

FINDING THE SEPARATION CONSTANT (λ) FOR (LPDEs) OF SECOND ORDER BY USING SEPARATION METHOD WITHOUT USING ANY (I.C) AND (B.C)

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ABSTRACT

Our aim in this search is to find the values of separation constant (λ) for the complete solution without using any initial and boundary conditions for some classified of the linear second order partial differential equations for all cases which contains the case $B = \frac{A^2}{4}$ since that is not easy to find the values (λ) without using any initial and boundary conditions for an application homogenous linear partial differential equations (LPDEs) which is solving by separation of variables method, by this method we found other complete solutions for some classified of the linear second order partial differential equations.

1. INTRODUCTION

One of the power full methods of generating asset of solutions for a given homogeneous (LPDEs) is the method of separation variables.

The dependent variable in separation variable is assumed to be a product of functions and by finding the partial derivatives in the original (PDEs) by this method we obtain ordinary differential equations are independent of each other, and each ordinary differential equation must be affixed to constant (separation of constant) which is supposed to be (λ) and by substituting initial and boundary conditions we obtain the values of separation constant (λ).

we could find in [4] the complete solution of second order (LPDEs) by using the assumption $Z(x, y) = e^{\int u(x)dx + \int v(y)dy}$, which is converted (LPDEs) of second order to linear ordinary differential equations (LODEs) of first order, and classified (LPDEs) to many cases.

In this rsearch we take all cases which is concerned the case $B = \frac{A^2}{4}$ for finding (λ) without using any initial and boundary conditions and finding other complete solutions for some classified of the linear second order partial differential equations.

2. BASIC DEFINITIONS

Definition 2.1, [3], [1]:- A partial differential equation is an equation that contains partial derivatives. In contrast to ordinary differential equation, where unknown function depends only on one variable, in partial differential equations; the unknown function depends on several variables.

Definition 2.2,[3],[5]:-A linear partial differential equation of second order in the independent variable x and y is an equation of the form

$$A_1(x, y)Z_{xx} + A_2(x, y)Z_{yy} + A_3(x, y)Z_{xy} + A_4(x, y)Z_x + A_5(x, y)Z_y + A_6Z = R(x, y) \quad \dots (1)$$

where $A_i(x, y), i = 1, 2, \dots, 6$ are functions of x and y or constants. If $R(x, y) = 0$ then the equation (1) is said to be homogenous and non-homogenous if $R(x, y) \neq 0$. If $A_i(x, y), i = 1, \dots, 6$ are constant functions then the equation (1) is called with constant coefficient, and if at least one of $A_i(x, y), i = 1, 2, \dots, 6$ are non-constants then the equation (1) is called with variable coefficients. And if $A_4 = A_5 = A_6 = 0$, then the equation (1) is said to be linear partial differential equation with homogenous terms.

3. EXTENSION OF LINEAR SECOND ORDER PARTIAL DIFFERENTIAL EQUATIONS [4],[2]

The linear second order partial differential equations, which have the general form:

$$A_1(x, y)Z_{xx} + A_2(x, y)Z_{yy} + A_3(x, y)Z_{xy} + A_4(x, y)Z_x + A_5(x, y)Z_y + A_6Z = 0 \dots (2)$$

In order to find the complete solution of the equation (2), in [4] search new functions $u(x)$ and $v(y)$ such that the assumption

$$\dots (3) \quad Z(x, y) = e^{\int u(x)dx + \int v(y)dy}$$

represent the complete solution of it, this assumption will transform the equation (2) to (ODEs) of first order by finding Z_x, Z_y, Z_{xy}, Z_{xx} and Z_{yy}

from the equation (3) and substitution the above partial derivatives in the equation (2) we get:

$$A_1(u' + u^2) + A_2u v + A_3(v' + v^2) + A_4u + A_5v + A_6 = 0 \dots (4)$$

In order to find the complete solution of the equation (4), take a special cases, which depend on the values of $A_i(x, y), i = 1, 2, \dots, 6$, there fore in [4] classified the equation (2) to many cases.

Note:- In [4] classified the equation (2) because the equation (4) cannot separate the variables $u(x)$ and $v(y)$.

