Some Kinds OF Jordan Centralizers on Completely Prime Gamma Rings

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Abstract:-

Let M be a non-commutative 2-torsion free completely prime Γ-ring, in section two of this paper, we prove that if $T:M\to M$ be an additive mapping such that $2T(x\alpha x)=T(x)\alpha x+x\alpha T(x)$ holds for all $x\in M$, $\alpha\in\Gamma$.in this case T is left and right centralizer (i.e centralizer).In section three of this paper we prove that, if $T:M\to M$ be an additive mapping satisfying $T(x\alpha y\beta x)=T(x)\alpha y\beta x$ for all $x,y\in M$, $\alpha,\beta\in\Gamma$ (resp., $T(x\alpha y\beta x)=x\alpha y\beta T(x)$ for all $x,y\in M$, $\alpha,\beta\in\Gamma$) then T is a left centralizer(resp., right centralizer).

1-Introduction

Throughout this paper,M will represent non –commutative Γ-ring.in[8]Zalar proved that any left(resp.,right)Jordan centralizer on a 2-torsion free semi-prime ring is a left (resp.,right)centralizer. In [3] authors proved that anyJordan left (resp.,right)σ- centralizer on a 2-torsion free R has a commutator right (resp., left) non- zero divisor is a left (resp.,right) σ-Centralizer. in [7]Vukman proved that if R is a 2-torsion free semi-prime ring and $T:R\to R$ be an additive mapping such that 2T(x)=T(x) x+xT(x)holds for all $x\in R$.in this case Tis left and right centralizer .

Let M and Γ be additive abelian groups,M is called a Γ-ring if for any $x,y,z\in M$ and $\alpha,\beta\in\Gamma$,the following conditions are satisfied

Key words:- ***Γ-ring,primeΓ-ring,semi-primeΓ-ring,Left centralizer,Right centralizer,centralizer,Jordan centralizer.***

(1) $x\alpha y\in M$
(2) $(x+y)\alpha z=x\alpha z+y\alpha z$

$X(\alpha+\beta)y=x\alpha y+x\beta y$

$x\alpha(y+z)=x\alpha y+x\alpha z$

(3)$(x\alpha y)\beta z=x\alpha(y\beta z)$

The notion of Γ-ring was introduced by Nobusawa[5]and generalized by Barnes[1],many properties of Γ-ring were obtained by many research such as[2]

A Γ-ring M is called a 2-torsion free if 2x=0 implies x=0 for all $x\in M$.A Γ-ring M is called prime if a $\Gamma M\Gamma$ b=0 implies a=0 or b=0 and M is called completely prime if a Γ b=0 implies a=0 or b=0(a,b $\in M$),since a $\Gamma b\Gamma$ a $\Gamma b\subset$ a $\Gamma M\Gamma b$ then every completely prime Γ-ring is prime .A Γ-ring M is called semi-prime if a $\Gamma M\Gamma$ a=0 implies a=0 and M is called completely semi-prime if a Γ a=0 implies a=0(a $\in M$).

In [6] Rajaa C.Shaheen define Jordan centralizer on Γ-ring

As follows:

<u>Definition1.1</u> :-Let M be a Γ-ring and $T:M\rightarrow M$ be an additive map,T is called

<u>left centralizer</u> of M ,if for any $a,b\in M$ and $\alpha\in\Gamma$,the following condition satisfies $T(a\alpha b)=T(a)\alpha b$,

<u>Right centralizer</u> of M ,if for any $a,b\in M$ and $\alpha\in\Gamma$,the following condition satisfies $T(a\alpha b)=a\alpha T(b)$,

<u>Jordan left centralizer</u> of M ,if for any $a\in M$ and $\alpha\in\Gamma$,the following condition satisfies $T(a\alpha a)=T(a)\alpha a$,

<u>Jordan Right centralizer</u> of M ,if for any $a\in M$ and $\alpha\in\Gamma$,the following condition satisfies $T(a\alpha a)=a\alpha T(a)$,

<u>Jordan centralizer</u> of M,if for any $a,b\in M$ and $\alpha\in\Gamma$,the following condition satisfies $T(a\alpha b+b\alpha a)=T(a)\alpha b+b\alpha T(a)=a\alpha T(b)+T(b)\alpha a$,

A centralizer of M is an additive mapping which is both left and right centralizer.An easy computation shows that every centralizer is also a Jordan centralizer but the converse is not true.in [6] Rajaa showed that the existence of a non-zero Jordan centralizer on a non-comutative 2-torsion free completely prime Γ-ring be centralizer. We should mentioned the reader that in this paper we shall suppose that $x\alpha y\beta z=x\beta y\alpha z$ for all $x,y,z\in M$,and for all $\alpha,\beta\in\Gamma$.

In this paper we define some kind of Jordan centralizers on gamma ring and study the relation between its and centralizer .

