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A New Globally Convergent of Spectral Hideaki - Yasushi Type Conjugate Gradient Method

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Abstract

This paper presents two new spectral conjugate gradient methods which are designed for solving nonlinear unconstrained optimization problems. These methods are based on the idea of the Hideaki – Yasushi method. Which produce sufficient descent search direction at every iteration. Experimental results indicate that the new proposed methods more efficient than the Hideaki – Yasush method.

الملخص

في هذا البحث تم اســـتحداث طريقتين جديدتين من طرائق التدرج المترافق الطيفية لحل مســـائل الأمثلية اللاخطية وغير المقيدة. هاتان الطريقتان معتمدتان في الأســـاس على فكرة طريقة Hideaki – Yasush. وقد أثبتت الطريقتان إن لهما اتجاه بحث ذو انحدار شـــديد عند كل تكرار. النتائج العددية أثبتت كفاءة الطريقتين مقارنة بطريقة Hideaki – Yasush.

Introduction

The nonlinear conjugate gradient (CG) method is designed to solve the following unconstrained optimization problem

$$\min\{f(x) \mid x \in R^n\} \qquad \text{..........(1)}$$

where $f : R^n \to R$ is a continuously differentiable nonlinear function whose gradient is denoted by g. Due to its simplicity and its very low memory requirement, the CG method has played a special rule for solving large scale nonlinear optimization problems. The iterative formula of the CG method is given by

$$x_{k+1} = x_k + \alpha_k d_k \qquad \text{..........(2)}$$

where $\alpha_k > 0$ is a step length which is computed by carrying out a line search and satisfies the standard Wolfe (SW) conditions

$$f(x_k + \alpha_k d_k) \le f(x_k) + \delta_1 \alpha_k d_k^T g_k \qquad \text{..........(3)}$$

$$g(x_k + \alpha_k d_k)^T d_k \ge \delta_2 d_k^T g_k \qquad \text{..........(4)}$$

with $0 < \delta_1 < \delta_2 < 1$, and d_{k+1} is the search direction defined by

$$d_{k+1} = \begin{cases} -g_1 & k = 1, \\ -g_{k+1} + \beta_k d_k & k > 1, \end{cases} \qquad \text{..........(5)}$$

The search direction d_k is generally required to satisfy :

$$g_{k+1}^T d_{k+1} < 0, \qquad \text{..........(6)}$$

which guarantees that d_k is a descent direction of $f(x)$ at x_k. In order to guarantee the global convergence, we sometimes require d_k to satisfy a sufficient descent condition :

$$g_{k+1}^T d_{k+1} \le -c \|g_{k+1}\|^2 \qquad \text{..........(7)}$$

where c is a constant [8]. Different CG methods correspond to different choices for the scalar β_k. The well known formulas for β_k such as β_k^{HS} (Hestenes, Stiefel [7]), β_k^{FR} (Fletcher, Reeves [6]), β_k^{PRP} (Polak, Ribiere and Polyak [9]), β_k^{CD} (Fletcher [5]), β_k^{LS} (Liu, Storey [8]), β_k^{DY} (Dai, Yuan [4]) can be found in many related literatures. Recently, Hideaki and Yasushi **[14]** proposed a new conjugate gradient method which was obtained by modifying the DY method and called HY method. A nice property of the HY method is that it generates sufficient descent directions. The parameter β_k in HY method is given by

$$\beta_k^{HY} = \frac{g_{k+1}^T g_{k+1}}{\frac{2}{\alpha_k}(f_k - f_{k+1})} \qquad \text{..........(8)}$$

performs more effective more details can be found in [14]. It is well know that the linear conjugate gradient methods generate a sequence of search directions d_{k+1} such that the following conjugacy condition holds :

$$d_i^T H d_j = 0, \ \forall \ i \neq j \ , \qquad \text{..........(9)}$$

where H is the Hessian of the objective function. For general nonlinear function f, we know by the mean value theorem that there exists some $t \in (0\ ,1)$ such that

$$d_{k+1}^T y_k = \alpha_k d_{k+1}^T \nabla^2 f(x_k + t\alpha_k d_k) \ . \qquad \text{..........(10)}$$

