MODIFIED QUASI SIMPS0N 'S 3/8 RULE FOR SOLVING SYSTEM OF INTEGRAL EQUATION OF THE SECOND KIND LINEAR

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A R T I C L E I N F O

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A B S T R A C T

Actually, it is possible to solve systems of integral equation by using many approaches. However, in this study, the modified quasi Simpson's 3/8 rule used to find the numerical solution of a system of linear Volterra integral equations of the second kind. This method solves systems of linear Volterra integral equations of the second kind in more accurate way than the modified trapezoidal rule. Some indicative examples are given to elaborate the accuracy of this method.

1. Introduction:

In the present study , the modified trapezoidal method used for solving system of linear integral equations of the second kind ,[7].Furthermore , some numerical rules are used for dimensional integral equations,[3].They introduce numerical solutions for one – dimensional volterra linear integral equations of the second kind , [2]. In this paper the repeated modified quasi Simpson's 3/8 rule

solving system of the one – dimensional linear volterra integral equations, [4].

Now we consider the following system of linear Volterra integral equations of the second kind :

$$u_r(x) = f_r(x) + \sum_{s=1}^{m} \lambda_{r,s} \int_a^x k_{r,s}(x,y) u_s(y) dy, \quad r = 1,2,\ldots,m \quad \ldots\ldots\ldots\ldots\ldots (1.1)$$

where $a \le x \le b$, $\lambda_{r,s}$ is a real number, f_r , $k_{r,s}$, are given continuous functions and u_1 , u_2 , … , u_m are function that must be determined by systems of linear volterra integral equations of the unknown the second kind.

There are many methods for solving the one – dimensional volterra integral equations. These methods depend on the structure of the one–

$$\int_a^b f(x)dx = \frac{3h}{80}\left[13f(a) + 27\sum_{j=1}^{\frac{n}{3}}[f(x_{3j-1}) + f(x_{3j-2})] + 26\sum_{j=1}^{\frac{n}{3}-1} f(x_{3j}) + 13f(b)\right] +$$

$$\frac{3h^2}{40}\left[f'(a) - f'(b)\right] \quad \ldots\ldots\ldots\ldots (1.2) \quad , \quad [1]$$

and repeated modified trapezoidal rule ….. (1.3)

$$\int_a^b f(x)dx = \frac{h}{2}\left[f(a) + 2\sum_{j=1}^{n-1} f(x_j) + f(b)\right] + \frac{h^2}{12}\left[f'(a) - f'(b)\right] - \frac{(b-a)h^{(4)}}{720} f^{(4)}(\xi)$$

to find the numerical solution for the system given by equation (1.1).To do this, one assumes the used and $f_r/(x)$ exist for all r, s = 1,2,…,m. For further

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Now , we approximate the integral that appear in the right hand side of equation (1.1) at x = xi , the repeated modified quasi Simpson's 3/8 rule to get : i = 0,1,…, n, by

information on formula (function $\frac{\partial k_{r,s}(x,y)}{\partial x},\frac{\partial k_{r,s}(x,y)}{\partial y}$ 1.3), See, [6],[5].

2.The Modified Quasi Simpson's 3/8 Rule.

$$u_r(x_0)=f_r(x_0),$$

$$u_r(x_i)=f_r(x_i)+\sum_{s=1}^{m}\lambda_{r,s}\left\{\frac{3h}{80}\Big[13k_{r,s}(x_i,x_0)u_s(x_0)+27\sum_{j=1}^{\frac{i}{3}}\{k_{r,s}(x_i,x_{3j-1})u_s(x_{3j-1})+k_{r,s}(x_i,x_{3j-2})u_s(x_{3j-2})\}+\right.$$

$$26\sum_{j=1}^{\frac{i}{3}-1}k_{r,s}(x_i,x_{3j})u_s(x_{3j})+13k_{r,s}(x_i,x_i)u_s(x_i)\Big]+$$

$$\left.\frac{3h^2}{40}\Big[k_{r,s}(x_i,x_0)u_s'(x_0)+J_{r,s}(x_i,x_o)u_s(x_o)-k_{r,s}(x_i,x_i)u_s'(x_i)-J_{r,s}(x_i,x_i)u_s(x_i)\Big]\right\},i=3,6,\dots,n$$