2-Result

<u>Theorem 2.1</u>:- Let M be a 2-torsion free completely prime Γ *-ring and let* $T:M\rightarrow M$ *be an additive mapping such that* $2T(x\alpha x)=T(x)\alpha x+x\alpha T(x)$ *holds for all* $x\in M$, $\alpha\in\Gamma$ *.In this case T is left and right centralizer.*

<u>Proof:</u>-Since $2T(x\alpha x)=T(x)\alpha x+x\alpha T(x)$ ……………………(*)

Replace x by x+y

$W=2T((x+y)\alpha(x+y))$

$= T(x+y)\alpha(x+y)+(x+y)\alpha T(x+y)$

$= T(x)\alpha x+T(x)\alpha y+T(y)\alpha x+T(y)\alpha y+x\alpha T(x)+x\alpha T(y)+y\alpha T(x)+y\alpha T(y).$

On the other hand

$W=2T((x+y)\alpha(x+y))$

$=2T(x\alpha x+x\alpha y+y\alpha x+y\alpha y)$

$=2T(x\alpha x)+2T(x\alpha y+y\alpha x)+2T(y\alpha y)$

By comparing these two expression of W ,we get

$2T(x\alpha y+y\alpha x)= T(x)\alpha y+T(y)\alpha x+x\alpha T(y)+y\alpha T(x)$ ……….(1)

Now, replace y by $2(x\beta y+y\beta x)$

$W=4T(x\alpha(x\beta y+y\beta x)+(x\beta y+y\beta x)\alpha x)$

$= 2T(x)\alpha(x\beta y+y\beta x)+2T((x\beta y+y\beta x))\alpha x$

$+2x\alpha T((x\beta y+y\beta x))+2(x\beta y+y\beta x)\alpha T(x).$

$=2T(x)\alpha(x\beta y+y\beta x)+x\alpha T(x)\beta y+x\alpha x\beta T(y)+2x\alpha T(y)\beta x+x\alpha y\beta T(x)+T(x)\alpha y\beta x+T(y)\alpha x\beta x+y\alpha T(x)\beta x+2((x\beta y+y\beta x)\alpha T(x)$

$=T(x)\alpha(2x\beta y+3y\beta x)+(3x\beta y+2y\beta x)\alpha T(x)+x\alpha T(x)\beta y+y\alpha T(x)\beta x+2x\alpha T(y)\beta x+x\alpha x\beta T(y)+T(y)\alpha x\beta x.$

On the other hand ,

$W=4T(x\alpha(x\beta y+y\beta x)+(x\beta y+y\beta x)\alpha x)$

$=4T(x\alpha x\beta y+y\beta x\alpha x)+8T(x\alpha y\beta x)$

$=T(x)\ \alpha x\beta y+y\alpha x\beta T(x)+x\alpha T(x)\ \beta y+y\alpha T(x)\ \beta x+2x\alpha x\beta T(y)+2\ T(y)\ \alpha x\ \beta x$ $+8T(x\alpha y\beta x)$

By comparing these two expression of W ,we get

$8T(x\alpha y\beta x)= T(x)\ \alpha\ (x\beta\ y+3y\beta x)+ (3x\beta\ y+y\beta x)\ \alpha\ T(x)+2x\ \alpha\ T(y)\ \beta x-x\ \alpha x\ \beta\ T(y)-T(y)\ \alpha x\ \beta x$……………………(2)

Since

$2T(x\alpha y+y\alpha\ x)= T(x)\ \alpha y+T(y)\ \alpha x + x\alpha T(y)+y\alpha T(x)$

Replace y by $8x\beta\ y\lambda x$

$W=16\ T(x\alpha (\ x\ \beta\ y\lambda x) +(\ x\ \beta\ y\lambda x\ )\alpha\ x)$

$=8[T(x)\ \alpha (\ x\ \beta\ y\lambda x)+T((\ x\ \beta\ y\lambda x))\ \alpha x + x\alpha T((\ x\ \beta\ y\lambda x))+ (\ x\beta\ y\lambda x)\alpha T(x)]$

$=8\ T(x)\ \alpha x\ \beta y\lambda\ x+\ x\alpha [T(x)\ \beta (x\lambda y+3y\lambda x)+(y\lambda x+3x\lambda y)\beta T(x)\ +2x\beta T(y)\ \lambda x-x\beta x\lambda T(y)-T(y)\ \beta\ x\ \lambda x]+[\ T(x)\ \beta (x\ \lambda y+3y\ \lambda x)+(y\ \lambda x +3x\ \lambda y)\ \beta\ T(x)+2x\ \beta\ T(y)\ \lambda x -x\ \beta x\ \lambda T(y)-T(y)\ \beta x\ \lambda x]\ \alpha x+8x\ \beta y\ \lambda x\ \alpha\ T(x).$

$=8\ T(x)\ \alpha x\ \beta\ y\ \lambda x+x\alpha\ T(x)\ \beta (x\lambda y+3y\lambda x)\ +(x\alpha y\lambda x+3x\alpha x\lambda y)\ \beta T(x)\ +2x\alpha x\beta T(y)\ \lambda x-x\alpha x\beta x\lambda\ T(y)-x\ \alpha\ T(y)\ \beta x\ \lambda x + T(x)\ \beta (x\ \lambda y\ \alpha x +3y\ \lambda x\ \alpha x)\ +(y\ \lambda x +3\ x\ \lambda y)\ \beta\ T(x)\ \alpha x +2x\ \beta\ T(y)\ \lambda x\ \alpha x-x\ \beta x\lambda\ T(y)\ \alpha x- T(y)\ \beta x\ \lambda x\ \alpha x +8x\ \beta y\ \lambda x\ \alpha\ T(x).$

