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Effect of Friction on the Dynamical Analysis of Three-Link Planar Robot Arm by Using Lagrange Approach

Abstract- *The dynamic analysis of Three-Link planar robot arm and control system with (PID) are presented and investigated. The dynamic analysis is very important in the design and control of the robot. The difference between the actual dynamic analysis and ideal dynamic analysis is the presence of friction in the robot joints. In this work, the frictional effect in the joints of three-link planar robot is inserting in the dynamic equations and that makes the dynamic analysis is more reality and difficult. The mathematical model that represent the friction consist of two types of friction (Coulomb and viscous friction). A Lagrange method is used and applied to evaluate the generalized forces in the two cases (without and with the effect of friction). Control system with (PID) controller is presented with Simulink block set to evaluate and show the dynamic response of each link in two cases (without and with friction). MATLAB software is used for programing and simulation the equations. In addition, with that, error signals are presented and analyzed for each link. It is concluded from the results that the values of generalized forces in case of presence of friction are more about (12%) than the values of the forces in case of without friction and the behaviors of the dynamic response is linear in case of without friction while the behavior become (non-linear) by inserting the frictional effect in the robot joints. The results indicate that the effect of friction is very important and must be not neglected.*

Keywords- *Three-Link planar, Robot, Coulomb Friction, Viscous Friction, Lagrange, Dynamic Analysis, PID Controller*

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1. Introduction

The analysis and design of any robot affected by the dynamic consideration require the evaluation of the desired forces that developed in the actuators (prismatic joints) to move the robot from one place to another. The reality of dynamic analysis means including the effect of friction of the robot joints into the equations of approach that used in the dynamic analysis.

A lot of approaches and methods are used to solve the problem of dynamic analysis of the robot. In 1998, Pratt and Pratt [1] presented the dynamic and control of seven-Link planar robot by using a simple control algorithm. In 1998, Dasgupta and Mruthyunjay [2] used Newton-Euler method for the dynamic analysis of parallel robot. In 2000, Shoval and Shoha [3] investigated the force analysis of mobile robot. In 2007, Yong Sheng Zhao et.al [4] used Lagrange method to derive the dynamic equations of 5-UPS/PRPU parallel robot. In 2009, Lopez [5] proposed the dynamic analysis of Gough-Stewart robot by using the momentum approach. . In 2010, Alwan and Hamza [6] used the principle of the virtual work to make the dynamic analysis of the Gough-Stewart manipulator. In 2012, Wang et al. [7] presented the dynamic analysis of 3-DOF serial-parallel robot by using Newton-Euler theorm. In 2014, Alwan and Ahmed [8] suggested a mathematical solution of parallel manipulator to evaluate the singularity. In 2014, Mustafa and Al-Sai [9] analyzed and studied the dynamics and control of 2-Revolute joint by using Denavit-Hartenberg and PID controller. In 2015 Chen and Zhang 10] used the transformation matrix method for solving the dynamic analysis problem of (2-DOF) planar robot.

Not all the previous researches take into account the effect of friction in the dynamic analysis. The aim of this paper is to make the dynamic analysis more reality by including the effect of friction of the robot joints. In this work, Three-link planar robot arm with 3-DOF as show in Figure 1 is adopted. This type of robot is classified as serial robot with open kinematic chain. The mathematical model of friction forces and friction moments includes two types of friction i.e. (Coulomb and Viscous friction). Also in this study, Lagrange equations are derived and used for dynamic analysis of the robot with considering the effect of friction.

