# Effect of Cutouts on the Behavior of Clamped Rectangular Plates 

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Received on : 26/9/2010
Accepted on : 15/10/2011


#### Abstract

. Plates with interior openings are often used in both modern and classical aerospace, mechanical and civil engineering. The understanding of the effects of two cutouts on the stress concentration factor, maximum stress and deflections in perforated clamped rectangular plates, were considered. Parameters such as location, size of cutout and the aspect ratio of plates are very important in designing of structures. These factors were presently studied and solved by finite element method (ANSYS) program. The results based on numerical solution were compared with the results obtained from different analytical solution methods. One of the main objectives of this study is to demonstrate the accuracy of the analytical solution for clamped square plate.

In general, the results of the square clamped plates with two cutouts come out in good agreement. The results presented here indicated that the maximum stress, deflection of perforated plates can be significantly changed by using proper cutouts locations and/or size. The results show that the rectangular plate containing two cutouts arranged along the width is stronger and stiffer than when arranged along the length at a given spacing, and the square plate is always stronger and stiffer than an equivalent rectangular plate for the same loading condition.


Keywords: cutout, isotropic plate, perforated plate, stress concentrations, ansys program.

## 1. INTRODUCTION.

It is well known that a rectangular plate with a rectangular hole has been widely used in industrial design such as ships, aircrafts, plants, bridges, machines, etc. Plates of various constructions find wide use as primary structural elements in both modern and classical structures. In recent years, the increasing need for lightweight efficient structures has led the structural engineer to the field of structural shape optimization. Different cutout ratio and location in structural elements are needed to reduce the weight of the system and provide access to other parts of the structure. In some cases, a cutout is not a part of initial design of the structure. In other words, structural elements are being damaged during their service life. It is well known that the presence of a cutout or hole in a stressed member creates highly localized stresses in the vicinity of the cutout. The ratio of the maximum stress at the cutout edge to the nominal stress is called the stress concentration factor (SCF).

The study of the importance of SCF of perforated isotropic plates is well established. The high stress concentration at the edge of a cutout is of practical importance in designing engineering structures. The SCFs of these types of cutouts are usually determined either experimentally or numerically using finite element methods. The simple analytical stress analysis presented in this study provides a numerical result for stress concentration factors for perforated isotropic plates with special shaped cutouts. Analytical and numerical studies were
conducted to investigate the effects of variation in cutout shape (triangular, square and pentagonal), cutout orientation, bluntness and load direction on the values of SCF in flat plate under uniaxial tension load [1]. Avalos and Laura [2] performed a series of numerical experiments on simply supported rectangular plates with two rectangular holes with free edges under vibration. Folias and Wang [3] presented most of the previous work on stress concentration in this subject. They presented a series solution for the stress field around circular holes in plates with arbitrary thickness. A wide range of hole diameter to plate thickness was considered. Theocaris and Petrou [4] used Schwarz-Christoffel transformation to evaluate the stress concentration factor for an infinite plate with central triangular cutout. Stress and strain distributions along the boundary of rectangular cutout in an infinite elastic plate were presented by Theocaris and Petrou [5]. Lasko et al. [6] used relaxation element method to determine the stress fields in a plate with three circular cutouts subjected to uniaxial tensile load. Singh and Paul [7] presented numerical results based on generalized work-energy method for rectangular plates with a circular cutout and circular plates with a rectangular cutout. Monahan et al. [8] applied the finite-element method to a clamped rectangular plate with a rectangular hole and verified the numerical results by experiments. Cutouts of regular geometric shapes are designed for various purposes [9] in diverse engineering fields. Sometimes a cutout is not a part of initial design [10]. Wu and Mu [11] investigated the SCF for isotropic plates under uniaxial and bi-axial loads. The SCF was also determined for isotropic and orthotropic cylindrical shells with circular cutout. A method based on the theory developed by Greszczuk [12] provides an analytical solution to the elastic stress distribution around the perimeter of a circular hole in a thin, single layer, homogeneous orthotropic plate under a combination of in-plane loadings.

