

## Load –Deflection Relationship for Clamped-Horizontally Restrained R.C. Polygonal Slabs

## ● Dr. Ali Flayeh Hassan- Asst Prof.

Department of Civil Engineering, College of Engineering, University of Duhok

Received: 25/01/2017 Accepted: 24/05/2017

DOI Link: https://doi.org/10.17656/sjes.10055

#### Abstract



A theoretical model is presented in the analysis and prediction of the complete load-deflection behavior of fixed-horizontally restrained reinforced concrete polygonal slabs

under uniform loading. The limiting cases of fixed reinforced concrete square and circular slabs are also deduced. The proposed model considers three stages, elastic, elasto-plastic and plastic. In the elastic and -elasto-plastic stages, the results of classical plate theory are used. The cracking of concrete and yielding of steel reinforcement are accounted for by suitably modifying flexural rigidity. Changes that occur in support conditions due to possible yielding are also considered. In the plastic stage a rigid plastic membrane analysis is presented. A method is proposed to predict the real ultimate load (including membrane action) and the deflection at the ultimate load.

**Key Words:** Circular slabs, polygonal slab, square slabs, flexural rigidity, membrane action, yield criterion, yield-line theory.

## 1. Introduction

Extensive experimental work on restrained slabs [1-5] have shown the load-deflection relationship is in the form of ABCD shown in Fig (1). The ultimate load is considerably higher than that suggested by Johansen yield line theory [6]. This enhancement in ultimate load has been attributed to the effect of induced compressive membrane action. The conventional yield line theory of Johansen based upon rigid-plastic approximation has been proven successful in predicting the initial collapse loads of reinforced concrete slabs negligible membrane forces. researchers for example, wood [1], park [7] and Al-Hassani [8] have used similar methods to obtain initial collapse load with membrane forces and give a load -deflection relationship of the form HCD shown in Fig (1).

In this paper, an attempt is made to predict the complete load-deflection behavior of restrained reinforced concrete polygonal slabs under uniform loading.

#### 2. Theoretical Analysis

A clamped (horizontally restrained) polygonal slab having q number of sides each of length  $L_1$  carrying a uniformly distributed load, is considered, isotropically at center and with the same amount of reinforcement in the top face only at supports, as shown in Fig (2) is analyzed theoretically under the action of uniform load of intensity P. The analysis is carried out in three stages:

- 1. First stage (elastic).
- 2. Second stage (elasto-plastic).
- 3. Third stage (plastic).

The stages of analysis are shown in Fig (3).

#### a. First stage (elastic stage)

The line OA in Fig (3) represents elastic behavior. The maximum bending moment and maximum deflection occurring at the slab center are calculated from classical plate theory  $^{[9,\ 10]}$  as

$$M_{max} = k_1 PL^2 \tag{1}$$

$$\omega = K_2 \frac{PL^4}{E_C I_g}$$
 (2)

Where L: is the side length of an equivalent square slab having the same area as that of the polygonal slab,
Thus:

$$L = \sqrt{\frac{q}{4 \tan \frac{180}{a}}} L_1 \tag{3}$$

For the extremity  $q=\infty$  (i.e the case of a circular slab of radius say R) L is simply evaluated as



$$L = \sqrt{\pi} R \tag{4}$$

This stage is terminated when first cracking appears at the slab center at an intensity of load equals per. This represented by point A in figure which can be estimated by equating Mmax to the cracking moment Mer such that

$$M_{\text{max}} = M_{\text{cr}} = \frac{f_{\text{r}}J_{\text{g}}}{y_{\text{t}}}$$
 (5)

A combination of Eq. (1) and (5) gives

$$p_{cr} = \frac{f_r J_g}{k_1 y_t L^2} \tag{6}$$

And when this load is substituted into Eq. (2), the cracking deflection w<sub>cr</sub> corresponding to point A will be

$$\omega_{\rm cr} = \frac{k_2}{k_1} \frac{f_{\rm r}L^2}{y_{\rm t} E_{\rm C}} \tag{7}$$