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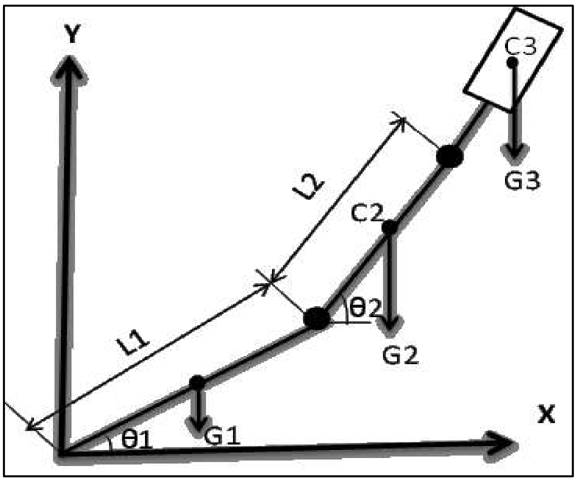

Figure 1: Mechanism of three- link planar robot configuration

The robot considered in this work is a Three-Link planar serial robot with open kinematic chain as show in Figure 1. The setup of this robot consists of three links with three joints (Revolute-Revolute- Prismatic). The robot has 3-DOF according to three angles (θ_1, θ_2, x_3). The angle (θ_1) is between the link (1) and the (Ox-axis) and (θ_2) is between the link (2) and (Ox-axis) and (x_3) is the linear movement of the prismatic joint. The lengths, masses and mass moment of inertia of the links (1,2) are denoted by (L_1,L_2) , (m_1,m_2) and (I_{C1},I_{C2}) respectively. The mass and mass moment of inertia of link three is denoted by (m_3, I_{C3}). The Coulomb friction coefficient between link(1,2) and links (2,3) is denoted by (μ) and the viscous friction coefficient between links (1,2) and links (2,3) is (C_s).

2. Kinematic Analysis

Kinematic of robotics relates with geometry to evaluate the position and the orientation of the robot. The emphasis on the kinematic of robot geometry leads to the links of the robot is modeled as rigid bodies and the joints of the robot are assumed to provide pure rotation or translation movement. In the present study, the robot that is chosen is Three Link Planar robot with three revolute joints. In this work, the problem of the kinematics is solved by using Denavit-Hartenberg (D-H) method. The parameters are presented in Table 1.

Table 1: Parameters of (D-H) method

Frame No.	a_i	α	d	θ_i
1	L_1	0	0	Θ_1
2	L_2	0	0	Θ_2

The kinematic equations that are used and written as follows:

$$X_1 = L_1 \cos\theta_1$$
(1)

$$Y_1 = L_1 \sin\theta_1$$
(2)

$$X_2 = L_1 \cos\theta_1 + L_2 \cos(\theta_1 + \theta_2)$$
(3)

$$Y_2 = L_1 \sin\theta_1 + L_2 \sin(\theta_1 + \theta_2)$$
(4)

3. Dynamic Model of the Robot

The problem of the dynamic analysis with the effect of friction of Three-Link planar robot is investigated and solved by using Lagrange method. The Lagrange equations are derived and applied in this work. The fixed coordinate system of the such robot is denoted by (X_o^O ,Y_o^O) and the local coordinate system is denoted by (x_i y_i) as show in Figure 1. The gravitational forces of each linkage are evaluated as follows:

$$F_G = G_i - m_i * g * (\sin\theta_i + \cos\theta_{ij}) \quad (5)$$

Where i=1, 2, 3.

The reaction forces that exerted by each linkage are evaluated as:-

F_{12}= NJ_1, F_{21}= -NJ_1 , F_{23}= NJ_2, F_{32}= -NJ_2 (6)

The general form of the Lagrangian of the dynamic system is defined as:

$$L = T - U \quad (7)$$

Where i= 1, 2, 3. The symbol (L) refers to Lagrangian, (T) refers to the total kinetic energy and (U) refers to potential energy.

