

# Equal Incremental Fuel Cost Approach for Multi Area Operation of Power System

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**Abstract** This paper presents a novel approach, Equal Incremental fuel cost ( $\lambda$ -Concept) approach, for solving Multi Area Economic Dispatch (MAED) problem. It is a simple approach and developed from the basic observation of incremental fuel cost of an area. The proposed approach has been tested on 4-area system with four generators in each area and a large 2-area system consists of 120 generators. The suggested algorithm has been tested extensively by considering the different tie line power transfer limits and useful recommendations are provided. Further, the impact of tie line power transfer limits on total fuel cost is also discussed. It is found from the test cases that the proposed method is shown to be robust, very fast and extensible to include a large class of problems. The simulation results of the proposed method have been compared with the existing methods.

**Index Terms**— Central economic dispatch; Equal incremental fuel cost; Multi area economic dispatch; Tie line constraints

## I. INTRODUCTION

Economic Dispatch (ED) is one of the important optimization tasks in power system operation. The aim of Multi Area Economic Dispatch (MAED) is to determine the amount of power that can be generated optimally in one area and transferred to other areas, without violating physical and tie line capacity constraints. It is a large scale, discontinuous, multi objective, non-linear optimization problem [1].

Several methods have been proposed for solving the MAED problem. Shoults et al. addressed the economic dispatch problem in [2]. The results obtained by Doty et al. [3] were global optimum and reflected transmission constraints with linear losses. The list of other techniques is provided in TABLE I.

TABLE I

METHODS TO SOLVE MAED PROBLEM

Category	Method used	Reference
Conventional	Sequential Method	[7]
	Nonlinear Network Flow Programming	[8]
	Direct Search Method	[13]
Heuristic/Bio Inspired	Nature Inspired Optimization	[5]
	Flower Pollination	[6]

Hybrid	Algorithm	
	Hopfield Neural Network	[11]
	Evolutionary programming	[12]
	Differential Evolution	[9]
	Particle Swarm Optimizer	
	Nonlinear optimization Neural Network	[14]
	Adaptive Shuffled Frog Leaping Algorithm	[15]
	Modified Iteration Particle Swarm Optimization	[16]

Multi area with multi fuel options is included into economic dispatch problem in [7]. Prasanna et al. [10] incorporated fuzzy logic strategy in Evolutionary programming and Tabu search, to solve security constrained multi area economic dispatch problem. In order to solve non-linear, discontinuous, multi-objective and global optimization problems, bio-inspired algorithms have been suggested by many researchers [17]. Most of these algorithms are iterative in nature and quality of the solution depends on user defined parameters. Hence, it is necessary to introduce a simple approach where it can provide a better solution without depending on user defined parameters. This is the motivation to introduce  $\lambda$ - concept in this paper.

The proposed method has been developed in MATLAB (R2012 A) on a personal computer (Intel Ri3, 2.1 GHz, 2 GB RAM). The remaining paper is arranged as follows: Section II describes MAED problem formulation. Section III explains description of the proposed methodology. Section IV presents simulation results of various test cases. Section V reports conclusion of the work.

## II. MULTI AREA ECONOMIC DISPATCH

A brief description about objective function and constraints are provided as follows:

### A. Objective Function

The objective function is minimization of fuel cost without violating physical constraints.

$$FC = \sum_{n=1}^N \sum_{m=1}^{M_n} (a_{nm} + b_{nm} P_{nm} + c_{nm} P_{nm}^2) \quad (1)$$

Where  $FC$  is Fuel cost of generators in \$,  $N$  is number of areas,  $M_n$  is number of generators in area  $n$ ,  $P_{nm}$  is output power of generator  $m$  in area  $n$  and  $a_{nm}$ ,  $b_{nm}$ ,  $c_{nm}$  are fuel cost coefficients.

Another operational cost involved in multi area operation is cost of tie line power transfer between areas. It is expressed as follows:

$$TC(P_T) = \sum_{n=1}^{N-1} \sum_{k=n+1}^N f_{nk} P_{Tie,nk} \quad (2)$$

Where  $TC(P_T)$  is Cost of Transmission in \$,  $P_{Tie,nk}$  is tie line power flow from area  $n$  to area  $k$  and  $f_{nk}$  is the transmission cost coefficient.

