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On nonlinear partial differential Equations

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Abstract:-

This paper introduces suggested method for solving the nonlinear partial differential equations which are homogeneous degree and its equal to degree 2 with the following general form:

$$\mathrm{A}(x,y,Z,Z_x,Z_y,Z_{xx},Z_{xy},Z_{yy})Z_{xx}+\mathrm{B}(x,y,Z,Z_x,Z_y,Z_{xx},Z_{xy},Z_{yy})Z_{xy}+$$
$$\mathrm{C}(x,y,Z,Z_x,Z_y,Z_{xx},Z_{xy},Z_{yy})Z_{yy}+\mathrm{D}(x,y,Z,Z_x,Z_y,Z_{xx},Z_{xy},Z_{yy})Z_x+$$
$$\mathrm{E}(x,y,Z,Z_x,Z_y,Z_{xx},Z_{xy},Z_{yy})Z_y+\mathrm{F}(x,y,Z,Z_x,Z_y,Z_{xx},Z_{xy},Z_{yy})Z=0$$

where A , B , C , D , E and F are linear functions of dependent variable Z and partial derivatives of dependent variable with respect to the independent variables x and y.

1-Introduction :

Many of researchers try to find new methods for solving (P.D.Es) , Kudaer [7],2006 studied the linear second order (O.D.Es) ,which have the form $y''+P(x)\,y'+Q(x)\,y=0$

and used the assumption $y(x)=\mathrm{e}^{\int Z(x)\,dx}$ to find the general solution of it , and the solution depends on the forms of $P(x)$ and $Q(x)$.

Abd Al-Sada [1],2006 studied the linear second order (P.D.Es) with constant coefficients and which have the form

$$\mathrm{A}\,Z_{xx}+\mathrm{B}\,Z_{xy}+\mathrm{C}\,Z_{yy}+\mathrm{D}\,Z_x+\mathrm{E}\,Z_y+\mathrm{F}\,Z=0,$$

where A , B , C , D , E and F are arbitrary constants .

she used the assumption $Z(x,y)=\mathrm{e}^{\int U(x)\,dx+\int V(y)\,dy}$

to find the complete solution of it, and the solution depends on the values of A , B , C , D , E and F .

Hani [5],2008, studied the linear second order (P.D.Es) which have three independent variables , and which have the form

$$AZ_{xx}+BZ_{xy}+CZ_{xt}+DZ_{yy}+EZ_{yt}+FZ_{tt}+$$
$$GZ_x+HZ_y+IZ_t+JZ=0$$

where A,B,C,…,J are arbitrary constants .

she used the assumption $Z(x,y,t)=e^{\int U(x)dx+\int V(y)dy+\int W(t)dt}$

to find the complete solution of it , and the solution depends on the values of A,B,C,…,J.

Finally, Hanon [6],2009, studied the linear second order (P.D.Es) , with variable coefficients which have the form

$$A(x,y)\ Z_{xx} + B(x,y)\ Z_{xy} + C(x,y)\ Z_{yy} + D(x,y)\ Z_x + E(x,y)\ Z_y + F(x,y)\ Z = 0$$

where some of $A(x,y), B(x,y), C(x,y), D(x,y), E(x,y)$ *and* $F(x,y)$ are functions of x or y or both x and y .

To solve this kind of equations ,she used the assumptions

$$Z(x,y) = e^{\int \frac{U(x)}{x}\,dx + \int V(y)\,dy} \quad , \quad Z(x,y) = e^{\int U(x)\,dx + \int \frac{V(y)}{y}\,dy} \quad \text{and}$$

$$Z(x,y) = e^{\int \frac{U(x)}{x}\,dx + \int \frac{V(y)}{y}\,dy}$$

These assumptions represent the complete solution of the above equation and the solution depends on the forms of $A(x,y), B(x,y), C(x,y), D(x,y), E(x,y)$ *and* $F(x,y)$