The kinetic energy of each joint is evaluated as [11]:

$$T_1 = \frac{1}{2} I_O\, \omega_1 \times \omega_1^T \quad (8)$$

$$T_2 = \frac{1}{2} I_O\, \omega_2 \times \omega_2^T \quad (9)$$

$$T_3 = \frac{1}{2} m_3 * v_3^2 + Q \quad (10)$$

Where (Q) is the actuator or generalized force generated from the prismatic joint to overcome the weight of linkages, reaction forces and friction forces developed in the joints. The total kinetic energy is:

$T = T_1 + T_2 + T_3$

The angular velocity (ω_i) of each linkage is evaluated as:

$$\omega_i = \dot{\theta}_i k_i \quad (11)$$

The symbol (rc_i) refers to the center of mass (C_i) of each linkage and it is evaluated as:-

$$rc_1 = \frac{L_1}{2} \;,\; rc_1 = \frac{L_2}{2} \;,\quad rc_1 = \frac{L_3}{2} \quad (12)$$

The velocity of the center mass of each link is written as:

$$vc_i = rc_i + \omega_i \times rc_i \quad (13)$$

Where i=1, 2, 3.

The velocity vp_1 of the point P attached to the link 1 is evaluated as follows:

$$v_1 = \omega_1 \times rp_1 \quad (14)$$

$$v_2 = \omega_2 \times rp_2 \quad ; \quad v_3 = \omega_3 \times rp_3 \quad (15)$$

The potential energy is evaluated as:-

$$P.E = m_1 * g * l_{g1}\sin\theta_1 + m_2 * g\left[l_1 * \sin\theta_1 + (l_{g2} * \sin(\theta_1 + \theta_2)\right] + m_3 * g\left[l_2 * \sin\theta_2 + (l_{g3} * \sin(\theta_2 + \theta_3))\right] \quad (16)$$

The mathematical model that used to evaluate the friction forces consist of two types of friction i.e.(Coulomb and viscous friction) is written as follows [12]:

$$Ff_{12} = -[c_s\omega_1 + \mu * N] \quad (17)$$

$$Ff_{23} = -[c_s\omega_2 + \mu * N] \quad (18)$$

Where:-

C_s: Viscous friction coefficient.

ω : Angular velocity of the joint.

N: Reaction force of the joint.

$$Ff_{21} = -Ff_{12} \quad \text{and} \quad Ff_{32} = -Ff_{23} \quad (19)$$

The generalized forces (Q) of e linkage (3) in case of neglecting the effect of friction are evaluated as:

$$Q_i = \frac{\partial vc_1}{\partial \dot{\theta}_i}.G_1 + \frac{\partial vc_2}{\partial \dot{\theta}_i}.G_2 + \frac{\partial vc_3}{\partial \dot{\theta}_i}.G_3 + \frac{\partial vp_1}{\partial \dot{\theta}_i}.F_{12} + \frac{\partial vp_2}{\partial \dot{\theta}_i}.F_{21} + \frac{\partial vp_3}{\partial \dot{\theta}_i}.F_{32} \quad (20)$$

The generalized force (Q_i for the linkage (i) in case of presence of friction in the robot joints is evaluated as:

$$Q_i = \frac{\partial vc_1}{\partial \dot{\theta}_i}.G_1 + \frac{\partial vc_2}{\partial \dot{\theta}_i}.G_2 + \frac{\partial vc_3}{\partial \dot{\theta}_i}.G_3 + \frac{\partial vp_1}{\partial \dot{\theta}_i}.(Ff_{12} + F_{12}) + \frac{\partial vp_2}{\partial \dot{\theta}_i}.(Ff_{21} + F_{21}) + \frac{\partial vp_3}{\partial \dot{\theta}_i}.(Ff_{32} + F_{32}) \quad (21)$$

The difference between equations (20) and (21) is that in equation (20) the effect of friction in the joints is neglected and that is called (Ideal Mechanism) but in equation (21), the effect of friction is considered and inserting in the equation and that is called (Actual Mechanism).

4. Control system of the robot

In this section, (PID) controller is applied and used to evaluate and show the dynamic response of each link of the robot in the two cases i.e.(without and with friction). This type of the PID controller is based on linear model technique.