Therefore, it is reasonable to replace (10) with the following conjugacy condition :

$$d_{k+1}^T y_k = 0 \ . \qquad \text{..........(11)}$$

Recently, extensions of (10) have been studied in [4,7] that are based on the standard secant equation

$$H_{k+1} y_k = v_k \ . \qquad \text{..........(12)}$$

from (12) and the search direction d_{k+1} can be calculated in the form

$$d_{k+1} = -H_{k+1} g_{k+1} \qquad \text{..........(13)}$$

we have

$$d_{k+1}^T y_k = -(H_{k+1} g_{k+1})^T y_k = -g_{k+1}^T H_{k+1} y_k = -g_{k+1}^T v_k . \qquad \text{..........(14)}$$

By introducing a scaling factor t, Dai and Liao considered a generalized

conjugate condition,

$$d_{k+1}^T y_k = -t g_{k+1}^T v_k \ , \ t \geq 0 \ , \qquad \text{..........(15)}$$

where t is a parameter. In the case $t = 0$, then (15) becomes (11). In case $t = 1$, (15) reduced to (14). Furthermore, if exact line search is used, then $g_{k+1}^T v_k = 0$ holds for all k. It follows that both (14) and (15) coincide with (9). More details can be found in [10]

Another popular method to solving problem (1) is the spectral gradient method, which was developed originally by Barzilai and Borwein in 1988. The direction d_{k+1} is given by the following way

$$d_{k+1} = -\theta_k g_{k+1} + \beta_k d_k \qquad \text{..........(16)}$$

where θ_k is scalar parameter which follows to be determined. More details can be found in [3].

The structure of the paper is as follows. In section (2) we present the new spectral conjugate gradient methods and descent algorithm. Section (3) show that the search direction generated by this proposed algorithm at each iteration satisfies the sufficient descent condition. Section (4) establishes the global convergence property for the new CG-method. Section (5) establishes some numerical results to show the effectiveness of the proposed CG-method and section (6) gives a brief conclusions and discussions.

2. A New Spectral Conjugate Gradient Methods

In this article we present a modification of the Hideaki and Yasushi rule, is defined on the basis of β_k^{HY} as follows :

$$\beta_k^{SAH} = \frac{g_{k+1}^T g_{k+1}}{\frac{2}{\alpha_k}(f_k - f_{k+1})} - \frac{g_{k+1}^T g_{k+1}(v_k^T g_{k+1})}{\left(\frac{2}{\alpha_k}(f_k - f_{k+1})\right)^2} \qquad \text{..........(17)}$$

Observe that

$$\beta_k^{SAH} = \frac{g_{k+1}^T g_{k+1}}{\frac{2}{\alpha_k}(f_k - f_{k+1})}\left[1 - \frac{v_k^T g_{k+1}}{\frac{2}{\alpha_k}(f_k - f_{k+1})}\right] = \beta_k^{HY} t_k$$

where

$$t_k = 1 - \frac{v_k^T g_{k+1}}{\frac{2}{\alpha_k}(f_k - f_{k+1})}$$

From the second Wolfe condition it follows that $v_k^T g_{k+1} \geq \delta_2 v_k^T g_k = \delta_2 \alpha_k d_k^T g_k$. In [14] Hideaki and Yasushi proved the $\frac{2}{\alpha_k}(f_k - f_{k+1}) \geq -2\delta_1 g_k^T d_k$. It follows that $\frac{v_k^T g_{k+1}}{\frac{2}{\alpha_k}(f_k - f_{k+1})} \geq \frac{\delta_2 \alpha_k d_k^T g_k}{-2\delta_1 g_k^T d_k} = \frac{\delta_2 \alpha_k}{-2\delta_1}$. Hence $t_k = 1 - \frac{v_k^T g_{k+1}}{\frac{2}{\alpha_k}(f_k - f_{k+1})} \leq 1 - \left[\frac{\delta_2 \alpha_k}{-2\delta_1}\right] = 1 + \frac{\delta_2 \alpha_k}{2\delta_1} = Z_k$

Therefore, $\beta_k^{SAH} \leq \beta_k^{HY} Z_k$.