In this paper, we will discuss the non-linear (P.D.Es) of homogeneous degree which are defined as the following " The partial differential equation that each degree of any term in it is equal in the degree with another terms in it's dependent variable and partial derivatives of dependent variable with respect to the independent variables x and y " . We will solve the nonlinear (P.D.Es) which are homogeneous of degree 2 that the general form is

$$A(x,y,Z,Z_x,Z_y,Z_{xx},Z_{xy},Z_{yy})Z_{xx} + B(x,y,Z,Z_x,Z_y,Z_{xx},Z_{xy},Z_{yy})Z_{xy} +$$
$$C(x,y,Z,Z_x,Z_y,Z_{xx},Z_{xy},Z_{yy})Z_{yy} + D(x,y,Z,Z_x,Z_y,Z_{xx},Z_{xy},Z_{yy})Z_x +$$
$$E(x,y,Z,Z_x,Z_y,Z_{xx},Z_{xy},Z_{yy})Z_y + F(x,y,Z,Z_x,Z_y,Z_{xx},Z_{xy},Z_{yy})Z = 0$$

where A, B, C, D, E and F are linear functions of dependent variable Z and partial derivatives of dependent variable with respect to the independent variables x and y. We will use the following assumptions

$$Z(x,y) = e^{\int U(x)dx + \int V(y)dy} \quad , \quad Z(x,y) = e^{\int \frac{U(x)}{x}dx + \int V(y)dy}$$

$$Z(x,y) = e^{\int U(x)dx + \int \frac{V(y)}{y}dy} \quad \text{and} \quad Z(x,y) = e^{\int \frac{U(x)}{x}dx + \int \frac{V(y)}{y}dy}$$

These assumptions help us to find the complete solutions of this kind of equations

2-DEFINITION

The partial differential equation that each degree of any term in it equal to the degree of another terms of it (in its dependent variable and partial derivatives of dependent variable with respect to the independent variables x and y) is called (P.D.Es) with homogeneous degree .

3-SOLVING SPECIAL KINDS OF THE NONLINEAR PARTIAL DIFFERENTIAL EQUATIONS WHICH ARE HOMOGENEOUS OF DEGREE 2.

The general form of them is:

$$\begin{aligned} &\mathrm{A}(x,y,Z,Z_x,Z_y,Z_{xx},Z_{xy},Z_{yy})Z_{xx} + \mathrm{B}(x,y,Z,Z_x,Z_y,Z_{xx},Z_{xy},Z_{yy})Z_{xy} + \\ &\mathrm{C}(x,y,Z,Z_x,Z_y,Z_{xx},Z_{xy},Z_{yy})Z_{yy} + \mathrm{D}(x,y,Z,Z_x,Z_y,Z_{xx},Z_{xy},Z_{yy})Z_x + \\ &\mathrm{E}(x,y,Z,Z_x,Z_y,Z_{xx},Z_{xy},Z_{yy})Z_y + \mathrm{F}(x,y,Z,Z_x,Z_y,Z_{xx},Z_{xy},Z_{yy})Z = 0 \end{aligned} \quad \ldots(1)$$

where $\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D}, \mathrm{E}$ and F are linear functions of dependent variable Z and partial derivatives of dependent variable with respect to the independent variables x and y. We will choose some kinds of nonlinear (P.D.Es) which are homogeneous of degree 2 to find the complete solution of them , and divide them ,according to the assumption that help us to solve them, into four kinds:

Kind (1)

The non-linear (P.D.Es) of homogeneous degree with constant coefficients (x and y are missing),as:-

$$\mathrm{A}_1 Z_{xx} Z_{yy} + \mathrm{A}_2 Z_x Z_y = 0 \quad \ldots(2)$$

where $\mathrm{A}_1 and \mathrm{A}_2$ are arbitrary constants and not identically zero. Here, we search new functions $U(x)$ and $V(y)$ such that the assumption

$$Z(x,y) = e^{\int U(x)dx + \int V(y)dy} \quad \ldots(3)$$

helps us to find the complete solutions of the equation (2). By finding Z_x , Z_y , Z_{xx} and Z_{yy} from (3), we get

$$Z_x = U(x)\ \mathrm{e}^{\int U(x)\,dx + \int V(y)\,dy}$$

$$Z_{xx} = [U'(x) + U^2(x)]\mathrm{e}^{\int U(x)\,dx + \int V(y)\,dy}$$

$$Z_y = V(y)\ \mathrm{e}^{\int U(x)\,dx + \int V(y)\,dy}$$

$$Z_{yy} = [V'(y) + V^2(y)]\mathrm{e}^{\int U(x)\,dx + \int V(y)\,dy}$$

by substituting Z_x , Z_y , Z_{xx} and Z_{yy} in the equation (2), we get

$$[\mathrm{A}_1(U'(x)+U^2(x))(V'(y)+V^2(y)) + \mathrm{A}_2 U(x)V(y)]e^{2[\int U(x)dx + \int V(y)dy]} = 0$$