4. CLASSIFIED AND THE COMPLETE SOLUTION (C.S) OF SECOND ORDER (LPDES) [4],[3]

Case(1):

a) $A_1 Z_{xx} = 0 \xrightarrow{c.s} Z(x, y) = B \Psi(y)(x - c)$

b) $A_2 Z_{xy} = 0 \xrightarrow{c.s} i) Z(x, y) = B_1 \Psi_1(y)$

ii) $Z(x, y) = B_2 \Psi_2(x)$

iii) $Z(x, y) = B_3$

c) $A_3 Z_{yy} = 0 \xrightarrow{c.s} Z(x, y) = B_4 \Psi_4(x)(y - c)$

where B_1, B_2, B_3, B_4, B and c arbitrary constants, and λ represent the separation of constant.

Case(2):

a) $A_1 Z_{xx} + A_2 Z_{xy} = 0 \xrightarrow{c.s} Z(x, y) = e^{\frac{A_1}{A_2} \lambda^2 y} (B_1 e^{-\lambda^2 x} + B_2)$

b) $A_2 Z_{xy} + A_3 Z_{yy} = 0 \xrightarrow{c.s} Z(x, y) = e^{\frac{A_3}{A_2} \lambda^2 x} (B_1 e^{-\lambda^2 y} + B_2)$

c) $A_1 Z_{xx} + A_3 Z_{yy} = 0 \xrightarrow{c.s} \rightarrow$

$Z(x, y) = \left(d_2 \cos \sqrt{\frac{1}{A_1}} \lambda x + b_2 \sin \sqrt{\frac{1}{A_1}} \lambda x \right) \left(d_3 \cosh \sqrt{\frac{1}{A_3}} \lambda y + b_3 \sinh \sqrt{\frac{1}{A_3}} \lambda y \right)$

d) $A_1 Z_{xx} + A_2 Z_{xy} + A_3 Z_{yy} = 0 \rightarrow$ If

i) $B \neq \frac{A^2}{4} \xrightarrow{c.s} Z(x, y) = e^{-\frac{A}{2} x + \lambda y} \left(d_1 \cos \sqrt{B - \frac{A^2}{4}} x + b_1 \sin \sqrt{B - \frac{A^2}{4}} x \right)$

ii) $B = \frac{A^2}{4} \xrightarrow{c.s} Z(x, y) = D e^{-\frac{A}{2} x + \lambda y} (x - c)$

where $A = \frac{A_2}{A_1} \lambda$ and $B = \frac{A_3}{A_1} \lambda^2$, and $B_1, B_2, d_2, d_3, b_2, b_3, D, c$ and

λ arbitrary constants.

Case(3):

a) $A_1 Z_{xx} + A_4 Z_x + A_5 Z_y + A_6 Z = 0 \rightarrow$ If

i) $B \neq \frac{A^2}{4} \xrightarrow{c.s} Z(x, y) = e^{-\frac{A}{2} x + \frac{\lambda^2 - A_6}{A_5} y} \left(d_1 \cos \sqrt{B - \frac{A^2}{4}} x + b_1 \sin \sqrt{B - \frac{A^2}{4}} x \right)$

ii) $B = \frac{A^2}{4} \xrightarrow{c.s} Z(x, y) = D e^{-\frac{A}{2} x + \frac{\lambda^2 - A_6}{A_5} y} (x - c)$

where $A = \frac{A_4}{A_1}$ and $B = \frac{\lambda^2}{A_1}$, and d_1, b_1, D, c and λ arbitrary constants.

b) $A_2 Z_{xy} + A_4 Z_x + A_5 Z_y + A_6 Z = 0 \xrightarrow{c.s} Z(x, y) = A e^{\lambda x - \left(\frac{A_4 \lambda + A_6}{A_5 + A_2 \lambda} \right) y}$

where A and λ arbitrary constants.

c) $A_3 Z_{yy} + A_4 Z_x + A_5 Z_y + A_6 Z = 0 \rightarrow$ If

$$i) B \neq \frac{A^2}{4} \xrightarrow{c.s} Z(x, y) = e^{\frac{-A}{2}y + \frac{\lambda^2 - A_6}{A_4}x} \left(d_1 \cos \sqrt{B - \frac{A^2}{4}}y + b_1 \sin \sqrt{B - \frac{A^2}{4}}y \right)$$

$$ii) B = \frac{A^2}{4} \xrightarrow{c.s} Z(x, y) = D e^{\frac{\lambda^2 - A_6}{A_4}x - \frac{A}{2}y} (y - c)$$

where $A = \frac{A_5}{A_3}$ and $B = \frac{\lambda^2}{A_3}$, and d_1, b_1, D, c and λ arbitrary constants.

