3429.7	3776.4
	Δ%	28.4	16.9	5.6	-8.7	-12.5	-3.1	6.9	1.8	-1.7
	IVBM.M	674.6	1066.9	1459.1	1851.4	2243.7	2636	3028.3	3420.6	3812.9
	Δ%	1.98	8.4	3.2	-8.2	-10.7	-1.4	8.0	2.0	-2.7
^{218}Ra										
I_i^+		2	4	6	8	10	12	14	16	18
$E_{exp.}$(keV)		388.9	741.1	1122.04	1546.7	1961.7	2390.8	2825.5	3285.1	3756
$E_{cal.}$ keV	BM	166.1	525.7	1017.2	1558.1	2065.8	2483.9	2816.7	3170.7	3803.4
	Δ%	57.3	29.1	9.3	-0.7	-5.3	-3.9	0.3	3.5	-1.3
	BM.M	367.6	746.8	1138	1541.4	1957.5	2386.9	2829.6	3285.9	3755.2
	Δ%	5.5	-0.8	-1.4	0.3	0.2	0.2	-0.1	-0.02	0.02
	IBM-1	366.9	746.2	1137.9	1542	1958.4	2387.2	2828.4	3282	3747.9
	Δ%	5.7	-0.7	-1.4	0.3	0.2	0.2	-0.1	0.1	0.2
	IVBM	366.9	746.2	1137.9	1542	1958.4	2387.2	2828.4	3282	3747.9
	Δ%	5.7	-0.7	-1.4	0.3	0.2	0.2	-0.1	0.1	0.2
^{220}Ra										

I_1^+		2	4	6	8	10	12	14	16	18
$E_{exp.}$(keV)		178.47	410.07	688.1	1001.2	1342.7	1711.2	2105.7	2523.5	2961.9
$E_{cal.}$ keV	BM	107.7	341.5	663.5	1025.4	1383.5	1719.2	2065.4	2537.5	3370.4
	Δ%	39.7	16.7	3.6	-2.4	-3.0	-0.5	1.9	-0.6	-13.8
	BM.M	180.1	410.6	686.4	1000.1	1344.1	1712.6	2103.9	2524	2990.2
	Δ%	-0.9	-0.1	0.3	0.1	-0.1	-0.1	0.1	-0.02	-0.9
	IBM-1	186.1	413.9	683.5	994.7	1347.6	1742.2	2178.6	2656.6	3176.3
	Δ%	-4.3	-0.9	0.7	0.7	-0.4	-1.8	-3.5	-5.3	-7.2
	IVBM	186.1	413.9	683.5	994.7	1347.6	1742.2	2178.6	2656.6	3176.3
	Δ%	-4.3	-0.9	0.7	0.7	-0.4	-1.8	-3.5	-5.3	-7.2
^{222}Ra										
I_1^+		2	4	6	8	10	12	14	16	18
$E_{exp.}$(keV)		111.12	301.39	550.3	843.3	1173.3	1537.2	1933.2	2358.7	2811
$E_{cal.}$ keV	BM	84.3	270.7	536.7	853.3	1193.2	1542.1	1913.4	2365.5	3022.2
	Δ%	24.1	10.2	2.5	-1.2	-1.7	-0.3	1.0	-0.3	-7.5
	BM.M	112.5	291	541.7	845.3	1182.3	1540.7	1924	2361.7	2921.8
	Δ%	-1.2	3.5	1.6	-0.2	-0.8	-0.2	0.5	-0.1	-3.9
	IBM-1	108.8	306.1	549.7	839.4	1175.4	1557.5	1985.9	2460.6	2981.4
	Δ%	2.1	-1.6	0.1	0.5	-0.2	-1.3	-2.7	-4.3	-6.1
	IVBM	125.4	306.1	542.3	833.9	1180.9	1583.2	2041	2554.2	3122.8
	Δ%	-12.9	-1.6	1.5	1.1	-0.7	-2.9	-5.6	-8.3	-11.1
	IVBM.M	108.8	306.1	549.7	839.4	1175.4	1557.5	1985.9	2460.6	2981.4
	Δ%	2.1	-1.6	0.1	0.5	-0.2	-1.3	-2.7	-4.3	-6.1
^{224}Ra										
I_1^+		2	4	6	8	10	12	14	16	18
$E_{exp.}$(keV)		84.37	250.78	479.12	754.88	1068.9	1413.7	1787.5	2187.7	2612.1
$E_{cal.}$ keV	BM	72.8	235.3	471.4	759.7	1079.3	1416.9	1776.7	2191.3	2735.6
	Δ%	13.7	6.2	1.6	-0.6	-0.9	-0.2	0.6	-0.2	-4.7
	BM.M	86.5	245.1	473.7	755.6	1073.9	1416.2	1781.8	2189.5	2686.8
	Δ%	-2.5	2.3	1.1	-0.1	-0.5	-0.2	0.3	-0.1	-2.9
	IBM-1	81.9	255.9	478.7	750.5	1071.2	1440.9	1859.5	2327.1	2843.6
	Δ%	2.9	-2.0	0.1	0.6	-0.2	-1.9	-4.0	-6.4	-8.9
	IVBM	98.8	255.8	471.2	744.8	1076.8	1467.1	1915.6	2422.5	2987.6
	Δ%	-17.1	-2.0	1.7	1.3	-0.7	-3.8	-7.2	-10.7	-14.4
	IVBM.M	81.9	255.9	478.7	750.5	1071.2	1440.9	1859.5	2327.1	2843.6
	Δ%	2.9	-2.0	0.1	0.6	-0.2	-1.9	-4.0	-6.4	-8.9
^{226}Ra										
I_1^+		2	4	6	8	10	12	14	16	18
$E_{exp.}$(keV)		67.67	211.54	416.5	669.4	959.9	1280.5	1625	1993	2382
$E_{cal.}$ keV	BM	62.8	204.2	412.3	671.5	965.4	1282	1619.7	1994.8	2450
	Δ%	7.2	3.5	1.0	-0.3	-0.6	-0.1	0.3	-0.1	-2.9
	IBM-1	62.3	219.4	420.2	664.9	953.3	1285.5	1661.4	2081.2	2544.7
	Δ%	7.9	-3.7	-0.9	0.7	0.7	-0.4	-2.2	-4.4	-6.8
	IVBM	86.2	224.2	413.9	655.3	948.5	1293.4	1690.1	2138.5	2638.7
	Δ%	-27.4	-5.9	0.6	2.1	1.2	-1.0	-4.0	-7.3	-10.8
	IVBM.M	62.3	219.4	420.2	664.9	953.3	1285.5	1661.4	2081.2	2544.7
	Δ%	7.9	-3.7	-0.9	0.7	0.7	-0.4	-2.2	-4.4	-6.8