I .PID Controller Based on Linear Model

The main characteristic of this method is that it applies for a wide range of the points in the robotics system. First, it must be defining new variables that used to convert the model to linear as:

$$x_1 = \theta_1 \; ; \; x_2 = \theta_2 \; ; \; x_3 = \dot{\theta}_1 ; \; x_4 = \dot{\theta}_2$$

$$\dot{x}_1 = \dot{\theta}_1 ; \; \dot{x}_2 = \dot{\theta}_2 ; \dot{x}_3 = \ddot{\theta}_1 \; ; \; \dot{x}_4 = \ddot{\theta}_2$$

Linearization of variable (x_1) with respect to other variable is:

$$\frac{\partial \dot{x}_1}{\partial x_1} = 0 ; \quad \frac{\partial \dot{x}_1}{\partial x_2} = 0 ; \quad \frac{\partial \dot{x}_1}{\partial x_3} = 1; \quad \frac{\partial \dot{x}_1}{\partial x_4} = 0.$$

$$\frac{\partial \dot{x}_2}{\partial x_1} = 0 ; \quad \frac{\partial \dot{x}_2}{\partial x_2} = 0 ; \quad \frac{\partial \dot{x}_2}{\partial x_3} = 0; \quad \frac{\partial \dot{x}_2}{\partial x_4} = 1.$$

$$\frac{\partial \dot{x}_3}{\partial x_1} = \frac{c_4 c_5}{c_1 c_5 - M_2}; \quad \frac{\partial \dot{x}_3}{\partial x_2} = \frac{c_2 c_6}{c_1 c_5 - M_2};$$

$$\frac{\partial \dot{x}_3}{\partial x_3} = 0; \quad \frac{\partial \dot{x}_3}{\partial x_4} = 0.$$

$$\frac{\partial \dot{x}_4}{\partial x_1} = \frac{-c_6}{c_5}; \quad \frac{\partial \dot{x}_4}{\partial x_2} = \frac{c_2 c_6}{c_1 c_5 - M_2};$$

$$\frac{\partial \dot{x}_4}{\partial x_3} = 0; \quad \frac{\partial \dot{x}_3}{\partial x_4} = 0.$$

Now, the state space model is written as:

$$\begin{bmatrix} \Delta \dot{x}_1 \\ \Delta \dot{x}_2 \\ \Delta \dot{x}_3 \\ \Delta \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{c_5 c_4}{c_1 c_5 - M_2} & \frac{c_6 c_2}{c_1 c_5 - M_2} & 0 & 0 \\ 0 & \frac{-c_6}{c_5} & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \\ \Delta x_3 \\ \Delta x_4 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{c_5}{c_1 c_5 - M_2} & \frac{-c_2}{c_1 c_5 - M_2} \\ 0 & \frac{1 - c_2}{c_5} \end{bmatrix} * \begin{bmatrix} \Delta M_1 \\ \Delta M_2 \end{bmatrix} \quad (22)$$

The general form of state –space matrix is:

$$\dot{X} = Ax + Bu$$

$$Y = Cx + Du$$

The values of constants (c_1 to c_6) are substitute in the state space model to evaluate the state space matrices as follows:

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -0.56 & 0.49 & 0 & 0 \\ 0.17 & 1.25 & 0 & 0 \end{bmatrix} ; B = \begin{bmatrix} 0 & 0 \\ -0.41 & 0.217 \\ 0.787 & -0.042 \\ 0.5 & -0.03 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad ; D = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

After evaluating the above matrices, MATLAB software is used and Simulink window is opened and the block diagram of the control system has been configured by the state space model and PID -controller linear model as shown in Figure 2.