Since $\mathrm{e}^{2[\int U(x)dx + \int V(y)dy]} \neq 0$

So ,

$$\mathrm{A}_1(U'(x) + U^2(x))(V'(y) + V^2(y)) + \mathrm{A}_2 U(x)V(y) = 0$$

This equation is variable separable [4] . We can solve the last equation as follows:

$$\frac{A_1\left(U'(x)+U^2(x)\right)}{U(x)}=\frac{-A_2V(y)}{V'(y)+V^2(y)}=\lambda^2 \quad ;\lambda \text{ =arbitrary constant}$$

Therefore $A_1U'(x)+A_1U^2(x)-\lambda^2U(x)=0$

$$U'(x)+U^2(x)-\frac{\lambda^2}{A_1}U(x)=0 \qquad \ldots(4)$$

also, $V'(y)+V^2(y)+\frac{A_2}{\lambda^2}V(y)=0$

…(5) The equations (4)and (5) are similar to Bernoulli equation [3], then the solution for them is given by:

$$U(x)=\frac{e^{\frac{\lambda^2}{A_1}x}}{\int e^{\frac{\lambda^2}{A_1}x}dx} \quad , \quad V(y)=\frac{e^{\frac{-A_2}{\lambda^2}y}}{\int e^{\frac{-A_2}{\lambda^2}y}dy}$$

Then the complete solution of the equation (2) is given by:-

$$Z(x,y)=e^{\int\left(\frac{e^{\frac{\lambda^2}{A_1}x}}{\int e^{\frac{\lambda^2}{A_1}x}dx}\right)dx+\int\left(\frac{e^{\frac{-A_2}{\lambda^2}y}}{\int e^{\frac{-A_2}{\lambda^2}y}dy}\right)dy}$$

$$=e^{\ln\left(\int e^{\frac{\lambda^2}{A_1}x}dx\right)+\ln\left(\int e^{\frac{-A_2}{\lambda^2}y}dy\right)+c_1}$$

$$=\mathrm{k}\left[\frac{A_1}{\lambda^2}e^{\frac{\lambda^2}{A_1}x}+c_2\right]\left[\frac{-\lambda^2}{A_2}e^{-\frac{A_2}{\lambda^2}y}+c_3\right]$$

; $\lambda\neq 0$ *and* $\mathrm{K}=\mathrm{e}^{c_1}$,where c_2,c_3 and λ are arbitrary constants.

<u>Example</u> :-To solve the (P.D.E):-

$$2Z_{xx}Z_{yy}-\frac{1}{4}Z_xZ_y=0$$

We will use the above formula, then the complete solution of the above (P.D.E.) is given by

$\lambda \neq 0$; $Z(x,y) = k\left[\frac{2}{\lambda^2} e^{\frac{\lambda^2}{2}x} + c_2\right]\left[4\lambda^2 e^{\frac{1}{4\lambda^2}y} + c_3\right]$

where K, λ, c_2 *and* c_3 are arbitrary constants .

Kind (2)

The non-linear (P.D.Es) of homogeneous degree with variable coefficients (variable x only, y is missing),as:-

$$A_1 Z_{yy}^2 + A_2 x Z_x Z_{yy} + A_3 Z^2 = 0 \qquad ...(6)$$

where A_1, A_2 and A_3 are arbitrary constants and not identically zero.

Here, we search new functions $U(x)$ and $V(y)$ such that the assumption

$$Z(x,y) = e^{\int \frac{U(x)}{x}dx + \int V(y)dy} \qquad \dots (7)$$

Helps us to find the complete solution of the equation (6). Using Z_{yy} from the kind (1) and finding Z_x from (7) , we get

$$Z_x = \frac{U(x)}{x} \, \mathrm{e}^{\int \frac{U(x)}{x}dx + \int V(y)dy}$$

by substituting Z , Z_x and Z_{yy} in the equation (6) , we get

$$\left[A_1\left(V'(y) + V^2(y)\right)^2 + A_2 U(x)\left(V'(y) + V^2(y)\right) + A_3\right] e^{2\left[\int \frac{U(x)}{x}dx + \int V(y)dy\right]} = 0$$