(Table 3) shows the best fitting parameters of IVBM, IVBM.M, BM and BM.M equations with the measured values of NPB. New parameters of BM and BM.M were used for $^{218-226}$Ra in this band, but the same GSB parameters of IVBM and IVBM.M were used since an additional two parameters were used to calculate the NPB states. (Table 4) shows the measured and calculated energy states of NPB for $^{218-226}$Ra, using BM, BM.M, IVBM and IVBM.M. There are no calculations for ^{216}Ra due to the poor information of NPB for this isotope. Reasonable results were obtained with low errors for all calculated states of the isotopes.

Table 3: BM, BM.M, IVBM and IVBM. M parameters of NPB in keV for $^{216-226}$Ra isotopes

Isotopes	Models	E_0	A_N	$B_N(10^{-2})$	$C_N(10^{-5})$	D_N
^{218}Ra	BM.M	793.21	20.27	-3.982	4.562	-69.493
^{220}Ra	BM.M	412.98	12.54	-1.795	2.066	-27.596
^{222}Ra	BM.M	242.11	9.99	-0.783	0.558	-33.365
^{224}Ra	BM.M	215.98	8.96	-0.596	0.377	-25.855
^{226}Ra	BM	253.73	6.66	0.044	-0.582	-------
Isotopes	**Models**	η		ξ		
^{218}Ra	IVBM	-37.622		307.75		
^{220}Ra	IVBM	-47.302		347.94		
^{222}Ra	IVBM.M	-33.151		231.84		
^{224}Ra	IVBM.M	-28.344		221.97		
^{226}Ra	IVBM.M	-22.589		259.63		

Table 4: The measured (Wu, 2007; Askor and Balraj, 2006; Brown and Tuli, 2011, Sukhjeet *et al*., 2011; Sukhjeet and Balraj, 2015; Akovali, 1996) **and calculated energy levels in keV of NPB for $^{216-226}$Ra isotopes**

