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Strongly irrsolute precontinuous functions in intuitionistic fuzzy special Topological spaces

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Abstract: The aim of this paper Is to generalize the concept of intuitionistic fuzzy strongly irresolute precontinuous functions due to B.Krsteska &S. Abbas , [2] to intuitionistic fuzzy special topological spaces. Also we study some of their properties and relations with intuitionistic fuzzy special strongly precontinuous functions .we investigate several characterizing theorem.

Keywords irrsolute , precontinuous functions , intuitionistic , fuzzy special ,Topological spaces

Introduction

The concept of fuzzy set was introduced by Zadeh in his paper [8] Uing the concept of fuzzy set Chang [4] introduced the fuzzy topological spaces ,since Atanassov[7] introduced the notion of intuitionistic fuzzy , Coker[5] defined the intuitionistic fuzzy topological spaces. The concept is used to define intuitionistic fuzzy special sets by Coker [10] and intuitionistic fuzzy special topological spaces are introduced.

In section 3 we introduced the intuitionistic fuzzy special strongly irresolute precontinuous function between intuitionistic fuzzy special topological spaces ,some of their properties are studied .we wil establish their properties and relationships with other classes of early defined form of intuitionistic fuzzy special continuous functions. Also we discuss the relationships between this and strongly precontinuous. Also give example show that the converse is not true in general .

Preliminaries.

We introduce some basic definition that are used in the sequel.

Definition 2.1 [11]

Let X be a nonempty set .An intuitionistic fuzzy special set A is an object having the form A =< x , A1, A2> , where A1 and A2 are subsets of X satisfying A1∩ A2=ϕ .The set A1 is called the set of members of A , while A2 is called the set of nonmembers of A.

Definition2.2 [10]

Let X be a nonempty set and the intuitionistic fuzzy special set A and B be in the form A =< x , A1, A2> , B = < x , B1, B2 > furthermore .Let {Ai: i∈j } be an arbitrary family of intuitionistic fuzzy special sets in X where Ai=<x, Ai(!), Ai(2)>

, then

$$A \subseteq B \Leftrightarrow A_1 \subseteq B_1 \,\&\, B_2 \subseteq A_2,$$

$$A = B \Leftrightarrow A \subseteq B \,\&\, B \subseteq A,$$

The complement of A is denoted by $\overline{A}$ and defined by

$$\overline{A} = \langle x, A_2, A_1 \rangle,$$

$$\bigcup A_i = \langle x, \cup A_i^{(1)}, \cap A_i^{(2)} \rangle, \bigcap A_i = \langle x, \cap A_i^{(1)}, \cup A_i^{(2)} \rangle,$$

$\tilde{\Phi}$ =< x, ϕ, X >, $\tilde{X}$ = < x,X , ϕ >

Defintion2.3 [10]

1. If B = < y , B1, B2 > is an intuitionistic fuzzy special set in Y ,then the preimage of B under f denoted by f -1(B) and defined f -1 (B)= < x , f -1(B1) , f -1(B2) > .
2. if A= < x, A1, A2 > is an intuitionistic fuzzy special set in X

, then the image of A under f denoted by f(A)and defined by $f(A) = \langle y, f(A_1), \underline{f}(A_2) \rangle$ where $\underline{f}(A) = (f(A_2^c))^c$.

Corollary2.4 [10]

Let A, Ai (i∈J) be an intuitionistic fuzzy special sets in X

B, Bj (j ∈K) an intuitionistic fuzzy special sets in Y, and

f: x→y be function then ,

A1 ⊆ A2 → f(A1) ⊆ f (A2),

B1⊆ B2→ f -1 (B1) ⊆ f -1(B2) ,

A ⊆ f -1 (f (A)) ,and if f is injective ,then A = f -1(f (A)) ,

f(f -1(B)⊆ B, and if f is surjective , then f(f -1(B)) =B,

f -1(∪ Bj)= ∪ f -1(Bj),

f -1(∩ Bj) = ∩ f -1(Bj),

f (∪ Ai) = ∪ f (Ai) ,

f (∩ Ai)⊆ ∩ f (Ai) , if f is injecthve , then f (∩ Ai) = ∩ f (A i) ,

f -1($\tilde{Y}$) = $\tilde{X}$, f -1($\tilde{\Phi}$) = $\tilde{\Phi}$,

if f is surjective f ($\tilde{X}$) = $\tilde{Y}$, , $f^{-1}(\overline{B}) = \overline{f^{-1}(B)}$.