## 2. GENERAL BEHAVIOR OF PLATES.

Consider a load-free plate, shown in Fig.(1a), in which the xy plane coincides with the midplane and hence the z deflection is zero. The components of displacement at a point, occurring in the $\mathrm{x}, \mathrm{y}$, and z directions, are denoted by $\mathrm{u}, \mathrm{v}$, and w , respectively. When, due to lateral loading, deformation takes place, the midsurface at any point A (xa,ya) has deflection w Fig.(1b). The fundamental assumptions of the small-deflection theory of bending or so-called classical or customary theory for isotropic, homogeneous, elastic, thin plates is based on the geometry of deformations. They may be stated as follows:

1. The deflection of the midsurface is small compared with the thickness of the plate. The slope of the deflection surface is therefore very small and the square of the slope is a negligible quantity in comparison with unity.
2. The midplane remains unstrained subsequent to bending.
3. Plane sections ( $m n$ ) initially normal to the midsurface remain plane and normal to that surface after bending. This means that the vertical shear strains $\gamma_{x z}$ and $\gamma_{y z}$ are negligible. The deflection of the plate is thus associated principally with bending strains. It is deduced therefore that the normal strain $\varepsilon_{z}$ resulting from transverse loading may also be omitted.
4. The stress normal to the midplane, $\sigma_{z}$, is small compared with order stress components and may be neglected. This supposition becomes unreliable in the vicinity of highly concentrated transverse loads.

The above assumptions, known as the Kirchhoff hypotheses, are analogous to those associated with the simple bending theory of beams. Small- and large-scale tests have shown their validity. In the vast majority of engineering applications, adequate justification may be found for the simplifications stated with respect to the state of deformation and stress. Because of the resulting decrease in complexity, a three-dimensional plate problem reduces to
one involving only two dimensions. Consequently, the governing plate equation can be derived in a concise and straightforward manner.

When the deflections are not small, the bending of plates is accompanied by strain in the midplane, and assumptions 1 and 2 are inapplicable. An exception, however, applies when a plate bends into a developable surface (e.g., surfaces of cones and cylinders). This type of surface can be bent back to a plane without variation in the distances between any two points on the surface. If the midsurface of a freely or simply supported and loaded plate has a developable form, it remains unstrained even for deflections that are equal to or larger than its thickness but are still small as compared with other dimensions of the plate. Only under this limitation on the deflections will the squares of slopes be small compared with unity; hence the approximate expression used for the curvatures is sufficiently accurate.

In thick plates, the shear stresses are important, as in short, deep beams. Such plates are treated by means of a more general theory owing to the fact that assumptions 3 and 4 are no longer appropriate [13].

## 3. THE GOVERNING EQUATION FOR DEFLECTION OF PLATES.

The basic differential equation for the deflection of plates is given by the equation[13]:

$$
\begin{equation*}
\frac{\partial M_{x}}{\partial x^{2}}+2 \frac{\partial^{2} M_{x y}}{\partial x \partial y}+\frac{\partial^{2} M_{y}}{\partial y^{2}}=-P \tag{1}
\end{equation*}
$$

where the first expressions for $M_{x}, M_{y}, M_{x y}$ represent the curvatures and the deflection and are given by[13]:

$$
\begin{align*}
& M_{x}=-D\left(k_{x}+v k_{y}\right)=-D\left(\frac{\partial^{2} w}{\partial x^{2}}+v \frac{\partial^{2} w}{\partial y^{2}}\right) \\
& M_{y}=-D\left(k_{y}+v k_{x}\right)=-D\left(\frac{\partial^{2} w}{\partial y^{2}}+v \frac{\partial^{2} w}{\partial x^{2}}\right)  \tag{2}\\
& M_{x y}=-D(1-v) k_{x y}=-D(1-v) \frac{\partial^{2} w}{\partial x \partial y}
\end{align*}
$$

where D is the flexural rigidity of the plate and is given by [13]:

$$
\begin{equation*}
D=\frac{E t^{3}}{12\left(1-v^{2}\right)} \tag{3}
\end{equation*}
$$

and the negative sign agrees with the convention for moment and curvature. It is noted that if a plate element of unit width and parallel to the x axis were free to move sidewise under transverse loading, the top and bottom surfaces would be deformed into saddle-shaped or anticlastic surfaces of curvature $k_{y}$. The flexural rigidity would then be $\mathrm{Et}^{2} / 12$, as in the case of a beam. The remainder of the plate however prevents the anticlastic curvature. Owing to this action, a plate manifests greater stiffness than a beam by a factor $1 / 12\left(1-v^{2}\right)$, approximately 10 percent.