^{218}Ra										
I_1^-		3	5	7	9	11	13	15	17	19
$E_{exp.}$(keV)		793.21	1038.32	1340.85	1694.35	2109.3	2526.3	2966.4	3388.8	3805.9
$E_{cal.}$ (keV)	BM.M	822.3	1019.2	1325	1702.7	2115.4	2534.9	2952.6	3393.1	3928.9
	Δ%	-3.7	1.8	1.2	-0.5	-0.3	-0.3	0.5	-0.1	-3.2
	IVBM	749.9	1060.2	1382.8	1717.8	2065.2	2425	2797.1	3181.6	3578.5
	Δ%	5.5	-2.1	-3.1	-1.4	-2.1	4.0	5.7	6.1	5.9
^{220}Ra										
I_1^-		1	3	5	7	9	11	13	15	17
$E_{exp.}$(keV)		412.98	474.17	634.8	873	1163.8	1496.1	1863.7	2262.5	2690.1
$E_{cal.}$ keV	BM.M	410.4	478.1	635.5	869.2	1162.5	1498.9	1865.7	2259.3	2691
	Δ%	0.6	-0.8	-0.1	0.4	0.1	-0.2	-0.1	0.1	-0.03
	IVBM	388.5	500.9	654.9	850.7	1088.2	1367.3	1688.2	2050.8	2455.6
	Δ%	5.9	-5.6	-3.2	2.6	6.5	8.6	9.4	9.4	8.7
^{222}Ra										
I_1^-		1	3	5	7	9	11	13	15	17
$E_{exp.}$(keV)		242.11	317.29	473.76	703.2	992.4	1330.8	1710.3	2125.3	2570.1
$E_{cal.}$ keV	BM.M	237.1	319.3	476.8	703.3	990.8	1329.8	1710.9	2126	2569.8
	Δ%	2.1	-0.6	-0.6	-0.01	0.2	0.1	-0.03	-0.03	0.01
	IVBM.M	226.2	334	488	688.1	934.4	1226.9	1565.5	1950.3	2381.2
	Δ%	6.6	-5.3	-3.0	2.2	5.8	7.8	8.5	8.2	7.4
^{224}Ra										
I_1^-		1	3	5	7	9	11	13	15	17
$E_{exp.}$(keV)		215.98	290.35	433.02	640.69	906.17	1220.7	1573.6	1964.7	2384.1
$E_{cal.}$ keV	BM.M	214.5	290.3	434.2	641.5	905.6	1219.3	1574.7	1964.7	2384
	Δ%	0.7	0.02	0.3	-0.1	0.1	0.1	-0.1	0.0	0.004
	IVBM.M	207	299.7	441.4	632	871.6	1160.1	1497.6	1884	2319.4
	Δ%	4.2	-3.2	-1.9	1.4	3.8	4.9	4.8	4.1	2.7

^{226}Ra									
I_i^-		1	3	5	7	9	11	13	15	17
$E_{exp.}$(keV)		253.73	321.54	446.3	626.7	857.6	1133.1	1446	1793	2170
$E_{cal.}$ keV	BM	267.1	333.8	453.9	627.3	852.9	1127.8	1446.3	1798.3	2167.9
	Δ%	-5.3	-3.8	-1.7	-0.1	0.6	0.5	-0.02	-0.3	0.1
	IVBM.M	237.2	327.1	460.8	638.3	859.5	1124.4	1433.2	1785.6	2181.9
	Δ%	6.5	-1.7	-3.3	-1.9	-0.2	0.8	0.9	0.4	-0.6

CONCLUSIONS

Knowing the number of protons and neutrons and their distribution on different shells and subshells as well as the positions of their first excited states and the ratio of the second to the first excited states of $^{216\text{-}226}$Ra isotopes are not enough to determine their properties among all their excited states. The back or up-bending, the E-GOS, the ratio of different successive states to those states proceeding them and the staggering of the difference between GSB and NPB states are helpful to determine the properties of the nuclei among their many excited states. From Tables (1-4), one can recognize that ^{216}Ra has a magic properties while ^{218,220}Ra have vibrational properties and $^{222\text{-}224}$Ra have the O(6)-SU(3) properties, where ^{226}Ra has a pure rotational properties. Using different models, the IBM-1, the IVBM and Bohr-Mottelson to calculate the GSB and NPB show different accuracy. The IBM-1 calculations were the best one, since using suitable limits of IBM-1 for the $^{216\text{-}226}$Ra isotopes; where the general form of IBM-1 was suitable for ^{216}Ra. On the other hand the U(5) limit was found for ^{218,220}Ra, while the SU(3)-O(6) was convenient for ^{222,224}Ra and the pure SU(3) appropriate for ^{226}Ra. A modification term was added to improve the calculations of IVBM and BM, where a gamma-soft term was added to the IVBM and a vibrational term was added to the BM model. An obvious improvement was observed which supports the modification used for each model.

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