The total operational cost is sum of fuel cost and power transmission cost. It is expressed as follows:

$$F(P_T) = FC + TC(P_T) \quad (3)$$

### B. Constraints

The constraints considered are as follows:

#### 1) Generator constraint:

$$P_{nm,min} \leq P_{nm} \leq P_{nm,max} \quad (4)$$

2) Area power balance constraint: Mathematically, it is expressed as:

$$\sum_{m=1}^{M_n} P_{nm} = P_{Ln} + \sum_{k,k \neq n} P_{Tie,nk} \quad (5)$$

Where  $P_{Ln}$  is power demand of area  $n$ .

#### 3) Tie line constraint:

$$P_{Tie,nk,min} \leq P_{Tie,nk} \leq P_{Tie,nk,max} \quad (6)$$

Where  $P_{Tie,nk,min}$  is the tie line minimum power transfer limit,  $P_{Tie,nk,max}$  is the tie line maximum power transfer limit.

#### 4) Power import/export constraint of area:

$$\sum_{n=1}^{M_n} P_n \geq P_{Ln} - I_n \quad (7)$$

$$\sum_{n=1}^{M_n} P_n \leq P_{Ln} + E_n \quad (8)$$

Where  $I_n$  and  $E_n$  are import and export of area  $n$  respectively.

## III. EQUAL AREA INCREMENTAL FUEL COST( $\lambda$ ) APPROACH

Equal incremental fuel cost concept has been developed in this paper from the basic observation of area incremental fuel costs of all areas. In MAED problem, different areas are connected with tie lines. Equal incremental fuel cost can be calculated for each area. Assume that two areas are connected with tie line as shown in Fig. 1.

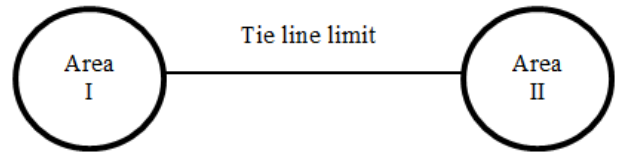


Fig. 1. Two area system connected with a tie line. Assume equal incremental fuel costs of area I is 5 \$/MW and area II is 7 \$/MW and tie line limit is 50 MW. It is clear that the area II yields more fuel cost compared to area I for the same load. Hence, it is more economical if power transfers from area I to II. In central economic dispatch (CED), ED is performed by combining generators of all areas at overall power demand. Here, overall power demand is sum of power demands of all areas. In MAED it is necessary to identify areas which can either import or export power through tie lines to other areas without violating tie line power transfer limit. CED plays a dominant role in deciding power import/export of area by comparing system lambda with area lambda.

