AL-Qadisiyha Journal For Science Vol.19 No. 2 Year 2014
ISSN 1997-2490 Eman A.\Muntah k.

The Solution of Diffusion And Exothermic Zero Equation By Using Newton – Kantorovich Method

Received : 9\10\2013 Accepted : 4\12\2013

Eman Ali Hussan
AL-Mustansiriyah university
College of science

Muntaha K. Abbas
Foundation Technical Education
Technical College Management

Abstract

This paper aims to a solution of a diffusion and exothermic equation by using Newton- Kantorovich method . The Newton- Kantorovich method is used to convert the nonlinear boundary value problem into linear boundary value problem , and then using numerical method to solve the linear boundary value problem .

The numerical method is a suitable based on a finite – difference method (the central-difference) scheme was considered here in since it gave a good results and thus obtaining small errors . The central- difference method is preferable over backward or forward difference scheme. Finally ,some numerical examples show that in this paper the results of proposed methods have a good agreement compared with that of exact solution . The use of finite- difference method produced a system of linear algebraic equations which are solved by using the Matlab computer software .

Keywords : Newton- Kantorovich method, , solution of nonlinear equation , two-point boundary value problems, central – difference method

1- Introduction

A large number of problems in engineering and physics can be described through the use of linear and nonlinear ordinary differential equations . When the boundary conditions which together with the differential equation describe the behavior of a particular physical system , the resulting problem is referred to as a linear or nonlinear boundary value problem [1] . One of the numerical methods will be introduced which is called Newton – Kantorovich method , in many aspects the Newton – Kantorovich method is essentially is generalized Newton – Raphson for functional equations and the Newton – Kantorovich technique not only linearizes the nonlinear equation but also provides a sequence of functions which is in general converges rapidly to the solution of the original nonlinear equation [1] .

This work is devoted to study the solution of nonlinear second order ordinary differential equations numerically by Newton – Kantorovich method and generalized this method to solve some nonlinear second order partial differential equations . At the last of this method , a system of linear algebraic equations will be obtained which can be ′solved by matlab computer software which become the tool of nearly all engineers and′applied mathematicians , so the users have an easier and more productive time in this matlab computer software [2].

Very few nonlinear systems can be solved explicitly , and so one must typically rely on a numerical scheme to accurately approximate the solution . Basic methods for initial value problems , beginning with the simple Euler scheme , and working up to the extremely popular Runge – Kutta fourth order method [3] , Donea , (1982)[4] applies the finite element method on nonlinear equation such as Naiver – Stokes equation , which is an important equation in many branches as physics and engineering which is difficult to find the solutions to it .Kubicek M. , (1983)[1] used the boundary conditions may be classified according to various criteria, such as nonlinear boundary conditions separated conditions , mixed conditions and two point, multipoint conditions and so on .

Burden R. , (1985)[5] used the shooting method and the difference method to solve linear and nonlinear second order differential equation . Al – Juburee , Amina (2005) [8] solved some of nonlinear equations by Newton – Kantorovich method .The nonlinear equations were solved using the finite –difference method , she found that the central different scheme gave a good results comparing with exact solution .

Al – Robeay , Bushra (2006) [9] used numerical solution of two points boundary value problem and solved both linear and nonlinear two points ordinary value problem by using some numerical and approximate methods .

2- Newton – Kantorovich Method

The Newton – Kantorovich method , some times also called the quasi – linearization technique , is worth while .The quasi – linearization technique was developed by Bellman and Kalaba in (1965) . Kantorovich and McGill studied the convergence properties of the generalized Newton – Raphson method , and gave algorithm for the numerical solution of nonlinear ordinary differential equation,[1,3] . Newton – Kantorovich – Raphson method is paid to explain the technique of Newton – Kantorovich and its implementation .