By using  $f_{\rm r}$  =0.7  $\sqrt{f_{c^{`}}}$  and  $E_{\rm c}$  =4700  $\sqrt{f_{c^{`}}}$  (as recommended by ACI code  $^{\rm [11]}$  and knowing that  $I_g = h^3/12$  and  $y_t=h/2$ , Eqs. (6) and (7) may be

$$P_{cr} = 0.1167 \frac{\sqrt{f_c'}}{k_1} (\frac{h}{L})^2$$
 (8)

$$\omega_{\rm cr} = 3 \times 10^{-4} \left(\frac{k_2}{k_1}\right) \left(\frac{L}{h}\right)^2 h$$
 (9)

The constants  $k_1$  and  $k_2$  (the moment and deflection coefficients corresponding to fixed edge conditions) are listed in Table (1) [9, 10]:

#### Second stage (Elasto-plastic)

This stage starts from the cracking load at point A to Johansen yield line theory load at point D of Fig (3). The effect of cracking of concrete and yielding of steel is included by choosing a decreasing moment of inertia function (Ie) analogous to that specified by section (9.5.3.4) of ACI code [11], but was modified and related to the load intensities as

$$I_{e} = (\frac{p_{cr}}{p_{a}})^{3} I_{g} + [1 - (\frac{p_{cr}}{p_{a}})^{3}] I_{cr} \le I_{g}$$
 (10)

The ACI code equation

$$I_{e} = \left(\frac{M_{cr}}{M_{a}}\right)^{3} I_{g} + \left[1 - \left(\frac{M_{cr}}{M_{a}}\right)^{3}\right] I_{cr}$$
 (11)

cannot be applied for region CD, since points C and D have the same moment capacity given by Eq. (18) Where

 $\begin{array}{lll} P_{cr} = {\rm cracking\ load} \\ P_{a} & = {\rm\ The\ intensity\ of\ uniformly} \end{array}$ distributed load at the stage for which the deflection is calculated

 $I_g$  = Gross moment of inertia of section

 $I_{cr}$  = moment of inertia of cracked transformed section

$$I_{cr} = \frac{b d^3}{3} \left[ k^3 + 3n\rho (1 - k)^2 \right]$$
 (12)

Where

$$k = \sqrt{(\rho n)^2 + 2\rho n} - \rho n \tag{13}$$

 $n = the modular ratio = E_S/E_C$ 

For continuous members, ACI code stipulates that Ie may be taken as the average values obtained from Eq. (11) for the critical positive and negative moment section. Thus, for clamped polygonal slabs, the fallowing expression may be used;

$$I_e = 0.5 (Ie_{(c)} + Ie_{(e)})$$
 (14)

Where the subscript c and e refer to center and the edges, respectively.

Referring to Fig (3), the elasto-plastic stage is subdivided into three intermediate stages represented by portions AB, BC and CD of the load -deflection curve.

#### **Portion AB**

During this portion of the load- deflection curve, the slab has tension cracks penetrating all the way to the neutral axis and spreading outward from slab center, When the terminating point B is reached, the steel bars commence to yield at the slab center and the moment capacity of the slab at this section is simply the yield moment  $M_y$  which can be easily calculated on the basis of the stress distribution shown in Fig (4) as

$$M_y = \rho \, f_y \, d^2 \, (1 - \frac{k}{3}) \tag{15}$$

The corresponding load and deflection are

$$p_{y} = \frac{M_{y}}{k_{1} L^{2}} \tag{16}$$



and

$$\omega_{y} = \omega_{cr} + k_{2} \frac{(p_{y} - p_{cr})_{L^{4}}}{E_{c} (I_{e})_{y}}$$
 (17)