11 12. if f is surjective then f(A)⊆ f(A)and if furthermore f is injective , we have (f (A)) = f (A))

Definition2.5 [10] An intuitionistic fuzzy special topology on a nonempty set X is family T of intuitionistic fuzzy special sets in X satisfying the following conditions.

1. $\tilde{\Phi}$, $\tilde{X}$ ∈ T.
2. T is closed under finite intersection.
3. T is closed under arbitrary unions.

The pair (X, T) is called an intuitionistic fuzzy special topological space and any intuitionistic fuzzy special set in T known open set in X.

From now the word space means an intuitionistic fuzzy special topological space.

The complement of an open set in a space (X, T) is called closed set.

Definition2.6 [9]

Let X be a nonempty set, P ∈ X a fixed element in X. The intuitionistic fuzzy special set $\tilde{P}$ defined by $\tilde{P}$ = < x, {p}, {pc} > is called an intuitionistic fuzzy special point in X.

Definition2.7 [6]

Let $\tilde{P}$ be an intuitionistic fuzzy special point of an intuitionistic fuzzy special topological space (X, T). An intuitionistic fuzzy special set A of X is called an intuitionistic fuzzy special neighborhood of $\tilde{P}$ if there exists an open set B in X such that P∈ B ⊆ A.

Definition2.8 [5]

Let A be an intuitionistic fuzzy special set in a space (X, T). Then

1. intA =∪ { G : G is open set in X and G ⊆ A } is called an intuitionistic fuzzy special interior of A ;
2. clA = ∩ { G : G is closed set in X and A ⊆ G } is called an intuitionistic fuzzy special closure of A .

Definition2.9 [3]

An intuitionistic fuzzy special set A in a space (X, T) is called an intuitionistic fuzzy special preopen set if A ⊆ intclA

The complement of an intuitionistic fuzzy special preopen set A is called an intuitionistic fuzzy special preclosed set in X.

Definition2.10 [1]

Let A be an intuitionistic fuzzy special set in a space (X, T). Then. Then

Pint A =∪ {B: B ⊆ A, B is preopen set of X} is called an intuitionistic fuzzy special preinterior of A.

pcl A = ∩ { B : B⊇ A , B is preclosed set of X} is called intuitionistic fuzzy special preclosure of A

Definition2.11 [1]

An intuitionistic fuzzy special set A in a space (X, T) is called an intuitionistic fuzzy special strongly peropen set if A ⊆ int(pclA).

The complement of an intuitionistic fuzzy special strongly peropen set A in a space (X, T) is called an intuitionistic fuzzy special strongly preclosed set of X. The family of strongly preopen set denoted SPO(X).

Definition2.12 [1]

Let A be an An intuitionistic fuzzy special set set in a space (X, T) . Then

1. spint A = ∪ { B : B is strongly preopen set of X and B⊆ A } is called an intuitionistic fuzzy special strongly preinterior of A.
2. spcl A = ∩ B : B is strongly preclosed set of X and B ⊇ A } is called an intuitionistic fuzzy special strongly preclosure of A

Theorem 2. 13 [1]

Let A be an intuitionistic fuzzy special set of a space (X, T), then

cl $\overline{A}$ = intA

2. int $\overline{A}$ = cl A

3. pcl $\overline{A}$ = pint A

4. pint $\overline{A}$ = pcl A

5. spcl $\overline{A}$ = spint A

6. spint $\overline{A}$ = spcl

Definition2.14 [1,6]

Let f: X → Y be a function from a space (X, T) into a space (Y,∂). The function f is called;

1. An intuitionistic fuzzy special continuous if f- -1(B) is open set in X, for each open set B in Y.
2. An intuitionistic fuzzy special strongly precontinuous if f -1(B) is strongly preopen set in X ,for each open set B in Y .