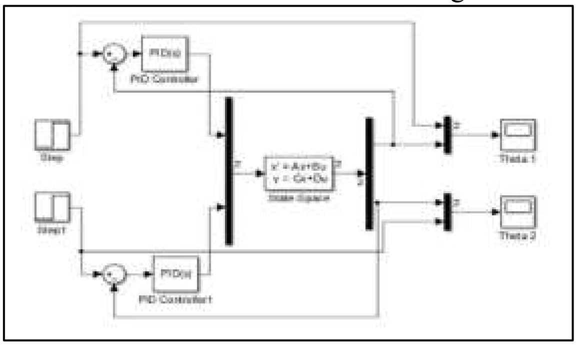

Figure 2: Simulink block diagram of PID controller

II. Error Analysis

In this section, error signals represent the difference in the dynamic response of each link in the two cases i.e.(without and with the effect of friction). The equations that are used to evaluate the error signals are written as:

$$e\theta_1 = \theta_{1f} - \theta_1 \quad ; \qquad (23)$$

$$e\theta_2 = \theta_{2f} - \theta_2 \qquad (24)$$

4. Results and Discussions

In this section, all the previous equations of the Three-Link planar robot are simulated by using (MATLAB) programing to represent the Cartesian displacement, velocity and acceleration and evaluate the generalized forces in two cases i.e. (without and with friction). Additional with that, (Simulink) is used to evaluate the dynamic response of each link and the error signals are determined.

First, the equations from (1) to (6) that related with the kinematic model are programed by MATLAB to evaluate the Cartesian displacement, velocity and acceleration of the robot according to the parametric study as:

- The lengths of linkages 1, 2 and 3 are (0.5 m). The initial condition is that time (θ_1, θ_2 =0) when time (t=0). The results of kinematic analysis are shown in Figures 3, 4, and 5 respectively.

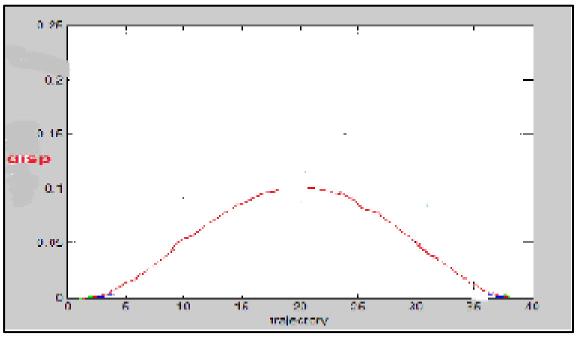

Figure 3: Displacement Profile

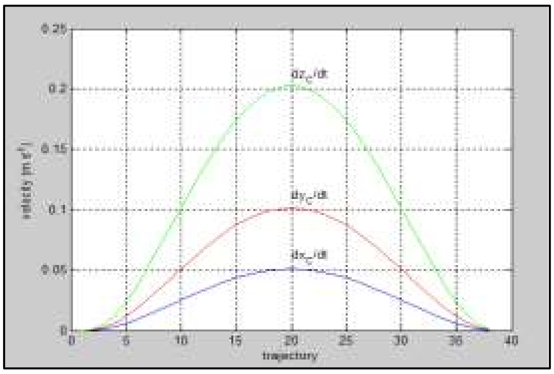

Figure 4: Velocity Profile

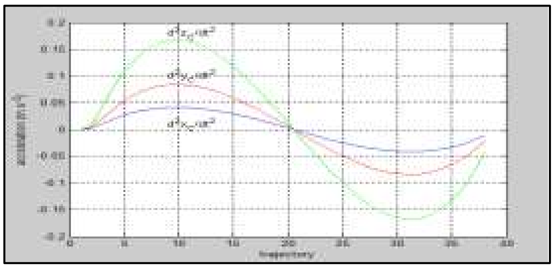

Figure 5: Acceleration Profile

From the results that represented above, it can be seen that the values of the displacement, velocity and acceleration are (real values) not complex and this indicate that there is no (singularity) in the path trajectory of the robot motion.

Second, equations from (7) to (25) are programed to evaluate the actuator or generalized force that developed from the prismatic joint which is responsible to move the robot from one position to another position according to the parametric study as:

- The masses of linkages 1 and 2 are (0.5 kg) and the mass of linkage 3 is (0.1 kg).
- The mass moment of inertia is (0.015 kg. m^2).
- The Coulomb friction coefficient (μ) is (0.1).
- The viscous friction coefficient (c_s) is (0.04 N.m.sec/rad).
- The angular velocity of joint (1) is (0.02 rad/sec) and for joint (2) is (0.05 rad/sec) and the

linear velocity for joint (3) (prismatic joint) is (0.01 m/sec).