#### Portion BC

Along portion BC, the slab continues to yield and a heavy spread of cracks may be witnessed accompanied by inelastic stress distribution of concrete until when point C is reached the section of the slab at center has reached its ultimate moment capacity  $(M_{\rm u})$  where

$$M_{\rm u} = \rho \, f_y \, d^2 \, \big( 1 - \, 0.59 \, \rho \frac{f_y}{f_{c'}} \big) \tag{18}$$

The corresponding load and deflection are

$$p_{\rm u} = \frac{M_{\rm u}}{k_1 L^2} \tag{19}$$

and

$$\omega_{\rm u} = \omega_{\rm y} + k_2 \; \frac{(p_{\rm u} - p_{\rm y)\,L^4}}{E_{\rm C}\,(I_{\rm e})_{\rm u}} \; (20)$$

#### **Portion CD**

Further increments in loading allow a complete yield line pattern to form in slab as shown in Fig. (5) with positive yield lines accompanied by circumferential negative yield lines. Johansen`s yield theory  $P_{\rm J}$ line load (corresponding to point D) for the q-sided restrained polygonal slab under consideration is determined using the equilibrium method and taking moment about axis of rotation of a typical slab element gives

$$P_{J} = \frac{24 \left(M_{u}^{+} + M_{u}^{-}\right)}{L_{I}^{2} \cot^{2} \frac{180}{2}}$$
 (21)

If the slab is square,

$$P_{J} = \frac{24 \left(M_{u}^{+} + M_{u}^{-}\right)}{L_{I}^{2}}$$
 (22)

When the slab is circular of radius R,

$$P_{J} = \frac{6(M_{u}^{+} + M_{u}^{-})}{R^{2}}$$
 (23)

And for the special case of hexagonal slab,

$$P_{J} = \frac{8 \left(M_{u}^{+} + M_{u}^{-}\right)}{L_{I}^{2}} \tag{24}$$

At point D in Fig (2) it is assumed that the slab has yield along the boundaries, thus resulting in

slightly altered edge conditions. Thus, the slab is assumed to be partially restrained and the deflection corresponding to point D is determined from

$$\omega_{J} = \omega_{u} + k_{3} \frac{(p_{J} - p_{u})}{E_{C}(I_{e})_{J}}$$
 (25)

And  $k_3$ = the average of the deflection coefficients corresponding to the fixed and simply supported edge condition are listed in table (2)  $^{[9,\ 10]}$  In this way, the load-deflection plot in the second stage is determined.

### c. Third stage

As the slab deflects more, the lower part of the slab at the boundary has a tendency to move outward against the boundary element resulting in the development of inplane compressive forces (compressive membrane forces) in the slab. The compressive membrane forces will cause the ultimate load of the slab to be considerably greater than the ultimate load calculated using Johansen's yield line theory. At larger deflections, the slab edges tend to move inward and tensile membrane forces will be induced which will enable the slab to carry further load by catenary action.

The analysis of membrane action is carried in two parts using the following assumption:

- 1- The materials are rigid- perfectly plastic
- 2- The form of failure is pyramid as determined by yield line theory method
- 3- The adopted failure mode does not change with deformations.

First part: The behavior of the slab under compressive membrane action. This is represented by portion DE of the load –deflection curve of Fig (3). The analysis is carried out by first establishing a yield criterion similar to that developed by wood [1] for a slab section under simultaneous action of a yield moment M and axial force N, each per-unit width, acting at the mid-depth of the slab (Fig. (4). The stress block parameters are based on ACI code instead of British standard code of practice CP114 (1957) [12] which were adopted by wood.