To determine the parameter θ_k in $d_{k+1} = -\theta_k g_{k+1} + \beta_k v_k$ we suggest the following two procedures, in order to satisfy both the descent condition and the conjugate condition in the frame of conjugate gradient methods :

1-The first procedure is based on the descent condition. Putting β_k^{SAH} into (16) with descent condition , we obtain :

$$d_{k+1}^T g_{k+1} = -\theta_k g_{k+1}^T g_{k+1} + \beta_k^{SAH} v_k^T g_{k+1} < 0$$

$$= -\theta_k g_{k+1}^T g_{k+1} + \left[\frac{g_{k+1}^T g_{k+1}}{\frac{2}{\alpha_k}(f_k - f_{k+1})} - \frac{g_{k+1}^T g_{k+1}(v_k^T g_{k+1})}{\left(\frac{2}{\alpha_k}(f_k - f_{k+1})\right)^2} \right] v_k^T g_{k+1} < 0 \qquad \text{..........(18)}$$

$$\left[\frac{g_{k+1}^T g_{k+1}}{\frac{2}{\alpha_k}(f_k - f_{k+1})} - \frac{g_{k+1}^T g_{k+1}(v_k^T g_{k+1})}{\left(\frac{2}{\alpha_k}(f_k - f_{k+1})\right)^2} \right] v_k^T g_{k+1} < \theta_k g_{k+1}^T g_{k+1}$$

From (18) we get :

$$\theta_k^{S1} > \left[\frac{g_{k+1}^T g_{k+1}}{\frac{2}{\alpha_k}(f_k - f_{k+1})} - \frac{g_{k+1}^T g_{k+1}(v_k^T g_{k+1})}{\left(\frac{2}{\alpha_k}(f_k - f_{k+1})\right)^2} \right] \frac{v_k^T g_{k+1}}{g_{k+1}^T g_{k+1}} \qquad \text{..........(19)}$$

2- The second procedure is based on the conjugate condition. Substituting (16) into (14) , we obtain :

$$d_{k+1}^T y_k = -\theta_k g_{k+1}^T y_k + \beta_k^{SAH} v_k^T y_k$$

$$d_{k+1}^T y_k = -\theta_k g_{k+1}^T y_k + \left[\frac{g_{k+1}^T g_{k+1}}{\frac{2}{\alpha_k}(f_k - f_{k+1})} - \frac{g_{k+1}^T g_{k+1}(v_k^T g_{k+1})}{\left(\frac{2}{\alpha_k}(f_k - f_{k+1})\right)^2} \right] v_k^T y_k \qquad \text{..........(20)}$$

$$\theta_k g_{k+1}^T y_k = \left[\frac{g_{k+1}^T g_{k+1}}{\frac{2}{\alpha_k}(f_k - f_{k+1})} - \frac{g_{k+1}^T g_{k+1}(v_k^T g_{k+1})}{\left(\frac{2}{\alpha_k}(f_k - f_{k+1})\right)^2} \right] v_k^T y_k + g_{k+1}^T v_k$$

we have :

$$\theta_k^{S2} = \left[\frac{g_{k+1}^T g_{k+1}}{\frac{2}{\alpha_k}(f_k - f_{k+1})} - \frac{g_{k+1}^T g_{k+1}(v_k^T g_{k+1})}{\left(\frac{2}{\alpha_k}(f_k - f_{k+1})\right)^2} \right] \frac{v_k^T y_k}{g_{k+1}^T y_k} + \frac{g_{k+1}^T v_k}{g_{k+1}^T y_k} \qquad \text{..........}(21)$$

As above, since $g_{k+1}^T v_k \to 0$ along the iterations, θ_k^{S1} obtained from the Newton direction paradigm is very similar to θ_k^{S2} based on the conjugacy condition.

Now we can obtain the new conjugate gradient algorithms.

New Algorithm :

Step 1. Select $x_1 \in R^n$ and the parameters $0 < \delta_1 < \delta_2 < 1$. Compute $f(x_1)$ and g_1. Consider $d_1 = -g_1$ and set the initial guess $\alpha_1 = 1/\|g_1\|$.

Step 2. Test for continuation of iterations. If $\|g_{k+1}\| \le 10^{-6}$, then stop.

Step 3. Line search. Compute $\alpha_{k+1} > 0$ satisfying the Wolfe line search condition (3) and (4) and update the variables $x_{k+1} = x_k + \alpha_k d_k$.