and

$$u_r(x_i)=f(x_i)+\sum_{s=1}^{m}\lambda_{r,s}\left\{\frac{h}{2}\left[k_{r,s}(x_i,x_0)u_s(x_0)+2\sum_{j=1}^{i-1}k_{r,s}(x_i,x_j)u_s(x_j)+k_{r,s}(x_i,x_i)u_s(x_i)\right]+\right.$$

$$\left.\frac{h^2}{12}\Big[k_{r,s}(x_i,x_0)u_s'(x_0)+J_{r,s}(x_i,x_0)u_s(x_0)-k_{r,s}(x_i,x_i)u_s'(x_i)-J_{r,s}(x_i,x_i)u_s(x_i)\Big]\right\},i\neq 3,6,\dots,n$$

i=1,2 , … ,n. the above system of equations consists of

and $\ell=0,i$, $J_{r,s}(x_i,y_\ell)=\frac{\partial k_{r,s}(x_i,y_\ell)}{\partial y}$ where

, r= 1,2,…,m. to do this , we must differentiate equation (1.1) with respect to x to get i=0,1,…,n

equations with 2m(n+1) unknowns namely, ur(xi) , i=0,1,…,n , u/r(xi). Next,we have to find u/r(xi), 2m(n+1)

$$u_s'(x)=f_r'(x)+\sum_{s=1}^{m}\lambda_{r,s}\left\{\int_a^x H_{r,s}(x,y)u_s(y)\,dy+k_{r,s}(x,x)u_s(x)\right\},\quad a\le x\le b,\ r=1,2,\dots,m\ \dots\dots\dots(2.1)$$

, r = 1,2,…,m. It is easy to check any solution of equation (1.1) is a solution $H_{r,s}(x,y)=\frac{\partial k_{r,s}(x,y)}{\partial x}$ where

equation (2.2) . By evaluating equation (2.1) at x = xi , i = 0,1,…,n , one can get of

$$u_s'(x_i)=f_r'(x_i)+\sum_{s=1}^{m}\lambda_{r,s}\left\{\int_a^{x_i}H_{r,s}(x_i,y)u_s(y)\,dy+k_{r,s}(x_i,x_i)u_s(x_i)\right\},\quad i=o,1,\dots,n,\quad r=1,2,\dots,m\dots\dots\dots(2.2)$$

in the right hand side of the integral equation (2.2) with the repeated modified quasi Simpson's 3/8 rule to obtain :

: Next, to solve equation (2.2), one must consider three cases :

exists for each r , s = 1,2,…,m , we approximate

the integral that $\frac{\partial^2 k_{r,s}(x,y)}{\partial x\partial y}$ Case (1):- If appear

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$$u_r'(x_0)=f_r'(x_0)+\lambda_{r,s}\ k_{r,s}(x_o,x_0)\ u_s(x_0),$$

$$u_r^{/}(x_i) = f_r^{/}(x_i) + \sum_{s=1}^{m} \lambda_{r,s} \left\{ \frac{3h}{80} \left[13 H_{r,s}(x_i, x_0) u_s(x_0) + 27 \sum_{j=1}^{\frac{i}{3}} \{ H_{r,s}(x_i, x_{3j-1}) u_s(x_{3j-1}) + H_{r,s}(x_i, x_{3j-2}) u_s(x_{3j-2}) \} + \right.\right.$$

$$26 \sum_{j=1}^{\frac{i}{3}-1} H_{r,s}(x_i, x_{3j}) u_s(x_{3j}) + 13 H_{r,s}(x_i, x_i) u_s(x_i) \Big] +$$

$$\left.\frac{3h^2}{40} \left[H_{r,s}(x_i, x_0) u_s^{/}(x_0) + L_{r,s}(x_i, x_o) u_s(x_o) - H_{r,s}(x_i, x_i) u_s^{/}(x_i) - L_{r,s}(x_i, x_i) u_s(x_i) \right] + k_{r,s}(x_i, x_i) u_s(x_i) \right\},$$