Case(4):

a) $A_1 Z_{xx} + A_2 Z_{xy} + A_4 Z_x + A_5 Z_y + A_6 Z = 0 \rightarrow$ If

$$i) B \neq \frac{A^2}{4} \xrightarrow{c.s} Z(x, y) = e^{\frac{-A}{2}x + \lambda^2 y} \left(d_1 \cos \sqrt{B - \frac{A^2}{4}}x + b_1 \sin \sqrt{B - \frac{A^2}{4}}x \right)$$

$$ii) B = \frac{A^2}{4} \xrightarrow{c.s} Z(x, y) = D e^{\lambda^2 y - \frac{A}{2}x} (x - c)$$

where $A = \frac{A_4 + A_2 \lambda^2}{A_1}$, $B = \frac{A_6 + A_5 \lambda^2}{A_1}$, and d_1, b_1, D, c and λ arbitrary constants.

b) $A_1 Z_{xx} + A_3 Z_{yy} + A_4 Z_x + A_5 Z_y + A_6 Z = 0 \rightarrow$ If

$$i) B \neq \frac{A^2}{4} \& D \neq \frac{C^2}{4} \xrightarrow{c.s} Z(x, y) = e^{\frac{-A}{2}x - \frac{C}{2}y} \left(d_1 \cos \sqrt{B - \frac{A^2}{4}}x + a_1 \sin \sqrt{B - \frac{A^2}{4}}x \right) \left(d_2 \cos \sqrt{D - \frac{C^2}{4}}y + a_2 \sin \sqrt{D - \frac{C^2}{4}}y \right)$$

$$ii) B \neq \frac{A^2}{4} \& D = \frac{C^2}{4} \xrightarrow{c.s} Z(x, y) = e^{\frac{-A}{2}x - \frac{C}{2}y} (y - c) \left(d_1 \cos \sqrt{B - \frac{A^2}{4}}x + b_1 \sin \sqrt{B - \frac{A^2}{4}}x \right)$$

$$iii) B = \frac{A^2}{4} \& D \neq \frac{C^2}{4} \xrightarrow{c.s} Z(x, y) = e^{\frac{-A}{2}x - \frac{C}{2}y} (x - c) \left(d_1 \cos \sqrt{D - \frac{C^2}{4}}y + b_1 \sin \sqrt{D - \frac{C^2}{4}}y \right)$$

$$iv) B = \frac{A^2}{4} \& D = \frac{C^2}{4} \xrightarrow{c.s} Z(x, y) = k e^{\frac{-A}{2}x - \frac{C}{2}y} (x - c_1)(y - c_2)$$

where $A = \frac{A_4}{A_1}$, $B = \frac{\lambda^2}{A_1}$, $C = \frac{A_5}{A_3}$ and $D = \frac{A_6 - \lambda^2}{A_3}$, $d_1, d_2, b_1, a_1, a_2, c_1, c_2$, and

λ arbitrary constants.

c) $A_2 Z_{xy} + A_3 Z_{yy} + A_4 Z_x + A_5 Z_y + A_6 Z = 0 \rightarrow$ If

$$i) B \neq \frac{A^2}{4} \xrightarrow{c.s} Z(x, y) = e^{\frac{-A}{2}y + \lambda^2 x} \left(d_1 \cos \sqrt{B - \frac{A^2}{4}}y + b_1 \sin \sqrt{B - \frac{A^2}{4}}y \right)$$

$$ii) B = \frac{A^2}{4} \xrightarrow{c.s} Z(x, y) = D e^{\lambda^2 x - \frac{A}{2} y} (y - c)$$

where $A = \frac{A_4}{A_1}$ and $B = \frac{\lambda^2}{A_1}$, d_1, b_1, D, c and λ arbitrary constants.