Therefore we have

$16\ T(x\alpha x\beta y\lambda x\ +x\alpha y\beta x\lambda x)= T(x)\alpha (9x\beta y\lambda x+3y\beta x\lambda x)+(9x\alpha y\beta x+3x\alpha x\beta y)\ \lambda T(x) +x\alpha T(x)\ \beta (x\lambda y+3y\lambda x)+(y\lambda x+3x\lambda y)\ \beta T(x)\ \lambda x+x\alpha x\beta T(x)\ \lambda x+x\alpha T(y)\ \lambda x\beta x-T(y)\ \lambda x\beta x\alpha x-x\alpha x\beta x\lambda T(y).$

On the other hand

$W=16\ T(x\alpha x\beta y\lambda x +x\alpha y\beta x\lambda x)$

$=16\ T(x\alpha (x\beta y)\lambda x) +16T(x\alpha (y\beta x)\lambda x)$

$=2T(x)\ \alpha (x\beta x\lambda y+3x\beta y\lambda x)\ +2(x\alpha y\beta x+3x\alpha x\beta y)\ \lambda T(x)+4x\alpha T(y\beta x)\lambda x-2T(x\alpha y)\ \beta x\lambda x-2x\alpha x\beta T(x\lambda y)\ +2T(x)\ \alpha (x\beta y\lambda x+3y\beta x\lambda x)+2(y\beta x\alpha x+3x\alpha y\beta x)\ \lambda T(x)+4x\alpha T(y\beta x)\lambda x-2x\alpha x\beta T(y\lambda x)-2T(y\beta x)\ \alpha x\lambda x.$

$=T(x)\ \lambda (2x\alpha x\beta y+6y\alpha x\beta x+8x\alpha y\beta x)\ +(8x\alpha y\beta x+2y\alpha x\beta x+6x\alpha x\beta y)\ \lambda T(x) +4x\lambda T(x\alpha y+y\alpha x)\ \beta x-2x\alpha x\beta T(x\lambda y+y\lambda x)-2T(x\alpha y+y\alpha x)\ \beta x\lambda x$

$=T(x)\ \alpha(2x\beta x\lambda y+6y\beta x\lambda x+8x\beta y\lambda x)\ +(8x\alpha y\beta x+2y\alpha x\beta x+6x\alpha x\beta y)\ \lambda T(x)\ +2x\alpha T(x)\ \beta y\lambda x+2x\alpha x\beta T(y)\ \lambda x+2x\alpha T(y)\ \beta x\lambda x+2x\alpha y\beta T(x)\ \lambda x-x\alpha x\beta T(x)\ \lambda y-x\alpha x\beta x\lambda T(x)-x\alpha x\beta T(y)\ \lambda x-x\alpha x\beta y\lambda T(x)-\ T(x)\ \alpha y\alpha\ \beta x\lambda x-x\alpha T(y)\ \beta x\lambda x-T(y)\ \alpha x\beta x\lambda x-y\alpha T(x)\ \beta x\lambda x$

Therefore we have

$W=16\ T(x\alpha x\beta y\lambda x\ +x\alpha y\beta x\lambda x)$

$=\ T(x)\ \alpha\ (2x\ \beta x\ \lambda y+5y\ \beta x\ \lambda x+8x\ \beta\ y\ \lambda x)+(2y\ \beta x\ \lambda x+5x\ \beta\ x\ \lambda\ y+8x\ \beta y\ \beta\ \lambda x)\ \alpha T(x)+2x\ \alpha\ T(x)\ \beta y\ \lambda x+2x\ \alpha y\ \beta\ T(x)\ \lambda x+x\ \alpha x\ \beta\ T(y)\ \lambda x+x\ \alpha\ T(y)\ \beta x\ \lambda x-x\ \alpha x\ \beta\ T(x)\ \lambda y-y\ \alpha\ T(x)\ \beta x\ \lambda x-x\ \alpha x\ \beta x\ \lambda\ T(y)-\ T(y)\ \alpha x\ \beta x\ \lambda x.$

By comparing these two expression of W ,we get

$T(x)\ \alpha(x\ \beta y\ \lambda x-2y\ \beta x\ \lambda x-2x\ \beta x\ \lambda y)+(x\ \alpha y\ \beta x-2x\ \alpha x\ \beta y-2y\ \alpha x\ \beta x)\ \lambda\ T(x)\ +x\ \alpha\ T(x)\ \beta(x\ \ \lambda y\ \ +y\ \ \lambda x)+(x\ \ \alpha y\ \ +y\ \ \alpha x)\ \ \beta\ \ T(x)\ \ \lambda x\ \ +x\ \ \alpha x\ \ \beta\ \ T(x)\ \ \lambda y\ \ +y\ \ \alpha\ \ T(x)\ \ \beta x\ \lambda x=0$ ……………………………….(3)

Replace y by $y\psi x$ *in (3),we obtain*

$T(x)\ \alpha(x\ \beta\ y\psi x\ \lambda x-2\ y\psi x\ \beta x\ \lambda x-2x\ \beta x\ \lambda\ y\psi x)+(x\ \alpha\ y\psi x\ \beta x-2x\ \alpha x\ \beta\ y\psi x\ -2\ y\psi x\ \alpha x\ \beta x)\ \lambda\ T(x)\ +x\ \alpha\ T(x)\ \beta(x\ \lambda\ y\psi x\ +\ y\psi x\ \lambda x)+(x\ \alpha\ y\psi x\ +\ y\psi x\ \alpha x)\ \beta\ T(x)\ \lambda x\ +x\ \alpha x\ \beta\ T(x)\ \lambda\ y\psi x\ +\ y\psi x\ \alpha\ T(x)\ \beta x\ \lambda x=0$ ……………………………….(4)