$where\ i = 3,6,\ldots,n,\ r = 1,2,\ldots,m,$

and

$$u_r(x_i) = f(x_i) + \sum_{s=1}^{m} \lambda_{r,s} \left\{ \frac{h}{2} \left[H_{r,s}(x_i, x_0) u_s(x_0) + 2 \sum_{j=1}^{i-1} H_{r,s}(x_i, x_j) u_s(x_j) + H_{r,s}(x_i, x_i) u_s(x_i) \right] + \right.$$

$$\left. \frac{h^2}{12} \left[H_{r,s}(x_i, x_0) u_s^{/}(x_0) + L_{r,s}(x_i, x_0) u_s(x_0) - H_{r,s}(x_i, x_i) u_s^{/}(x_i) - L_{r,s}(x_i, x_i) u_s(x_i) \right] + k_{r,s}(x_i, x_i) u_s(x_i) \right\},$$

$where\ \ i \neq 3,6,\ldots,n\ ,\ r = 1,2,\ldots,m\ ,$

$$L_{r,s}(x_i, y_\ell) = \frac{\partial H_{r,s}(x_i, y_\ell)}{\partial y}\ , \quad \ell = 0, i\ , \quad i = 1,2,\ldots,n\ , \quad r, s = 1,2,\ldots,m.$$

and

Case (2) :- if $\frac{\partial^2 k_{r,s}(x,y)}{\partial x \partial y}$ does not exist for each r, s = 1 , 2 , …. ,m, we approximate the integral appear in the right hand side of the integral equation (2.2) with the repeated Simpson's 3/8 rule to that obtain:

$$u_r^{/}(x_0) = f_r^{/}(x_0) + \lambda_{r,s}\ k_{r,s}(x_o, x_0)\ u_s(x_0),$$

$$u_r^{/}(x_i) = f_r^{/}(x_i) + \sum_{s=1}^{m} \lambda_{r,s} \left\{ \frac{3h}{8} \left[H_{r,s}(x_i, x_0) u_s(x_0) + 3 \sum_{j=1}^{\frac{i}{3}} \{ H_{r,s}(x_i, x_{3j-1}) u_s(x_{3j-1}) + H_{r,s}(x_i, x_{3j-2}) u_s(x_{3j-2}) \} + \right.\right.$$

$$\left. 2 \sum_{j=1}^{\frac{i}{3}-1} H_{r,s}(x_i . x_{3j}) u_s(x_{3j}) + H_{r,s}(x_i, x_i) u_s(x_i) \Big] + k_{r,s}(x_i, x_i) u_s(x_i) \right\}, i = 3,6,\ldots,n, r = 1,2,\ldots,m.$$

By solving the above equations with the equation (1.1) altogether, one can get the system of linear equations.

Case(3):- if $\frac{\partial^2 k_{r,s}(x,y)}{\partial x \partial y}$ does not exist for some p, q =1 ,2 , … ,m , we approximate the integral $\int_a^{x_i} \frac{\partial k_{p,q}(x,y)}{\partial x \partial y} u_q(y)\,dy$ that appear in the right hand side of the integral equation(2.2) with the repeated Simpson's 3/8 rule and the other integrals that appear in the right hand side of the integral equation (2.2) with the repeated modified quasi Simpson's 3/8 rule to obtain:

$$u_r^{/}(x_0) = f_r^{/}(x_0) + \lambda_{r,s}\ k_{r,s}(x_o, x_0)\ u_s(x_0),$$

$$u_p^{/}(x_0) = f_p^{/}(x_0) + \lambda_{p,s}\ k_{p,s}(x_o, x_0)\ u_s(x_0),$$

By solving the above equations with the equation (1.1) altogether, one can get the system of linear equations.