Referring to the stress distribution at ultimate stage shown in the Figure,

$$N = 0.85 f'_{c} a - A_{s} f_{v}$$
 (26)

$$M = 0.85 f'_c a \left(\frac{h}{2} - \frac{a}{2}\right) + A_s f_y \left(d - \frac{h}{2}\right)$$
 (27)



A substitution of (a) obtained from Eq. (26) into Eq. (27) leads to the fallowing non-dimensional yield criterion

$$\frac{M}{M_a} = 1 + \alpha \left(\frac{N}{T}\right) - \beta \left(\frac{N}{T}\right)^2 \tag{28}$$

Where  $M_{\rm u}$ : is as given by Eq.(18)

$$T = A_s f_v = \rho df_v$$
 (29)

$$\alpha = \frac{\frac{1}{2} \frac{h}{d} - 1.18 \rho \frac{f_y}{f_c'}}{1 - 0.59 \rho \frac{f_y}{f_c'}}$$
(30)

and

$$\beta = \frac{0.59 \,\rho \,\frac{f_{y}}{f_{c}'}}{1 - 0.59 \,\rho \,\frac{f_{y}}{f_{c}'}} \tag{31}$$

The maximum moment is reached when

$$\frac{N}{T} = \frac{\alpha}{2\beta} \tag{32}$$

giving 
$$\frac{M_{max}}{M_u} = 1 + \frac{\alpha^2}{4\beta}$$
 (33)

Which is an enhancement factor for horizontally restrained slab. This implies that the ultimate load which corresponds to zero deflection since it is based on assuming that the slab behaves in a rigid- perfectly plastic manner will be

$$\frac{P_{\text{mi}}}{p_{\text{I}}} = 1 + \frac{\alpha^2}{4\beta} \tag{34}$$

From which 
$$P_{mi} = \left[1 + \frac{\alpha^2}{4\beta}\right] P_j$$
 (35)

Where  $p_{mi}$  = initial ultimate load including membrane action

This correspond to point H in the Fig (3)

It is worth re-emphasizing here that the value of the ultimate load according to Eq. (35) is somewhat exaggerated. If elastic strains, for instance had been introduced in the analysis, the value of the ultimate load would have been reduced and will be given as

$$P_{ma} = F P_{mi}$$
 (36)

where  $P_{ma} = Actual$  ultimate load including membrane action

F = Reduction factor

From the results of the extensive series of test, Wood [1] suggested the following reduction factors for the ultimate load.

$\% \ reinforcement \ \rho$	reduction factor F
$\rho > 0.8$	0.70
$0.8 > \rho > 0.4$	0.60
$\rho < 0.4$	0.50

Eq. (28) is valid for deflections between  $\omega_J$  and  $\omega'$ , where  $\omega'$  represent the limiting deflection at which the slab is cracked throughout its depth at center and is found to be

$$\omega' = \omega_{\rm J} + \frac{4}{3} \, \text{h} \tag{37}$$

The second part of this stage belongs to all values of the deflection greater than  $\,\omega'$  where the slab will be cracked throughout its depth in the central region. Thus, for the cracked –through slab section, the yield criterion becomes

$$\frac{N}{T} = -1 \tag{38}$$

$$\frac{M}{M_{\rm D}} = 1 - \alpha - \beta \tag{39}$$

Using the plastic potential theory with total strain approach [6], the load -deflection relationship for the first part of this stage is established by determining first the position of the neutral axis along the yield lines through a combination of geometrical consideration and inplane equilibrium. Thereafter, the yield criteria  $^{[\mathrm{Eq.~(28)}]}$ together with Eq. (26) are used to evaluate moments and membrane forces along the yield lines. The yield load is then found from moment equilibrium of one of the (q) triangular elements of the collapsed slab. The corresponding loaddeflection relationship is found as follows:

For 
$$\omega_s \leq \omega \leq \omega'$$

$$\frac{P}{P_{I}} = 1 + \frac{\alpha^{2}}{4\beta} - \frac{\alpha}{4} \left(\frac{\alpha}{\beta} + 2\right) \frac{\omega}{h} + \frac{5}{64} \beta \left(\frac{\alpha}{\beta} + 2\right)^{2} \left(\frac{\omega}{h}\right)^{2}$$
(40)