Since $e^{2\left[\int \frac{U(x)}{x}dx + \int V(y)dy\right]} \neq 0$

Then ,

$$A_1\left(V'(y) + V^2(y)\right)^2 + A_2 U(x)\left(V'(y) + V^2(y)\right) + A_3 = 0$$

This equation is variable separable [4].We can solve it as follows:-

$$\frac{A_1\left(V'(y) + V^2(y)\right)^2 + A_3}{\left(V'(y) + V^2(y)\right)} = -A_2 U(x) = \lambda^2$$

Therefore , $U(x) = \dfrac{-\lambda^2}{A_2}$

Also,

$$A_1\left(V'(y) + V^2(y)\right)^2 - \lambda^2\left(V'(y) + V^2(y)\right) + A_3 = 0$$

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$$\left(V'(y)+V^2(y)\right)=\frac{\lambda^2\pm\sqrt{\lambda^4-4A_1A_3}}{2A_1}$$

Let $\quad B=\sqrt{\dfrac{\lambda^2\pm\sqrt{\lambda^4-4A_1A_3}}{2A_1}}$

Then the last equation becomes:

$$V'(y)+V^2(y)-B^2=0$$

This equation is (O.D.E.) with constant coefficient , we can solve it as follows:-

$$\frac{dV}{V^2(y)-B^2}+dy=0 \quad \Rightarrow \frac{-1}{B}\tanh^{-1}\left(\frac{V(y)}{B}\right)=c-y$$

$$V(y)=B\tanh(By-cB)$$

Then the solution of the equation (6) is given by:-

$$Z(x,y)=e^{-\int\left(\frac{\lambda^2}{A_2 x}\right)dx+\int(B\tanh(By-cB))dy}$$

$$=e^{\frac{-\lambda^2}{A_2}\ln x+\ln(\cosh(By-cB))+c_1}$$

$$=Kx^{\frac{-\lambda^2}{A_2}}(\cosh(By-cB)) \quad ;K=e^{c_1}$$

$$=x^{\frac{-\lambda^2}{A_2}}\left[d_1\cosh\sqrt{\frac{\lambda^2\pm\sqrt{\lambda^4-4A_1A_3}}{2A_1}}\,y-\right.$$

$$\left.d_2\sinh\sqrt{\frac{\lambda^2\pm\sqrt{\lambda^4-4A_1A_3}}{2A_1}}\,y\right] \qquad ;K=e^{c_1}$$

Where ,

$$d_1=K\cosh\sqrt{\frac{\lambda^2\pm\sqrt{\lambda^4-4A_1A_3}}{2A_1}}\,c$$

$$d_2=K\sinh\sqrt{\frac{\lambda^2\pm\sqrt{\lambda^4-4A_1A_3}}{2A_1}}\,c$$

c *and* λ are arbitrary constants. And the complete solution of the equation (6) is given by:-

$$\left(Z(x,y) - x^{\frac{-\lambda^2}{A_2}} \left[K_1 \cosh \sqrt{\frac{\lambda^2 + \sqrt{\lambda^4 - 4A_1A_3}}{2A_1}}\, y - \right.\right.$$

$$\left.\left. K_2 \sinh \sqrt{\frac{\lambda^2 + \sqrt{\lambda^4 - 4A_1A_3}}{2A_1}}\, y \right]\right)$$

$$\left(Z(x,y) - x^{\frac{-\lambda^2}{A_2}} \left[K_3 \cosh \sqrt{\frac{\lambda^2 - \sqrt{\lambda^4 - 4A_1A_3}}{2A_1}}\, y - \right.\right.$$

$$\left. K_4 \sinh \sqrt{\frac{\lambda^2 - \sqrt{\lambda^4 - 4A_1A_3}}{2A_1}}\, y \right) = 0$$

where

$$K_1 = K \cosh \sqrt{\frac{\lambda^2 + \sqrt{\lambda^4 - 4A_1A_3}}{2A_1}}\, c$$

$$, K_2 = K \sinh \sqrt{\frac{\lambda^2 + \sqrt{\lambda^4 - 4A_1A_3}}{2A_1}}\, c$$

$$, K_3 = K \cosh \sqrt{\frac{\lambda^2 - \sqrt{\lambda^4 - 4A_1A_3}}{2A_1}}\, c$$

$$, K_4 = K \sinh \sqrt{\frac{\lambda^2 - \sqrt{\lambda^4 - 4A_1A_3}}{2A_1}}\, c$$

and λ are arbitrary constants.

