

# STUDYING THE NUCLEAR STRUCTURE OF EVEN-EVEN DEFORMED GERMANIUM $^{70}_{32}\text{Ge}_{38}$ NUCLEUS

ALI KALAF AOBaid

DEPT. OF PHYSICS-COLLEGE OF EDUCATION-AL-ANBAR UNIVERSITY



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## ABSTRACT

In the present work, the interacting boson model(IBM-1) for s and d boson is used to calculate the energy levels of the positive parity bands, electric quadrupole moments, E2-transition probabilities B(E2)in particular between states which may be affected by the finite boson number  $N=N_{\pi}+N_{\nu}=2+5=7$  of even-even  $^{70}_{32}\text{Ge}_{38}$  nucleus . This nucleus is belong to SU(5)-SU(3) dynamical symmetry .

The calculated results of this study are compared with the available experimental data and they found to be in a very good agreement.

## Introduction

In 1974 Arima A. and Iachello F. have proposed a new nuclear model, called ; Interacting Boson Model (IBM); [1],which attempts to describe some nuclear properties such as spins and energies of the lowest levels of intermediate and heavy mass nuclei, except those near closed shells[2,3].The simplest versions of IBM called; IBM-1; has to be able to describe the even-even deformed nuclei as an inert core combined with bosons which represent pairs of fermions , which can occupy one of two levels: a ground state with (L=0)called s-boson or an excited state with(L=2)called d-bosons[4].

The dynamical symmetries in this model are depending on the main group U(6) of unitary transformations in 6-dimansions and 36 generators. The analysis of this group leads to three subgroups called chains that end in O(2). They can be written as[3]:

(I) The vibrational SU(5)chain

$$U(6) \supset SU(5) \supset O(5) \supset O(3) \supset O(2) \dots (1)$$

(II ) The rotational SU(3)chain:

$$U(6) \supset SU(3) \supset O(3) \supset O(2) \dots (2)$$

(III) The  $\gamma$  -unstable O(6)chain:

$$U(6) \supset O(6) \supset O(5) \supset O(3) \supset O(2) \dots, (3)$$

One of the basic properties of deformed nuclei is the presence of specific collective rotational states SU(3) in their excitation spectra. These states can only be observed in nuclei with a non-spherical shape [5].

The collective motion of deformed nuclei depends on two facts [6,7]:

- 1-All deformed nuclei have an electric quadrupole moments in their energy states .
- 2-The changing in the deformation shape depends on the direction of nucleons and nuclear motion causing rotation of some nucleons around an axis different from nuclear symmetry axis , for this reason, we can explain two types of deformation as follows:

### 1-Prolate deformation:

In this deformation, the nucleus rotate around an axis perpendicular to the nuclear symmetry axis . This type has positive quantity of electric quadrupole moment (+QL).

### 2-Oblete deformation:

In this deformation, the nucleus is rotating in axis parallel to the nuclear symmetric axis. This rotating

\* Corresponding author at: DEPT. OF PHYSICS-COLLEGE OF EDUCATION-AL-ANBAR UNIVERSITY, Iraq,E-mail address: [scicoll@yahoo.com](mailto:scicoll@yahoo.com)

happened in spherical nuclei , and it has negative quantity of electric quadrupole moment (-QL).Some of the previous studies about this subject are:

Iman T. AL-Alawy (2000)[8], had been calculate the (EO,E2,B(E2),QL) transitions of collective levels of Pt-196 nucleus.

Iman T. AL- Alawy, et al (2005) [9]had studied the electric quadrupole moments, and g-factor for deformed Yb(A=176)nucleus of dynamical symmetry SU(3)-SU(5).

Aobaid A.K.(2006)[10] studied the nuclear structure of some even-even

(<sup>56</sup>Fe<sub>30</sub>, <sup>74</sup>Se<sub>40</sub>, <sup>88</sup>Zr<sub>48</sub>, <sup>106</sup>Pd<sub>60</sub>, <sup>156</sup>Gd<sub>92</sub>, <sup>180</sup>Hf<sub>108</sub>, <sup>186</sup>W<sub>112</sub>) nuclei. He found a good agreement results with available experimental data.