From equation (3) it is easy to see that

$T(x)\alpha(x\beta y\lambda x\psi x\ -2y\beta x\lambda x\psi x\ -2x\beta x\lambda y\psi x)+(x\alpha y\beta x-2x\ \alpha x\ \beta y-2y\ \alpha x\ \beta x)\ \lambda\ T(x)\ \psi x\ +x\ \alpha\ T(x)\ \beta(x\ \lambda y\psi x\ +y\ \lambda x\psi x)+(x\ \alpha y\ +y\ \alpha x)\ \beta\ T(x)\ \lambda x\ \psi x\ +x\ \alpha x\ \beta\ T(x)\ \lambda y\psi x\ +y\ \alpha\ T(x)\ \beta x\ \lambda x\psi x\ =0$ ……………………………….(5)

By subtracting (5) from (4),we obtain

$(x\beta y\psi x-2x\beta x\psi y-2y\beta x\psi x)\ \lambda x\alpha\ \ T(x)-(x\ \alpha y\ \beta x-2x\ \alpha x\ \beta y-2y\ \alpha x\ \beta x)\ \psi\ \ T(x)\ \lambda x+\ x\ \psi y\ \lambda\,[x,T(x)]_{\beta}\ \ \alpha x\ +\ y\ \psi x\ \ \lambda\,[x,T(x)]_{\beta}\ \alpha x+y\alpha\,[x,T(x)]_{\beta}\ \psi x\lambda x=0$

And so

$x\alpha y\beta x\lambda\,[x,\ \ T(x)]_{\psi}\ +2x\alpha x\beta y\lambda\,[\ \ T(x),x]_{\psi}\ +2y\alpha x\beta x\lambda\,[\ \ T(x),x]_{\psi}\ +x\alpha y\beta\,[x,\ \ T(x)]_{\lambda}$

$\psi x+y\alpha x\beta\,[x,T(x)]_{\lambda}\ \psi x+y\alpha\,[x,T(x)]_{\beta}\ \psi x\lambda x=0$

which reduces after collecting the first and the fourth term together to

$x\alpha y\beta[x\lambda x,T(x)]_{\psi}+2x\alpha x\beta y\lambda[T(x),x]_{\psi}+2y\alpha x\beta x\lambda[T(x),x]_{\psi}$

$+y\alpha x\beta[x,T(x)]_{\lambda}\psi x+y\alpha[x,T(x)]_{\beta}\lambda x\psi x=0$…………………(6)

Substituting $T(x)\Gamma y$ *for y in the above relation gives*

$x\alpha T(x)\beta y\psi[x\lambda x,T(x)]_{\Gamma}+2x\alpha x\beta T(x)\lambda y\psi[T(x),x]_{\Gamma}$

$+2T(x)\alpha y\beta x\lambda x\psi[T(x),x]_{\Gamma}+T(x)\alpha y\beta x\lambda[x,T(x)]_{\psi}\Gamma x$

$+T(x)\alpha y\beta[x,T(x)]_{\lambda}\psi x\Gamma x=0$………(7)

from (6) it is easy to see that

$T(x)\alpha x\beta y\lambda[x\psi x,T(x)]_{\Gamma}+2T(x)\alpha x\beta x\lambda y\psi[T(x),x]_{\Gamma}$

$+2T(x)\alpha y\beta x\lambda x\psi[T(x),x]_{\Gamma}+T(x)\alpha y\beta x\lambda[x,T(x)]_{\psi}\Gamma x$

$+T(x)\alpha y\beta[x,T(x)]_{\lambda}\psi x\Gamma x=0$…………………(8)

Subtracting (8) from (7) we arrive at

$[T(x),x]_{\alpha}\beta y\lambda[T(x),x\psi x]_{\Gamma}-2[T(x),x\alpha x]_{\psi}\beta y\lambda[T(x),x]_{\Gamma}=0$

Let $a=[T(x),x]_{\alpha}$, $b=[T(x),x\psi x]_{\Gamma}$, $c=-2[T(x),x\alpha x]_{\psi}$

Then the above relation becomes

$a\beta y\lambda b+c\beta y\lambda a=0$……………….(9)

putting in (9) $y\alpha a\psi z$ *for y ,we get*

$a\beta y\alpha a\psi z\lambda b+c\beta y\alpha a\psi z\lambda a=0$………….(10)

from equation (9) it is easy to see that

$a\alpha y\beta a\lambda z\psi b+a\alpha y\beta c\lambda z\psi a=0$………..(11)

subtracting (11) from (10) ,we obtain

$(a\alpha y\beta c-c\alpha y\beta a)\lambda z\psi a=0$………………(12)

let z be $z\Gamma c\Omega y$*, we obtain*

$(a\alpha y\beta c\text{-}c\alpha y\beta a)\ \lambda\ z\Gamma c\Omega y\ \psi\ a=0$………….(13)

From (12) it is easy to see that

$(a\alpha y\beta c\text{-}c\alpha y\beta a)\ \lambda z\psi\ a\Omega y\Gamma c=0$…………(14)

subtracting (13) from (14) ,we obtain

$(a\alpha y\beta c\text{-}c\alpha y\beta a)\ \lambda z\psi\ (a\alpha y\beta c\text{-}c\alpha y\beta a)=0$