3. An intuitionistic fuzzy special strongly preopen (preclosed) function if f (A) is strongly preopen set (preclosed set) in Y , for each open set (closed set) A in X .

Intuitionistic fuzzy special strongly irresolute precontinuous

Definition3.1

A function f :X → Y from a space (X, T) into a space (Y, ∂) is called an intuitionistic fuzzy special strongly irresolute precontinuous if f -1(B) is strongly preopen set in X , for each strongly preopen set B in Y .

Remark 3.2

If f is an intuitionistic fuzzy special strongly irresolute precontinuous, then f is intuitionistic fuzzy special strongly precontinuous .

The following example shows that the converse of remark [3.2] is not true in general.

Example 3.3

Let X = {a, b, c}, T = { $\tilde{\Phi}$, $\tilde{X}$, A , B } where

A= < X, {a}, {b} > , B = <x, {a,b } ,ϕ >,

SPO(X)= { $\tilde{\Phi}$, $\tilde{X}$, A, B , Ki } where i = 1,2,3,4,5,-----------17

k1 = < x,{a},{c}> , k2= < x, {a,b} ,{c} > , k3=< x, {a,c} , {b}> , k4=< x, {a}, ϕ > k5=< x, {b} , {c}> , k6 = < x, {b} , ϕ > K7= < x, {b,c} ,{a}> , k8=< x, {c},{a}> , k9=< x, {c},{b} >

k10= < x,{ a,c}, ϕ > , k11= < x, {b,c} , ϕ > , k12= < x, ϕ ,{a,c}> ,k13= < x, ϕ , {b}> , k14=< x,{c},{a,b}> , k15=<x,{a},{b,c}> , k16= < x, ϕ ,{b,c}> ,k17= <x, {c},ϕ >

And let Y= {1, 2, 3}, ∂ = { $\tilde{\Phi}, \tilde{Y}$, C, D} where

C=< y, {1}, {3}>,

D= <y, {1, 2},ϕ >,

SPO(Y) = { $\tilde{\Phi}, \tilde{Y}$, C, D Lj} where j=1, 2, 3, ------17}

L1=< y,{1},{2}> , l2=< y, {1,2},{3}>, l3= <y,{1},ϕ> ,l4= <y{1,3},ϕ >, l5= <y,{2},{1}> ,l6= <y,{2},{3}>,l7=<y,{2,3},{1}>

L8=< y,{2},ϕ >, l9= <y, {3},{2} >, l10= <y,{1,3},{2}>, l11=< y,{2,3},ϕ > , l12=<y,{3},ϕ > , l13= < y, ϕ ,{3} >, l14=<y, ϕ ,{2,3}>,

L15= <y, ϕ ,{1,3}> ,l16= <y, ϕ ,{2}> , l17=<y,{2} ,{1,3}>,

f:X→ Y defined as f(a)=1 ,f(b)=2 , f(c)= 3 ,

f -1(c) =< x, {a}, {c} > and f -1(D) = < x, {a,b},ϕ >

f is strongly precontinuous since for every open set in Y the iverse image is strongly preopen in X.

But f is not strongly irresolute precontinuous since

l5=< y, {2},{1}> is strongly preopen set in Y but

f -1(l5)= < x, {b},{a} > is not strongly preopen in X .