Theoretical Part:

1-The Hamiltonian Operators In IBM-1:

Scholten, Iachello and Arima (1978) [11] proposed a simplest Hamiltonian form as follows:

$$\hat{H} = \varepsilon \hat{n}_d + b_0 (\hat{P}^\dagger \cdot \hat{P}) + b_1 \hat{L}^2 + b_2 \hat{Q} + b_3 \hat{T}_3^2 + b_4 \hat{T}_4^2 \dots \dots \dots (4)$$

Where:

$\varepsilon$  is the energy parameter of d-boson operator ( $\hat{n}_d$ ),  $b_0$  is the parameter of the pairing operator ( $\hat{P}$ ),  $b_1$  is the parameter of the angular momentum operator ( $\hat{L}$ ),  $b_2$  is the parameter of the quadrupole momentum operator ( $\hat{Q}$ ),  $b_3$  is the parameter of the octupole operator ( $\hat{T}_3$ ),  $b_4$  is the parameter of the hexadecapole operator ( $\hat{T}_4$ ).

The Hamiltonian in equation (4) depends on the type of dynamical symmetry as in equations (1,2,3) and takes the following form[3,4]:

for chain ( I ) :

$$\hat{H}^{(I)} = \varepsilon \hat{n}_d + b_1 \hat{L}^2 + b_3 \hat{T}_3^2 + b_4 \hat{T}_4^2 \dots (5)$$

for chain ( II ) :

$$\hat{H}^{(II)} = b_1 \hat{L}^2 + b_2 \hat{Q}^2 \dots (6)$$

for chain ( III ) :

$$\hat{H}^{(III)} = b_0 (\hat{P}^\dagger \cdot \hat{P}) + b_1 \hat{L}^2 + b_3 \hat{T}_3^2 \dots (7)$$

The electric transition operator, which is associated with collective states can be written as [4]:

$$\hat{T}(E2) = \delta_0 [\hat{d}^\dagger \times \hat{s} + \hat{s}^\dagger \times \hat{d}]^{(2)} + \delta_1 [\hat{d}^\dagger \times \hat{\tilde{d}}]^{(2)} \dots (8)$$

This operator has two parts [ $\hat{d}^\dagger \times \hat{s} + \hat{s}^\dagger \times \hat{d}$ ] which satisfies the selection rule  $\Delta n_d = \pm 1$ , and the term [ $\hat{d}^\dagger \times \hat{\tilde{d}}$ ] which satisfies the  $\Delta n_d = 0$  selection rule .

The parameters  $\delta_0$  and  $\delta_1$  depend on the limit involved or the appropriate intermediate structure .

The electric transition operator T(E2) in equation (8) can be written down as[3] :

$$\hat{T}(E2) = e_B [(\hat{d}^\dagger \times \hat{s} + \hat{s}^\dagger \times \hat{d}) + \chi (\hat{d}^\dagger \times \hat{\tilde{d}})]^{(2)} = e_B Q \dots (9)$$

Where  $e_B$  is a boson effective charge , $\chi$  is any parameter.

The electric transition probability B(E2) determined from the following reduced matrix element [4]:

$$\langle L M_L | \hat{T}_K^{(K)} | L' M'_L \rangle = (-1)^{L-M} \begin{pmatrix} L & K & L' \\ -M_L & K & M'_L \end{pmatrix} \langle L | \hat{T}^{(K)} | L' \rangle \dots (10)$$

The calculation of electric quadrupole moments for three chains depended on selected rules as shown in table (1), and can be written as:

$$Q_L = 0.676 \begin{pmatrix} L & 2 & L \\ -L & 0 & L \end{pmatrix} \langle L_f | \hat{T}^{(E2)} | L_i \rangle \dots (11)$$

While the electric transition probability  $B(E2)$  can be written as:

$$B(E2; L_i \rightarrow L_f) = \frac{1}{(2L+1)} |\langle L_f || \hat{T}^{(E2)} || L_i \rangle|^2 \dots (12)$$

**Results and Discussions:**

**1-The energy levels , and Their Transitions:**

In order to calculate the nuclear structure of even-even  $^{70}_{32}\text{Ge}_{38}$  deformed nucleus, we must find the dynamical symmetry of this nucleus. The dynamical symmetry for any nucleus was determined according to the following[4]:

- 1-Energy ratios as shown in table(2).
- 2-Experimental energy spectrum see references [3,4].
- 3-Energy band arrangement(g, $\beta$ , $\gamma$  )

Table (2) listed the calculated energy ratios (pw) in comparison with experimental [12,13] and ideal [3] values . This table shows the nucleus under study lying in the transitional region SU(5)-SU(3), also this table gives very reasonable description of the behavior  $^{70}_{32}\text{Ge}_{38}$  nucleus.

The parameters of Hamiltonian equation (4) for  $^{70}_{32}\text{Ge}_{38}$  nucleus are shown in table (3).