Since M is a completely prime Γ*-ring, then we have*

$a\alpha y\beta c=c\alpha y\beta a$……………………………….(15)

combining (9) with (15) ,we arrive at

$a\alpha y\beta (b+c)=0$

in other words

$[T(x),x]_{\alpha}\ \beta y\lambda\ [T(x),x\ \psi\ x]_{\Gamma}=0$……………..(16)

From the above relation one obtains easily

$([T(x),x]_{\alpha}\ \Omega x+x\Omega\ [T(x),x]_{\alpha})\ \beta y\lambda\ [T(x),x\ \psi\ x]_{\Gamma}=0$

We have therefore

$[\,T(x),x\ \Omega x_{\Gamma}]_{\alpha}\ \beta y\lambda [\,T(x),x\ \psi\ x]_{\Gamma}=0$

And so

$[\,T(x),x\ \psi\ x]_{\Gamma}\ \beta y\lambda [\,T(x),x\ \psi\ x]_{\Gamma}=0$

Since M is a completely prime gamma ring, we have

$[\,T(x),x\ \psi\ x]_{\Gamma}=0$……………………………………(17)

Substitution x+y for x in (17) gives

$[\,T(x),y\ \psi\ y]_{\Gamma}+[\,T(y),x\ \psi\ x]_{\Gamma}+[\,T(x),x\psi\ y+y\psi\ x]_{\Gamma}-[T(y),\,x\ \psi\ y+y\psi\ x]_{\Gamma}=0$……(18)

Putting in the above relation –x for x

$-[\,T(x),y\ \psi\ y]_{\Gamma}+[\,T(y),x\ \psi\ x]_{\Gamma}+[\,T(x),x\ \psi\ y+y\psi\ x]_{\Gamma}-[\,T(y),\,x\ \psi\ y+y\psi\ x]_{\Gamma}=0$……(19)

And by comparing (18) with (19)

$[T(x),x\psi y+y\psi x]_\Gamma+[T(y),x\psi x]_\Gamma=0$………….(20)

Putting in the above relation $2(x\alpha y+y\alpha x)$ for y

$[T(x),x\psi 2(x\alpha y+y\alpha x)+2(x\alpha y+y\alpha x)\psi x]_\Gamma+[T(2(x\alpha y+y\alpha x)),x\psi x]_\Gamma=0$

$0=2[T(x),x\psi x\alpha y+y\alpha x\psi x+2x\alpha y\psi x]_\Gamma+[T(x)\alpha y+x\alpha T(y)+T(y)\alpha x+y\alpha T(x),x\psi x]_\Gamma$

And we obtain according to (7) and (17)

$2x\alpha x\psi[T(x),y]_\Gamma+2[T(x),y]_\Gamma\psi x\alpha x+4[T(x),x\alpha y\psi x]_\Gamma+T(x)\alpha[y,x\psi x]_\Gamma+x\alpha[T(y),x\psi x]_\Gamma+[T(y),x\alpha x]_\Gamma\psi x+[y,x\alpha x]_\Gamma\psi T(x)=0$…………..(21)

By replacing y by x ,we get

$2x\alpha x\psi[T(x),x]_\Gamma+2[T(x),x]_\Gamma\psi x\alpha x+4[T(x),x\alpha x\psi x]_\Gamma+T(x)\alpha[x,x\psi x]_\Gamma+x\alpha[T(x),x\psi x]_\Gamma+[T(x),x\alpha x]_\Gamma\psi x+[x,x\alpha x]_\Gamma\psi T(x)=0$…….(22)

By (17),we get

$2x\alpha x\psi[T(x),x]_\Gamma+2[T(x),x]_\Gamma\psi x\alpha x+4[T(x),x\alpha x\psi x]_\Gamma=0$

So

$x\alpha x\psi[T(x),x]_\Gamma+[T(x),x]_\Gamma\psi x\alpha x+2[T(x),x\alpha x\psi x]_\Gamma=0$

$x\alpha x\psi[T(x),x]_\Gamma+[T(x),x]_\Gamma\psi x\alpha x+2x\alpha[T(x),x\psi x]_\Gamma+2[T(x),x]_\Gamma\alpha x\psi x=0$

$x\alpha x\psi[T(x),x]_\Gamma+3[T(x),x]_\Gamma\psi x\alpha x+2x\alpha[T(x),x\psi x]_\Gamma=0$

by (17) ,we get

$x\alpha x\psi[T(x),x]_\Gamma+3[T(x),x]_\Gamma\psi x\alpha x=0$

since $[T(x),x\alpha x]_\Gamma=0$ (see 17)

then $[T(x),x]_\Gamma\alpha x+x\alpha[T(x),x]_\Gamma=0$

one can replace in the above relation

$x\alpha x\psi[T(x),x]_\Gamma$ by $[T(x),x]_\Gamma\alpha x\psi x$

which gives

$4[T(x),x]_{\Gamma}\alpha x\psi x=0$

Since M is 2-torsion free

$[T(x),x]_{\Gamma}\alpha x\psi x=0$ ………(23)

And

$x\alpha x\psi [T(x),x]_{\Gamma}=0$ ……………(24)

and we have also

$x\alpha [T(x),x]_{\Gamma}\psi x=0$ ………………(25)

because of (20),one can replace in (21)