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$$u_r^{\prime}(x_i)=f_r^{\prime}(x_i)+\sum_{s=1}^{m}\lambda_{r,s}\left\{\frac{3h}{80}\left[13H_{r,s}(x_i,x_0)u_s(x_0)+27\sum_{j=1}^{\frac{i}{3}}\{H_{r,s}(x_i,x_{3j-1})u_s(x_{3j-1})+H_{r,s}(x_i,x_{3j-2})u_s(x_{3j-2})\}+\right.\right.$$

$$26\sum_{j=1}^{\frac{i}{3}-1}H_{r,s}(x_i,x_{3j})u_s(x_{3j})+13H_{r,s}(x_i,x_i)u_s(x_i)]+$$

$$\left.\frac{3h^2}{40}\left[H_{r,s}(x_i,x_0)u_s^{\prime}(x_0)+L_{r,s}(x_i,x_o)u_s(x_o)-H_{r,s}(x_i,x_i)u_s^{\prime}(x_i)-L_{r,s}(x_i,x_i)u_s(x_i)\right]+k_{r,s}(x_i,x_i)u_s(x_i)\right\},$$

$where\ i=3,6,\ldots,n,\ r=1,2,\ldots,m,\ r\neq p,$

$$u_p^{\prime}(x_i)=f_p^{\prime}(x_i)+\sum_{\substack{s=1\\ s\neq q}}^{m}\lambda_{p,s}\left\{\frac{3h}{80}\left[13H_{p,s}(x_i,x_0)u_s(x_0)+27\sum_{j=1}^{\frac{i}{3}}\left[H_{p,s}(x_i,x_{3j-1})u_s(x_{3j-1})+H_{p,s}(x_i,x_{3j-2})u_s(x_{3j-2})\right]+\right.\right.$$

$$\left.26\sum_{j=1}^{\frac{i}{3}-1}H_{p,s}(x_i,x_{3j})u_s(x_{3j})+13H_{p,s}(x_i,x_i)u_s(x_i)\right]+$$

$$\left.\frac{3h^2}{40}\left[H_{p,s}(x_i,x_0)\,u_s^{\prime}(x_0)+L_{p,s}(x_i,x_0)\,u_s(x_0)-H_{p,s}(x_i,x_i)\,u_s^{\prime}(x_i)-L_{p,s}(x_i,x_i)\,u_s(x_i)\right]+k_{p,s}(x_i,x_i)u_s(x_i)\right\}+$$

$$\frac{3h}{8}\lambda_{p,q}\left[H_{p,q}(x_i,x_0)u_q(x_0)+3\sum_{j=1}^{\frac{i}{3}}\left[H_{p,q}(x_i,x_{3j-1})u_q(x_{3j-1})+H_{p,q}(x_i,x_{3j-2})u_q(x_{3j-2})\right]+\right.$$

$$\left.26\sum_{j=1}^{\frac{i}{3}-1}H_{p,q}(x_i,x_{3j})u_q(x_{3j})+H_{p,q}(x_i,x_i)u_q(x_i)\right]+\lambda_{p,q}\,k_{p,q}(x_i,x_i)u_q(x_i),\ i=3,6,\ldots,n,$$

and

$$u_p^{\prime}(x_i)=f_p^{\prime}(x_i)+\sum_{\substack{s=1\\ s\neq q}}^{m}\lambda_{p,s}\left\{\frac{h}{2}\left[H_{p,s}(x_i,x_0)u_s(x_0)+2\sum_{j=1}^{i-1}H_{p,s}(x_i,x_j)u_s(x_j)+H_{p,s}(x_i,x_i)u_s(x_i)\right]+\right.$$

$$\left.\frac{h^2}{12}\left[H_{p,s}(x_i,x_0)u_s^{\prime}(x_0)+L_{p,s}(x_i,x_0)u_s(x_0)-H_{p,s}(x_i,x_i)u_s^{\prime}(x_i)-L_{p,s}(x_i,x_i)u_s(x_i)\right]+k_{p,s}(x_i,x_i)u_s(x_i)\right\}+$$

$$\lambda_{p,q}\left\{\frac{h}{2}\left[H_{p,q}(x_i,x_0)u_q(x_0)+2\sum_{j=1}^{i-1}H_{p,q}(x_i,x_j)u_q(x_j)+H_{p,q}(x_i,x_i)u_q(x_i)\right]+k_{p,q}(x_i,x_i)u(x_i)\right\},i\neq 3,6,\ldots,n.$$

$$where\ \ L_{r,s}(x,y_\ell)=\frac{\partial H_{r,s}(x,y_\ell)}{\partial y}=\frac{\partial^2 k_{r,s}(x,y_\ell)}{\partial x\partial y}\ \ and\ \ L_{p,s}(x,y_\ell)=\frac{\partial H_{p,s}(x,y_\ell)}{\partial y}=\frac{\partial^2 k_{p,s}(x,y_\ell)}{\partial x\partial y},$$

$\ell=0,i\ ,\ i=1,2,\ldots,n\ ,\ r,s=1,2,\ldots,m.$

By solving the above equations with the equation (1.1) altogether, one can get the system of linear equations.