In the case of a three-dimensional state of stress, stresses and strains are related by the generalized Hook's law, valid for an isotropic homogeneous material

$$
\begin{array}{ll}
\varepsilon_{x}=\frac{1}{E}\left[\sigma_{x}-v\left(\sigma_{y}+\sigma_{z}\right)\right] & \gamma_{x y}=\frac{\tau_{x y}}{G} \\
\varepsilon_{y}=\frac{1}{E}\left[\sigma_{y}-v\left(\sigma_{x}+\sigma_{z}\right)\right] & \gamma_{x z}=\frac{\tau_{x z}}{G}  \tag{4}\\
\varepsilon_{z}=\frac{1}{E}\left[\sigma_{z}-v\left(\sigma_{x}+\sigma_{y}\right)\right] & \gamma_{y z}=\frac{\tau_{y z}}{G}
\end{array}
$$

where $\tau_{\mathrm{ij}}=\tau_{\mathrm{ij}}(\mathrm{i}, \mathrm{j}=\mathrm{x}, \mathrm{y}, \mathrm{z})$. the constant $\mathrm{E}, \mathrm{v}$, and G represent the modulus of elasticity, Poisson's ratio, and the shear modulus of elasticity, respectively. The relationship between these elastic constants is:

$$
\begin{equation*}
G=\frac{E}{2(1+v)} \tag{5}
\end{equation*}
$$

Substitution of $\varepsilon_{z}=\gamma_{y z}=\gamma_{x z}=0$ into Eqs. (4) yields the following stress-strain relations for a thin plate

$$
\begin{align*}
\sigma_{x} & =\frac{E}{1-v^{2}}\left(\varepsilon_{x}+v \varepsilon_{y}\right) \\
\sigma_{y} & =\frac{E}{1-v^{2}}\left(\varepsilon_{y}+v \varepsilon_{x}\right)  \tag{6}\\
\tau_{x y} & =G \gamma_{x y}
\end{align*}
$$

The two-dimensional stress components are found from substituting Eqs. (2) and by employing Eq. (3) into the equation of plate curvatures. In this way, the obtained equations are:

$$
\begin{equation*}
\sigma_{x}=\frac{12 M_{x Z}}{t^{3}} \quad \sigma_{y}=\frac{12 M_{y z}}{t^{3}} \quad \tau_{x y}=\frac{12 M_{x y Z}}{t^{3}} \tag{7}
\end{equation*}
$$

The maximum stresses occur on the bottom and top surfaces (at $Z= \pm \mathrm{t} / 2$ ) of the plate. The above expresses Eq. (1), the plate equilibrium, in terms of the curvatures. An alternate form is determined by inserting the definition of curvatures form[13]:

$$
\begin{equation*}
\frac{\partial^{4} w}{\partial x^{4}}+2 \frac{\partial^{4} w}{\partial x^{2} \partial y^{2}}+\frac{\partial^{4} w}{\partial y^{4}}=\frac{P}{D} \tag{8a}
\end{equation*}
$$

This equation, first derived by Lagrange in 1811, can also be written in a concise form:

$$
\begin{equation*}
\nabla^{4} w=\frac{P}{D} \tag{8b}
\end{equation*}
$$

in which $\nabla^{4}=\nabla^{2} \nabla^{2}=\left(\nabla^{2}\right)^{2}$. When there is no lateral load acting on the plate,

$$
\begin{equation*}
\frac{\partial^{4} w}{\partial x^{4}}+2 \frac{\partial^{4} w}{\partial x^{2} \partial y^{2}}+\frac{\partial^{4} w}{\partial y^{4}}=0 \tag{9}
\end{equation*}
$$

Expression (8) is the governing differential equation for deflection of thin plates. To determine $w$, it is required to integrate this equation with the constants of integration dependent upon the appropriate boundary conditions. In case of a clamped rectangular plate, the largest deflection also occurs at the center and the largest moments are found at the middle of the fixed edges. The strip method, introduced by H.Grashof, presents a simple approximate approach for computing deflection in a clamped rectangular plate of sides a and $b$, carrying a uniform load P Fig.(2), with the following boundary conditions[13]:

$$
\begin{array}{lll}
w=0 & \frac{\partial^{2} w}{\partial^{2} x}=0 & (x=0, y=a) \\
w=0 & \frac{\partial^{2} w}{\partial^{2} y}=0 & (y=0, y=b) \tag{10}
\end{array}
$$

In this so-called strip method, the plate is assumed to be divided into two systems of strips at right angles to one another, each strip is regarded as functioning as a beam. Consider a rectangular plate under a uniform load $\mathrm{p}_{0}$, and assume that the plate is divided into strips of spans a and $b$, carrying the uniform loads $p_{a}$ and $p_{b}$, respectively. The loaded system of beams will be impossible to arrange in such a way as to compose the plate unless the following conditions are met:

$$
\begin{equation*}
w_{a}=w_{b} \quad p_{0}=p_{a}+p_{b} \quad(x=y=0) \tag{11}
\end{equation*}
$$