Step 4. β_k conjugate gradient parameter which defined in (17).

Step 5. θ_k is computed as in (19) and (21) where $\theta_k > 1/4$.

Step 6. Direction computation. Compute $d_{k+1} = -\theta_k g_{k+1} + \beta_k v_k$. If the restart criterion of Powell $\left|g_{k+1}^T g_k\right| \ge 0.2 \|g_{k+1}\|^2$, is satisfied, then set $d_{k+1} = -\theta_k g_{k+1}$, else set $k = k+1$ and continue with step2 .

3. The sufficient descent condition

In this section we shall introduce the new theorem which is ensure the sufficient descent of the new methods given in (17) with (19) and (21).

Theorem (3.1)

If $\theta_k^{S1} > 1/4$, then the direction $d_{k+1} = -\theta_k^{S1} g_{k+1} + \beta_k^{SAH} v_k$ satisfies the sufficient descent direction.

$$g_{k+1}^T d_{k+1} \le -\left[\theta_k^{S1} - \frac{1}{4}\right] \|g_{k+1}\|^2 \ . \qquad \text{..........}(22)$$

Proof.

Since $d_0 = -g_0$, we have $g_0^T d_0 \le -\|g_0\|^2 < 0$. Assume by induction that

$$g_k^T d_k \le -c\|g_k\|^2 < 0 \text{ where } 0 < c < 1 \qquad \text{..........}(23)$$

which is a sufficient descent direction. To complete the proof, we have to show that the theorem is true for all $k+1$.. Multiplying (16) by g_{k+1}^T we have :

$$g_{k+1}^T d_{k+1} = -\theta_k^{S1}\|g_{k+1}\|^2 + \beta_k^{SAH} g_{k+1}^T v_k$$

$$= -\theta_k^{S1}\|g_{k+1}\|^2 + \left[\frac{g_{k+1}^T g_{k+1}}{\frac{2}{\alpha_k}(f_k - f_{k+1})} - \frac{g_{k+1}^T g_{k+1}(v_k^T g_{k+1})}{\left(\frac{2}{\alpha_k}(f_k - f_{k+1})\right)^2}\right] g_{k+1}^T v_k \qquad \text{..........}(24)$$

Now, using the inequality $u^T v \le 1/2(\|u\|^2 + \|v\|^2)$ to the second term of the right hand side of the above equality, with $u = ((2/\alpha_k)(f_k - f_{k+1}))g_{k+1}$ and $v = (g_{k+1}^T d_k)y_k$ we get :

$$\frac{g_{k+1}^T g_{k+1}(v_k^T g_{k+1})}{\frac{2}{\alpha_k}(f_k - f_{k+1})} = \frac{\left[\left(\frac{1}{\alpha_k}\{2(f_k - f_{k+1})\}\right) g_{k+1}/\sqrt{2}\right]^T \left[\sqrt{2}\,(v_k^T g_{k+1}) g_{k+1}\right]}{\left(\frac{2}{\alpha_k}(f_k - f_{k+1})\right)^2}$$

$$\le \frac{\frac{1}{2}\left[\frac{1}{2}\left(\frac{1}{\alpha_k}\{2(f_k - f_{k+1})\}\right)^2 \|g_{k+1}\|^2 + 2(v_k^T g_{k+1})^2\|g_{k+1}\|^2\right]}{\left(\frac{2}{\alpha_k}(f_k - f_{k+1})\right)^2} \qquad \text{..........}(25)$$

$$= \frac{1}{4}\|g_{k+1}\|^2 + \frac{(v_k^T g_{k+1})^2\|g_{k+1}\|^2}{\left(\frac{2}{\alpha_k}(f_k - f_{k+1})\right)^2}$$

from (24) and (25) we have

$$g_{k+1}^T d_{k+1} \le -\theta_k^{S1}\|g_{k+1}\|^2 + \frac{1}{4}\|g_{k+1}\|^2 \qquad \text{..........}(26)$$

$$g_{k+1}^T d_{k+1} \le -\left[\theta_k^{S1} - \frac{1}{4}\right]\|g_{k+1}\|^2 \ . \qquad \text{..........}(27)$$

To conclude, the sufficient descent condition from (22), the quantity $\theta_k^{S1} - 1/4$ is required to be nonnegative. Supposing that $\theta_k^{S1} - 1/4 > 0$, then the direction given by (16) and (17) is a descent direction more details can be found in [2].