Case(5):

$$A_1 Z_{xx} + A_2 Z_{xy} + A_3 Z_{yy} + A_4 Z_x + A_5 Z_y + A_6 Z = 0 \rightarrow \text{If}$$

$$i) B \neq \frac{A^2}{4} \xrightarrow{c.s} Z(x, y) = e^{-\frac{A}{2} x + \lambda y} \left(d_1 \cos \sqrt{B - \frac{A^2}{4}} x + b_1 \sin \sqrt{B - \frac{A^2}{4}} x \right)$$

$$ii) B = \frac{A^2}{4} \xrightarrow{c.s} Z(x, y) = D e^{-\frac{A}{2} x + \lambda y} (x - c)$$

where $A = \frac{A_2 \lambda + A_4}{A_1}$, $B = \frac{A_3 \lambda^2 + A_5 \lambda + A_6}{A_1}$, d_1, b_1, D, c and λ arbitrary

constants.

Note:- We take the cases that concerned the case $B = \frac{A^2}{4}$ for finding the separation of constant (λ) without using any initial and boundary conditions.

5.HOW TO FIND THE SEPARATION OF CONSTANT (λ) WITHOUT USING ANY (I.C) AND (B.C) FOR (LPDEs) BY USING SEPARATION METHOD

Case (2):

$$d) A_1 Z_{xx} + A_2 Z_{xy} + A_3 Z_{yy} = 0$$

If $B = \frac{A^2}{4}$, when $B = \frac{A_3}{A_1} \lambda^2$ and $A = \frac{A_2}{A_1} \lambda$

Proof:-

$$\text{since } B = \frac{A^2}{4} \text{ hence } \frac{A_3}{A_1} \lambda^2 - \frac{A_2^2}{4 A_1^2} \lambda^2 = 0 \Rightarrow \lambda^2 \left(\frac{A_3}{A_1} - \frac{A_2^2}{4 A_1^2} \right) = 0$$

since $A_1, A_2, A_3 \neq 0$ therefore ($\lambda = 0$), then the (C.S) becomes

$$Z(x, y) = D(x - c)$$

where D and c are arbitrary constants.

Case(3):

$$a) A_1 Z_{xx} + A_4 Z_x + A_5 Z_y + A_6 Z = 0$$

If $B = \frac{A^2}{4}$, when $B = \frac{\lambda^2}{A_1}$ and $A = \frac{A_4}{A_1}$

proof:-

since $B = \frac{A^2}{4}$ hence $\frac{\lambda^2}{A_1} - \frac{A_4^2}{4A_1^2} = 0$ therefore $\lambda^2 = \frac{A_4^2}{4A_1}$, then the (C.S) becomes

$$Z(x, y) = D e^{\frac{-A_4 x + A_4^2 - 4A_1 A_5 y}{4A_1 A_5} (x - c)}$$

where D and c are arbitrary constants.

c) $A_3 Z_{yy} + A_4 Z_x + A_5 Z_y + A_6 Z = 0$

If $B = \frac{A^2}{4}$, when $A = \frac{A_5}{A_3}$ and $B = \frac{\lambda^2}{A_3}$

proof:-

since $B = \frac{A^2}{4}$ hence $\frac{\lambda^2}{A_3} - \frac{A_5^2}{4A_3^2} = 0$ therefore $\lambda^2 = \frac{A_5^2}{4A_3}$ then the (C.S) becomes

$$Z(x, y) = D e^{\frac{A_5^2 - 4A_3 A_6 x - A_5 y}{4A_3 A_4} (y - c)}$$

where D and c are arbitrary constants.

Case(4):

a) $A_1 Z_{xx} + A_2 Z_{xy} + A_4 Z_x + A_5 Z_y + A_6 Z = 0$

If $B = \frac{A^2}{4}$, when $A = \frac{A_4 + A_2 \lambda^2}{A_1}$ and $B = \frac{A_6 + A_5 \lambda^2}{A_1}$

proof:-

since $B = \frac{A^2}{4}$ hence $\frac{A_6 + A_5 \lambda^2}{A_1} - \frac{(A_4 + A_2 \lambda^2)^2}{4A_1^2} = 0$ therefore

$$\lambda^4 + \left(\frac{2A_2 A_4 - 4A_1 A_5}{A_2^2} \right) \lambda^2 + \frac{A_4^2 - 4A_1 A_6}{A_2^2} = 0 \dots (5)$$

let $\alpha = \frac{2A_2 A_4 - 4A_1 A_5}{A_2^2}$ and $\beta = \frac{A_4^2 - 4A_1 A_6}{A_2^2}$

then the equation (5) becomes

$$\lambda^4 + \alpha \lambda^2 + \beta = 0 \dots (6)$$

and by solving the equation (6) we can know the value of (λ)