Eq. (40) reduced to Eq. (34) with  $\omega = 0$ 

For  $\omega > \omega'$ 

$$\begin{split} \frac{P}{P_J} &= 1 + \beta + \frac{\alpha}{2} \left( \frac{\alpha}{\beta} + 2 \right) + \beta \left( \frac{\alpha}{\beta} + 2 \right) \left( \frac{\alpha}{\beta} + 2.5 \right) \left( \frac{\omega}{h} \right) + \\ \frac{1}{3} \beta (\frac{\alpha}{\beta} + 2)^2 \left[ (\frac{\omega}{h})^2 - \sqrt{(\frac{\omega}{h}) \times (2 + \frac{\omega}{h})^3} \right] \end{split} \tag{41}$$

In Eqs. (40) and (41), the effect of the number of sides (q) on the load –deflection behavior of the fixed polygonal slab is included in the load term  $p_{\rm J}\,.$ 

The load - deflection relationship given by Eqs. (40) and (41) will have the shape shown by the dotted curve HEFG shown in Fig (3). Point H of



the curve corresponds to the theoretical peak load. The coordinates of point E corresponds to the actual peak load and determined by first calculating  $p_{ma}$  using Eq. (36), then the corresponding deflection  $\omega_{ma}$  may be obtained by the substituting into Eq. (40), thus

$$\frac{\omega_{\text{ma}}}{h} = \frac{B - \sqrt{B^2 - 4AC}}{2A} \tag{42}$$

where 
$$A = \frac{5}{64} \beta \left(\frac{\alpha}{\beta} + 2\right)^2 \tag{43}$$

$$B = \frac{\alpha}{4} \left( \frac{\alpha}{\beta} + 2 \right) \tag{44}$$

$$C = 1 + \frac{\alpha^2}{4\beta} - \frac{P_{ma}}{p_J} \tag{45}$$

#### 3. Summary of theoretical analysis

A complete load –deflection diagram for a horizontally restrained polygonal slab can be obtained with reference to Fig (3) as follows

The values of the coordinates of the points A, B, C, D and E are calculated using the appropriate equation as indicated below

Point	No. of Eq. used to evaluate the		
	(deflection)	(load)	
Α	9	8	
В	17	16	
C	20	19	
D	25	21	
E	42	36	

- 2- The referred points O, A, B, C, D and E are connected by straight lines to obtain part OABCD of the load- deflection diagram
- 3- The remaining part (the rigid- perfectly plastic curve) of the load-deflection diagram represented by EFG is drawn using Eqs. (40) and (41) for  $\omega \geq \omega_{ma}$  given by Eq (42)

# 4. Illustration of the procedure and discussion of results:

To illustrate the theoretical procedure, Fig (6) is constructed which shows the difference in the load-deflection curves between different shapes of model concrete slabs (square, hexagonal and circular) all of which are fixed, uniformly loaded, isotropically reinforced at the bottom face and the

same reinforcement percentage is used as negative (top) steel around the edges. They have the following identical properties,

Area of slab =1 $m^2$ , h=30mm, d=26 mm,  $\rho$ = 0.25% fc=20 MPa, fy=414 MPa and Es = 200 000 MPa

A complete load-deflection diagram for each shape can be obtained as follows:

The values of the coordinates of the points A, B, C, D and E are calculated using the appropriate equation as indicated previously. The results are as shown below.

			slab s	shape		
Point	squ	are	cir	cle	hexa	agon
	Def.	load	Def.	load	Def.	load
Α	0.52	20.9	1.78	18.6	1.2	19.25
В	0.96	28.4	3.3	25.3	2.22	26.2
C	1.03	29.4	3.54	26.2	2.4	27.2
D	1.65	32.6	3.54	26.2	2.7	28.3
Н	0	110.2	О	88.66	О	96.0
E	29.12	56.4	31.0	45.3	30.17	48.9
$\mathbf{F}$	41.65	48.1	43.54	38.65	42.7	41.71
G	90	60.6	90	48.7	90	52.6

The referred points are connected by straight lines to obtain a complete load-deflection relationship for each shape similar to that of Fig (3). These are shown in Figs (6).