2 – 1 Derivation of Newton – Kantorovich Method

New steps can be applied for an operator equation

$$F(y) = 0 \qquad \text{...(1)}$$

The development of this method will be given for a single nonlinear second- order differential equation [5] .

$$F(y) = y'' + f(x ,y , y')=0 \quad ; \quad x \varepsilon (a ,b) \qquad \text{…(2)}$$

Subject to the linear homogeneous two – points boundary condition

$$\alpha_0 y(a) + \beta_0 y'(a) = 0$$
$$\alpha_1 y(b) + \beta_1 y'(b) = 0 \qquad \ldots(3)$$

Where α_0 , β_0 , α_1 and β_1 are constants

After expanding the function f(x , y , y′) in a Taylor series expansion around the solution y_k we get

$$f(x, y_{k+1}, y'_{k+1'}) = f(x, y_k, y'_k) + \frac{\partial f(x, y_k, y'_k)}{\partial y}(y_{k+1} - y_k) + \frac{\partial f(x, y_k, y'_k)}{\partial y'}(y'_{k+1} - y'_k) + \ldots(4)$$ A

nd substituting the equation(4) in the following equation

y″+ f(x ,y , y′)=0
We get

$$y''_{k+1} + f(x, y_k, y'_k) + \frac{\partial f(x, y_k, y'_k)}{\partial y}(y_{k+1} - y_k) + \frac{\partial f(x, y_k, y'_k)}{\partial y'}(y'_{k+1} - y'_k). = 0 \ldots(5)$$

Add the term y″ on both sides of equation (5) we get

$$(y_{k+1} - y_k)'' + \frac{\partial f}{\partial y'}(\delta_y)' + \frac{\partial f}{\partial y}\delta_y = -y''_k - f(x, y, y'_k) \qquad \ldots(6)$$

The main formula of Newton method on the operator equation(2) is given by the following :

$$F'_y(y_k)\delta_y = -F(y_k)$$

Where …(7)

$$\delta_y = y_{k+1} - y_k$$
$$F'_y(y)\delta_y = (\delta_y)'' + \frac{\partial f}{\partial y'}(\delta_y)' + \frac{\partial f}{\partial y}\delta_y \qquad \ldots(8)$$

Now , equation (2) could be written as :

$$(\delta_y)'' + \frac{\partial f}{\partial y'}(\delta_y)' + \frac{\partial f}{\partial y}\delta_y = -y''_k - f(x, y, y'_k) \qquad \ldots.(9)$$

Now the boundary conditions can be rewrite to the following forms .

$$\alpha_0 \delta_y(a) + \beta_0 \delta_{y'}(a) = 0,$$
$$\alpha_0 y_{k+1}(a) + \beta_0 y'_{k+1}(a) = 0$$
$$\alpha_1 \delta_y(b) + \beta_1 \delta_{y'}(b) = 0$$
$$\alpha_1 y_{k+1}(b) + \beta_1 y'_{k+1}(b) = 0 \qquad \ldots(10)$$

Since $\delta_y = y_{k+1} - y_k$ and by equation (3) ,we have

$$\alpha_0[y_{k+1}(a) - \delta_y(a)] + \beta_0[y_{k+1}\ (a) - \delta_{y'}(a)] = 0 \qquad \ldots(11)$$

$$\alpha_1[y_{k+1}(b) - \delta_y(b)] + \beta_{11}[y'_{k+1}(b) - \delta_y(b)] = 0 \qquad \ldots(12)$$

For equation(11) , We can obtain

$$\alpha_1 y_{k+1}(a) + \beta_{01} y'_{k+1}(a) = \alpha_0 \delta_y(a) \ + \beta_0 \delta_y(a) = 0 \qquad \ldots(13)$$

Also , from equation (12) , We can obtain

$$\alpha_1 y_{k+1}(b) + \beta_{11} y'_{k+1}(b) = \alpha_1 \delta_y(b) \ + \beta_1 \delta_y(b) = 0. \qquad \ldots(14)$$

Hence the linearized equation and boundary conditions became in the following form :

$$\alpha_0 \delta_y(a) + \beta_0 \delta_{y'}(a) = 0 \qquad \alpha_0 \delta_y(a) + \beta_0 \delta_{y'}(a) = 0$$

$$\alpha_0 y_{k+1}(b) + \beta_0 y'_{k+1}(b) = 0 \qquad \alpha_0 y_{k+1}(b) + \beta_0 y'_{k+1}(b) = 0 \qquad \ldots(15)$$

3- The Application (Diffusion and Exothermic Zero) of Newton-Kantorovich method [1,6]

Consider boundary value problem

$y'' = \delta e^Y$, 0=δ0<δ<1 ...(16)

Subject to the boundary condition

y(0) = 0 , y(1) =0 . ...(17)

Since f(x , y , y′) = -δe^y ...(18)

Now, apply the Newton – Kantorovich method on equation (16) yields:

1- $\dfrac{\partial f}{\partial y} = -\delta e^y, \dfrac{\partial f}{\partial y'} = 0$

2- By using equation (9) we get the form

$$(\delta_y)'' + \delta e^y \delta y = -y''_k + \delta e^{y_k} \qquad \ldots(19)$$

3- Subject to homogenous linear boundary conditions

δ(0) = 0 , δ(1)= 0 …(20)

where y_k is a pervious iteration which is considered to be known function and ($\delta y = y_{k+1} - y_k$)

4- Let us choose a zero iteration ($y_0 = 0$) so equation (19) and equation(20) are in the form :

$$\delta'' = \delta \qquad ...(21)$$

$$\delta(0) = 0 \ , \qquad \delta(1) = 0 \qquad ...(22)$$

and use the central- finite – difference approximate yield for, [1,7].
h =0.1 , [$\delta i = \delta(xi) = \delta(0+ih)$] , we get the form

$$\frac{\delta_{i+1} - 2\delta_i + \delta_{i-1}}{h^2} = \delta \qquad , \quad \delta=0.3$$

$$\therefore \delta_{i+1} - 2\delta_i + \delta_{i-1} = h^2\delta \qquad i= 1,2,\ldots,9$$

$$\therefore \ \delta_{i+1} - 2\delta_i + \delta_{i-1} = 0.003 \qquad i=1,2,\ldots,9 \qquad .(23)$$

5- The boundary conditions after discretization became :

$$\delta_0 = 0 \qquad \text{jf } i = 0$$

$$\delta_{10} = 0 \qquad \text{if } i=10 \qquad \ldots.(24)$$

for equation (22) a system of nine linear algebraic equation will be obtained [2,8].

$$\begin{aligned}
&\delta_0 - 2\delta_1 + \delta_2 = 0.003 \\
&\delta_1 - 2\delta_2 + \delta_3 = 0.003 \\
&\delta_2 - 2\delta_3 + \delta_4 = 0.003 \\
&\delta_3 - 2\delta_4 + \delta_5 = 0,003 \\
&\delta_4 - 2\delta_5 + \delta_6 = 0,003 \\
&\delta_5 - 2\delta_6 + \delta_7 = 0,003 \\
&\delta_6 - 2\delta_7 + \delta_8 = 0,003 \\
&\delta_7 - 2\delta_8 + \delta_9 = 0,003 \\
&\delta_8 - 2\delta_9 + \delta_{10} = 0,003
\end{aligned} \qquad \ldots.(25)$$

And we can write the above system in matrix form :

$$\begin{bmatrix} -2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -2 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -2 \end{bmatrix} \begin{bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \\ \delta_4 \\ \delta_5 \\ \delta_6 \\ \delta_7 \\ \delta_8 \\ \delta_9 \end{bmatrix} = \begin{bmatrix} 0.003 \\ 0.003 \\ 0.003 \\ 0.003 \\ 0.003 \\ 0.003 \\ 0.003 \\ 0.003 \\ 0.003 \\ 0.003 \end{bmatrix}$$

The resulting set of linear algebraic equations can be easily solved by direct method . It is convenient to write equation (25) in the following matrix form :

$$AX=B \quad \text{....(26)}$$

Where A represent the tridiagonal matrix

$$A = \begin{bmatrix} -2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -2 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -2 \end{bmatrix}$$

X and B represent the following column vectors

$$X = \begin{bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \\ \delta_4 \\ \delta_5 \\ \delta_6 \\ \delta_7 \\ \delta_8 \\ \delta_9 \end{bmatrix} \quad \text{….(27)} \qquad , \quad B = \begin{bmatrix} 0.003 \\ 0.003 \\ 0.003 \\ 0.003 \\ 0.003 \\ 0.003 \\ 0.003 \\ 0.003 \\ 0.003 \\ 0.003 \end{bmatrix} \quad \text{….(28)}$$

Since the matrix A and B are completely known into equation (29) can be solve as follows , provided that the matrix A is non singular

$$X = A^{-1}B \qquad \text{….(29)}$$

Thus the problem of solving linear differential boundary type is reduced the problem of evaluating the inverse of the matrix A [5,9] .
Since equation (25) is linear , the resulting difference equations are also linear . Thus , equation (26) can be solve d by using MATLAB computer software and obtain the following results :

The inverse of the matrix A (A^{-1}) is:

-0.1000	-0.2000	0.3000-	0.4000-	0.5000-	0.6000-	7000.0-	8000.0-	-0.9000
-2000.0	- 0.4000	0.6000-	8000.0-	1.0000-	1.2000-	1.4000-	1.6000-	-0.8000
-0.3000	- 0.6000	0.9000-	1.2000-	1.5000-	1.8000-	2.1000-	1.4000-	-0.7000-
-0.4000	- 0.8000	1.2000-	1.6000-	2.0000-	2.4000-	1.8000-	1.2000-	-0.6000
-5000.0	- 1.0000	1.5000-	2.0000-	2.5000-	0000.2-	1.5000-	1.0000-	0.5000
-0.6000	- 1.2000	1.8000-	2.4000-	0000.2-	6000.1-	2000.1-	8000.0-	0.4000-
-0.7000	- 1.4000	1000.2-	1.8000-	1.5000-	1.2000-	9000.0-	6000.0-	3000.0-
-0.8000	- 1.6000	1.4000-	1.2000-	1.0000-	0.8000-	0.6000-	4000.0-	0.2000-
-0.9000	- 0.8000	0.7000-	0.6000-	0.5000-	0.4000-	0.3000-	0.2000-	1000.0-

$$X=\begin{bmatrix}\delta_1\\\delta_2\\\delta_3\\\delta_4\\\delta_5\\\delta_6\\\delta_7\\\delta_8\\\delta_9\end{bmatrix}=\begin{bmatrix}-0.0135\\-0.0240\\-0.0315\\-0.0360\\-0.0375\\-0.0360\\-0.0315\\-0.0240\\-0.0135\end{bmatrix}$$

Now , $\delta i = y_{k+1}(x_i) - y_k(x_i)$ to find $y_1(x_i)$, i=1,2,…,9
and in this case k =0

So , $\delta_i = y_1(x_i) - y_0(x_i)$, from pervious equation we can be obtain $y_1(x_1)$, …, $y_1(x_9)$. such that $y_0(x_i)$ are all zero . To find the second approximation , let k=1 then equation(18) take the form :

$$(\delta_y)'' + \delta e^{y_1}\delta y = -y_1'' + \delta e^{y_1} \qquad \ldots(30)$$

And after substituting $y_1(x_1)$, … , $y_1(x_9)$ and by using the finite- difference method in equation (30), we obtain the same a system of linear algebraic equations as follow in system (25) ,except the vector B is :

$$B = \begin{bmatrix}0.0296\\0.0296\\0.0296\\0.0296\\0.0296\\0.0296\\0.0296\\0.0296\\0.0296\\0.0296\end{bmatrix} \qquad \ldots(31)$$

And solving the above matrix by the method in MATLAB computer software and obtain the following results :

.

$$\begin{bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \\ \delta_4 \\ \delta_5 \\ \delta_6 \\ \delta_7 \\ \delta_8 \\ \delta_9 \end{bmatrix} = \begin{bmatrix} -0.0133 \\ -0.0237 \\ -0.0311 \\ -0.0355 \\ -0.0370 \\ -0.0355 \\ -0.0311 \\ -0.0237 \\ -0.0133 \end{bmatrix} \qquad ...(32)$$

After this we can easily find $y_2(x_1)$, …, $y_2(x_9)$., $\delta_i= y_2(x_i) – y_1(x_i)$

And the results record in the Table 1 :

$y_2(x_1) = -0.0268$ $\qquad$ $y_2(x_6) = -0.0775$
$y_2(x_2) = -0,0477$ $\qquad$ $y_2(x_7) = -0.06266$
$y_2(x_3) = -0.0626$ $\qquad$ $y_2(x_8) = -0.0477$
$y_2(x_4) = -0.0775$ $\qquad$ $y_{2(}(x_9) = -0.0268$
$y_2(x_5) = -0.0745$

Finally to find third approximation at k=2 , then the equation(19) take the form:

$$(\delta_y)'' + \delta e^{y_2} \delta y = -y_2'' + \delta e^{y_2} \qquad \ldots(33)$$