Remark : we use similarly technique to classical algorithm θ_k^{S2}.

4. Convergence analysis

In this section we analyze the convergence of the algorithm (2) and (16), where θ_k and β_k are given by (17), (19) and (21) respectively. In the following we consider that

$$g_{k+1}. \neq 0 \ , \ \forall \ k \geq 1 . \qquad \text{..........(28)}$$

Otherwise, a stationary point is at hand. We make the following basic assumptions on the objective function.

Definition 4.1

A twice continuously differentiable function f is said to be uniformly convex on the nonempty open convex set S if and only if there exists $M > 0$ such that

$$(g(x) - g(y))^T.(x-y) \geq M\|x-y\|^2 \ , \ x, y \in S \qquad \text{..........(29)}$$

or, equivalently, there exists $r > 0$ such that

$$z^T \nabla^2 f(x) z \geq r\|z\|^2 \ , \forall \, x \in S \ , \ \forall z \in R^n \ . \qquad \text{..........(30)}$$

see [12].

Assumption 1

The level set $l = \{ \, x : f(x) \leq f(x_1) \, \}$ is bounded ; that is, there exists a constant $B > 0$ such that

$$\|d\| \leq B, \ \forall \ x \in l . \qquad \text{..........(31)}$$

Assumption 2

In some neighborhood N of l $(l \subseteq N)$, f is continuously differentiable, and its gradient is Lipschitz continuous ; that is, there exists a constant $L > 0$ such that

$$\|\nabla f(x) - \nabla f(y)\| \leq L\|x-y\|, \ \forall \ x, y \in N . \qquad \text{..........(32)}$$

The following proposition is now immediate [12-13].

Proposition 4.1

Under Assumptions 1 and 2 on f, there exists a constant $\bar{\gamma} > 0$ such that

$$\|\nabla f(x)\| \leq \bar{\gamma}\ , \ \forall x \in l. \qquad ....(33)$$

Lemma 4.1.

Suppose that Assumptions 1 and 2 hold. Consider any conjugate gradient method in the form $(2)-(14)$, where d_{k+1} is a descent direction and α_k is computed using the strong Wolfe line search conditions. If

$$\sum_{k\geq 0} \frac{1}{\|d_{k+1}\|^2}. = \infty, \qquad ....(34)$$

then we have

$$\liminf_{k\to\infty} \|g_k\| = 0. \qquad ....(35)$$

Theorem 4.2

Suppose that Assumptions 1 and 2 and the descent condition hold. Consider a conjugate gradient method in the form $(17)-(19)$ with θ_k^{S1} and β_k^{SAH} , where α_k is computed from the standard Wolfe line search conditions $(3)-(4)$. Suppose that there exists the positive constant τ such that $1/4 \leq \theta_k \leq \tau$ for all $k \geq 1$. If the objective function is uniformly convex on S, then $\lim_{k\to\infty} \|g_k\| = 0.$

Proof :

Now, from $\beta_k^{SAH} \leq \beta_k^{HY} Z_k$ with θ_k^{S1} it follows that f is a uniformly convex function. Because the descent condition hold, we have $d_{k+1} \neq 0.$ Also, from Assumptions 1 and 2, **Proposition 4.1**, **Lemma 4.2**, we have

$$
\begin{aligned}
\|d_{k+1}\| &= \left\|-\theta_k^{S1} g_{k+1} + \beta_k^{SAH} d_k\right\| \\
&\le \left|\theta_k^{S1}\right| \|g_{k+1}\| + \left|\beta_k^{HY} Z_k\right| \|d_k\| \\
&\le \tau \|g_{k+1}\| + \left(\frac{\|g_{k+1}\|^2}{\frac{2}{\alpha_k}(f_k - f_{k+1})} Z_k \right) \|d_k\| \\
&\le \tau \bar{\gamma} + Z_k \frac{\left[\bar{\gamma}\right]^2}{2c\delta_1 \|g_k\|^2} B \qquad \text{..........(45)}
\end{aligned}
$$

$$
\|d_{k+1}\| \le \left(\tau + C\bar{\gamma}\right)\bar{\gamma} \quad , \quad C = Z_k \frac{B}{2c\delta_1 \|g_k\|^2}
$$