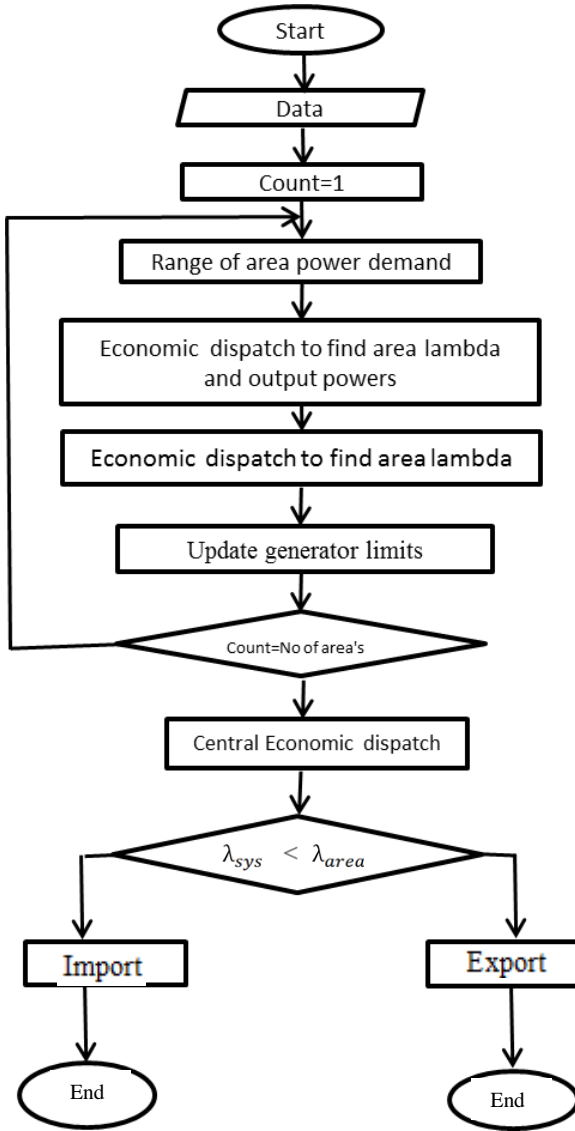


Fig. 2 Flowchart of the proposed approach.

The entire procedure of the algorithm depends on the lambda and hence it is named as “λ-concept”. The flowchart of proposed method is given in Fig.2. Detailed steps of incremental fuel cost concept are available in [18].

#### IV. SIMULATION RESULTS

The proposed method has been tested on four area and two area systems. The algorithm was tested extensively at different tie line power transfer limits and results are compared with the existing methods.

##### A. Case 1: Four Area System

In this case, the proposed approach has been tested on 4-area system with four generators in each area. The data of fuel cost and power demand in each area is adopted from [8].

The simulation results of updated generator limits are shown in Table II. From the economic dispatch, the equal incremental fuel costs of four areas are 8.5556 \$/MW, 7.4615 \$/MW, 22.0498 \$/MW and 6.4579 \$/MW at the given power demands of 400 MW, 200 MW, 350 MW and 300 MW respectively. The overall power demand is 1250 MW and the incremental fuel cost of CED is 9.6906 \$/MW.

TABLE II

UPDATED GENERATOR POWER LIMITS (MWs)

Area	Unit	Pmin	Pmax	Pmin	Pmax
I	1	50	150	122.8972	150
	2	25	100	74.29907	100
	3	25	100	34.57944	100
	4	25	100	68.2243	100
II	1	50	150	50	68.64408
	2	25	100	25	100
	3	25	100	25	49.15255
	4	25	100	25	82.2034
III	1	50	150	62.4911	150
	2	25	100	60.40925	100
	3	25	100	67.4911	100
	4	25	100	59.60854	100
IV	1	50	150	65.78947	150
	2	25	100	55.26316	100
	3	25	100	25	55.5559
	4	25	100	53.94737	94.44488

It is understood from the area equal incremental fuel costs and incremental fuel cost of CED that area I, II and IV exports the power to area III in order to get economic benefits. The simulation results of the proposed method at the tie line power transfer limit of 100 MW are provided in Table III.

The transfer of power through tie line depends on its tie line power transfer limit. The proposed algorithm has been tested extensively by considering the different tie line power transfer limits. Power generation by each area at different tie line power transfer limits is shown in Table IV. There is no change in the power generation in each area, if the tie line power transfer limit exceeds 210 MW. Variation of power generation at different tie line power transfer limits is shown in Fig. 3.

It is clear from Fig. 3 that power generation is reducing in Area III with increased tie line power transfer limit. Area III imports maximum power through the tie line from the other areas. It is not economical if the tie line limit exceeds 200 MW.