Since from Table B.6 [13] it is found that $p_{a} a^{4}=p_{b} b^{4}$ for $w_{a}=w_{b}$. the largest deflection of the plate is obtained in case of clamped square plate $(a=b)$ :

$$
\begin{equation*}
w_{\max }=0.0013 \frac{p_{0} a^{4}}{D} \quad(\mathrm{x}=0, \mathrm{y}=0) \tag{12}
\end{equation*}
$$

For the same case and using the method of superposition, the largest deflection obtained occurring at the center [13]:

$$
\begin{equation*}
w_{\max }=0.00126 \frac{p_{0} a^{4}}{D} \tag{13}
\end{equation*}
$$

The ritz method can by applied by representing the work done by the lateral surface loading $\mathrm{p}(\mathrm{x}, \mathrm{y})$ as:

$$
\begin{equation*}
w=\iint_{A} w p d x d y \tag{14}
\end{equation*}
$$

where $A$ is the area of plate surface. For the case of a clamped rectangular plate of sides a and b , and by solving Eq. (14) and assuming a deflection expression of the form

$$
\begin{equation*}
w=\sum_{m=1}^{\infty} \sum_{n=1}^{\infty} a_{m n}\left(1-\cos \frac{2 m \pi x}{a}\right)\left(1-\cos \frac{2 n \pi y}{b}\right) \tag{15}
\end{equation*}
$$

The boundary conditions for Eq. (10) are satisfied. For a square plate $(a=b)$ the maximum deflection occurs at the center of the plate [13]:

$$
\begin{equation*}
w_{\max }=0.00128 \frac{p_{0} a^{4}}{D} \tag{16}
\end{equation*}
$$

This is approximately 1.5 percent greater than the value given by Eq. (13). If the deflection $w$ is expressed by retaining seven parameters $\mathrm{a}_{11}, \mathrm{a}_{12}, \mathrm{a}_{21}, \mathrm{a}_{22}, \mathrm{a}_{13}, \mathrm{a}_{31}$, and $\mathrm{a}_{33}$, the result for maximum deflection is found to be the same value given by Eq. (13) [13].

## 4. NUMERICAL WORK.

The material of the plate was chosen to have the mechanical properties defined by elastic modulus ( 70 MPa ), density ( $2600 \mathrm{Kg} \cdot \mathrm{m}^{-3}$ ) and Poisson's ratio ( 0.33 ). The element used is SOLID 95 which is defined by 20 nodes having three degrees of freedom per node, namely translations in the nodal $\mathrm{x}, \mathrm{y}$ and z directions.

To validate the results obtained by the FEM software (ANSYS), a special case for square plate $(\mathrm{a}=\mathrm{b})$ clamped at all sides and under a distributed load was chosen Fig.(2) The value of maximum deflection was computed by the methods listed in Table (1). As noted in this table, the value obtained by the FEM is within an error of $2.4 \%$ from the values obtained by the other methods.

In the numerical program performed, the plate was chosen to be of uniform thickness, clamped at all sides, loaded by a distributed load in the area of the plate and having two symmetrical cutouts with respect to the center of the plate. The parameters studied are:

1. The spacing (s) between the two cutouts, both horizontally and diagonally, Figs.(3 and 4) respectively.
2. The aspect ratio of the plate, i.e. a and b. these parameters were examined to show their influence on the maximum stress, maximum deflection and the stress concentration factor (SCF).
In addition, the effect of size of cutout ( $a_{1}$ and $b_{1}$ ) on the maximum stress was also examined.

## 5. RESULTS AND DISCUSSION.

Figs.(5,6 and 7) show the effect of horizontal spacing on the maximum stress, maximum deflection and SCF respectively, for different aspect ratios of the plate, namely a/b $=1,2 / 3$ and 3/2. It is apparent from Fig.(5) that the square plate exhibits less normal stresses then other plates, hence the square plate has the higher strength among the rectangular plates regardless of the spacing between cutouts. in the other hand, a rectangular plate would have higher strength when the spacing between cutouts is increased; the strength is more increased when the spacing is increased in the direction of the plate width.