Example :- To solve the (P.D.E.)

$$\frac{1}{2} Z_{yy}^2 + 2xZ_x Z_{yy} - Z^2 = 0$$

We use the above formula ,so the complete solution of the given (P.D.E.) is given by :-

$$\left(Z(x,y) - x^{\frac{-\lambda^2}{2}} \left[K_1 \cosh \sqrt{\lambda^2 + \sqrt{\lambda^4 + 2}}\, y - \right.\right.$$

$$\left.\left. K_2 \sinh \sqrt{\lambda^2 + \sqrt{\lambda^4 + 2}}\, y \right]\right)$$

$$\left(Z(x,y) - x^{\frac{-\lambda^2}{2}} \left[K_3 \cosh \sqrt{\lambda^2 + \sqrt{\lambda^4 + 2}}\, y - \right.\right.$$

$$\left.\left. K_4 \sinh \sqrt{\lambda^2 + \sqrt{\lambda^4 + 2}}\, y \right]\right) = 0$$

where K_1, K_2, K_3, K_4 *and* λ are arbitrary constants.

Kind (3)

The non-linear (P.D.Es) of homogeneous degree with variable coefficients (variable y only, x is missing),as:-

$$A_1 y^2 Z_{xy}^2 + A_2 y Z Z_{xy} + A_3 Z^2 = 0 \qquad \ldots(8)$$

where A_1, A_2 and A_3 are arbitrary constants and not identically zero. Here, we search new functions $U(x)$ and $V(y)$ such that the assumption

$$Z(x,y) = e^{\int U(x)dx + \int \frac{V(y)}{y} dy} \qquad \ldots(9)$$

Helps us to find the complete solution of the equation (8). By finding Z_{xy} from (9) , we get

$$Z_{xy} = \frac{U(x)V(y)}{y} e^{\int U(x)\, dx + \int \frac{V(y)}{y} dy}$$

by substituting Z *and* Z_{xy} in the equation (8),we get

$$\left[A_1 U^2(x) V^2(y) + A_2 U(x) V(y) + A_3\right] e^{2\left[\int U(x)dx + \int \frac{V(y)}{y} dy\right]} = 0$$

Since $e^{2\left[\int U(x)dx + \int \frac{V(y)}{y} dy\right]} \neq 0$

So , $A_1 U^2(x) V^2(y) + A_2 U(x) V(y) + A_3 = 0$

Here although we can separate the variables, but we suppose that $V(y) = \lambda$ (because we get the same result) where λ is an arbitrary constant, and then the last equation becomes:-

$$A_1 \lambda^2 U^2(x) + A_2 \lambda U(x) + A_3 = 0$$

$$U(x) = \frac{-A_2 \pm \sqrt{A_2^2 - 4A_1A_3}}{2\lambda A_1} ; \lambda \neq 0$$

$$Z(x,y) = e^{\int \left(\frac{-A_2 \pm \sqrt{A_2^2 - 4A_1A_3}}{2\lambda A_1} \right) dx + \int \left(\frac{\lambda}{y} \right) dy}$$ So,

$$= e^{\frac{-A_2 \pm \sqrt{A_2^2 - 4A_1A_3}}{2\lambda A_1} x + \lambda \ln y + c_1}$$

$$= \mathrm{K} y^{\lambda} e^{\frac{-\mathrm{A}_2 \pm \sqrt{\mathrm{A}_2^2 - 4\mathrm{A}_1\mathrm{A}_3}}{2\lambda \mathrm{A}_1} x} \quad , \mathrm{K} = e^{c_1}$$

and the complete solution of (8) is given by:-

$$\left(Z(x,y) - \mathrm{K} y^{\lambda} e^{\frac{-\mathrm{A}_2 + \sqrt{\mathrm{A}_2^2 - 4\mathrm{A}_1\mathrm{A}_3}}{2\lambda \mathrm{A}_1} x} \right)\left(Z(x,y) - \mathrm{K} y^{\lambda} e^{\frac{-\mathrm{A}_2 - \sqrt{\mathrm{A}_2^2 - 4\mathrm{A}_1\mathrm{A}_3}}{2\lambda \mathrm{A}_1} x} \right) = 0$$

$;\lambda \neq 0$

where K *and* λ are arbitrary constants.