$[T(y),x\alpha x]_{\Gamma}$ *by* $-[T(x),x\alpha y+y\alpha x]_{\Gamma}$ *which gives*

$2x\alpha x\psi [T(x),y]_{\Gamma}+2[T(x),y]_{\Gamma}\alpha x\psi x+4[T(x),x\alpha y\psi x]_{\Gamma}+T(x)\alpha [y,x\psi x]_{\Gamma}+[y,x\alpha x]_{\Gamma}\psi T(x)-x\alpha [T(x),x\psi y+y\psi x]_{\Gamma}-[T(x),x\alpha y+y\alpha x]_{\Gamma}\psi x=0$

And so

$2x\alpha x\psi [T(x),y]_{\Gamma}+2[T(x),y]_{\Gamma}\alpha x\psi x+4[T(x),x]_{\Gamma}\alpha y\psi x+4x\alpha [T(x),y]_{\Gamma}\psi x+4x\alpha y\psi [T(x),x]_{\Gamma}+T(x)\alpha [y,x\psi x]+[y,x\alpha x]\psi T(x)-x\alpha [T(x),x]_{\Gamma}\psi y-x\alpha x\psi [T(x),y]_{\Gamma}-x\alpha [T(x),y]_{\Gamma}\psi x-x\alpha y\psi [T(x),x]_{\Gamma}-[T(x),x]_{\Gamma}\alpha y\psi x-x\alpha [T(x),y]_{\Gamma}\psi x-[T(x),y]_{\Gamma}\alpha x\psi x-y\alpha [T(x),x]_{\Gamma}\psi x=0$

We have therefore

$x\alpha x\psi [T(x),y]_{\Gamma}+[T(x),y]_{\Gamma}\alpha x\psi x+3[T(x),x]_{\Gamma}\alpha y\psi x+2x\alpha [T(x),y]_{\Gamma}\psi x+3x\alpha y\psi [T(x),x]_{\Gamma}+T(x)\alpha [y,x\psi x]_{\Gamma}+[y,x\alpha x]_{\Gamma}\psi T(x)-x\alpha [T(x),x]_{\Gamma}\psi y-y\alpha [T(x),x]_{\Gamma}\psi x=0$ ……………………………(26)

by replacing y by $y\beta x$ *,we get*

$x\alpha x\psi [T(x),y\beta x]_{\Gamma}+[T(x),y\beta x]_{\Gamma}\alpha x\psi x+3[T(x),x]_{\Gamma}\alpha y\beta x\psi x+$

$2x\alpha [T(x),y\beta x]_{\Gamma}\psi x+3x\alpha y\beta x\psi [T(x),x]_{\Gamma}+T(x)\alpha [y\beta x,x\psi x]_{\Gamma}$

$+[y\beta x,x\alpha x]_{\Gamma}\psi T(x)-x\alpha [T(x),x]_{\Gamma}\psi y\beta x-y\beta x\alpha [T(x),x]_{\Gamma}\psi x=0$

And $x\alpha x\psi [T(x),y]_{\Gamma}\beta x+x\alpha x\psi y\beta [T(x),x]_{\Gamma}+[T(x),y]_{\Gamma}\alpha x\psi x\beta x+$

$y\alpha [T(x),x]_\Gamma \psi x\beta x+3[T(x),x]_\Gamma \alpha y\psi x\beta x+3x\alpha y\beta x\psi [T(x),x]_\Gamma$

$+2x\alpha [T(x),y]_\Gamma \psi x\beta x+2x\alpha y\beta [T(x),x]_\Gamma \psi x +T(x) \alpha [y ,x\psi x]_\Gamma \beta x$

$+[y,x\alpha x]_\Gamma \psi x\beta T(x)-x\alpha [T(x),x]_\Gamma \beta y\psi x-y\alpha x\beta [T(x),x]_\Gamma \psi x=0$

which reduces because of (23) and (25) to

$x\alpha x\psi [T(x),y]_\Gamma \beta x+x\alpha x\psi y\beta [T(x),x]_\Gamma +[T(x),y]_\Gamma \alpha x\psi x\beta x+3[T(x),x]_\Gamma \alpha y\psi x\beta x+3x\alpha y\beta x\psi [T(x),x]_\Gamma +2x\alpha [T(x),y]_\Gamma \psi x\beta x+2x\alpha y\beta [T(x),x]_\Gamma \psi x+T(x)\alpha [y,x\psi x]_\Gamma \beta x+[y,x\alpha x]_\Gamma \psi x\beta T(x)-x\alpha [T(x),x]_\Gamma \beta y\psi x=0$ ………(27)

we should mentioned the reader that, if a=0 then $a\alpha x=0\ \forall x\in M,\ \alpha \in \Gamma$*.then the equation (26)becomes*

$x\alpha x\psi [T(x),y]_\Gamma \beta x+[T(x),y]_\Gamma \alpha x\psi x \beta x +3[T(x),x]_\Gamma \alpha y\psi x \beta x +2x\alpha [T(x),y]_\Gamma \psi x \beta x +3x\alpha y\psi [T(x),x]_\Gamma \beta x +T(x) \alpha [y,x\psi x]_\Gamma \beta x +[y,x\alpha x]_\Gamma \psi T(x) \beta x -x\alpha [T(x),x]_\Gamma \psi y \beta x =0$……………………….(28)

subtracting (28) from (27),we obtain

$x\alpha x\psi y\beta [T(x),x]_\Gamma +3x\alpha y\psi [x,[T(x),x]_\beta]_\Gamma +2x\alpha y \beta [T(x),x]_\Gamma \psi x$