Fig.(6) shows again that the square plate is more stiffness than the rectangular plates. Moreover, two cutouts, arranged along the length of a given rectangular plate, seems to decrease the stiffness of the plate more than when arranged along the width of the rectangular plate. In general, the stiffness increases although a little as the spacing is increased for all types of plates.

The same trend of Fig.(5) is found in Fig.(7), regarding the strength of the plates having two cutouts, as the maximum stress increase is attributed to an equal increase in the stress concentration factor.

Figs. (8,9 and 10) reveal the effect of diagonal spacing on the maximum stress, maximum deflection and SCF respectively, for different aspect ratios of the plate, namely $\mathrm{a} / \mathrm{b}$ $=1,2 / 3$ and $3 / 2$. It should be noted that the rectangular plates of aspect ratios of $2 / 3$ and $3 / 2$ are identical when the two cutouts are arranged diagonally so that their behavior coincide in these figures. Again, the square plate shows higher strength and stiffness than the rectangular plates upon increasing the spacing between cutouts diagonally, although this effect diminishes as the two cutouts approach the edges of plate.

Figs. ( 11,12 and 13 ) reveal the influence of the size of cutouts arranged horizontally on the maximum stress induced in the plates of different aspect ratios for different spacing between cutouts.

As shown in Fig.(11), the strength of square plate worsens upon increasing the size of cutouts to a critical value after which the effect is reversed. This trend of behavior is more apparent when the cutouts are so close. Therefore, the strength of a square plate decreases if the size of cutouts is less than $0.3 \mathrm{and} /$ or the spacing between cutouts is less than 20 mm .

Fig.(12), indicated that a rectangular plate loses strength if the cutouts are arranged along width direction no matter how the cutouts are spaced. However, the 10 mm - spaced cutouts of size 0.2 represents the maximum strength of the plate.

Fig.(13), like Fig.(11), records maximum loss in plate strength if the plate contains two cutouts of size 0.1 arranged along the plate major axis. Also, the plate seems to restore its strength for cutout size values of 0.3.

Figs. ( 14,15 and 16) show how the maximum stress would be affected by the size of cutouts and the diagonal spacing between them for different types of plates.

In Fig.(14), the 14.4 mm and 28.2 mm diagonal spacings have almost no effect on the maximum stress induced in square plates at all sizes of cutouts. Nevertheless, when the cutouts are spaced at 56.5 mm , the size of cutouts has shown a notable influence on the maximum stress; while the 0.1 size of cutout maximize the stress, higher sizes of cutout seem to minimize it even below those corresponding to 14.4 mm and 28.2 mm spacing.

For a rectangular plate of $2 / 3$ aspect ratio, see Fig.(15), the effect of size of cutout is consistent at all values of spacings. In other words, the maximum stress is decreased as a result to the increase of size of cutout. A noteworthy point is the high effectiveness of the 72.1 mm spacing compared with other spacings.

In Fig.(16), the larger size of cutout results in increasing relatively the maximum stress regardless of the extent of diagonal spacing between cutouts. However, the 72.1 mm spacing is showing the lowest effect on the maximum stress.

## 6. CONCLUSIONS.

The strength and stiffness of plates are examined in terms of maximum stress (or SCF) and maximum deflection, respectively. This study comes to some conclusions which can be summarized as: -

1. The numerical solution carried out by (ANSYS) is comparable to those computed by analytical methods.
2. The strength and stiffness of all plates are increased as a result of increasing the spacing between cutouts both horizontally and diagonally..
3. A rectangular plate containing two cutouts arranged along the width is stronger and stiffer than when arranged along the length at a given spacing.
4. A square plate is always stronger and stiffer than an equivalent rectangular plate for the same loading condition.
5. Plate with horizontally spaced cutouts are stronger and stiffer than those with equivalent diagonally spaced cutouts.
6. Diagonally - arranged cutouts larger than 0.1 size ratio are performed especially for $2 / 3$ aspect ratio plate.