Theorem 3.4

Let f: X→ Y be a function from a space (X,T) into a space (Y,∂) ,the following statements are equivalent ,

1. f is an intuitionistic fuzzy special strongly irresolute precontinuous function ,
2. f -1(B) is strongly preclosed in X ,for each strongly preclosed set B in Y ,
3. spcl f -1(B) ⊆ f -1(spclB),for each set B of Y ,
4. f -1(spintB) ⊆ spint f -1(B) , for each set B of Y ,

Proof

1→2 Let B any closed set in Y , then $\overline{B}$ is open set in Y , since f is strongly irresolute percontinuous,then f -1($\overline{B}$) is strongly preopen set in X (def.3.1)

But f -1($\overline{B}$) = f -1(B) [(coro. 2.4)11], then f -1(B) is strongly preclosed set in X .

2 → 3 Let B any set in Y from B⊆ spclB

, follows that

f -1(B) ⊆ f -1(spclB) .Since spclB is strongly preclosed set in Y

According to the assumption we have that f -1(spclB) is strongly preclosed in X.

Therefore spcl f -1(B) ⊆ f -1(spclB).

3→ 4 It can be proved by using the complement.

4 →1 Let B be any strongly preopen set in Y, then spintB =B

According to the assumption we have

f -1 (B) = f -1(spintB) ⊆ spint f -1(B) ,so f -1(B) is strongly preopen set in X .

Hence f is an intuitionistic fuzzy special strongly irresolute precontinuous function.

Theorem 3.5

Let f: X → Y be a function from a space (X,T) into a space (Y,∂) ,then the following statements are equivalent .

1. f is an intuitionistic fuzzy special strongly irresolute precontinuous function .

2. cl(pint f -1(B))$\subseteq$ f -1(spclB), for each closed set B in Y ;
3. f -1(spintB) $\subseteq$ int(pcl f -1(B)), for each open set B in Y ;
4. f (cl(pintA))$\subseteq$ spclf(A) , for each set A in X .

Proof

1 $\rightarrow$ 2 Let B any closed set in Y. According to the assumption we have that f -1(spclB) is strongly preclosed set in X.

Hence f -1 (spclB)$\supseteq$ cl(pint f -1(spclB))$\supseteq$ cl(pint f -1(B)).

2 $\rightarrow$3 It can be proved by using the complement.

3 $\rightarrow$ 4 Let A be any set in X, we put B =f (A)

Then A $\subseteq$ f -1(B). According to the assumption we have

Int(pcl($\overline{A}$) $\subseteq$ int(pcl f -1($\overline{B}$)$\subseteq$ f -1(spint($\overline{B}$))

Thus

cl(pintA)$\subseteq$ cl(pint f -1(B)) $\subseteq$ f -1(spclB) .

Hence

f (cl(pintA)) $\subseteq$ f f -1(spclB) $\subseteq$ spclB =spcl f(A).

4 $\rightarrow$1 Let B any strongly perclosed set in Y .According to the assumption we obtain,

f(cl(pint f -1(B)) $\subseteq$ spclf f -1(B) $\subseteq$ spclB =B .

Then

cl(pint f -1(B))$\subseteq$ f -1f (cl(pint f -1(B))$\subseteq$ f -1(B).

Thus

f -1(B) is strongly preclosed set in X.

So

f is an intuitionistic fuzzy special strongly irresolute precontinuous function .

Theorem 3.6

Let f : X $\rightarrow$ Y be an intuitionistic fuzzy special strongly irresolute precontinuous function from a space (X,T) into a space (Y, ∂) .

Then f -1(B) $\subseteq$ spint f -1(int(pcl(B))), for each strongly preopen set B in Y .

Proof

Let f be any intuitionistic fuzzy special strongly irresolute precontinuous function, and let B any strongly preopen set in Y,

Then f -1(B) $\subseteq$ f -1(int(pcl(B))) .

Since f -1(int(pcl(B))) is an intuitionistic fuzzy special strongly preopen set in X . It follows that,

f -1(B) $\subseteq$ spint f -1(int(pcl(B))).