b) $A_1 Z_{xx} + A_3 Z_{yy} + A_4 Z_x + A_5 Z_y + A_6 Z = 0$

If $B = \frac{A^2}{4}$ & $D = \frac{C^2}{4}$ when $D = \frac{A_6 - \lambda^2}{A_3}$ and $A = \frac{A_4}{A_1}$, $B = \frac{\lambda^2}{A_1}$, $C = \frac{A_5}{A_3}$

proof:-

since $B = \frac{A^2}{4}$ and $D = \frac{C^2}{4}$ hence $\frac{\lambda_1^2}{A_1} - \frac{A_4^2}{4A_1^2} = 0$ and

$$\frac{A_6 - \lambda_2^2}{A_3} - \frac{A_5^2}{4A_3^2} = 0, \text{ therefore}$$

$$\lambda_1^2 = \frac{A_4^2}{4A_1} \text{ and } \lambda_2^2 = A_6 - \frac{A_5^2}{4A_3}, \text{ then the (C.S) becomes}$$

$$Z(x, y) = k e^{\frac{-A_4 x - A_5 y}{2A_1 x - 2A_3 y}} (x - c_1)(y - c_2)$$

c) $A_2 Z_{xy} + A_3 Z_{yy} + A_4 Z_x + A_5 Z_y + A_6 Z = 0$

If $B = \frac{A^2}{4}$, when $A = \frac{A_2 \lambda^2 + A_5}{A_3}$ and $B = \frac{A_4 \lambda^2 + A_6}{A_3}$

proof:-

since $B = \frac{A^2}{4}$ hence $\frac{A_4 \lambda^2 + A_6}{A_3} - \frac{(A_2 \lambda^2 + A_5)^2}{4A_3^2} = 0$, therefore

$$\lambda^4 + \left(\frac{2A_2 A_5 - 4A_3 A_4}{A_2^2} \right) \lambda^2 + \frac{A_5^2 - 4A_3 A_6}{A_2^2} = 0 \dots (7)$$

let $\alpha = \frac{2A_2 A_5 - 4A_3 A_4}{A_2^2}$ and $\beta = \frac{A_5^2 - 4A_3 A_6}{A_2^2}$ then the equation (7) becomes

$$\lambda^4 + \alpha \lambda^2 + \beta = 0 \dots (8)$$

and by solving the equation(8) we can know the value of (λ) .

Case(5):

$$A_1 Z_{xx} + A_2 Z_{xy} + A_3 Z_{yy} + A_4 Z_x + A_5 Z_y + A_6 Z = 0$$

If $B = \frac{A^2}{4}$, when $A = \frac{A_2 \lambda + A_4}{A_1}$ and $B = \frac{A_3 \lambda^2 + A_5 \lambda + A_6}{A_1}$

proof:-

since $B = \frac{A^2}{4}$ hence $\frac{A_3 \lambda^2 + A_5 \lambda + A_6}{A_1} - \frac{(A_2 \lambda + A_4)^2}{4A_1^2} = 0$, therefore

$$\lambda^2 + \left(\frac{2A_2 A_4 - 4A_1 A_5}{A_2^2 - 4A_1 A_3} \right) \lambda + \frac{A_4^2 - 4A_1 A_6}{A_2^2 - 4A_1 A_3} = 0 \dots (9)$$

let $\alpha = \frac{2A_2 A_4 - 4A_1 A_5}{A_2^2 - 4A_1 A_3}$ and $\beta = \frac{A_4^2 - 4A_1 A_6}{A_2^2 - 4A_1 A_3}$ then the equation (9) becomes

$$\lambda^2 + \alpha \lambda + \beta = 0 \dots (10)$$

and by solving the equation(10) we can know the value of (λ)

6.EXAMPLES

Example 6.1:- to solve the (PDE):- $Z_{xx} + 2Z_x + Z_y + 2Z = 0$

by using the (case (3), a) then $\lambda^2 = 1$, we get the (C.S) which has the form

$$Z(x, y) = D e^{-2x-y} (x - c)$$

where D and c are arbitrary constants.

Example 6.2:- to solve the (PDE):- $Z_{yy} + 2Z_x - 2Z_y + Z = 0$

by using the (case(3),c) then $\lambda^2 = 1$, we get the (C.S) which has the form

$$Z(x, y) = D e^y (y - c)$$

where D and c are arbitrary constants.