And equation(20) becomes δ(0)=0 , δ(2)=0 And after substituting $y_2(x_1)$, … , $y_2(x_9)$ and using the finite- difference method in equation (23)
The following system of linear algebraic equations could be obtain :

$$\begin{bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \\ \delta_4 \\ \delta_5 \\ \delta_6 \\ \delta_7 \\ \delta_8 \\ \delta_9 \end{bmatrix} = \begin{bmatrix} -0.0109 \\ -0.0194 \\ -0.0254 \\ -0.0290 \\ -0.0302 \\ -0.0290 \\ -0.0254 \\ -0.0194 \\ -0.0109 \end{bmatrix} \qquad \ldots(34)$$

The MATAB computer software will be use to solve matrix and we obtain the following results

δ1 =-0,0109 , δ2=-0.0194 , δ3 = -0.0254 , δ4 = -0.0290 , δ5 = -0.0302
δ6 = -0.0290 , δ7= -0.0254 , δ8 = -0.0194 , δ9 =-0,0109

The results of the first , second and third approximation with its

$\delta_i = y_3(x_i) - y_2(x_i)$

And the results record in the Table 1:

$y_3(x_1) = -0.0377$ $\qquad$ $y_3(x_6) = -0.1065$
$y_3(x_2) = -0.0671$ $\qquad$ $y_3(x_7) = -0.0880$
$y_3(x_3) = -0.088$ $\qquad$ $y_3(x_8) = -0.0671$
$y_3(x_4) = -0.1065$ $\qquad$ $y_3(x_9) = -0.0377$
$y_3(x_5) = -0.1047$

Table(1) The values of $Y_n(x)$ to the problem and their absolute error

X	Y0	Y1	Y2	Y3	Rate of error	Absolute error
0	0.000	0.000	0.000	0.000	0.000	0.000
0.1	0.000	-0.0135	-0.0268	-0.0377	-0.0109	0.0109
0.2	0.000	-0.0240	-0.0477	-0.0671	-0.0194	0.0194
0.3	0.000	-0.0315	-0.0626	-0.0880	-0.0254	0.0254
0.4	0.000	-0.0360	-0.0775	-0.1065	-0.0290	0.0290
0.5	0.000	-0.0375	-0.0754	-0.1047	-0.0293	0.0293
0.6	0.000	-0.0360	-0.0775	-0.1065	-0.0290	0.0290
0.7	0.000	-0.0315	-0.0626	-0.0880	-0.0254	0.0254
0.8	0.000	-0.0240	-0.0477	-0.0671	-0.0194	0.0194
0.9	0.000	-0.0135	-0.0268	-0.0377	-0.0109	0.0109
1	0.000	0.000	0.000	0.000	0.000	0.000

4- Conclusions

- Newton – Kantorovich method transforms the nonlinear ordinary differential equation into linear differential equation .
- If the problem is simple it will be solved analytic or it can be solved in numerical methods .
- In is paper finite- difference method was used to solve the obtained linear differential equation after transformed the nonlinear ordinary differential equation to linear differential equation .
- The use of central – difference scheme , as a type of the finite –difference method , gave good results into table (1) . Thus , the central difference method is preferable over forward and backward difference schemes .

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AL-Qadisiyha Journal For Science Vol.19 No. 2 Year 2014
ISSN 1997-2490 **Eman A.\Muntah k.**

حل معادلة الانتشار والباعثة للحرارة (صفر) باستخدام طريقة نيوتن – كانتروفيتش

تاريخ الاستلام : 2013\10\9 **تاريخ القبول : 2013\12\4**

إيمان علي حسين **منتهى خضير عباس**
كلية العلوم \ الجامعة المستنصرية

الخلاصة

يهدف هذا البحث الى حل معادلة الانتشار والباعثة للحرارة (الحرارة صفر) باستخدام طريقة نيوتن – كانتروفيتش وتستند هذه الطريقة الى تحويل مسألة القيم الحدودية اللاخطية الى مسالة قيم حدودية خطية . ومن ثم استخدام طريقة عددية للحل مسالة القيم الحدودية الخطية . والطريقة مناسبة وتعتمد على طريقة الفروقات المنتهية (طريقة الفروقات المركزية) التي استخدمت لانه اعطت حلول جيدة واخطاء صغيرة . وطريقة الفروقات المركزية مفضلة على طريقة الفروقات الامامية والخلفية ،واخيرا . بعض الامثلة العددية المطروحة في هذا البحث اظهرت ان الطريقة المقترحة كان لها نتائج مطابقة مع الحل المضبوط . وطريقة الفروقات المنتهية اعطى نظام من المعادلة الجبرية والتي حلت باستخدام (Matlab) .