These parameters have been obtained by fits them with experimental energy levels values for low spin and used to calculate the energy levels and their transitions as in table(4)

Table(4) shows a good comparison between calculated energy levels and available experimental values ,specially at low-lying of the ground states  $(0_1^+, 2_1^+, 4_1^+, 6_1^+, 8_1^+)$ ,and in other states  $(2_3^+, 0_3^+, 2_4^+, 4_2^+, 2_5^+, 4_3^+, 4_4^+, 4_5^+, 2_7^+, 4_7^+)$  while at

high angular momentum, some theoretical values are somehow larger than experimental because:

- 1-These values need large time to be measured.
- 2-The IBM-1 does not distinguish between neutrons and protons bosons.
- 3-The fitted of equation (4) was very difficult to get same values as experimental data for all energy levels.

**2-The electric Transitions Probability B(E2):**

In this work, the equations (10,12) are used to calculate the electric transition probability and their reduced matrix element for this deformed nucleus .The parameters for these equations are ( $\delta_0=0.10498(\text{eb})$ , and  $\delta_1=0.068(\text{eb})$ ). These parameters have been obtained by fitting them with experimental values of  $B(E2)$ .

The calculated results are listed in table (4). This table contain the comparison between calculated  $B(E2)$  and available experimental data. The comparison shows an acceptable agreement, specially at transitions  $(2_1^+ \rightarrow 0_1^+), (4_1^+ \rightarrow 2_1^+), (4_2^+ \rightarrow 2_2^+), (4_3^+ \rightarrow 2_1^+)$ ,and $(6_1^+ \rightarrow 4_1^+)$ .

Figure (1) shows the relation between the electric transitions probability  $B(E2)$  for the transitions  $(2_1^+ \rightarrow 0_1^+), (4_1^+ \rightarrow 2_1^+), (6_1^+ \rightarrow 4_1^+), (8_1^+ \rightarrow 6_1^+)$  as a function of angular momentum  $L+2 \rightarrow L$ . It is noticed in this figure that the highest probability happened at transition  $(6_1^+ \rightarrow 4_1^+)$ . This means that the strong  $B(E2)$  happened at this transition ,also this figure shows very good agreement between present work and experimental data .

**3-The electric quadrupole moments:**

The electric quadrupole moments can be calculated from equation (8)by using the same

parameters ( $\delta_0$  and  $\delta_1$ ) of B(E2). The calculated values of (QL) show that these values are depending on  $\delta_1$  -value more than ( $\delta_0$ ) and both of them depends on total number of bosons(N).

Table (5) shows the values of QL<sub>i</sub> where L=2 to 6, i=1 to 10.

These values have been compared with experimental data and gives very perfect agreement of the first excited state ( $2_1^+$ ) and somehow small in second excited state ( $2_2^+$ ).

Figure (2) explain the shape deformation of angular momentum  $2_1^+ \rightarrow 2_{10}^+$  for nucleus understudy, which has a prolate type of deformation at state ( $2_1^+, 2_3^+, 2_5^+, 2_7^+, 2_8^+,$  and  $2_9^+$ ), This is agreeing with experimental shape and value in the state ( $2_1^+$ ), while it has an oblate type at states ( $2_2^+, 2_4^+, 2_6^+,$  and  $2_{10}^+$ ).

Figure (3) same as in figure (2) but for  $L=3_1^+ \rightarrow 3_{10}^+$ . This figure shows the nucleus has an oblate type in ( $3_1^+, 3_2^+, 3_3^+$ ) states, but the ( $3_4^+$ ) state has prolate type, and has QL=0 in other states. While in the figure (4) has prolate type in states ( $4_1^+, 4_3^+, 4_6^+, 4_7^+, 4_8^+,$  and  $4_9^+$ ) and an oblate type in another state .

Figure (5) shows that it has prolate type in states ( $5_1^+, 5_4^+, 5_5^+$ ), while it has an oblate type in state ( $5_2^+, 5_3^+$ ).

Figure (6) shows the nucleus has an oblate type in all states except has an oblate type in ( $6_3^+$ ) state.

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**Table(1):Selection rules and quadrupole moments in different dynamical symmetries[23]**