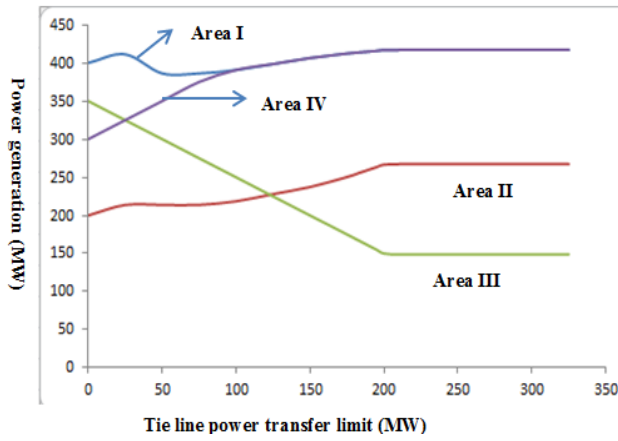


Fig. 3 Variation of power generation at different tie line power transfer limits

TABLE IV

POWER GENERATION AT DIFFERENT TIE LINE POWER TRANSFER LIMITS

Tie line power Limit(MW)	Power generation (MW) in each area			
	I	II	III	IV
50	386.2684	213.7317	300	350
100	390.7515	218.4971	250	390.7515
150	406.3584	237.2833	200	406.3584
200	416.7153	266.5694	150	416.7153
210	416.9044	267.2707	148.92	416.9044
225	416.9044	267.2707	148.92	416.9044
325	416.9044	267.2706	148.9205	416.9044

Powers in each area at different tie line power transfer limit is shown in Table V.

Export/ Import in each area at different tie line power transfer limits are provided in Table VI.

It is observed from the Table VI that if the tie line limit exceeds 200 MW, there is no change in export/ import of powers.

Total fuel cost at different tie line power transfer limits is given in Table VII.

TABLE VII

TOTAL FUEL COST AT DIFFERENT TIE LINE POWER TRANSFER LIMITS (MWs)

S. No	Tie line power transfer limit (MW)	Fuel cost (\$)
1	0	8799.7354
2	50	8159.9411
3	100	7693.6
4	150	7404.00
5	175	7338.2223
6	185	7328.3306
7	190	7326.9235
9	200	7331.1869
10	225	7332.2113

Simulation results of the proposed approach have been compared with the methods available in literature and shown in Table VIII.

TABLE VIII

COMPARISON OF OUTPUT POWERS WITH DIFFERENT METHODS

Area	Unit	Output powers(MW) by each method		
		INFP[12]	EP[12]	Proposed
I	1	150	150	150
	2	100	100	100
	3	66.97	65.66	64.9635
	4	100	99.9	100
II	1	56.97	57.88	54.964
	2	96.25	93.02	93.70
	3	41.87	42.89	40.6022
	4	75.52	71.48	70.803
III	1	50	50.01	50
	2	36.27	36.98	35.591
	3	38.49	40.36	37.7094
	4	37.32	38.14	36.7
IV	1	150	149.98	150
	2	100	100	100
	3	57.05	56.12	64.9635
	4	96.27	97.68	100
Fuel cost(\$)		7337	7338	7326.9235

It is clear from the Table VIII that the proposed method yields best solution compared with the existing methods available in the literature. The best solution has been achieved at the tie line power transfer limit of 190 MW.

#### B. Case II: Two Area System

The fuel cost data is obtained from [11]. The output powers of the proposed method are given in Table IX. Fuel costs of the proposed method at different tie line power transfer limits are provided in Table X.

TABLE X

TOTAL FUEL COST AT DIFFERENT TIE LINE POWER TRANSFER LIMITS

S. No.	Tie line power transfer limit (MW)	Fuel cost (\$)
1	100	377986.3
2	120	377979.5
3	200	377961.9
4	250	377958.5
5	300	377958.3
6	500	377958.3

It is clear that the algorithm provides best fuel cost, when the tie line power transfer limit is beyond 250 MW. However, there is no economic benefit if the tie line power transfer limit exceeds more than 250 MW at the given power demand of 28000 MW.

It is clear from Table XI that the proposed method yields best solution compared with the existing methods. During the execution of the proposed approach, Secant method is applied for solving ED problem.

**TABLE XI**  
TOTAL FUEL COST OF 120 UNIT SYSTEM FOR  
DIFFERENT METHODS

S. No.	Method	Fuel cost (k\$/hr)	Computational time
1	CM [11]	399.13	-
2	PHN [11]	400.93	-
3	Proposed Method	377.9583	1.56 seconds

**TABLE III**  
SIMULATION RESULTS OF THE PROPOSED METHOD AT A TIE LINE LIMIT OF 100 MW

Area	Output powers(MW)				Flow (\$/MW)	Flow (MW)	Tie line flow
	1	2	3	4			
I	150	100	66.9	100	8.5556	16.906	Export
II	56.9	96.13	41.81	72.42	7.4615	67.275	Export
III	50	32.04	33.45	33.42	22.0498	-201.0	Import
IV	150	100	66.9	100	6.4579	116.9	Export