3. Numerical Examples:

In this section we give some numerical examples to illustrate the above method for solving systems of linear volterra integral equations of the second kind . Also, we compare this method with modified trapezoidal rule method for solving system of linear volterra integral equations

Example (1),[7]:

Consider the following system of linear volterra integral equations of the second kind:

.

$$u_1(x) = (-\frac{1}{2}x^2 + \frac{1}{4}x + 1)e^{2x} + (x + \frac{1}{4})e^{-2x} - \frac{3}{4}x - \frac{1}{4} + \int_o^x x\,y\; u_1(y)\; dy + \int_o^x (x+y)\; u_2(y)\,dy,$$

$$u_2(x) = (2x^2 + x + \frac{5}{4})e^{-2x} - \frac{1}{4}e^{2x} - \frac{1}{2}x^2 + \int_0^x (x-y)u_1(y)dy + \int_0^x (x+y)^2 u_2(y)dy,\; 0 \le x \le 1$$

we solve this example numerically by using the composite modified quasi Simpson's 3/8 rule . To do this , , we subdivide the interval [0,1] into 9 subintervals such that xi = $\frac{i}{9}$, i = 0, 1, … ,9 . $k_{11}(x, y)$ = x y, $k_{12}(x, y)$ = x + y, $k_{21}(x, y)$ = x - y , $k_{22}(x, y)$ = $(x + y)^2$,

$$\frac{\partial^2 k_{11}(x,y)}{\partial x \partial y} = 1, \quad \frac{\partial^2 k_{12}(x,y)}{\partial x \partial y} = 0, \quad \frac{\partial^2 k_{21}(x,y)}{\partial x \partial y} = 0, \quad and \quad \frac{\partial^2 k_{22}(x,y)}{\partial x \partial y} = 2$$

Table (1) represents the exact and the numerical solutions of example (1) at specific points for n = 9

Nodes x_i	Exact Solutions		Modified Trapezoidal		Modified Quasi Simpson's 3/8 Rule	
	$u_1(x)$	$u_2(x)$	$u_1(x)$	$u_2(x)$	$u_1(x)$	$u_2(x)$
x = 0	1	1	1	1	1	0
x = 0.11111111111111	1. 24882886900168	0. 80073740291681	1. 24884939142863	0. 80073742524729	1. 24884939142863	0.80073742524729
x = 0.22222222222222	1. 55962349760678	0. 64118038842995	1. 55962408801108	0. 61181101976591	1. 55962408801108	0.64118110197659
x = 0.33333333333333	1. 94773404105468	0. 51341711903259	1. 9477342626525	0. 51341909763927	1. 94773404433254	0. 3134171767883
x = 0.44444444444444	2.43242545428721	0. 41111229050719	2. 43242462502882	0. 41111616404039	2. 43242480184683	0.41111602722440
x = 0.55555555555556	3.03773177751748	0. 32919298780791	3. 03772951997607	0. 32919964926517	3. 03772974835500	0.32919944641561
x = 0.66666666666667	3.79366789468318	0. 26359713811573	3. 7936639166085	0. 26360807970410	3. 7936639166085	0.26359909125401
x = 0.77777777777778	4.73771785964308	0. 21107208779109	4. 73771138295678	0. 21109004083595	4. 73771138295678	0.21108734673334
x = 0.88888888888889	5.91669359066433	0. 16901331540607	5. 91668175312201	0. 16904361933690	5. 91668440476481	0.16903939481635
x = 1	7. 38905609893065	0. 13533528323661	7. 38904532827186	0. 13538908681802	7. 38905746371846	0.13535405723657