Chain	operators	selection rule	Quadrupole moments
SU(5)	$[\hat{d}^\dagger \times \hat{s} + \hat{s}^\dagger \times \hat{d}]^{(2)}$ $[\hat{d}^\dagger \times \hat{d}]^{(2)}$	$\Delta n_d = \pm 1, \Delta v = \pm 1$ $\Delta n_d = 0, \Delta v = 0, \pm 2$	$Q_L = 0.676 \delta_1 \sqrt{L}$ 14
SU(3)	$[\hat{d}^\dagger \times \hat{s} + \hat{s}^\dagger \times \hat{d}]^{(2)}$ $[\hat{d}^\dagger \times \hat{d}]^{(2)}$	$\Delta \sigma = \pm 1, \Delta \gamma = \pm 1$ $\Delta \sigma = 0, \Delta \gamma = 0, \pm 2$	$Q_L = -1.121 \delta_0 \cdot (L / (2L+3)) (4N+3)$
O(6)	$[\hat{d}^\dagger \times \hat{s} + \hat{s}^\dagger \times \hat{d}]^{(2)}$ $[\hat{d}^\dagger \times \hat{d}]^{(2)}$	$\Delta \lambda = \Delta \mu = 0$ $\Delta \lambda = \Delta \mu = 0$ or $\Delta \lambda = \pm 4, \Delta \mu = \pm 4$	$Q_L = 0$

**Table(2):Ideal energy ratios of three chains[3] compared with experimental[12,13] and theoretical values .**

Valeus	$E(4_1^+) / E(2_1^+)$	$E(6_1^+) / E(2_1^+)$	$E(8_1^+) / E(2_1^+)$
SU(5)	2	3	4
SU(3)	3.33	7	12
O(6)	2.5	4.5	7
Exp.,[12,13]	2.0721	3.1735	4.0459
Present work(pw)	2.0072	3.0234	4.0501

**Table (3): The parameters of Hamiltonian operators for  $^{70}_{32}Ge_{38}$  nucleus.**

The Parameters	$N_r$	$N_o$	$N$	$E$ (MeV)	$\hat{p}^i, \hat{p}^j$ (MeV)	$\hat{L}^2$ (MeV)	$\hat{Q}^2$ (MeV)	$\hat{T}_3^2$ (MeV)	$\hat{T}_4^2$ (MeV)	$X$ (MeV)

Values
2
5
7
0.9740
0.0000
0.0090
0.0200
0.0500
-0.1000
0.8200

**Table (4): Theoretical energy levels, transition energy, reduced matrix element  $\langle L_f || \hat{T}^{(E2)} || L_i \rangle$ , and probabilities of electric transitions B(E2) compared with available experimental data**

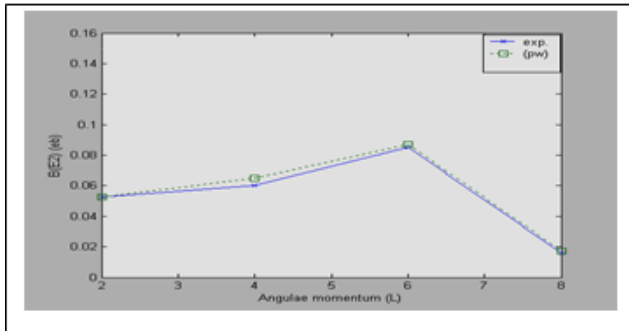
$L_i$	Energy level E(L) (MeV)		Spin sequence $L_i^+ - L_f^+$	Transition energy $E_\gamma$ (MeV)		$\langle L_f    \hat{T}^{(E2)}    L_i \rangle$ (eb)	B(E2) (eb) <sup>2</sup>	
	Exp.[12]	IBM-1		Exp.[12]	IBM-1		Exp.[12]	IBM-1
$0_1^+$	0.00	0.00	-	-	-	-	-	-
$2_1^+$	1.0392	1.0536	$2_1^+ \rightarrow 0_1^+$	1.0392	1.0536	0.5130	0.0525	0.0526
$0_2^+$	1.2154	1.4608	$0_2^+ \rightarrow 2_1^+$	0.1762	0.4072	0.2825	0.1200	0.0798
			$0_2^+ \rightarrow 0_1^+$	1.2154	1.4608	0.0000	0.0000	0.0000
$2_2^+$	1.7079	1.9443	$2_2^+ \rightarrow 0_2^+$	0.4925	0.4835	-0.1470	0.0625	0.0043
			$2_2^+ \rightarrow 2_1^+$	0.6687	0.8907	0.4915	0.2775	0.0482
			$2_2^+ \rightarrow 0_1^+$	1.7079	1.9443	-0.0133	0.0025	0.0003
$4_1^+$	2.1534	2.1148	$4_1^+ \rightarrow 2_2^+$	0.4455	0.1705	-0.1279	-	0.0018











Figure(1): Electric transition probability B(E2) with angular momentum

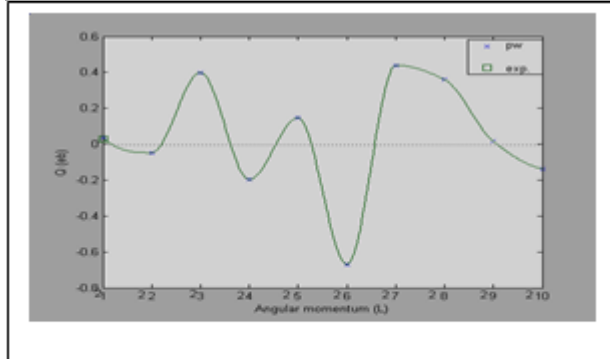


Figure (2): The electric quadrupole moment as a function of angular momentum L=2<sub>1</sub> to 2<sub>10</sub>.