Theorem 3.7

A function f : X $\rightarrow$ Y from a space (X,T) into a space (Y, ∂) is an intuitionistic fuzzy special strongly irresolute precontinuous if and only if for each point $\tilde{P}$ in X and intuitionistic fuzzy special strongly preopen set B in Y such that f($\tilde{P}$) there exists an intuitionistic fuzzy special strongly preopen set A in X such that $\tilde{P}\in$ A and f(A) $\subseteq$ B .

Proof

Let f be an intuitionistic fuzzy special strongly irresolute precontinuous function, $\tilde{P}$ is an intuitionistic fuzzy special point in X and B any strongly preopen set in Y such that f($\tilde{P}$) $\in$B , then $\tilde{P}\in$ f -1(B) =spint f -1(B),we put A = spintf -1(B) .

Then A is strongly preopen set in X which containing point $\tilde{P}$ and f(A) = f (spint f -1(B)) $\subseteq$ f f -1(B)) $\subseteq$ B .

Conversely

Let B any intuitionistic fuzzy special strongly preopen set in Y and $\tilde{P}$ an intuitionistic fuzzy special point in X,

Such that $\tilde{P}\in$f -1(B) . According to the assumption there exists strongly preopen set A in X such that $\tilde{P}\in$A and f(A)$\subseteq$ B

Therefore $\tilde{P}\in$A $\subseteq$ f -1(B) and $\tilde{P}\in$ A = spintA $\subseteq$ spint f -1(B).

Since $\tilde{P}$ is an arbitrary point and f -1(B) is union of all point containing in f -1(B) we obtain that f -1(B) = spint f -1(B),

So f is is an intuitionistic fuzzy special strongly irresolute precontinuous function.

Corollary3.8

A function f : X $\rightarrow$ Y from a space (X,T) into a space (Y, ∂) is an intuitionistic fuzzy special strongly irresolute precontinuous if and only if for each point $\tilde{P}$ in X and intuitionistic fuzzy special strongly preopen set B in Y such that f($\tilde{P}$) $\in$ B there exists intuitionistic fuzzy special strongly preopen set A in X such that $\tilde{P}\in$A and A $\subseteq$ f -1(B) .

Theorem 3.9

A function f : X $\rightarrow$ Y from a space (X,T) into a space (Y, ∂) is an intuitionistic fuzzy special strongly irresolute precontinuous if and only if for each point $\tilde{P}$ in X and intuitionistic fuzzy special strongly preopen set B in Y such that f($\widetilde{P}$)∈ B ,pclf -1(B) is Neighbrhood of point $\widetilde{P}$ in X .

Proof

Let f be any intuitionistic fuzzy special strongly irresolute precontinuous function , $\widetilde{P}$ is an intuitionistic fuzzy special point in X and B any strongly preopen set in Y such that f($\widetilde{P}$) ∈B , then $\widetilde{P}$∈ f -1(B)⊆ int(pcl f -1(B))⊆ pcl f -1(B),so pcl f -1(B) is an intuitionistic fuzzy special Neighborhood of point $\widetilde{P}$ in X .

Conversely

Let B be any strongly preopen set in Y and $\widetilde{P}$ is an intuitionistic fuzzy special point in X such that f ($\widetilde{P}$)∈ B, then $\widetilde{P}$ ∈ f -1(B).

According to the assumption pcl f -1(B) is Neighborhood of point $\widetilde{P}$ in X. Thus $\widetilde{P}$∈int(pcl f -1(B)),so f -1(B)⊆ int(pcl f -1(B)).

Therefore

f is any intuitionistic fuzzy special strongly irresolute precontinuous.

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الدوال المستمرة قبليا المحيرة القوية في الفضاءات التبولوجية المضببة الحدسية الخاصة

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الخلاصة

ان الهدف من هذا البحث هو تعميم مفهوم الدوال المستمرة قبليا المحيرة القوية في الفضاءات التبولوجية المضببة الحدسية في المصدر [2] الى الفضاءات التبولوجية المضببة الحدسية الخاصة كذلك تم دراسة بعض خواصها وعلاقتها بالدوال المستمرة قبليا القوية وقد تم تحقيق معظم النظريات المكافئة.