- 2- It is seen that the predicted load -deflection relationships are fairly comparable with typical load -deflection relationships of restrained slabs as indicated in Fig (1). They have one common characteristic feature. Unfortunately, there is no experimental data available in hand to show the discrepancies.
- 3- The proposed method enables the determination of the deflections at the ultimate load [Eq. (36)]. It is seen from the above table that the computed deflection at ultimate load for the square, circular and polygonal slabs was almost equal to the slab thickness. This prediction agrees with that obtained by Hopkins and Park [2] through testing a ¼ scale nine panel (three by three) reinforced concrete slab and beam floor.
- 4- Comparing the predicted actual ultimate load (peak load) with that of Johansen's yield line theory, there is 73% increase in the ultimate load for this particular case.

Τ.

 $M_{\rm a}$ 

 $\mathbf{M}_{11}$ 



#### 5. Conclusions:

A method is presented for prediction of the complete load-deflection behavior of regular q-sided reinforced concrete polygonal slabs that are fully restrained on all sides and subjected to uniform loading.

The main conclusions to be drawn from the present study are:

- 1- The predicted load-deflection curves are very comparable in shape with the typical experimental load –deflection curves of restrained slabs. Unfortunately, there is no experimental data available in hand to show the discrepancies.
- 2- The method of analysis is applicable to any restrained regular q-sided polygonal slabs including the limiting cases of square slabs (q=4) and circular slabs  $(q=\infty)$
- 3- The actual ultimate load including the compressive membrane action is much higher than predicted by Johansen's yield line theory and ACI code.
- 4- The proposed method enables the prediction of the deflection at ultimate (peak) load. According to this method, the predicted deflection at ultimate load is almost equal to the slab thickness. This result confirms with that obtained from tests.

#### Notation

- $\begin{array}{ll} A_s & & = Area \ of \ tensile \ reinforcement \ per \ unit \\ & width \ of \ slab \end{array}$
- a = Depth of the equivalent rectangular compression block of concrete
- d = Effective depth of slab
- $E_{\rm c}$  = Modulus of elasticity of concrete
- $E_{\rm s}$  = Modulus of elasticity of steel
- F = Load reduction factor
- fc' = Concrete cylinder strength
- $f_y$  = Yield stress of steel reinforcement
- h = Overall depth of slab
- $I_{\rm g}$  = Moment of inertia of the slab gross section per unit width
- $I_{\rm cr}$  = Moment of inertia of the slab cracked transformed section per unit width
- $I_e$  = Effective moment of inertia per unit width for computation of deflection
- $I_{e(c)} \hspace{1.5cm} = \hspace{-.1cm} I_e \hspace{1mm} at \hspace{1mm} slab \hspace{1mm} center$
- I<sub>e(e)</sub> =I<sub>e</sub> at slab edges
- k = Ratio of neutral axis depth to effective pth, defined by Eq. (13).
- $\label{eq:k1} \textbf{k}_1 \hspace{1cm} = \text{Constant in equation for load for given fixed}$  slab

- $k_2$  = Constant in equation for deflection for given fixed slab
- k<sub>3</sub> = Constant in equation for deflection for given partially fixed edge
- $L_1$  = Side length of polygonal slab
  - = Side length of an equivalent square slab having the same area as that of the polygonal slab
- M = Bending moment per unit width of slab
  - = Bending moment per unit width of slab at stage deflection is calculated
- M<sub>cr</sub> = Bending moment at first cracking
- $M_y$  = Bending moment at yielding at slab center
  - = Ultimate bending moment without membrane action
  - = Axial force at mid-depth per unit width of
- N slab