The period allocated for the movement of the robot starting with initial time is (0 sec) to the final time (20 sec). The results are shown in Figure 6.

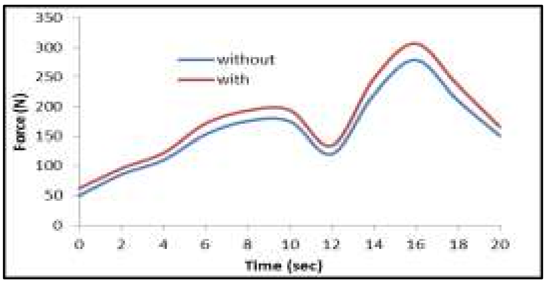

Figure 6: Generalized Forces Developed by the Prismatic Joint

From the figure that mentioned above, the curve with the blue color represent the variation of the of the generalized forces with the time in case of ideal dynamic analysis i.e. (without inserting the effect of friction) and the curve with the red color represent the variation of the values of the generalized forces with the time in case of actual dynamic analysis i.e. (with inserting the effect of friction). It is seen that from the curve that the behavior of the variation of the forces with the time in the real case is same and parallel to the variation of the forces with the time in the ideal case but there is a different in the values of the forces in the two cases i.e.(with and without friction). The values of the generalized forces with the presence of friction are greater about twenty percent (12%) than the values of the forces in the case of without friction and that different because of inserting the values of friction forces and of each joint in the dynamic analysis to make the analysis more reality.

Third, the equations from (26) to (30) and additional with that, the state space matrices (A,B,C,D) are simulated in the MATLAB/Simulink to show and represent the dynamic response of each link in the two cases i.e.(without and with the friction) that evaluated from the control system which represent by (PID) controller are shown in Figures 7, 8, and 9 respectively.

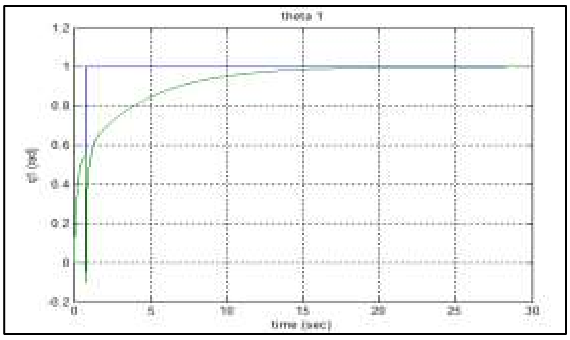

Figure 7: Dynamic response of link (1)

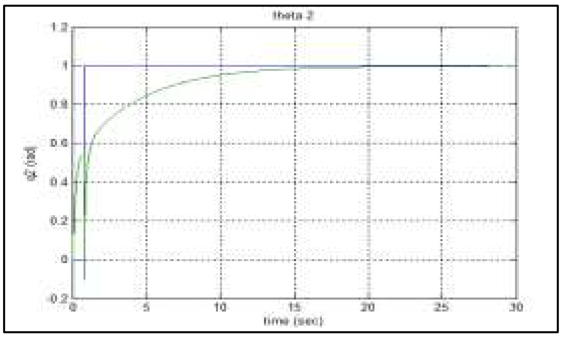

Figure 8: Dynamic response of Link (2)

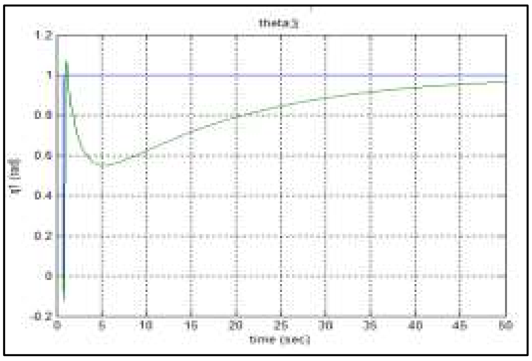

Figure 9: Dynamic response of link 3

Form Figures 7,8,9, it can see that the results that represent by with blue color represent the behavior of the dynamic response of each link in case of without friction and that behavior is linearity. However, the behavior of the results with green color is non-linear because of inserting the frictional effect in the control system.