Example (2),[7]:
Consider the following system of linear volterra integral equations of the second kind:

$$u_1(x) = \sin x - \frac{2}{5}x^{\frac{5}{2}} + x + \int_0^x (x-y)^{\frac{3}{2}} \left(u_1(y) + u_2(y)\right) dy,$$

$$u_2(x) = -\sin x + \frac{2}{5}(1-x)^{\frac{5}{2}} - x + \frac{3}{5} + \int_0^x (1-x+y)^{\frac{3}{2}} \left(u(y) + u(y)\right) dy, \quad 0 \le x \le 1$$

we solve this example numerically by using the composite modified quasi Simpson's 3/8 rule . to do this , we subdivide the interval [0,1] into 9 subintervals such that xi = $\frac{i}{9}$, i = 0, 1, … ,9 . Here $k_{11}(x, y)$ = $k_{12}(x, y)$ = $(x - y)^{3/2}$ and $k_{21}(x, y)$ = $k_{22}(x, y)$ = $(1-x + y)^{3/2}$,

$$\frac{\partial^2 k_{11}(x,y)}{\partial x \partial y} = \frac{\partial^2 k_{12}(x,y)}{\partial x \partial y} = \frac{-3}{4(x-y)^{\frac{1}{2}}} \quad and \quad \frac{\partial^2 k_{21}(x,y)}{\partial x \partial y} = \frac{\partial^2 k_{22}(x,y)}{\partial x \partial y} = \frac{-3}{4(1-x+y)^{\frac{1}{2}}}$$

Table (2) represents the exact and the numerical solutions of example (2) at specific points for n = 9

Nodes x_i	Exact Solutions		Modified Trapezoidal		Modified Quasi Simpson's 3/8 Rule	
	$u_1(x)$	$u_2(x)$	$u_1(x)$	$u_2(x)$	$u_1(x)$	$u_2(x)$
x = 0	0	1	0	1	0	1
x = 0.11111111111111	0. 2219937392106	0. 77800626037894	0. 22189085896263	0. 77801048748531	0. 22189085896263	0. 77801048748531

x = 0.22222222222222	0.44261996567834	0. 55738003432166	0. 44251542458257	0. 55737508783952	0. 44251542458257	0. 55737508783952
x = 0.33333333333333	0.66052803012949	0. 339471969987051	0. 66042196245848	0. 33945813898064	0. 66044068349694	0. 33945760912229
x = 0.44444444444444	0.87440080797280	0. 12559919202720	0. 87429231971092	0. 12557707003956	0. 8742939457174	0. 12557886508212
x = 0.55555555555556	1.08297094132742	-0. 08297094132742	1. 08285873285773	-0. 08300073194773	1. 08285895229119	-0. 08299907007052
x = 0.66666666666667	1. 28503646973640	-0. 28503646973640	1. 28491892585430	-0. 38511007708479	1. 28493825316104	-0. 28507340973712
x = 0.77777777777778	1.47947565392451	-0. 47947565392451	1. 47935085918747	-0. 47951953684871	1. 47935144859839	-0. 47952005566370
x = 0.88888888888889	1.66526081018955	-0. 66526081018955	1. 66512654146858	-0. 66531266536194	1. 66512727868642	-0. 6653131707032
x = 1	1.84147098480790	-0. 84147098480790	1. 84132468733486	-0. 84164753917704	1. 8413447359289	-0. 84162671487513

Example (3), [7]:

Consider the following system of linear volterra integral equations of the second kind:

$$u_1(x) = -\frac{1}{60}x^6 - \frac{5}{4}x^4 + \frac{1}{5}(3-3x)^{\frac{5}{2}} - \frac{1}{5}(3-x)^{\frac{5}{2}} - 1 + \int_o^x (3-x-2y)^{\frac{3}{2}} u_1(y)\, dy + \int_o^x (-x+y)^3\, u_2(y)\,dy,$$

$$u_2(x) = -\frac{1}{6}x^2 + \frac{1}{8}x^4 + \frac{25}{18}x^3 - \frac{1}{4}x^2 + 10x - 5 + \int_0^x \frac{(x^2+y^2)}{3} u_1(y)dy + \int_0^x \frac{(x^2-y+4)}{2} u_2(y)dy,\ 0 \le x \le 1$$

we subdivide the interval [0,1] into 9 subintervals such that xi = $\frac{i}{9}$ i = 0, 1, … ,9 .