$+[y,x\alpha x]_\Gamma \psi [x, T(x)]_\beta =0$

which reduces because of (24) to

$x\alpha x\psi y\beta [T(x),x]_\Gamma +3x\alpha y\psi x\beta [T(x),x]_\Gamma -3x\alpha y\beta [T(x),x]_\Gamma \psi x+2 x\alpha y\psi [T(x),x]_\Gamma \beta x +y\alpha x\psi x\beta [x, T(x)]_\Gamma - x\alpha x\psi y\beta [x, T(x)]_\Gamma =0$

so

$2x\alpha x\psi y\beta [T(x),x]_\Gamma + 3x\alpha y\psi x\beta [T(x),x]_\Gamma - x\alpha y\beta [T(x),x]_\Gamma \psi x=0$

Replacing in the above relation

$-[T(x),x]_\Gamma \beta x$ *by* $x\beta [T(x),x]_\Gamma$*,we obtain*

$2x\alpha x\psi y\beta [T(x),x]_\Gamma + 4x\alpha y\psi x\beta [T(x),x]_\Gamma =0$

Since M is 2-torsion free ,then

$x\alpha x\psi y\beta [T(x),x]_\Gamma + 2x\alpha y\psi x\beta [T(x),x]_\Gamma =0$

because of (17),(23),(24) and (25) the relation (6) reduce to

$x\alpha x\psi y\beta\ [T(x),x]_\Gamma=0$

which gives together with the above relation

$x\alpha y\psi x\beta\ [T(x),x]_\Gamma=0$

whence it follows

$[T(x),x]_\Gamma\ \beta x\alpha y\psi x\beta\ [T(x),x]_\Gamma=0$

Then

$x\beta\ [T(x),x]_\Gamma\ \alpha y\psi x\beta\ [T(x),x]_\Gamma=0$

$x\beta\ [T(x),x]_\Gamma=0$ ……………….(29)

of course we have ,also

$[T(x),x]_\Gamma\ \beta x=0$ ………………(30)

From (29) one obtains [see the proof of (20)]

$y\beta\ [T(x),x]_\Gamma+x\beta\ [T(x),y]_\Gamma+x\beta\ [T(y),x]_\Gamma=0$

and it is easy to see that

$[T(x),x]_\Gamma\ \alpha y\beta [T(x),x]_\Gamma+[T(x),x]_\Gamma\ \alpha x\beta\ [T(x),y]_\Gamma+[T(x),x]_\Gamma\ \alpha x\beta\ [T(y),x]_\Gamma=0$

By (30)

$[T(x),x]_\Gamma\ \alpha y\beta\ [T(x),x]_\Gamma=0$ ……………(31)

Whence it follows

$[T(x),x]_\Gamma=0$ for all $\Gamma\in\Gamma$ ………………………………..(32)

And so by combining (32) with (*)

$T(x\ \alpha x)=T(x)\ \alpha x$ for all $x\in M,\ \alpha\in\Gamma$.

And also $T(x\ \alpha x)=x\ \alpha T(x)$ for all $x\in M,\ \alpha\in\Gamma$.

Which means that T is Jordan left centralizer and also Jordan right centralizers

And so by [6 ,theorem 1.4 and theorem 1.5],we getT is both left centralizer and also right centralizers. Then the proof of theorem is complete.

3-On Jordan Triple centralizers on completely prime gamma ring

In this section we shall defined Jordan triple left (resp.,right centralizer) on gamma ring and study the relation between it and left(right)- centralizer.

Definition 3.1:- An additive mapping $T:M \to M$ is called Jordan triple left (resp., right)centralizer on Γ-ringM if it satisfy

$T(x\alpha y\beta x)=T(x)\alpha y\beta x$ (resp., $T(x\alpha y\beta x)=x\alpha y\beta T(x)$) for all $x,y \in M$ and $\alpha, \beta \in \Gamma$.

It is easy to see that every left (resp ., right) centralizer be a Jordan triple left (resp., right)centralizer but the converse is not true .in this section we study this problem.

Theorem 3.2 :- Let M be a 2-torsion free completely prime Γ-ring, if $T:M \to M$ be an additive mapping satisfying

$T(x\alpha y\beta x)=T(x)\alpha y\beta x$ for all $x,y \in M$, $\alpha, \beta \in \Gamma$. then T is a left centralizer.

