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The Jacobian Conjecture and Kernel Conjecture

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Abstract

In this paper we study the Jacobian conjecture from the viewpoint of derivations and relate it to a conjecture about the kernel of a derivation.

1. Derivations and the Jacobian condition.

The aim of this section is to study the Jacobian conjecture by means of derivations therefore we reformulate the Jacobian conjecture in terms of the kernel of a special derivation.

Let $F = (F_1,\dots,F_n): \not\subset^n \to \not\subset^n$ **be a polynomial map satisfying the Jacobian condition ,** $\det(JF) \in \not\subset^* = \not\subset - \{0\}$**.To such a map we associate an n-tuple of derivations on** $\not\subset[X]$ **denoted by** $\frac{\partial}{\partial F_1},\dots,\frac{\partial}{\partial F_n}$ **as follows**

$$\begin{pmatrix} \frac{\partial}{\partial F_1} \\ \frac{\partial}{\partial F_n} \end{pmatrix} = \left((JF)^{-1}\right)^T \begin{pmatrix} \frac{\partial}{\partial X_1} \\ \frac{\partial}{\partial X_n} \end{pmatrix} \quad \dots [1]$$

2- The Jacobian conjecture and kernel conjecture

Let $F = (F_1,\dots,F_n)$ **be polynomial map with** $\det(JF) \in \not\subset^*$ **and assume that the jacobian conjecture is true .**

Then $\not\subset[X_1,\dots,X_n] = \not\subset[F_1,\dots,F_n] \dots [2]$

Hence ker $\left(\frac{\partial}{\partial F_n}, \not\subset[x]\right) = \ker\left(\frac{\partial}{\partial F_n}, \not\subset[F_1,\dots,F_n]\right) = \not\subset[F_1,\dots,F_{n-1}]$

This leads us to

Conjecture (kernel conjecture KC(n)):if det $(JF) \in \not\subset^*$ **,then**

$$\ker\left(\frac{\partial}{\partial F_n}, \not\subset[x]\right) = \not\subset[F_1,\dots,F_{n-1}] .$$

The observation above then states that JC(n) implies KC(n), for all $n \ge 1$**.**

Conversely we have

Proposition : KC(n+1) implies JC(n) for all $n \ge 1$ **.**

proof: Let $F = (F_1,\dots,F_n): \not\subset \to \not\subset$ **with det** $(JF) \in \not\subset^*$**.**

We need to show that $\not\subset \lfloor F_1,\dots,F_n \rfloor = \not\subset \lfloor X_1,\dots,X_n \rfloor$.

Therefore put $F^* = (F\ , X_{n+1}) : \not\subset^{n+1} \rightarrow \not\subset^{n+1}$.

Then det(JF^* **)** $\in \not\subset$ *** .so by KC(n+1) we get**

$$\ker\left(\frac{\partial}{\partial F}, \not\subset [X_1,\dots,X_{n+1}]\right) = \not\subset [F_1,\dots,F_n] .. \text{(*)}$$

Howevere $\frac{\partial}{\partial F_{n+1}} = \frac{\partial}{\partial X_{n+1}}$ **(since they coincide on** X_{n+1} **and on each** F_i **, they coincide on** $\not\subset (F_1,\dots,F_n, X_{n+1})$ **and consequently on** $\not\subset (X_1,\dots,X_{n+1})$ **which is an algebric extention of** $\not\subset (F_1,\dots,F_n, X_{n+1})$ **so**

$$\ker\left(\frac{\partial}{\partial F_{n+1}}, \not\subset [X_1,\dots,X_{n+1}]\right) = \ker\left[\frac{\partial}{\partial X_{n+1}}, \not\subset [X_1,\dots,X_n]\right] = \not\subset [X_1,\dots,X_n].$$

Combining this with (*)we get
$\not\subset (F_1,\dots,F_n) = \not\subset \lfloor X_1,\dots,X_n \rfloor$ **so to study the jacobian conjecture we can as well study the kernel conjecture.**

References

[1] .H.Bass,E.connell ,and D.Wright,the jacobian conjecture:Reducation of degree and formal Expansion of the inverse, Bulletin of the American Mathematical Socity 7 (1982),287-330 .

[2] SusumuODA,on the Jacobian conjecture and its Generalization,arXiv:math.AC/040733242v4 23 Jul 2004.

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الخلاصة

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