<u>Example</u> :- To solve the (P.D.E.):-

$$2y^2 Z_{xy} - 3yZZ_{xy} + \frac{1}{4} Z^2 = 0$$

We will use the above formula ,then, the complete solution of the given (P.D.E .)is

$$\left(Z(x,y) - \mathrm{K} y^{\lambda} e^{\frac{3+\sqrt{7}}{4\lambda} x} \right)\left(Z(x,y) - \mathrm{K} y^{\lambda} e^{\frac{3-\sqrt{7}}{4\lambda} x} \right) = 0 \qquad ;\lambda \neq 0$$ where

K *and* λ are arbitrary constants .

<u>*Kind (4*</u>)

The non-linear (P.D.Es) of homogeneous degree with variable coefficient (both of variables x and y),as:-

$$\mathrm{A}_1 y^2 ZZ_{yy} + \mathrm{A}_2 x^2 Z_x^2 = 0 \qquad \ldots(10)$$

where A_1 *and* A_2 are arbitrary constants and not identically zero. Here, we will search new functions $U(x)$ and $V(y)$ such that the assumption

$$Z(x,y) = e^{\int \frac{U(x)}{x} dx + \int \frac{V(y)}{y} dy} \qquad \ldots (11)$$

helps us to find the complete solutions of the equation (10) .

Using Z_x and Z_{yy} from the above kinds and substituting them in the equation, (10) we get

$$\left[\mathrm{A}_1 \left(yV'(y) + V^2(y) - V(y) \right) + \mathrm{A}_2 U^2(x) \right] e^{2\left[\int \frac{U(x)}{x} dx + \int \frac{V(y)}{y} dy \right]} = 0$$

Since $\quad e^{2\left[\int \frac{U(x)}{x} dx + \int \frac{V(y)}{y} dy \right]} \neq 0$

So,

$$\mathrm{A}_1 \left(yV'(y) + V^2(y) - V(y) \right) + \mathrm{A}_2 U^2(x) = 0$$

This equation is variable separable [4]. We can solve it as follows:-

$$A_1\left(yV'(y)+V^2(y)-V(y)\right)=-A_2U^2(x)=\lambda^2$$

So, $U(x)=\pm i\sqrt{\frac{\lambda^2}{A_2}}$

Also,

$$yV'(y)+V^2(y)-V(y)-\frac{\lambda^2}{A_1}=0$$

i) If $\frac{\lambda^2}{A_1}\neq\frac{-1}{4}$, then

$$\frac{dV}{\left(V(y)-\frac{1}{2}\right)^2-d^2}+\frac{dy}{y}=0 \quad ;d=\sqrt{\frac{\lambda^2}{A_1}+\frac{1}{4}}$$

$$\frac{-1}{d}\tanh^{-1}\left(\frac{V-\frac{1}{2}}{d}\right)=-\ln(cy) \quad \Rightarrow V(y)=d\tanh(d\ln(cy))+\frac{1}{2}$$

Then ,

$$Z(x,y)=e^{\int\left(\frac{\pm i\sqrt{\frac{\lambda^2}{A_2}}}{x}\right)dx+\int\left(\frac{d\tanh(d\ln(cy))+\frac{1}{2}}{y}\right)dy}$$

$$=e^{\pm i\sqrt{\frac{\lambda^2}{A_2}}\ln x+\ln(\cosh(d_1\ln cy))+\frac{1}{2}\ln y+c_1}$$

$$=K\,x^{\pm i\sqrt{\frac{\lambda^2}{A_2}}}\,y^{\frac{1}{2}}\left(\cosh\left(\sqrt{\frac{\lambda^2}{A_1}+\frac{1}{4}}\ln cy\right)\right);cy>0$$

And the complete solution of (10) is given by:-

$$\left(Z-K\,x^{i\sqrt{\frac{\lambda^2}{A_2}}}\,y^{\frac{1}{2}}\left(\cosh\left(\sqrt{\frac{\lambda^2}{A_1}+\frac{1}{4}}\ln cy\right)\right)\right)\left(Z-K\,x^{-i\sqrt{\frac{\lambda^2}{A_2}}}\,y^{\frac{1}{2}}\left(\cosh\left(\sqrt{\frac{\lambda^2}{A_1}+\frac{1}{4}}\ln cy\right)\right)\right)=0$$