 $\mathbf{R}$ 

- = Modular ratio = E<sub>s</sub> / E<sub>c</sub>
- P = Intensity of uniform load
- $\label{eq:Pa} \textbf{P}_{a} \hspace{1cm} = \textbf{Intensity of uniform load at stage deflection} \\ \hspace{1cm} \textbf{is calculated}$
- P<sub>cr</sub> = Intensity of uniform load at first cracking
- P<sub>J</sub> = Johanse's yield line theory load
- $P_{ma} \qquad \quad = Actual \ ultimate \ load \ including \ membrane \\ action$
- $P_{mi}$  = Initial ultimate load including membrane
- Pu = ultimate load without membrane action
- P<sub>y</sub> = Intensity of load at yielding at slab center
- q = Number of sides of polygon slab
  - = Radius of circular slab
- $y_t$  = Distance from centroidal axis of gross
- $\propto$ ,  $\beta$  = Constants in equation for yield criterion
- $\rho$  = Ratio of steel area to effective area of a slab section of unit width =  $A_s/d$
- $\omega$  = Vertical deflection at center of slab
- $\omega_{cr}$  = Vertical deflection at cracking at center of slab
- $\omega_J$  = Vertical deflection at Johanse's load at
- $\omega_{max} \qquad = \text{Vertical deflection at actual ultimate load} \\ (P_{ma}) \text{ at center of slab}$
- $\omega_u$  = Vertical deflection at ultimate load  $(P_\mathrm{u})$  at center of slab
- $\omega_y \qquad \quad = \text{Vertical deflection at first yielding of steel} \\$  at center of slab



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# العلاقة بين الحمل والاود للبلاطات الخرسانية المسلحة المضلعة الشكل ذات الحافات المثبتة والمقيدة جانبيا

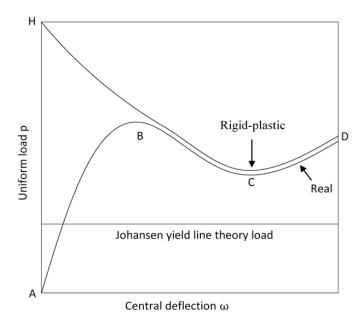
أ .م .د . علي فليح حسن قسم الهندسة المدنية - كلية الهندسة - جامعة دهوك

#### المستخلص:

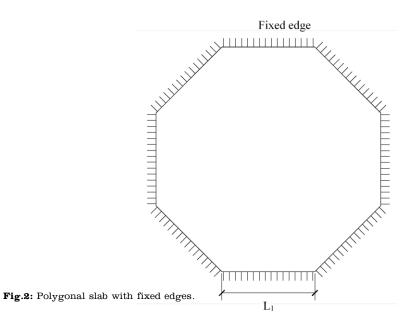
يقدم البحث نموذجا نظريا لايجاد العلاقة المتكاملة بين الحمل المنتظم والاود للبلاطات الخرسانية المضلعة الشكل ذات الحافات المثبتة والمقيدة جانبيا . يشكل البحث ايضا استنباط الحالات الخاصة المتمثلة بالبلاطات الخرسانية المسلحة المربعة والدائرية الشكل . يغطي النموذج المراحل الثلاثة الرئيسية والمتمثلة بمراحل المرونة ، اللامرونة واللدونة وفي مرحلة اللاونة الاخيرة يتم الاخذ بنظر الاعتبار التأثير المهم للفعل الغشائي في البلاطة . وتم تقديم طريقة لاحتساب الحمل الاقصى الحقيقي مع الاود عند هذا الحمل .

الكلمات المفتاحية: البلاطات الدائرية ، البلاطات المضلعة ، جساءة الانثناء ، الفعل الغشائي ، معايير الخضوع ، نظرية خط الخضوع .