Example 6.3:- to solve the (PDE):- $Z_{xx} + Z_{xy} + Z_x + Z_y + Z = 0$

by using the (case(4),a) then

$$\lambda_1^2 = 3, \lambda_2^2 = -1 \quad \lambda^4 - 2\lambda^2 - 3 = 0 \Rightarrow \text{hence } \alpha = -2, \beta = -3$$

If $\lambda_1^2 = 3$, we get the (C.S), which has the form

$$Z(x, y) = D e^{3y-2x} (x - c)$$

If $\lambda_2^2 = -1$, we get the (C.S), which has the form

$$Z(x, y) = D e^{-y} (x - c)$$

where D and c are arbitrary constants.

Example 6.4:- to solve the (PDE):- $Z_{xx} + Z_{yy} + 2Z_x - 2Z_y + 3Z = 0$

by using the (case(4),b) then $\lambda_1^2 = 1, \lambda_2^2 = 2$, we get the (C.S) which has the form

$$Z(x, y) = k e^{-(x-y)} (x - c_1)(y - c_2)$$

where c_1, c_2 and k are arbitrary constants.

Example 6.5:- to solve the (PDE):- $Z_{xy} + Z_{yy} + 2Z_x + Z_y - Z = 0$

by using the (case(4),c) then $\alpha = -6, \beta = 5$

$$\text{hence } \lambda^4 - 6\lambda^2 + 5 = 0 \Rightarrow \lambda_1^2 = 5, \lambda_2^2 = 1$$

If $\lambda_1^2 = 5$, we get the (C.S) which has the form

$$Z(x, y) = D e^{5x-3y} (y - c)$$

If $\lambda_2^2 = 1$, we get the (C.S) which has the form

$$Z(x, y) = D e^{x-y} (y - c)$$

where D and c are arbitrary constants.

Example 6.6:- to solve the (PDE):- $Z_{xx} + 3Z_{xy} + 2Z_{yy} + Z_x + Z_y + Z = 0$

by using the (case(5)) then $\alpha = 2, \beta = -3$ we get $\lambda^2 + 2\lambda - 3 = 0 \Rightarrow \lambda_1 = -3, \lambda_2 = 1$

If,we get the (C.S) which has the form $\lambda_1 = -3$

$$Z(x, y) = D e^{4x-3y} (x-c)$$

If $\lambda_2 = 1$,we get the (C.S) which has the form

$$Z(x, y) = D e^{-2x+y} (x-c)$$

where D and c are arbitrary constants.

REFERENCES

- [1] A.D.Polyanin, Handbook of "Linear Partial Differential Equations for Engineers and Scientists", Chapman &Hall/ CRC Press, Boca Raton, 2002.
- [2] A.D.Polyanin, V.F.Zaitsev, and A.Moussiaux, Handbook of "First Order Partial differential Equations", Taylor &Francis, London, 2002.
- [3] Georgef.Carrler, "Partial Differential Equations Theory and Technique", 1976.
- [4] Nada Zuhair Abd AL-sada, "The Complete Solution of Second Order Linear Partial Differential Equations", Msc,thesis,University of Kufa,College of Education,Department of Mathematies,2006.
- [5] Stanley J.Farlow, "Partial Differential Equations for Scientists and Engineers", New York, 1989.

ايجاد ثابت الفصل للمعادلة التفاضلية الاعتيادية الخطية من الرتبة الثانية باستخدام

فصل المتغيرات بدون شروط ندى زهير عبد السادة

جامعة القادسية كلية التربية

الخلاصة

هدفنا في هذا البحث هو إيجاد قيمة ثابت الفصل (λ) للحل الكامل (التام) بدون أي شروط ابتدائية وحدودية لبعض تصنيفات المعادلة التفاضلية الجزئية الخطية من الرتبة الثانية لجميع الحالات التي يكون فيها

$B = \frac{A^2}{4}$ ، حيث انه ليس من السهولة إيجاد قيمة (λ) بدون أي شروط ابتدائية وحدودية لأي تطبيق حول

المعادلات التفاضلية الجزئية الخطية من الرتبة الثانية التي تحل بطريقة فصل المتغيرات، بواسطة هذه الطريقة أوجدنا حلول كاملة (تامة) أخرى لبعض من تصنيفات المعادلة التفاضلية الجزئية الخطية من الرتبة الثانية .