**TABLE V**  
OUTPUT POWERS AT DIFFERENT TIE LINE POWER TRANSFER LIMITS (MWs)

Tie line power transfer limit	Area	Generators			
		1	2	3	4
0	I	150	100	55.5556	94.444
	II	50	68.2692	27.8846	53.846
	III	90.249	83.5409	95.2491	80.961
	IV	122.9	74.2991	34.5794	68.224
100	I	150	100	51.4451	89.306
	II	50	76.8064	32.1532	59.538
	III	62.491	60.4093	67.4911	59.609
	IV	150	100	51.4451	89.306
190	I	150	100	64.9635	100
	II	54.964	93.70	40.6022	70.803
	III	50	35.5911	37.7094	36.7
	IV	150	100	64.9635	100
250	I	150	100	66.9044	100
	II	56.904	96.1306	41.8153	72.42
	III	50	32.0435	33.4522	33.425
	IV	150	100	66.9044	100
300	I	150	100	66.904	100
	II	56.9	96.131	41.815	72.42
	III	50	32.044	33.452	33.42
	IV	150	100	66.904	100

**TABLE VI**  
EXPORT/IMPORT IN EACH AREA AT DIFFERENT TIE LINE POWER TRANSFER LIMITS (MWs)

Tie line Power Limit	Areas			
	I	II	III	IV
50	-13.7316	13.7317	-50	50
100	-9.2485	18.4971	-100	90.7515
185	14.0876	56.8249	-185	114.0876
190	14.9635	60.0730	-190	114.9635
195	15.8394	63.3212	-195	115.8394
200	16.7153	66.5694	-200	116.7153
300	16.9044	67.2706	-201.0795	116.9044
325	16.9044	67.2706	-201.0795	116.9044

**TABLE IX**  
**OUTPUT POWERS OF 120 UNITS BY THE PROPOSED METHOD**

Generator	1	2	3	4	5
Power (MW)	80	120	190	42	40.98605
Generator	6	7-9	10	11	12
Power (MW)	140	300	131.988	147.48	150.18
Generator	13	14	15	16	17
Power (MW)	238.01	376.5861	377.6522	377.6522	377.6522
Generator	18-19	20-27	28-30	31-34	35-36
Power (MW)	500	550	10.7146	20	18
Generator	37	38-40	41	42	43
Power (MW)	20	25	80	120	190
Generator	44	45	46	47-49	50
Power (MW)	42	40.98605	140	300	131.988
Generator	51	52	53	54	55
Power (MW)	147.48	150.181	238.0113	376.5861	377.6522
Generator	56	57	58-59	60-67	68-70
Power (MW)	377.6522	377.6522	500	550	10.7146
Generator	71-74	75-76	77	78-80	81
Power (MW)	20	18	20	25	80
Generator	82	83	84	85	86
Power (MW)	120	190	42	40.98605	140
Generator	87-89	90	91	92	93
Power (MW)	300	131.988	147.48	150.181	238.0113
Generator	94	95-97	98-99	100-107	108-110
Power (MW)	376.5861	377.6522	500	550	10.7146
Generator	111-114	115-116	117	118-120	
Power (MW)	20	18	20	25	

## V.CONCLUSIONS

The paper has presented a novel technique for Multi-Area Economic Dispatch using equal incremental fuel cost ( $\lambda$ ) concept. The significant features of the algorithm include:

- Incremental fuel cost ( $\lambda$ ) concept is developed based on a simple observation from the equal incremental fuel costs of areas.
- Tie line constraints can be incorporated effectively.
- Any technique available in literature can be used for economic dispatch along with the proposed approach to evaluate equal incremental fuel costs of all areas.
- The proposed approach is robust and fast because it will not depend on any user defined parameters.

The proposed method has achieved accurate and better solutions for 2-area system with 120 units

and 4-area system with 16 units. The adopted method has been tested extensively by considering the different tie line power transfer limits and useful recommendations are provided regarding the tie line power transfer limit. Extensive analysis has been made on this test case to show the impact of tie line power transfer limit on the variation of total fuel cost.

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