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## 8. NOMENCLATURE.

a length of the plate in $x$ direction (m).
$\mathbf{a}_{1}, \mathbf{a}_{2}$ length of the cutouts in $x$ direction (m).
b width of the plate in $y$ direction (m).
$\mathbf{b}_{1}, \mathbf{b}_{\mathbf{2}} \quad$ width of the cutouts in $y$ direction (m).
D flexural rigidity $\left[\mathrm{D}=\mathrm{Et}^{3} / 12\left(1-\mathrm{v}^{2}\right)\right]$ (N.m).
E modulus of elasticity (N.m ${ }^{-2}$ ).
G modulus of elasticity in shear ( $\mathrm{N} . \mathrm{m}^{-2}$ ).
M bending moment per unit distance (N.m/m).
$\mathbf{P} \quad$ concentrated force (N).
t thickness (m).
$\mathbf{w} \quad$ deflection of the plate (m).
$\mathbf{x}, \mathbf{y}, \mathbf{z}$ distance, rectangular coordinates (m).
$\gamma \quad$ Shear strain.
$\boldsymbol{\varepsilon}$ normal strain.
$v$ poisons ratio.
$\boldsymbol{\sigma} \quad$ normal stress $\left(\mathrm{N} . \mathrm{m}^{-2}\right)$.
$\boldsymbol{\tau} \quad$ shear stress $\left(\mathrm{N} \cdot \mathrm{m}^{-2}\right)$.

Table 1: Verification to Numerical Solution for Clamped Square Plate $\mathrm{a}=\mathrm{b}$.

| Solution <br> Method | FEM | Strip method <br> (Eq.12) | Ritz method <br> (Eq. 16) | Superposition and <br> Ritz methods (Eq. 13) |
| :---: | :---: | :---: | :---: | :---: |
| Maximum <br> Deflection <br> $(\boldsymbol{m m})$ | 347.964 | 367.75 | 362.09 | 356.44 |



Figure (1a): A plate of constant thickness.
Figure (1b): part of the plate before and after deflection.


Figure (3): Case of holes displaced along the middle horizontal axis of the plate.


Figure (4): Case of holes displaced along the plate diagonal.


Figure (5): Variation of stress with distance between two cutouts along the horizontal middle line.


Figure (6): Variation of deflection with distance between two cutouts along the horizontal middle line.


Figure (7): Variation of stress concentration factor (SCF) with distance between two cutouts along the horizontal middle line.


Figure (8): Variation of stress with distance between two cutouts along the diagonal.


Figure (9): Variation of deflection with distance between two cutouts along the diagonal.


Figure (10): Variation of stress concentration factor (SCF) with distance between two cutouts along the diagonal.


Figure (11): Variation of maximum stress with the size of cutouts along the horizontal middle line for square plate.


Figure (12): Variation of maximum stress with the size of cutouts along the horizontal middle line for rectangular plate of aspect ratio $2 / 3$.


Figure (13): Variation of maximum stress with the size of cutouts along the horizontal middle line for rectangular plate of aspect ratio $3 / 2$.


Figure (14): Variation of maximum stress with the size of cutouts along the diagonal for square plate.


Figure (15): Variation of maximum stress with the size of cutouts along the diagonal for rectangular plate of aspect ratio $2 / 3$.


Figure (16): Variation of maximum stress with the size of cutouts along the diagonal for rectangular plate of aspect ratio $3 / 2$.

# تأثير الثقوب على سلوك الصفائح المستطيلة المقيدة 

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الخلاصـة:
الصفائح ذات الثقوب الداخليـة غالبـا مـا تستخدم حديثا وقديما في التطبيقـات الهندسية الـدنيـة و الميكانيكية. ولفهم تأتثبر وجود الثقبين على معامل تركيز الإجهاد والإجهاد الأعظم والهطول في الصفائح المستطيلة و المقيدة تم إجر اء هذا البحث. إن المتغيرات مثل موقع وحجم الثقب أو النسبة بين الطول إلى العرض للصفيحة من الضروري دراستها وذاللك لأهمينها في تصـيم الأجزاء. جميع هذه العو امل تمت

 المختلفةِ. من إحدى الأهدافِ الرئبسيةِ في هذه الار اســةِ أَنْ تَعْرضَ دقـةَ الحَلِّ التحليلِي للصـفائحِ المربّعـة المقيدة.
عُموماً، ثبتّت إن نَنائِجَ الصفائح المربعة المقيدة ذات ثقبين متطابقة بدرجة جيدةِ. كذلك النَتائِج قدّمتٌ هنا الإجهادَ الأقصى، الهطول في الصفائح المنقبة والتي يمكن أن نتغير بشكل ملحوظ بتغير المو اقع و / أَو الحجم. النَتائِجُ تبين إن الصفائح المستطيلةُ والتي تَحتوي على ثقفين على العرضِ أقوى وأصلبُ مِنْ عندما


التحميل.

كلمـات رئيسية : ثقوب داخلية ، الإجهاد الأعظم ، برنامج (Ansys) ، صفائح ذات ثقوب داخلية .