Fourth, the equations (31, 32, 33) are programed and simulated to show the results of the error analysis of each link as shown in Figures 10, 11, and 12.

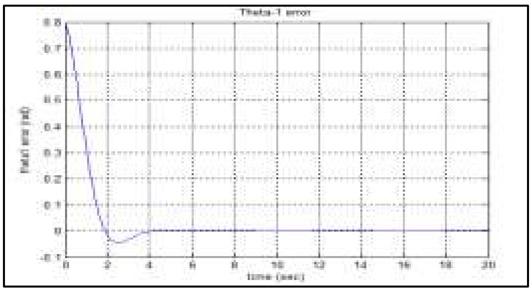

Figure 10: Error of linkage (1)

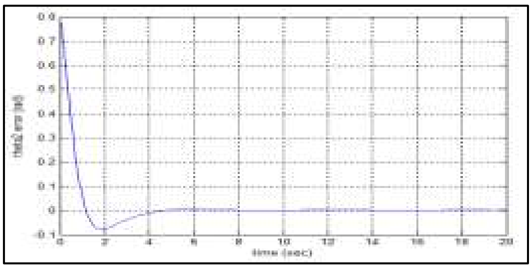

Figure 11: Error of Linkage (2)

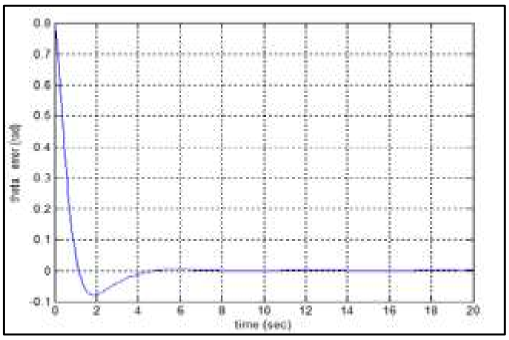

Figure 12: Error of Linkage (3)

5. Conclusions

The difference between the actual and ideal dynamic analysis of the robot is by inserting the effect of the friction of the robot joints in the dynamics equations and the control system of the robotic mechanism. The problem of the dynamic analysis and the dynamic response with presence of friction forces in the joints of the Three-Link planar robot is presented. Two types of friction i.e. (Coulomb and viscous friction) is taken into account and inserted into the dynamic equations of the Lagrange method and in the block diagram of the control system with (PID) controller of the robot. Lagrange method is applied to solve the dynamic analysis of the robot with including the effect of friction in the robot joints. The control system is used to show and represent the dynamic response of each link in two cases i.e. (without and with the effect of friction). MATLAB programing and Simulink block set are used to program and simulate the set of dynamic equations and the block diagram with (PID) controller for getting and show the results. It is concluded from the results of the dynamic analysis that obtained from the Lagrange equations that the values of generalized forces with presence of friction in the robot joints is grater about (12%) than the values of the forces in the ideal case i.e.(without effect of friction). Also, it is concluded from the results of the dynamic response of each link in the two cases (without and with the effect of friction) that the behavior of each link is linearity in case of no friction in the robot joints but the behaviors become curvature (non-linear) when the effect of friction is inserting in the block diagram of the control system with PID controller. Error analysis is presented and the results show that the curves of the error are return to the steady values after short duration and that is lead to the control system with PID controller is effective and gives good results. All the previous discussions in the conclusion sections leads to very important point that is that the frictional effect plays very important rule in the dynamic analysis and in the control system of the robotic mechanism and this effect must be not neglect.

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