$;cy>0$

Where K , λ and c are arbitrary constants .

ii)If $\frac{\lambda^2}{A_1}=\frac{-1}{4}$,we get

$$\frac{dV}{\left(V(y)-\frac{1}{2}\right)^2}+\frac{dy}{y}=0 \quad \Rightarrow \frac{-1}{V(y)-\frac{1}{2}}=-\ln(cy)$$

$V(y)=\dfrac{1}{\ln(cy)}+\dfrac{1}{2}$ then ,

$$Z(x,y)=e^{\int\left(\frac{\pm i\sqrt{\frac{\lambda^2}{A_2}}}{x}\right)dx+\int\left(\frac{\ln(cy)+\frac{1}{2}}{y}\right)dy}$$

$$=e^{\pm i\sqrt{\frac{\lambda^2}{A_2}}\ln x+\ln(\ln(cy))+\frac{1}{2}\ln y+c_1}$$

$$=K\,x^{\pm i\sqrt{\frac{\lambda^2}{A_2}}}\,y^{\frac{1}{2}}(\ln(cy)) \quad ;K=e^{c_1}$$

and the complete solution of (10) is given by:-

$$\left(Z-K\,x^{i\sqrt{\frac{\lambda^2}{A_2}}}\,y^{\frac{1}{2}}(\ln(cy))\right)\left(Z-K\,x^{-i\sqrt{\frac{\lambda^2}{A_2}}}\,y^{\frac{1}{2}}(\ln(cy))\right)=0\;;cy>0$$

Where K , λ and c are arbitrary constants .

<u>Example</u>:- To solve the(P.D.E.)

$$5y^2ZZ_{yy}+2x^2Z_x^2=0$$

1- If $\lambda^2\neq\dfrac{-1}{4}A_1$, then the complete solution of the above (P.D.E .) is given by:-

$$\left(Z-K\,x^{i\sqrt{\frac{\lambda^2}{2}}}\,y^{\frac{1}{2}}\left(\cosh\left(\sqrt{\frac{\lambda^2}{5}+\frac{1}{4}}\ln cy\right)\right)\right)$$
$$\left(Z-K\,x^{-i\sqrt{\frac{\lambda^2}{2}}}\,y^{\frac{1}{2}}\left(\cosh\left(\sqrt{\frac{\lambda^2}{5}+\frac{1}{4}}\ln cy\right)\right)\right)=0$$

$;cy>0$,Where K,λ *and* c are arbitrary constants .

2- If $\lambda^2=\dfrac{-1}{4}A_1$, then the complete solution of the above (P.D.E .) is given by:-

$$\left(Z-K\,x^{i\sqrt{\frac{\lambda^2}{5}}}\,y^{\frac{1}{2}}(\ln(cy))\right)\left(Z-K\,x^{-i\sqrt{\frac{\lambda^2}{5}}}\,y^{\frac{1}{2}}(\ln(cy))\right)=0$$

$;cy > 0$, where are arbitrary constants λ and K,c

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حول المعادلات التفاضليه الجزئيه اللاخطيه

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الخلاصة :-

هدف البحث تقديم طريقه مقترحه لحل المعادلات التفاضليه الجزئيه اللاخطيه المتجانسه الدرجه والتي درجتها اثنين وصيغتها العامه :-

$$\mathrm{A}(x,y,Z,Z_x,Z_y,Z_{xx},Z_{xy},Z_{yy})Z_{xx}+\mathrm{B}(x,y,Z,Z_x,Z_y,Z_{xx},Z_{xy},Z_{yy})Z_{xy}+$$
$$\mathrm{C}(x,y,Z,Z_x,Z_y,Z_{xx},Z_{xy},Z_{yy})Z_{yy}+\mathrm{D}(x,y,Z,Z_x,Z_y,Z_{xx},Z_{xy},Z_{yy})Z_x+$$
$$\mathrm{E}(x,y,Z,Z_x,Z_y,Z_{xx},Z_{xy},Z_{yy})Z_y+\mathrm{F}(x,y,Z,Z_x,Z_y,Z_{xx},Z_{xy},Z_{yy})Z=0$$

حيث ان $\mathrm{F,E,C,D,B,A}$ دوال خطيه للمتغير التابع y ومشتقاته الجزئيه بالنسبه للمتغيرين المستقلين x, y .