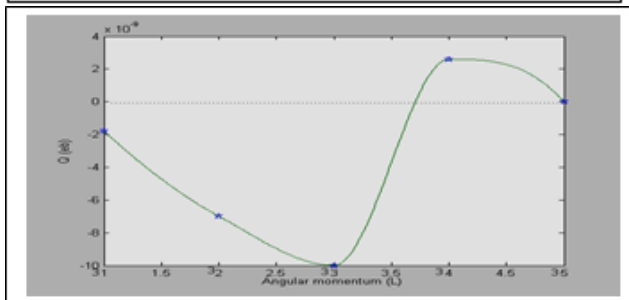
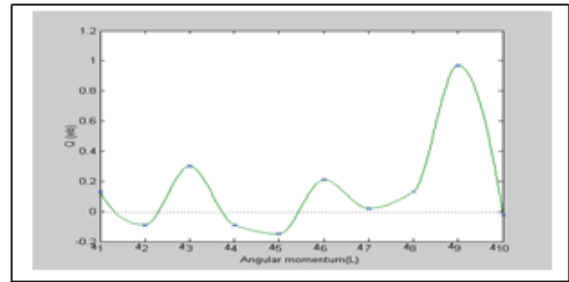
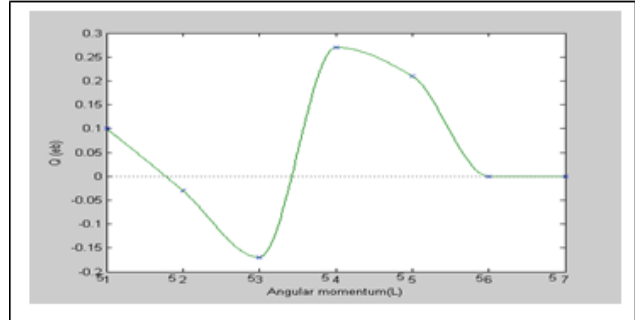


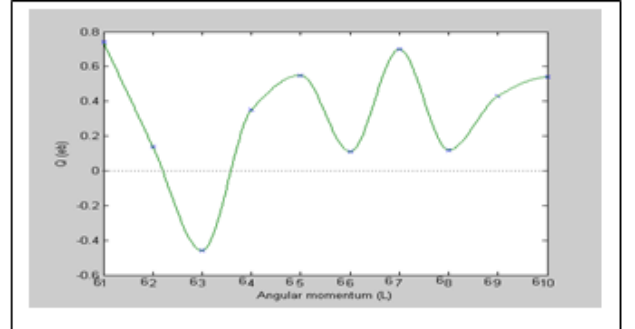
Figure (3): Same as in fig. (2) but for L=3<sub>1</sub> to 3<sub>5</sub>.



Figure(4): The electric quadrupole moment as a function of angular momentum L=4<sub>1</sub> to 4<sub>10</sub>



Figure(5): The electric quadrupole moment as a function of angular momentum L=5<sub>1</sub> to 5<sub>7</sub>



Figure(6): The electric quadrupole moment as a function of angular momentum L=6<sub>1</sub> to 6<sub>10</sub>

## دراسة التركيب النووي لنواة الجرمانيوم $^{70}_{32}Ge_{38}$ الزوجية-الزوجية المشوهة

علي خلف عبيد

قسم الفيزياء - كلية التربية - جامعة الانبار

الخلاصة

في البحث الحالي تم استخدام نموذج البيوزونات المتفاعله الاول ( IBM-1 ) لحساب مستويات الطاقة الموجبة التناظر وانتقالاتها الكامية كداله للزخم الزاوي والعزوم الكهربائية رباعية القطب واحتمالية الانتقالات الكهربائية رباعية القطب التي تعتمد على عناصر المصفوفة المختزلة لنواة الجرمانيوم الزوجية- الزوجية التي تنتمي الى التناظر الديناميكي SU(5)-SU(3) لقد تم مقارنة هذه النتائج مع النتائج العملية المتوفرة وكانت النتائج متوافقة بشكل جيد.