Proof :- Since $T(x\alpha y\beta x)=T(x)\alpha y\beta x$……………..(33)

Replace x by x+z

$T(x\alpha y\beta z+z\alpha y\beta x)=T(x)\alpha y\beta z+T(z)\alpha y\beta x$………..(34)

Replace z by $x\lambda x$ *in (34) ,we get*

$T(x\alpha y\beta (x\lambda x)+(x\lambda x)\alpha y\beta x)=T(x)\alpha y\beta (x\lambda x)+T(x\lambda x)\alpha$

$y\beta x$…………………(35)

replace y by $x\lambda y+y\lambda x$ *in (33)and use (34),to get*

$T(x\alpha (x\lambda y+y\lambda x)\beta x)=T(x)\alpha (x\lambda y+y\lambda x)\beta x$

$T(x\alpha x\lambda y\beta x+x\alpha y\lambda x\beta x)=T(x)\alpha x\lambda y\beta x+T(x)\alpha y\lambda x\beta x$…………(36)

By comparing (35) and (36),we get

$(T(x\lambda x)- T(x)\lambda x)\beta y\alpha x=0$

Suppose that

$A(x)= T(x\lambda x)- T(x)\lambda x$

Then

$A(x)\ \beta y \alpha x=0$………………………..(37)

Replace y by $x \alpha z \lambda A(x)$

$A(x)\ \beta x \alpha z \lambda A(x)\ \alpha x=0$

$A(x)\ \alpha x \beta z \lambda A(x)\ \alpha x=0$

Since M is a completely prime Γ*-ring,then*

$A(x)\ \alpha x=0$……………………(38)

Now replace x by x+y

$A(x+y)\ \alpha (x+y)=0$

$A(x+y)\ \alpha x + A(x+y)\ \alpha y=0$…………..(39)

Now we compute

$A(x+y)= (T(x\ \alpha\ y+y\ \alpha\ x)- T(x)\ \alpha\ y-T(y)\ \alpha x)+(T(x \alpha x)-T(x)\ \alpha x)$

$+(T(y \alpha y)-T(y)\ \alpha y)$

$A(x+y)=B(x,y)+A(x)+A(y)$…………………………………….(40)

Where $B(x,y)= T(x\ \alpha\ y+y\ \alpha\ x)- T(x)\ \alpha\ y-T(y)\ \alpha x$

Thus in view of (39) and (40) implies that

$A(x)\ \alpha x+A(y)\ \alpha x+ B(x,y)\ \alpha x+ A(x)\ \alpha y+ A(y)\ \alpha y+ B(x,y)\ \alpha y=0$

Then by using (38),we have

$A(x)\ \alpha y+ A(y)\ \alpha x+ B(x,y)\ \alpha x+ B(x,y)\ \alpha y=0$………….(41)

Again replace x by –x, in (41) , to get

$A(x)\ \alpha y- A(y)\ \alpha x+ B(x,y)\ \alpha x+ B(x,y)\ \alpha y=0$

$A(x)\ \alpha y+ B(x,y)\ \alpha x- A(y)\ \alpha x+ B(x,y)\ \alpha y=0$

$2(A(x)\ \alpha y+ B(x,y)\ \alpha x)=0$……………………….(42)

Since M is 2-torsion free,then

$A(x)\ \alpha\ y + B(x,y)\ \alpha\ x = 0$……………………….(43)

Since (if x=0 then $x\ \alpha\ y=0$*)*

$A(x)\ \alpha\ y\ \lambda\ A(x) + B(x,y)\ \alpha\ x\ \lambda\ A(x) = 0$……………….(44)

From equation (37),we get

$x\ \alpha\ A(x)\ \lambda\ z\ \beta\ x\ \alpha\ A(x) = 0$…………………………..(45)

Since M is prime Γ*-ring,then*

$x\ \alpha\ A(x) = 0$………………………………………..(46)

by (46),the equation (44)becomes

$A(x)\ \alpha\ y\ \lambda\ A(x) = 0$

Since M is a completely prime Γ*-ring, then*

A(x)=0 and so

$T(x\ \alpha\ x) = T(x)\ \alpha\ x$ *for all* $x,y \in M$*, for all* $\alpha \in \Gamma$.

i.e T is Jordan left centralizers

and so by [6 ,theorm 1.4],we get the result.

<u>Theorem 3.3</u> :- Let M be a 2-torsion free completely prime Γ*-ring, if* $T:M \to M$ *be an additive map satisfies* $T(x\ \alpha\ y\ \beta\ x) = x\ \alpha\ y\ \beta\ T(x)$ *for all* $x,y \in M$, $\alpha, \beta \in \Gamma$*. then T is a right centralizer.*

<u>Proof</u>:- by the same technique of the above theorem and by using [6 , Theorem 1.5] we get the result.

<u>References</u>

[1]W.E.Barnes(1966)"On the Γ*-ring of Nabusawa",Pacific J.Math.,18,411-422.*

[2]Y.Ceven(2002),"Jordan left derivations on completely prime Gamma rings ",Fenbilimleri dergisicilit 23 say 12.

[3] W.Cortes and C.Haetinger(2009)"On Lie ideals and left Jordan σ*- centralizers of 2-torsion free rings",.Math.J.Okayama Univ.51,111-119.*

[4]I.N.Herstien(1969) "Topics in ring theory",University of Chicago press .

[5]N.Nabusawa(1964)"On a generalization of the ring theory",

",Osaka J.Math.,1.

[6]Rajaa C.Shaheen,(2007)"On centralizers on some gamma ring"Journal Al-Qadisiya for pure science.Vol.12,No.2.

[7]J.Vukman(1999)"An identity related to centralizers in semi-prime rings",Comment.Math.Univ.Carolinae,40,3, 447-456.

[8] B.Zalar(1991)"On centralizers of semi-prime rings" Comment .Math .univ.Carolinae, 32 ,609-614.

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رجاء جفات شاهين

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الخلاصة:-

قدمنا في هذا البحث دراسة حول بعض انواع التطبيقات المركزية على الحلقات الاولية من نوع كاما.