This relation shows that

$$
\sum_{k\ge 1} \frac{1}{\|d_{k+1}\|^2} . \ge \left(\frac{1}{\left(c_1 + C\bar{\gamma}\right)\bar{\gamma}} \right) \sum_{k\ge 1} 1 = \infty . \qquad \text{..........(46)}
$$

Therefore, from Lemma 4.1 we have $\liminf_{k\to\infty} \|g_k\| = 0$, which for uniformly convex function is equivalent to $\lim_{k\to\infty} \|g_k\| = 0$.

Remark : we use similarly technique to classical algorithm θ_k^{S2} .

5. Numerical Results :

In this section, we reported some numerical results obtained with the implementation of the new methods on a set of unconstrained optimization test problems taken from (Andrie, 2008) [1].

We selected (15) large scale unconstrained optimization test problems. For each test function we have considered 10 numerical experiments with number of variables n=100, 200, ……1000. We use $\delta_1 = 10^{-4}$ and $\delta_2 = 0.9$ in the line search routine $(3)-(4)$. All these methods terminate when the following stopping criterion is met $\|g_{k+1}\| \le 10^{-6}$.

All codes are written in double precision FORTRAN Language with F90 default compiler settings. We record the number of iterations calls (NOI), and the number of restart calls (IRS) for the purpose our comparisons.

Table (5.1) : Comparison of methods for n= 100

Test problems	β_k^{HY}		θ_k^{S1}		θ_k^{S2}	
	NOI	IRS	NOI	IRS	NOI	IRS
Extended Rosenbrock	20	13	20	13	20	12
Extended While & Holst	54	20	46	17	46	17
Extended PSC 1	34	17	34	16	33	17
Extended Maratos	14	9	19	10	21	11
Quadratic QF2	48	16	36	12	45	15
Arwhead	19	14	12	7	12	7
Nondia	39	19	37	17	36	16
Partial Perturbed Quad.	593	199	213	74	213	74
Liarwhd	29	14	33	16	33	15
Denschnc	23	21	22	15	20	16
Extended Block Diagonal	36	17	37	15	35	14
Generalized Quad. GQ1	51	20	47	18	51	20
Sincos	25	18	25	10	25	10
Liarwhd (CUTE)	F	F	777	284	796	286
Generalized Quad. GQ2	162	60	159	57	137	52
Total	**1147**	**457**	**740**	**297**	**727**	**296**

Table (5.2) : Comparison of methods for n= 1000

Test problems	β_k^{HY}		θ_k^{S1}		θ_k^{S2}	
	NOI	IRS	NOI	IRS	NOI	IRS
Extended Rosenbrock	39	21	33	21	33	21
Extended While & Holst	55	20	49	18	51	19
Extended PSC 1	F	F	237	200	301	263
Extended Maratos	75	67	23	18	53	47
Quadratic QF2	51	17	42	14	51	17
Arwhead	24	21	32	20	25	22
Nondia	43	20	42	19	42	19

Partial Perturbed Quad.	F	F	F	F	F	F
Liarwhd	98	85	64	50	36	25
Denschnc	21	19	13	11	21	19
Extended Block Diagonal	47	18	37	17	39	18
Generalized Quad. GQ1	52	21	52	21	50	18
Sincos	38	19	35	14	33	14
Liarwhd (CUTE)	F	F	1566	906	701	252
Generalized Quad. GQ2	1420	474	1427	479	1393	469
Total	**1963**	**802**	**1849**	**702**	**1827**	**708**

Conclusions

Form the numerical results of the above tables, we say that the results of Table (5.1) and Table (5.2) give a general comparison between HY and two new spectral CG-methods taking non linear test function with n=100,1000. This table indicates that the modified methods saves (16-17) % NOI and (20) % IRS. The Percentage Performance of the improvements of the Table (5.1) and Table (5.2) are given by the following table (5.3).

Table(5.3): Relative efficiency of the new Algorithm

Tools	NOI	IRS
M Dai-Yuan method	100 %	100 %
New Algorithm with θ_k^{S1}	83.24 %	79.34 %
New Algorithm with θ_k^{S2}	82.12 %	79.74 %

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