we solve this example numerically by using the composite Simpson's 3/8 rule to integral, $\int_0^x (3-x-2y)^{\frac{3}{2}} u_1(y)\,dy$ and other integrals by using the modified quasi Simpson's 3/8 rule . To do this ,

$$k_{11}(x,y) = (3-x-2y)^{\frac{3}{2}},\ k_{12}(x,y) = (y-x)^3,\ k_{21}(x,y) = \frac{(x^2+y^2)}{3}\ \ and\ \ k_{22}(x,y) = \frac{(x^2-y+4)}{2},$$

$$\frac{\partial^2 k_{11}(x,y)}{\partial x \partial y} = \frac{-3}{2(3-x-2y)^{\frac{1}{2}}},\quad \frac{\partial^2 k_{12}(x,y)}{\partial x \partial y} = 6(x-y),\quad and \quad \frac{\partial^2 k_{21}(x,y)}{\partial x \partial y} = \frac{\partial^2 k_{22}(x,y)}{\partial x \partial y} = 0$$

Table (3) represents the exact and the numerical solutions of example (3) at specific points for n = 9

Nodes x_i	**Exact Solutions**		**Modified Trapezoidal**		**Modified Quasi Simpson's 3/8 Rule**	
	$u_1(x)$	$u_2(x)$	$u_1(x)$	$u_2(x)$	$u_1(x)$	$u_2(x)$
x = 0	1	-5	1	-5	1	-5
x = 0.11111111111111	1	-4. 98765432098765	0. 999999744137100	-4. 9876543214144	0.999999744137100	-4. 9876543242144
x = 0.22222222222222	1	-4. 95061728395062	0. 99999959181404	-4.25061728853062	0.99999954181404	-4.25061728853062
x = 0.33333333333333	1	-4. 88888888888889	0. 99999930971351	-4.88888890650520	0.99999943468232	-4. 88888890876938
x = 0.44444444444444	1	-4. 80246913580247	0. 99999933973798	-4.80240919473750	0.99999938236548	-4. 80246919615093
x = 0.55555555555556	1	-4. 64135802469136	0. 999999913827138	-4.69135816995950	0.99999939065581	-4. 69135816650165
x = 0.66666666666667	1	-4. 55555555555557	0. 99999936627144	-4. 5555586007242	0.99999990322345	-4. 55555583969343
x = 0.77777777777778	1	-4. 39506172839506	0. 999999456046670	-4.39506239588574	0.9999995628623	-4. 39506236200500
x = 0.88888888888889	1	-4. 20987654320989	0. 9999984549989	-4.20987800641883	0.99999812206309	-4. 20987806190103
x = 1	1	-4	0. 9997569473680	-4.00002419438480	0.99970570014155	-4. 00002033667935

Note:- one can notice in the above examples that (n=9) but it is not fixed value . so that , it is possible to take (n) to equal any number of multi (3) .

4. Conclusions and Recommendations:

From the present study , we can conclude that the composite modified quasi Simpson's 3/8 rule can be used to solve systems of the one – dimensional volterra integral equations of the second kind and also the repeated modified quasi Simpson's 3/8 rule gave more accurate results than the repeated modified trapezoidal rule . We will use this method to solve the multi – dimensional integral equations.

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قاعدة شبه سمبسون 8\3 المطورة لحل أنظمة من المعادلات التكاملية الخطية من النوع الثاني

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الخلاصة

يمكن حل منظومة من المعادلات التكاملية في عدة طرق . لكن في هذا البحث , سوف نستخدم قاعدة شبه سمبسون 8\3 المطورة لإيجاد الحل العددي لمنظومة من معادلات فولتيرا التكاملية الخطية من النوع الثاني . هذه الطريقة تقوم بحل منظومة من معادلات فولتيرا التكاملية الخطية من النوع الثاني بدقة أكثر من طريقة شبه المحرف المطورة . ثم إعطاء بعض الأمثلة الإيضاحية لبيان دقة هذه الطريقة .