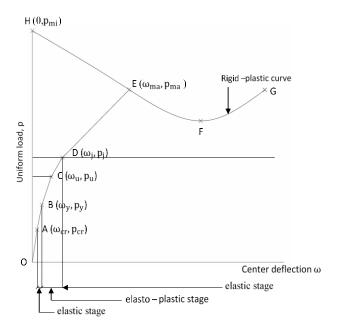


 $\label{Fig.1:} \textbf{Fig.1:} \ \ \text{Load-deflection relationship for horizontally restrained R.C. slab.}$ 



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 $\textbf{Fig.3:} \ \textbf{Idealized load --deflection curve for a horizontally restrained polygonal slab}.$ 

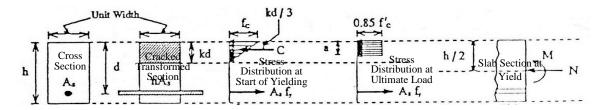
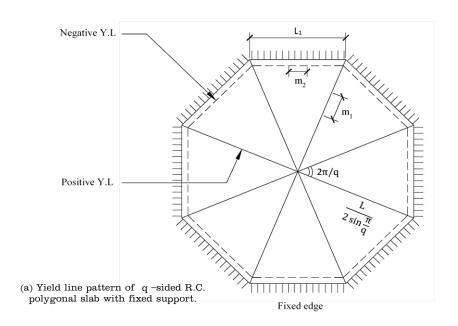
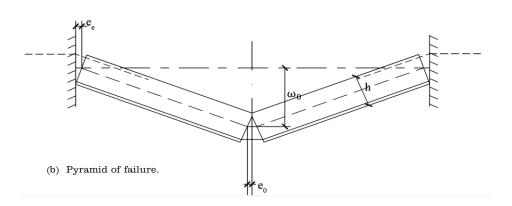


Fig.4: Stress distribution on slab section.







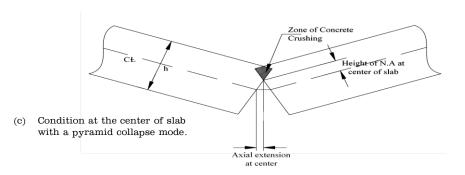
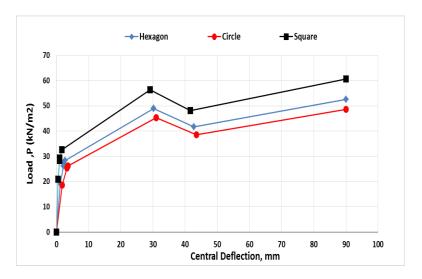


Fig.5, (a,b,c): Yield line pattern for q-sided fixed polygonal slab with pyramid collapse mode.





 $\textbf{Fig.6:} \ \ \textbf{Theoretical prediction for different types of R.C. polygonal slabs}.$ 

**Table1:** Factors  $k_1$  and  $k_2$  for different Shapes of fixed polygonal slabs.

Type of polygonal slab	$\mathbf{k_1}$	$\mathbf{k}_2$
Square slab	0.0231	0.0012
q=4		
Circular slab	0.0259	0.0046
$_{ m d=} \infty$		0.0010
Hexagonal slab	0.025	0.0030
q= 6 *	0.025	0.0030

 $<sup>^{\</sup>ast}$  The values of  $k_{1}$  and  $k_{2}$  for hexagonal slabs are suggested, since they are not available in literature.

 $\begin{tabular}{ll} \textbf{Table 2:} Factors $k_1$ and $k_2$ for different Shapes of partially restrained polygonal slabs \\ \end{tabular}$ 

Type of polygonal slab	$\mathbf{k}_1$	$k_2$
Square slab n=4	0.0326	0.0027
Circular slab n= ∞	0.0445	0.0058
Hexagonal slab n= 6	0.0388	0.0045
<u> </u>		

In this way, the load-deflection plot in the second stage is determined.